

Theory input for $t\bar{t}j$ experimental analyses at the LHC

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Studying the top quark at the LHC

fundamental particle with the largest mass in the SM:

$m_t = 172.76 \pm 0.30 \text{ GeV}$ (direct measurement)

$m_t(m_t) = 162.5^{+2.1}_{-1.5} \text{ GeV}$ ($\overline{\text{MS}}$ mass, cross section measurements)

$m_t = 172.5 \pm 0.7 \text{ GeV}$ (pole mass, cross section measurements)

[P.A. Zyla et al. (PDG) 2020]

LHC = 'top quark factory':

high statistics enable precise measurements of the top quark processes
(precision SM measurements, Higgs-Yukawa, BSM, ...)

Why is the measurement of the top quark mass interesting?

Radiative corrections to M_W

consistency check of the SM
through radiative corrections
involving m_t & m_H

Stability of EW vacuum

effective Higgs potential depends on
relations between m_H , m_t and α_s

Motivation for study of $t\bar{t}j$

→ **substantial fraction of $t\bar{t}$ -events contain a jet**

for jet cut $p_T^j > 40$ GeV & $\sqrt{s} = 13$ TeV: 40 % of $t\bar{t}$ -events accompanied by a jet
[Kraus [hep-ph/1608.05296](#)]

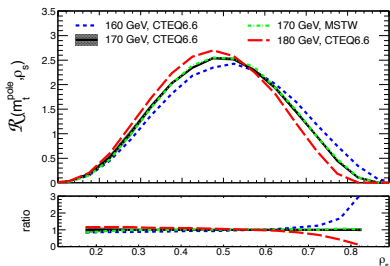
→ **dominant background to Higgs production in VBF**

signal: $pp \rightarrow Hjj \rightarrow W^+W^-jj$ with jj separated in y

background: $pp \rightarrow t\bar{t}j \rightarrow b\bar{b}W^+W^-j$ with b produced centrally, but j more evenly in y

→ **top quark mass determination through ρ distribution**

$\rho = 2m_0/\sqrt{m_{t\bar{t}}^2}$ with $m_0 = 170$ GeV [Alioli, Fernandez, Fuster, Irlles, Moch, Uwer, Vos
[hep-ph/1303.6415](#)]



ATLAS [“Measurement of the top-quark mass in $t\bar{t} + 1$ -jet events collected with the ATLAS detector in pp collisions at $\sqrt{s} = 8$ TeV”, [hep-ex/1905.02302](#)]

$$m_t^{\text{pole}} = 171.1 \pm 0.4(\text{stat}) \pm 0.9(\text{sys}) {}^{+0.7}_{-0.3}(\text{th}) \text{ GeV}$$

theory uncertainty dominated by scale variation uncertainty $(+0.6, -0.2)$ GeV
(PDF and α_s uncertainty lead to ± 0.2 GeV)

NLO calculations and POWHEG (Positive Weight Hardest Emission Generator) implementation of $t\bar{t} + \text{jet}$

- NLO calculation of $t\bar{t} + \text{jet}$
stable tops [Dittmaier, Uwer, Weinzierl [hep-ph/0810.0452](#)]
LO top-quark decay [Melnikov, Schulze [hep-ph/1004.3284](#)]
NLO QCD off-shell effects in fully leptonic decay [Bevilacqua, Hartanto, Kraus, Worek [hep-ph/1509.09242](#)]
- POWHEG (=HELAC-NLO + POWHEG-BOX) [Kardos, Papadopoulos, Trócsányi [hep-ph/1101.2672](#)]
- POWHEG-BOX $t\bar{t}b\bar{a}rj$ [V1 Alioli, Moch, Uwer [hep-ph/1110.5251](#)]

Method used in this study: POWHEG-BOX $t\bar{t}b\bar{a}rj$ V2
most important differences for this study to previous V1 version
[[hep-ph/1110.5251](#)]

- all amplitudes calculated with OpenLoops2
(V1: Born and real squared amplitudes from MadGraph, virtual from [hep-ph/0810.0452](#))
- able to parallelize calculation

Motivation: high-energy tails of NLO differential cross sections in $pp \rightarrow t\bar{t}j + X$ in fully leptonic decay better described through dynamical scale definition
[Bevilacqua, Hartanto, Kraus, Worek [hep-ph/1609.01659](https://arxiv.org/abs/hep-ph/1609.01659)]

fixed scale $\mu_0 = m_t$

dyn. scale $\mu_0 \in \{H_T^B/2, H_T^B/4\}, H_T^B = \left(\sqrt{p_{T,t}^B{}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^B{}^2 + m_t^2} + p_{T,j}^B \right)$

