On the On-Shell Renormalization of Fermion Masses, Fields and Mixing Matrices at 1-loop

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- *Usually* fermion masses and fields are renormalized as

$$m_i^0 \to m_i + \underbrace{\delta m_i}_{\text{diagonal ct}} \qquad\qquad \psi_j^0 \to \underbrace{\tilde{Z}_{ji}}_{\text{non-diagonal}} \psi_i$$

- *No-mixing conditions on external legs* [1]

$$\frac{1}{\not{p}-m_j}\Sigma_{ji}(p^2)u_i = 0, \quad \bar{u}_j\Sigma_{ji}(p^2)\frac{1}{\not{p}-m_i} = 0, \qquad i \neq j$$

$\Longrightarrow \delta\tilde{Z}_{L,R}$ *and* $\delta\tilde{Z}^{\dagger}_{L,R}$ in terms of self-energies (**no** $\dagger$ on Σ)!

- ***Overspecification*** of field renormalization constants [2]

$$\Sigma \neq \gamma^0 \Sigma^\dagger \gamma^0 \qquad \Longrightarrow \qquad \left(\tilde{Z}_{L,R}\right)^\dagger \neq \tilde{Z}^\dagger_{L,R}$$

  - (pseudo-)Hermiticity violated by **absorptive parts**
  - Different in/out LSZ factors $\Longrightarrow$ add a set of constants $\bar{Z}$
    - $\mathcal{L} \neq \mathcal{L}^\dagger$, but OK for external legs
  - OR relax the no-mixing conditions (e.g. $\widetilde{Re}$)

- CKM counterterm in Denner and Sack [3]

$$\delta V \sim -\delta \tilde{Z}_L^{A,u} V + V \delta \tilde{Z}_L^{A,d}$$

  - Correctly cancels UV in the Wud vertex ✓
  - But $\boldsymbol{\partial_\xi \delta V \neq 0}$ [4]
    - Various other approaches that deal with ξ...[2, 4, 5, 6, 7, 8, 9]
- In Kniehl and Sirlin [10, 11, 12], relaxed no-mixing condition and $\partial_\xi \delta V = 0$, but
  - Self-energies only for the SM
  - No explicit field renormalization
  - External leg "formalism"; discussion rather removed from the Lagrangian

Setup I

- Want a hermitian Lagrangian $\Longrightarrow$ **no-mixing** condition only for **incoming** particles

$$\boxed{\frac{1}{\not{p} - m_j}\Sigma_{ji}(p^2)u_i = 0}, \quad i \neq j$$

  - Outgoing particles *still mix*, but only due to absorptive parts!
- Want to separate $\xi \Longrightarrow$ **off-diagonal mass counterterms**
  - Also, no *need* for the mass counterterms to be diagonal

$$\boxed{\delta m_i \to \delta m_{\boldsymbol{ji}}}$$

- Want *universal* mass, field, and mixing matrix ct's in terms of self-energies $\Longrightarrow$ ...?

Setup II

- Renormalization

$$\bar{\psi}\left(m + \delta m^L P_L + \delta m^R P_R\right)\psi \quad \xrightarrow{Hermiticity} \quad \left(\delta m^L\right)^\dagger = \delta m^R$$

$$\psi^0_{L,R} \to Z^{1/2}_{L,R}\psi_{L,R}, \quad \bar{\psi}^0_{L,R} \to \bar{\psi}_{L,R} Z^{1/2\dagger}_{R,L}$$