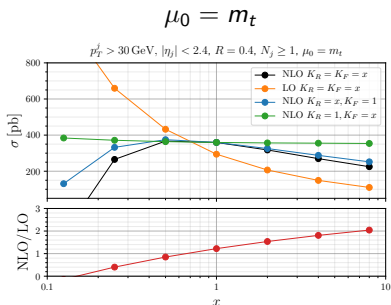
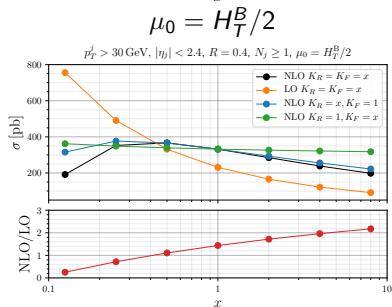
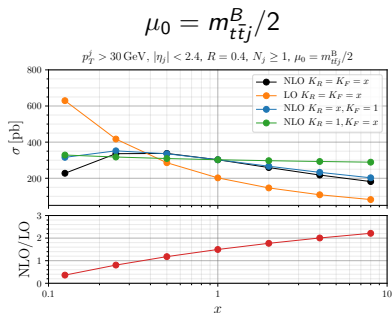
$$\mu_0 = m_{t\bar{t}j}^B/2 = \frac{1}{2} \sqrt{(p_t^B + p_{\bar{t}}^B + p_j^B)^2}$$

B : underlying Born variables

Simulation details: NLO accuracy, $\sqrt{s} = 13$ TeV, $m_t = 172$ GeV, stable top quarks, (PDF+ α_s) CT18NLO, 7 point scale variation with $\mu_{R/F} = K_{R/F}\mu_0$, $(K_R, K_F) \in \{(0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2)\}$

Analysis details: $N_j \geq 1$, $p_T^j > 30$ GeV, $|\eta_j| < 2.4$, anti- k_T jet clustering algorithm with $R = 0.4$

Integrated cross section with analysis cuts

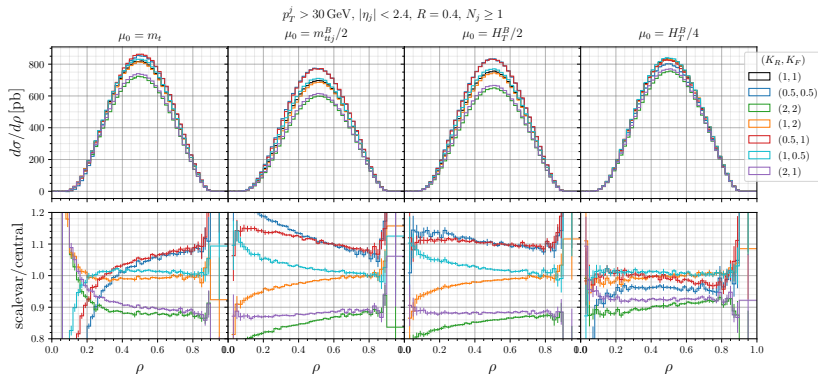


scale variation driven by renormalization
scale dependence ($K_R = x, K_F = 1$)

Observation:

$\text{NLO/LO} \sim 1$ for $\mu_R = \mu_F = H_T^B/4$
→ additional study of this central scale

Scale variation in ρ distribution

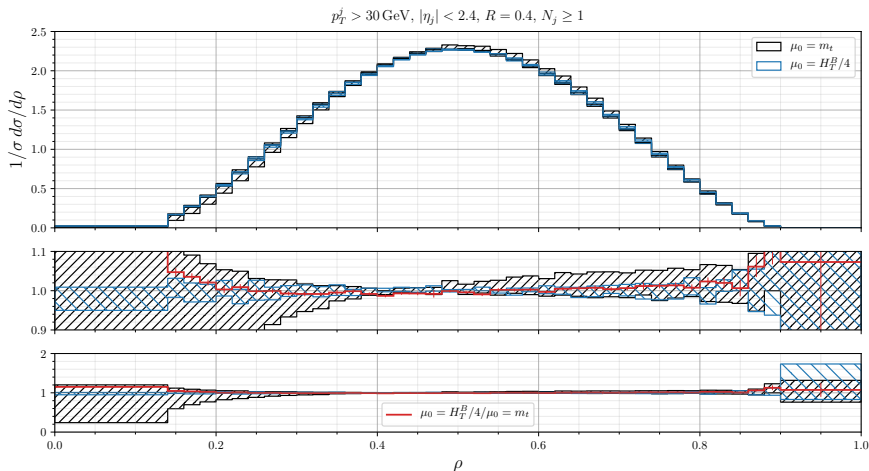


- large increase in width of scale variation band in high energy tail (\Leftrightarrow small ρ) and crossing of scale variation graphs using fixed scale not seen using dynamical scale
- varying the scale does not induce large shape variation using a dynamical scale