- Self-energy decomposition

$$\begin{aligned}\Sigma_{ji}(p^2) = {}& \Sigma^L_{ji}(p^2)\not{p}P_L + \Sigma^R_{ji}(p^2)\not{p}P_R + \Sigma^{sL}_{ji}(p^2)P_L + \Sigma^{sR}_{ji}(p^2)P_R \\ & \underbrace{+\frac{1}{2}\left(\delta Z^\dagger_{Lji} + \delta Z_{Lji}\right)\not{p}P_L + \frac{1}{2}\left(\delta Z^\dagger_{Rji} + \delta Z_{Rji}\right)\not{p}P_R}_{\text{standard piece}} \\ & - \left(\boxed{\delta m^L_{\boldsymbol{ji}}} + \frac{1}{2}\delta Z^\dagger_{Rji} m_i + \frac{1}{2} m_j \delta Z_{Lji}\right) P_L \\ & - \left(\boxed{\delta m^R_{\boldsymbol{ji}}} + \frac{1}{2}\delta Z^\dagger_{Lji} m_i + \frac{1}{2} m_j \delta Z_{Rji}\right) P_R\end{aligned}$$

Setup III

(Hereinafter $i \neq j$!)
No-mixing $\Longrightarrow$ *relation* between field and mass renormalization

$$\left(m_i^2 - m_j^2\right) \delta Z_{Lji} \underbrace{-2m_j \delta m^L_{ji} - 2m_i \delta m^R_{ji}}_{\text{new contributions}} = -2\left(m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2)\right.$$

$$\underbrace{\left. + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2)\right)}_{\text{standard piece}}$$

$$\left(m_i^2 - m_j^2\right) \delta Z_{Rji} \underbrace{-2m_j \delta m^R_{ji} - 2m_i \delta m^L_{ji}}_{\text{new contributions}} = -2\left(m_i^2 \Sigma^R_{ji}(m_i^2) + m_i m_j \Sigma^L_{ji}(m_i^2)\right.$$

$$\underbrace{\left. + m_j \Sigma^{sR}_{ji}(m_i^2) + m_i \Sigma^{sL}_{ji}(m_i^2)\right)}_{\text{standard piece}}$$

- $(\delta m^L)^\dagger = \delta m^R \implies$ relation to the **anti-hermitian part** of field renormalization

$$\boxed{(m_i^2 - m_j^2)}\delta Z^A_{Lji} - 2m_j \delta m^L_{ji} - 2m_i \delta m^R_{ji} = -\left(m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2)\right.$$
$$\left. + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2)\right) + \boxed{H.C.}$$

- Analogous equation for δZ^A_{Rji}
- 2 equations and 3 unknowns $\implies \mathbf{3 > 2}$!
- Very *distinct mass structure* in front of field renormalization

Let us explore the properties of $\boxed{m_i^2 - m_j^2}$ terms

Exploration: Gauge-dependence

From Nielsen Identities [13, 14, 15]

$$\partial_\xi \Sigma_{ji}(p^2) = \Lambda_{jj'}\Sigma_{j'i}(p^2) + \Sigma_{ji'}(p^2)\bar{\Lambda}_{i'i}$$

$$\xrightarrow{1-\text{loop}} \partial_\xi \Sigma_{ji}(p^2) = \Lambda_{ji}\left(\not{p} - m_i\right) + \left(\not{p} - m_j\right)\bar{\Lambda}_{ji}$$

- $\Lambda' s$ have Dirac structure and contain all ξ-dependence at 1-loop

$$\boxed{\left(m_i^2 - m_j^2\right)}\partial_\xi \delta Z^A_{Lji} - 2m_j \partial_\xi \delta m^L_{ji} - 2m_i \partial_\xi \delta m^R_{ji} =$$

$$- \boxed{\left(m_i^2 - m_j^2\right)}\left(m_i \bar{\Lambda}^R_{ji}(m_i^2) + \bar{\Lambda}^{sL}_{ji}(m_i^2)\right) + H.C.$$

$\Longrightarrow$ $\boxed{m_i^2 - m_j^2}$ **mass structure carried by ξ-dependent terms!**

Exploration: UV divergences I

- At 1-loop contributions to self-energies in terms of PV functions [16]
- For boson contributions we have

$$\Sigma^{L,R}(p^2) = f_{L,R} B_1(p^2, m^2_{\psi\text{loop}}, m^2_{\text{bos.}}) \quad \overset{\text{UV}}{\Longrightarrow} \quad \frac{1}{D-4}$$

$$\Sigma^{sL,(sR)}(p^2) = m_{\psi\text{loop}} f_s^{(\dagger)} B_0(p^2, m^2_{\psi\text{loop}}, m^2_{\text{bos.}}) \quad \overset{\text{UV}}{\Longrightarrow} \quad -\frac{2}{D-4}$$