$$\rho = \frac{2m_0}{m_{t\bar{t}j}}$$

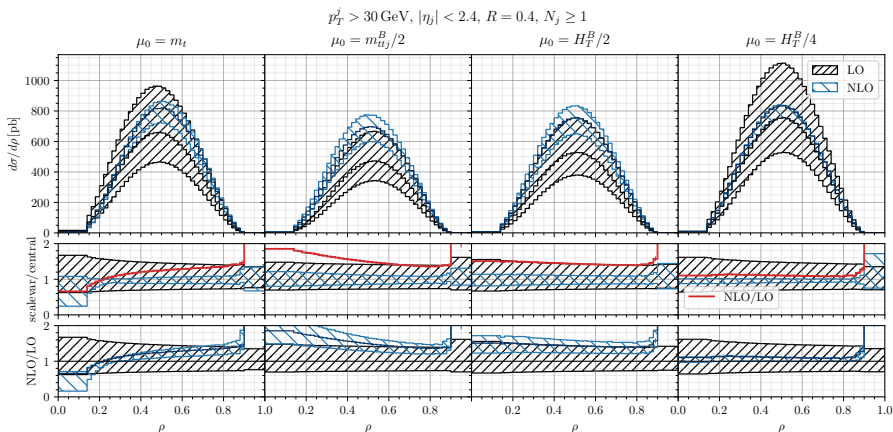
with $m_0 = 170 \text{ GeV}$

Scale variation in normalized ρ distribution at NLO



→ strongly reduced scale uncertainty with $\mu_0 = H_T^B/4$ w.r.t. $\mu_0 = m_t$ due to reduced shape variation of the ρ distribution through scale variation with the dynamical scale

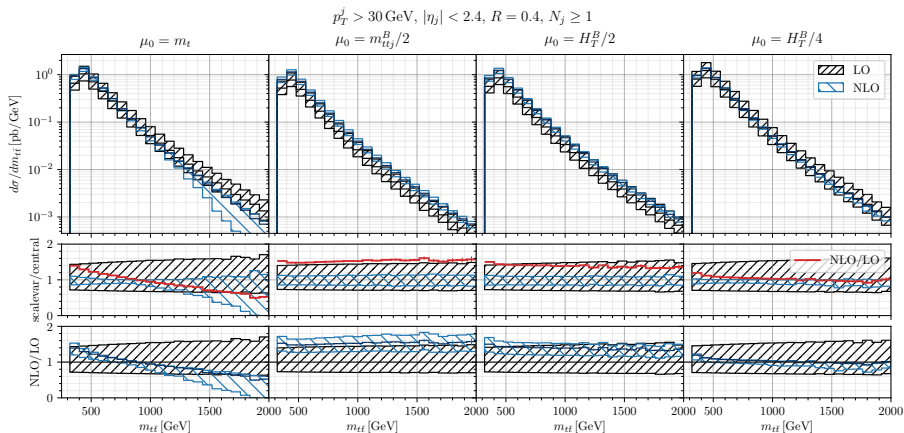
Comparison of the ρ distribution at NLO and LO



- shape variation comparing LO and NLO central scale prediction using $\mu_0 = m_t$ and $\mu_0 = m_{tj}^B/2$
- more uniform differential (NLO/LO) \mathcal{K} -factor using $\mu_0 \in \{H_T^B/2, H_T^B/4\}$

Comparison of the $m_{t\bar{t}}$ distribution at NLO and LO

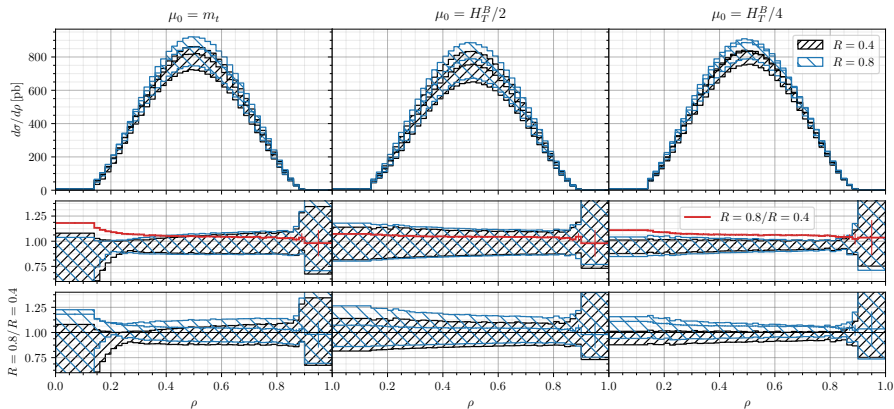
→ reduced width of scale variation uncertainty bands in high energy tails using a dynamical scale instead of a fixed scale already found in [Bevilacqua, Hartanto, Kraus, Worek [hep-ph/1609.01659](https://arxiv.org/abs/hep-ph/1609.01659)]



Effect of variation of the R -parameter in ρ distribution

R parameter in anti- k_T algorithm ($R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$)

$$p_T^j > 30 \text{ GeV}, |\eta_j| < 2.4, N_j \geq 1$$

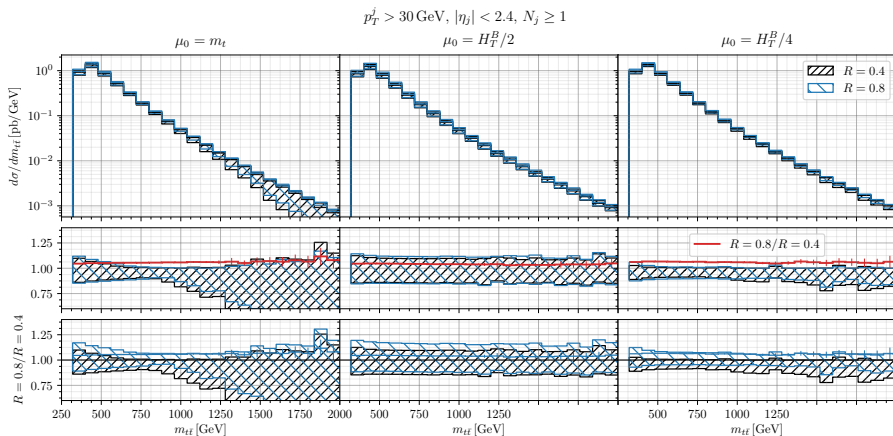


→ larger diff. cross section with $R = 0.8$ compared to $R = 0.4$

→ similar size of scale uncertainty for both R -values with dynamical scale

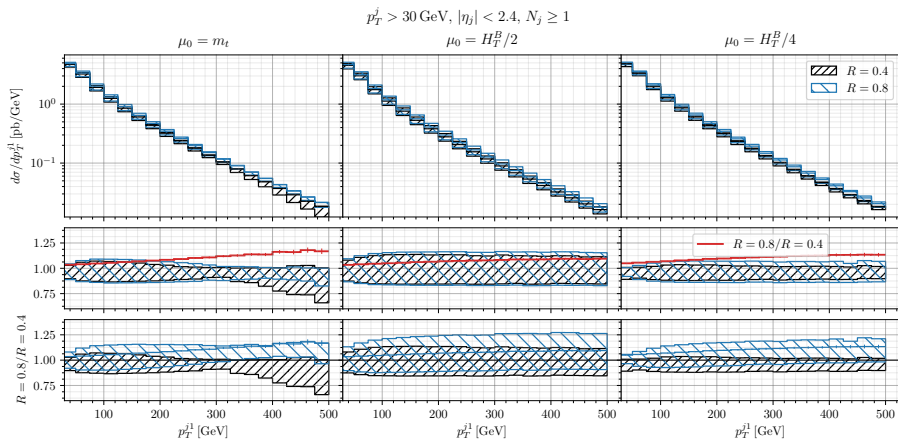
Effect of variation of the R -parameter in $m_{t\bar{t}}$ distribution

Example: same behaviour observed in other differential distributions



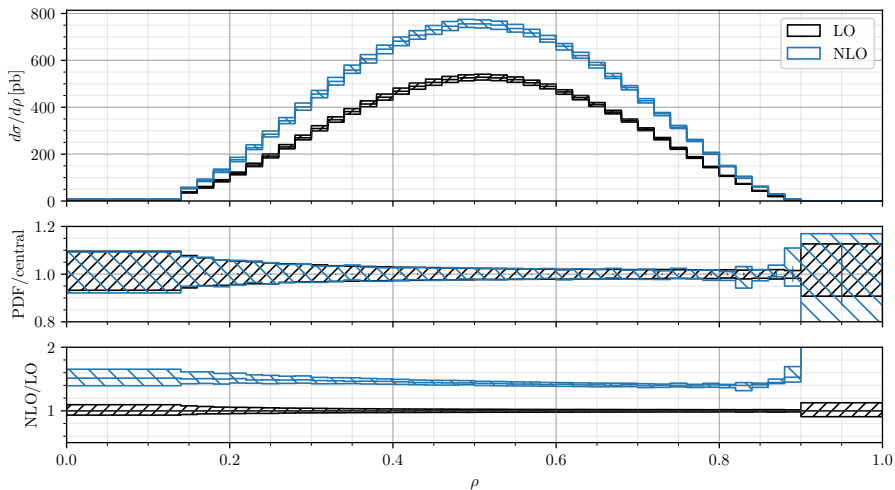
Effect of variation of the R -parameter in p_T^j distribution