- For fermion tadpoles we have

$$\Sigma^{sL,(sR)}(p^2) = f_T^{(\dagger)} m_{\psi\text{loop}} A_0(m^2_{\psi\text{loop}}) \quad \overset{\text{UV}}{\Longrightarrow} \quad -\frac{2m^2_{\psi\text{loop}}}{D-4}$$

- f's are appropriate couplings and $f^\dagger_{L,R} = f_{L,R}$
- D is the spacetime dimension ($4-\epsilon$ or $4-2\epsilon$)

Exploration: UV divergences II

- We have

$$\left[\left(m_i^2 - m_j^2\right)\delta Z^A_{Lji} - 2m_j\delta m^L_{ji} - 2m_i\delta m^R_{ji}\right]_{\text{div.}} = \frac{1}{D-4}\Big(-f_L\boxed{\left(m_i^2+m_j^2\right)} - f_R\boxed{2m_im_j}$$

$$+ 4m_{\psi\text{loop}}f_s\boxed{m_j} + 4m_{\psi\text{loop}}f_s^\dagger\boxed{m_i}$$

$$+ 4m^3_{\psi\text{loop}}f_T\boxed{m_j} + 4m^3_{\psi\text{loop}}f_T^\dagger\boxed{m_i}\Big)$$

- No UV divergences with $m_i^2 - m_j^2$!
- Only $(m_i^2+m_j^2)$, $2m_im_j$, m_i and m_j on the r.h.s and no $m_i^2 - m_j^2$ mass structure
  - Rewriting **mass ct's** on l.h.s.:

$$\Longrightarrow \quad -2\boxed{m_j}\delta m^L_{ji} - 2\boxed{m_i}\delta m^R_{ji} - 2\boxed{\left(m_i^2+m_j^2\right)}\delta m^-_{ji} - 2\cdot\boxed{2m_im_j}\delta m^+_{ji}$$

$\Longrightarrow$ **UV structure mimics mass counterterms!**

- **Gauge dependence** always comes with $\boxed{m_i^2 - m_j^2}$ factors
  - Gauge-*independent* terms may also have this structure
  - All the other mass structures are gauge-independent
- **No UV** divergences with $\boxed{m_i^2 - m_j^2}$ factors
- UV divergences comes with $m_i^2 + m_j^2$, $2m_i m_j$, m_j, and m_i mass structures
- $m_i^2 - m_j^2$ accompanies field ct's
- $m_i^2 + m_j^2$, $2m_i m_j$, m_j, and m_i accompanies mass ct's

Define the anti-hermitian part of field renormalization as the coefficient of $\boxed{m_i^2 - m_j^2}$ $(i \neq j)$

$$\begin{aligned}
\delta Z^A_{Lji} = - \Big[& m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2) \\
& + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2) + H.C. \Big]_{(\boldsymbol{m_i^2 - m_j^2})} \\
\delta Z^A_{Rji} = - \Big[& m_i^2 \Sigma^R_{ji}(m_i^2) + m_i m_j \Sigma^L_{ji}(m_i^2) \\
& + m_j \Sigma^{sR}_{ji}(m_i^2) + m_i \Sigma^{sL}_{ji}(m_i^2) + H.C. \Big]_{(\boldsymbol{m_i^2 - m_j^2})}
\end{aligned}$$

- Only ***finite*** logarithmic terms
- Contains all possible **gauge dependence**
- **Universal** definition in terms of self-energies and restrictions
- The hermitian part is unchanged w.r.t. the usual approach

Definitions: Mass renormalization

Now SOLVE for $\delta m^{L,R}$!