Example: same behaviour observed in other differential distributions



NLO PDF uncertainty within a NLO or LO simulation

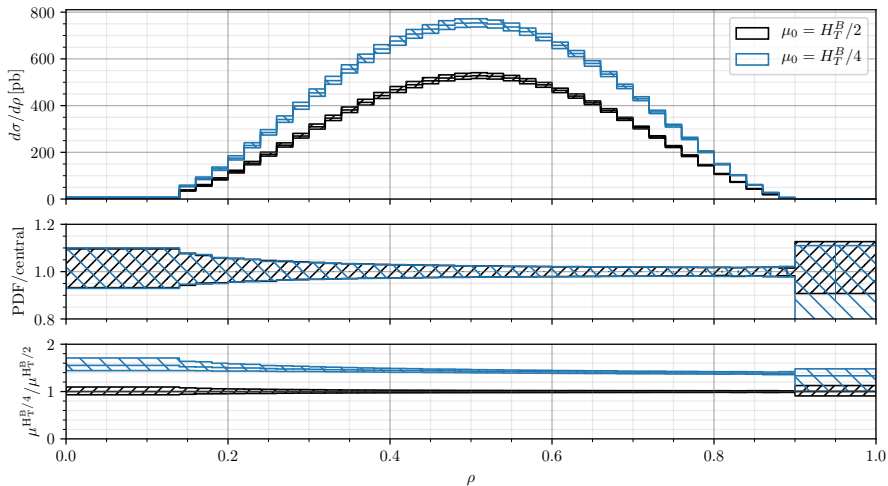
Predictions of the PDF uncertainty using the CT18NLO PDF set in association with a **NLO** or LO partonic cross section using the central scale $\mu_0 = H_T^B/2$



→ very similar size of PDF uncertainty using either a NLO or LO matrix element

NLO PDF uncertainty using $\mu_0 = H_T^B/2$ or $\mu_0 = H_T^B/4$

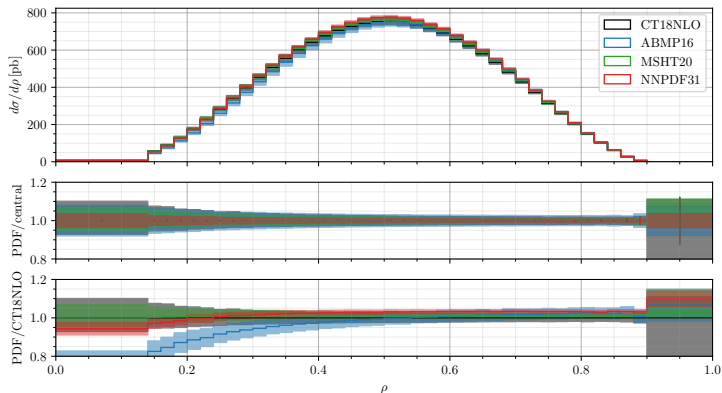
Predictions of the NLO PDF uncertainty using the CT18NLO PDF set based on a LO partonic cross section and setting either $\mu_0 = H_T^B/2$ or $\mu_0 = H_T^B/4$



→ very similar size of PDF uncertainty using either dynamical scale

PDF uncertainty in ρ distribution

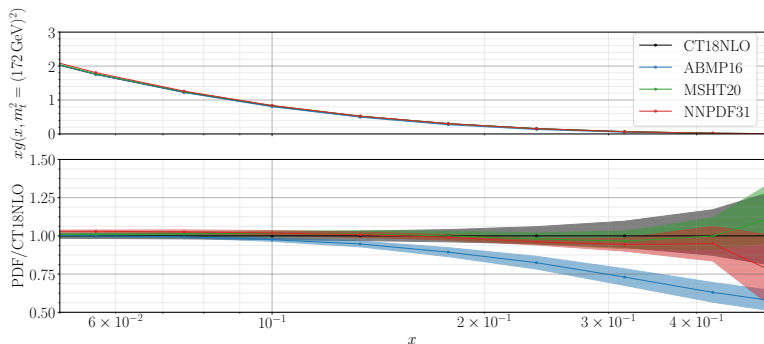
→ LO partonic cross section using $\mu_0 = H_T^B/4$



- PDF variation of the order of the scale variation in low ρ tails using the dynamical scale $\mu_0 = H_T^B/4$
- similar behaviour found with all PDF sets in the bulk of the distributions, differences in the high energy tails

Gluon PDF as a function of x

differences between predictions of ρ distribution obtained with different PDF sets
found in high energy tails (small ρ) \rightarrow region of large x
 \Rightarrow investigate gluon PDF as function of x for $Q^2 = m_t^2$
first bin $\rho \in [0, 0.14]$: peak at $x_{\min} = 0.15$, $x_{\max} = 0.25$



higher values of $\rho \rightarrow$ lower values of x
($\rho \in [0.14, 0.65]$: peak at $x_{\min} = 0.02$, $x_{\max} = 0.07$)
 \Rightarrow better agreement between central values of different PDF sets

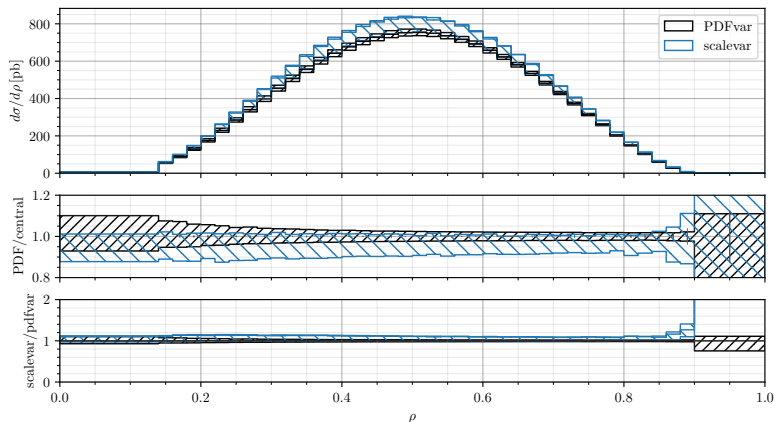
- dynamical scale preferable to fixed scale:
 - (i) smaller width of scale variation uncertainty band in the high-energy tails
 - (ii) strongly reduced scale variation uncertainty in normalized ρ distribution
- $\mu_0 \in \{H_T^B/2, H_T^B/4\}$ preferable to $\mu_0 = m_{t\bar{t}j}^B/2$:
 - (i) nearly constant differential (NLO/LO) \mathcal{K} -factor
 - (ii) NLO and LO scale variation bands overlap in low ρ region
 - (iii) lower scale variation uncertainty in ρ distribution at NLO
(here $\mu_0 = H_T^B/4$ preferable to $\mu_0 = H_T^B/2$)
- negligible influence of variation of R -parameter on the scale variation uncertainty using $\mu_0 = H_T^B/2$ or $\mu_0 = H_T^B/4$
- for dynamical scale $\mu_0 = H_T^B/4$ PDF uncertainty becomes as important as scale uncertainty in high-energy tails of the ρ distribution
(and in the normalized ρ distribution)

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- for dynamical scale $\mu_0 = H_T^B/4$ PDF uncertainty becomes as important as scale uncertainty in high-energy tails of the ρ distribution
(and in the normalized ρ distribution)

Thank you for your attention!

Comparing PDF uncertainty and scale uncertainty

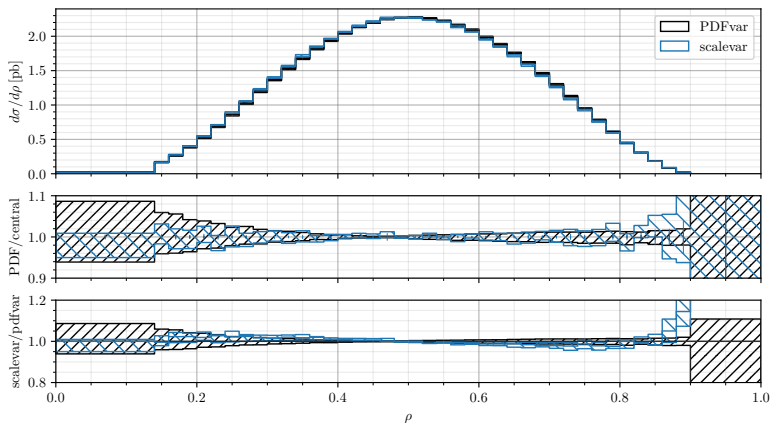
calculation of ρ distribution: using CT18NLO PDF set and $\mu_0 = H_T^B/4$, scale uncertainty: NLO partonic cross section and NLO scale variation, PDF uncertainty: LO partonic cross section and NLO PDF variation



Comparing PDF uncertainty and scale uncertainty

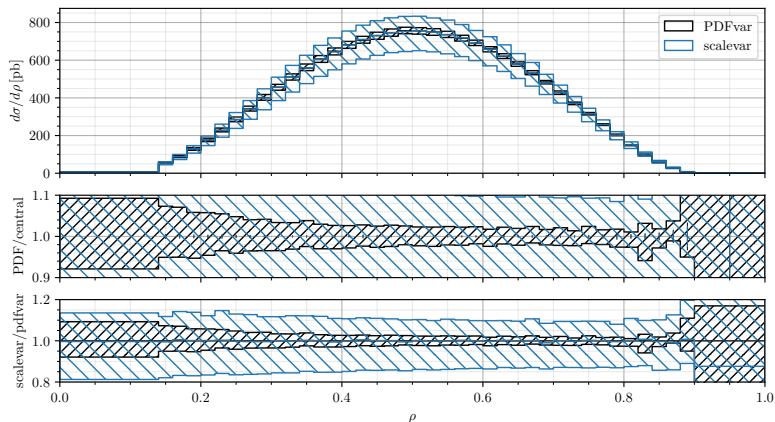
calculation of normalized ρ distribution: using CT18NLO PDF set and $\mu_0 = H_T^B/4$, scale uncertainty: NLO partonic cross section and NLO scale variation, PDF uncertainty: LO partonic cross section and NLO PDF variation

normalization: each scale variation graph by its total cross section and each ρ distribution obtained with different PDF eigenvector by its total cross section