$$\delta m^L_{ji} = \frac{1}{2}\Big(m_i \Sigma^R_{ji}(m_i^2) + \Sigma^{sL}_{ji}(m_i^2) + m_j \Sigma^{L\dagger}_{ji}(m_j^2) + \Sigma^{sR\dagger}_{ji}(m_j^2)\Big) + \frac{1}{2}\left(m_i \delta Z^A_{Rji} - m_j \delta Z^A_{Lji}\right)$$

$$\delta m^R_{ji} = \frac{1}{2}\Big(m_i \Sigma^L_{ji}(m_i^2) + \Sigma^{sR}_{ji}(m_i^2) + m_j \Sigma^{R\dagger}_{ji}(m_j^2) + \Sigma^{sL\dagger}_{ji}(m_j^2)\Big) + \underbrace{\frac{1}{2}\left(m_i \delta Z^A_{Lji} - m_j \delta Z^A_{Rji}\right)}_{\text{cancels gauge-dependence}}$$

- **Gauge dependence canceled** by field renormalization
- Hermiticity relation $\left(\delta m^L\right)^\dagger = \delta m^R$ holds by construction
- **Universal** expression in terms of self-energies
- No $\widetilde{\mathrm{Re}}$ or Re
  - *Exceptions* for Majorana particles
- Can ***extend the real part to the diagonal***, then $\mathrm{Re}\left(\delta m^L_{ii}\right) = \mathrm{Re}\left(\delta m^R_{ii}\right)$ and also matches the results in [2]

The *Need* to Renormalize CKM? I

	Usual approach(es)	Proposed scheme
On-shell propagator	Overspecified δZ *or* non-diag.	"diag." **in** *or* **out**
Field ct; hermitian part	$\xi, \epsilon_{\mathrm{UV}}$	$\xi, \epsilon_{\mathrm{UV}}$
diagonal mass ct	ϵ_{UV}	ϵ_{UV}
off-diagonal mass ct	$\times$	$\boldsymbol{\epsilon_{\mathrm{UV}}}$
Field ct; **anti**-hermitian part	$\xi, \boldsymbol{\epsilon_{\mathrm{UV}}}$	$\xi, \cancel{\boldsymbol{\epsilon_{\mathrm{UV}}}}$
Wud vertex	$\boldsymbol{\epsilon_{\mathrm{UV}}}$	$\cancel{\boldsymbol{\epsilon_{\mathrm{UV}}}}$
CKM ct	$\delta V \sim -\delta \tilde{Z}_L^{A,u} V + V \delta \tilde{Z}_L^{A,d}$ $\boldsymbol{-\epsilon_{\mathrm{UV}}}$ and sometimes ξ	$\boldsymbol{\delta V = 0}$ $\cancel{\xi, \epsilon_{\mathrm{UV}}}$

- UV divergences stay in the mass term and do *not migrate* to other terms
- Usual CKM ct only needed to **cancel the migration**!
  - That initially included ξ-dependent terms...

The *Need* to Renormalize CKM? II

Is it consistent to **not** renormalize mixing matrices?

- Mixing matrices are due to diagonalization of mass terms
- Scenario 1:
    - Diagonlize the mass matrix
    - Renormalize the theory — $V^0 \to V + \delta V$
    - Rotate back (undiagonalize) — $V + \delta V \to \cancel{V} + \delta V'$
    - Counterterm, but no associated parameter ×

The *Need* to Renormalize CKM? III

- Scenario 2:
  - Stay in the non-diagonal basis, such that there $\cancel{V^0}$
  - Renormalize the theory — $\cancel{V^0}$ means $\cancel{\delta V}$
  - Rotate to a diagonal basis — $V + \cancel{\delta V}$
  - Parameter, but no associated counterterm $\times$

The *Need* to Renormalize CKM? IV

- Mixing matrices are *basis artefacts*
- There is **no need to renormalize** them
- Our scheme gives an explicit example of such non-renormalization!