Comparing PDF uncertainty and scale uncertainty

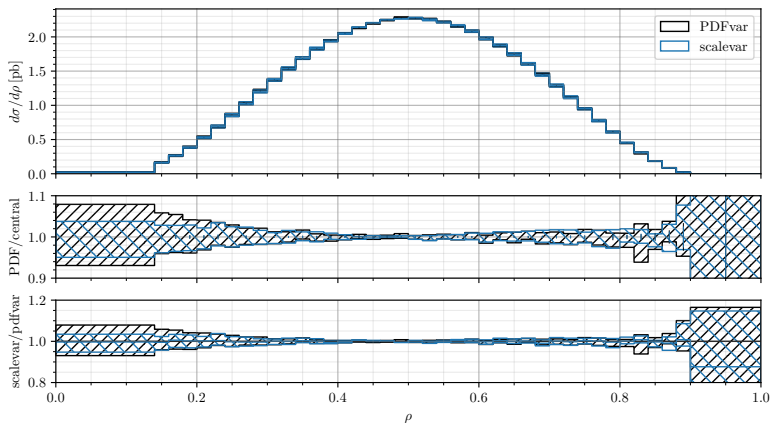
calculation of ρ distribution: using CT18NLO PDF set and $\mu_0 = H_T^B/2$, scale uncertainty: NLO partonic cross section and NLO scale variation, PDF uncertainty: NLO partonic cross section and NLO PDF variation



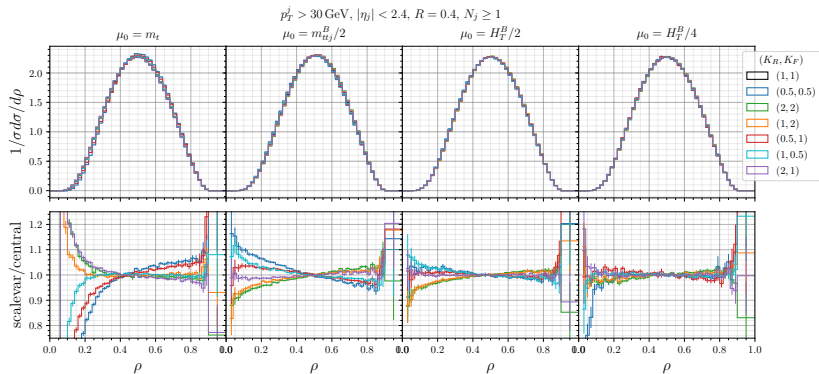
Comparing PDF uncertainty and scale uncertainty

calculation of normalized ρ distribution: using CT18NLO PDF set and $\mu_0 = H_T^B/2$, scale uncertainty: NLO partonic cross section and NLO scale variation, PDF uncertainty: NLO partonic cross section and NLO PDF variation

normalization: each scale variation graph by its total cross section and each ρ distribution obtained with different PDF eigenvector by its total cross section



Scale variation in normalized ρ -distribution

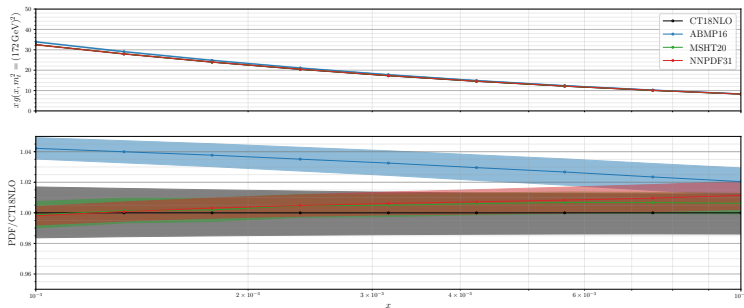


Gluon PDF as a function of x

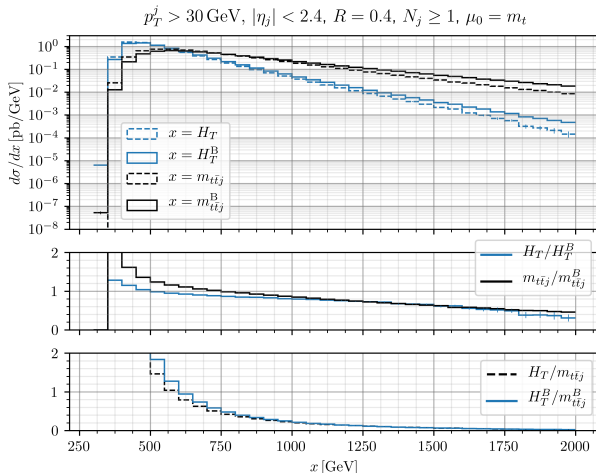
Observation in ρ distribution at $\rho \sim 1$ for CT18NLO: large uncertainty

Explanation: large statistical uncertainty and increasing PDF uncertainty of CT18NLO compared to ABMP16, MSHT20, NNPDF3.1

smallest x -value: 10^{-3}



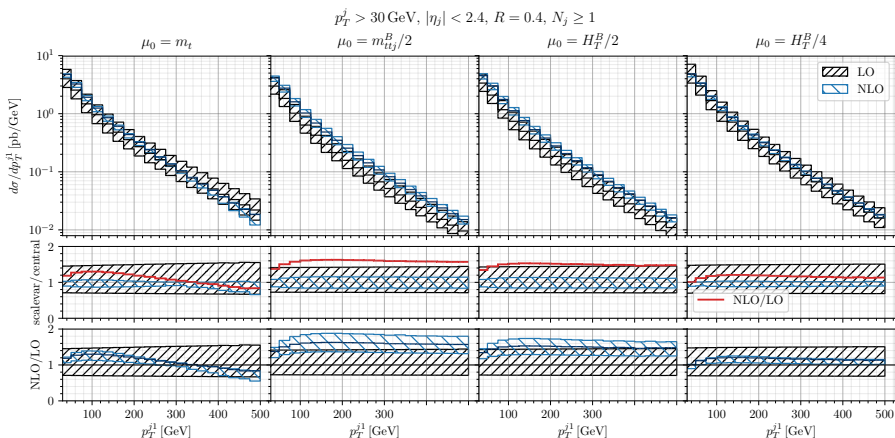
H_T and $m_{t\bar{t}j}$ distributions (real and underlying Born)



- harder spectrum of the $m_{t\bar{t}j}^B$ distribution compared to H_T^B distribution
- softer spectrum of distributions evaluated in real emission configuration, since additional parton carries away energy

Comparison of the p_T^j distribution at NLO and LO

Example: similar behaviour observed in other differential distributions



Extraction of the top quark mass through the normalized ρ distribution by the ATLAS collaboration

$$m_t^{\text{pole}} = 171.1 \pm 0.4(\text{stat}) \pm 0.9(\text{sys}) {}^{+0.7}_{-0.3}(\text{theo}) \text{ GeV (scale: } {}^{+0.6}_{-0.3} \text{ GeV, PDF and } \alpha_s: \pm 0.2 \text{ GeV)}$$

$$m_t(m_t) = 162.9 \pm 0.5(\text{stat}) \pm 1.0(\text{sys}) {}^{+2.1}_{-1.2}(\text{theo}) \text{ GeV (scale: } {}^{+2.1}_{-1.2} \text{ GeV, PDF and } \alpha_s: \pm 0.4 \text{ GeV)}$$

["Measurement of the top-quark mass in $t\bar{t}$ + 1-jet events collected with the ATLAS detector in pp collisions at $\sqrt{s} = 8 \text{ TeV}$ ", hep-ex/1905.02302]

General jet analysis cuts: $p_T^j > 25 \text{ GeV}$ and $|\eta_j| < 2.5$ reconstructed with the anti- k_T jet clustering algorithm with $R = 0.4$ + additional cuts on separation criteria and cuts on the leptons from the top-quark decay

data unfolded to parton level defined as including initial- and final-state radiation from quarks and gluons before the top-quark decay \rightarrow NLO+PS simulation with cut-off scale of the PS varying on an event-by-event basis

top quark mass extracted through a least-squares method (χ^2 fit)

dominant systematic uncertainties:

- simulation uncertainties: modelling of the PS and hadronization (0.4 GeV) and colour reconnection (0.4 GeV)
- detector response uncertainties: jet energy scale (0.4 GeV)

total systematic uncertainties $\pm 0.9 \text{ GeV}$

Technical parameters in the POWHEG BOX

ncall1 100000 ! number of calls for initializing the integration grid
itmx1 1 ! number of iterations for initializing the integration grid
(automatically set to 1 for parallel runs, see bbinit.f)
ncall2 200000 ! number of calls for computing the integral and finding upper
bound
itmx2 2 ! number of iterations for computing the integral and finding upper
bound
→ 100 parallel runs: 40 M phase space points
bornsuppfact 100d0 ! (default 0d0) mass param for Born suppression factor
(generation cut) If < 0 suppfact = 1

$$F(p_T^2) = \frac{p_T^2}{p_T^2 + \text{bornsupp}^2}$$

withdamp 1 ! (default 0, do not use) use Born-zero damping factor
hdamp 237.8775

$$F(k_T^2) = \frac{\text{hdamp}^2}{\text{hdamp}^2 + k_T^2}$$

$$R = R_s + R_f = F(k_T^2)R + (1 - F(k_T^2))R$$

bornktmin 0.2d0 ! (default 0d0) kt min at Born level for jet in ttbar+jet