Conclusions

We defined a new fermion renormalization scheme that

- ✓ Is *universal*
- ✓ Relies on (incoming) no-mixing conditions and *mass structures*
- ✓ Does *not rely on dropping the absorptive parts*
- ✓ Has *gauge–independent* mass counterterms
- ✓ Has *finite* anti-hermitian part of field counterterms
- ✓ Avoids *migrating* UV divergences and keeps the Lagrangian Hermitian

Also

- ✓ There is no need to renormalize mixing matrices!

Not in the presentation...

- Masless particles and radiative masses, explicit computations in the Grimus-Neufeld model...

References I

[1] K.-i. Aoki, Z. Hioki, R. Kawabe, M. Konuma, and T. Muta, "Electroweak Theory", Progress of Theoretical Physics Supplement **73**, 1–226 (1982).

[2] D. Espriu, J. Manzano, and P. Talavera, "Flavour Mixing, Gauge Invariance and Wave-function Renormalisation", Physical Review D - Particles, Fields, Gravitation and Cosmology **66**, 076002 (2002).

[3] A. Denner and T. Sack, "Renormalization of the quark mixing matrix", Nuclear Physics B **347**, 203–216 (1990).

[4] P. Gambino, P. A. Grassi, and F. Madricardo, "Fermion Mixing Renormalization and Gauge Invariance", Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics **454**, 98–104 (1998).

[5] A. Barroso, L. Brücher, and R. Santos, "Renormalization of the Cabibbo-Kobayashi-Maskawa matrix", Physical Review D **62**, 096003 (2000).

[6] K.-P. Diener and B. Kniehl, "On-mass-shell renormalization of fermion mixing matrices", Nuclear Physics B **617**, 291–307 (2001).

[7] A. Pilaftsis, "Gauge and scheme dependence of mixing matrix renormalization", Physical Review D **65**, 115013 (2002).

[8] Y. Zhou, "Renormalization of the Cabibbo–Kobayashi–Maskawa matrix in standard model", Physics Letters B **577**, 67–70 (2003).

[9] A. Denner, E. Kraus, and M. Roth, "Physical renormalization condition for the quark-mixing matrix", Physical Review D **70**, 033002 (2004).

[10] B. A. Kniehl and A. Sirlin, "Simple on-shell renormalization framework for the Cabibbo-Kobayashi-Maskawa matrix", Physical Review D **74**, 116003 (2006).

[11] B. A. Kniehl, A. Sirlin, and M. L. Marshak, "Novel formulations of CKM matrix renormalization", in Aip conference proceedings, Vol. 1182 (June 2009), pp. 327–330.

[12] A. A. Almasy, B. A. Kniehl, and A. Sirlin, "On-shell renormalization of the mixing matrices in Majorana neutrino theories", Nuclear Physics B **818**, 115–134 (2009).

[13] N. Nielsen, "On the gauge dependence of spontaneous symmetry breaking in gauge theories", Nuclear Physics B **101**, 173–188 (1975).

References II

[14] P. Gambino and P. A. Grassi, "The Nielsen Identities of the SM and the definition of mass", Physical Review D - Particles, Fields, Gravitation and Cosmology **62**, 1–7 (1999).

[15] Y. Yamada, "Gauge dependence of the on-shell renormalized mixing matrices", Physical Review D **64**, 036008 (2001).

[16] F. Staub, "Automatic Calculation of supersymmetric Renormalization Group Equations and Self Energies", Computer Physics Communications **182**, 808–833 (2010).

[17] W. Grimus and H. Neufeld, "Radiative neutrino masses in an SU(2) $\times$ U(1) model", Nuclear Physics B **325**, 18–32 (1989).

[18] V. Dūdėnas, T. Gajdosik, and D. Jurčiukonis, "Pole masses of neutrinos in the Grimus–Neufeld model", Acta Physica Polonica B **50**, 1737–1748 (2019).

[19] V. Dūdėnas, *Renormalization of neutrino masses in the Grimus-Neufeld model*, (Vilniaus universiteto leidykla, Vilnius, 2019), p. 125.

[20] S. Draukšas, V. Dūdėnas, T. Gajdosik, A. Juodagalvis, P. Juodsnukis, and D. Jurčiukonis, "The Grimus-Neufeld model with FlexibleSUSY at one-loop", Symmetry **11**, 1418 (2019).

BACKUP

Lightning Fast Intro to the Grimus-Neufeld Model

The Grimus-Neufeld model [17, 18, 19, 20] is the SM extended with

- Heavy Singlet Majorana neutrino $\Longrightarrow$ seesaw mechanism

$$\begin{pmatrix} 0_{3\times 3} & M_D \\ M_D^T & M_R \end{pmatrix} \Longrightarrow \mathrm{diag}\left(0, 0, m_3, m_4\right)$$

- Second Higgs doublet $\Longrightarrow$ radiative mass
  - General CP conserving THDM potential + Higgs basis
  - Additional physical Higgs particles: H, A, $H^{\pm}$
  - Yukawa couplings to the second doublet $G_{u,d,l}$ and G_ν
    - For neutrinos $(Y_\nu)_i \bar{N}.(L_i \tilde{H}_1) + (G_\nu)_i \bar{N}.(L_i \tilde{H}_2)$

Main features:

- 2 massless neutrinos at tree-level
- 1 heavy and 1 light neutrino at tree-level
- Radiative mass for 1 massless tree-level neutrino at 1-loop

Majorana Particles

- The *bare* Majorana condition $\nu = \nu^c$ at 1-loop implies

$$\nu = \nu_L + \nu_L^c \quad \Longrightarrow \quad Z_L \nu_L + Z_R \nu_L^c \quad \Longrightarrow \quad \delta Z_L = \delta Z_R^\star$$

- *But this does not hold due to absorptive parts*

$$\boxed{\delta Z_L \neq \delta Z_R^\star}$$

  - The same problem of overspecification
- The *Majorana condition* is not compatible with no-mixing already at 1-loop

Massless Particles and Radiative Masses I

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} ? & ? \\ \hline ? & \begin{array}{c} h, \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark \end{array} \end{array}\right)$$

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c}\delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}(0)\end{array} & h, \delta Z, \delta m, m_i \neq 0 \\ \hline h, \delta Z, \delta m, m_j \neq 0 & \begin{array}{c} h, \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark \end{array} \end{array}\right)$$

$$\delta m^L_{ji} = \frac{1}{2}\Big(m_i \Sigma^R_{ji}(m_i^2) + \Sigma^{sL}_{ji}(m_i^2) + m_j \Sigma^{L\dagger}_{ji}(m_j^2) + \Sigma^{sR\dagger}_{ji}(m_j^2)\Big) + \frac{1}{2}\boxed{m_i \delta Z^A_{Rji} - m_j \delta Z^A_{Lji}}$$

Massless Particles and Radiative Masses III

How to use mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \text{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c} \delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}(0) \Leftrightarrow \cancel{m} \text{ limit} \end{array} & \begin{array}{c} \Leftarrow \\ h, \delta Z, \delta m, m_i \neq 0 \\ \cancel{m_j} \text{ limit} \end{array} \\ \hline \begin{array}{c} \Uparrow \\ h, \delta Z, \delta m, m_j \neq 0 \\ \cancel{m_i} \text{ limit} \end{array} & \begin{array}{c} \Leftarrow\!\Uparrow \\ h, \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark \end{array} \end{array} \right)$$

$$\delta Z^A_{Lji} = -\left[m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2) \right.$$
$$\left. + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2) + H.C. \right]_{(m_i^2 - m_j^2)}$$

Massless Particles and Radiative Masses IV

How to use mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\left(0, 0, m_3, m_4\right)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c}\delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}\left(0\right)\end{array} & \ldots \\ \hline \ldots & \ldots \end{array}\right)$$

- No 1-loop $\implies$ $\Sigma^{sL,sR}\left(0\right)$ diagonalized with leftover freedom from the tree-level

$\implies$ *Radiative mass!* [17]