Low seesaw scale in the Grimus-Neufeld model

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Motivation for Grimus-Neufeld model

- Neutrino masses=BSM physics
- Neutrino masses:
  - Seesaw (Type I,II,III)
  - Radiative (e.g. scotogenic model)
  - Both seesaw+radiative (2HDM + seesaw)
- Adressing neutrino masses and mixings "minimally":
  - SM + seesaw $\Rightarrow$ minimally 2 sterile neutrinos required for seesaw
    - add 2nd Higgs doublet $\Rightarrow$ radiative mass possible $\Rightarrow$ 1 sterile neutrino is enough
  - 2HDM + Z_2 (scotogenic) $\Rightarrow$ minimally 2 sterile neutrinos for radiative mass generation.
    - Break Z_2 $\Rightarrow$ seesaw mass possible $\Rightarrow$ 1 sterile neutrino is enough
- General, but truly minimal: 2HDM + 1 seesaw neutrino $\rightarrow$ Grimus–Neufeld model [GN '89].

Motivation for low seesaw

- Original Type I seesaw – natural Yukawa values:

$$m_\nu = \frac{y^2 v^2}{2M}\,,\; M \approx M_{\mathrm{GUT}} \Rightarrow y = O(1)\,,$$

  where $M-$ heavy neutrino mass, but...
- fine tuning in the scalar sector ($\delta m_h \geq m_h$) when $M > 10^7$GeV:

$$\delta m_h \sim \frac{y^2}{16\pi^2} M^2$$

- $M < v$ gives "unnaturally" small Yukawa coupling y, but...
  - $y \to 0$ increases the symmetry of the Lagrangian.
- M of keV, MeV, GeV not excluded.

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- Neutrino Yukawa couplings in GN Y^1_ν and Y^2_ν are 2 general complex 3-vectors:

$$\mathscr{L}_{\mathrm{Yuk}+M} = -Y^1_{\nu_i} \ell_i \tilde{H}_1 N - Y^2_{\nu_i} \ell_i \tilde{H}_2 N - \frac{1}{2} M N^2 + H.c.\,, i = e, \mu, \tau\,;$$
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- By a unitary transformation V we can pick a basis:

$$\nu^F = V \nu\,,\ Y^1_\nu = (0, 0, y)\, V^\dagger\,,\ Y^2_\nu = \left(0, d, d'\right) V^\dagger\,,\ y, d \in \mathbb{R}^+, d' \in \mathbb{C}\,.$$

Neutrino masses

- Tree level neutrino massess:

$$m_1 = m_2 = 0\,,$$
$$m_4 > m_3 > 0\,.$$

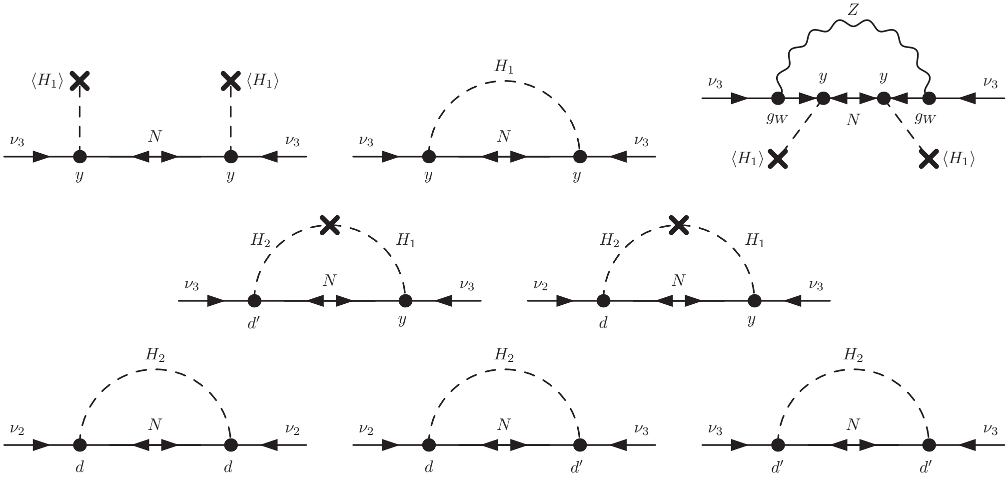
- 1-loop neutrino masses:

$$m_1^{\text{pole}} = 0\,,$$
$$m_4^{\text{pole}} > m_3^{\text{pole}} > m_2^{\text{pole}} > 0\,.$$

- Grimus-Lavoura (GL) approximation:
  - Large mass expansion for $m_4 \gg m_3$
    - All the neglected diagrams have $\mathrm{O}(> y^2)$ in Yukawa couplings
  - Evaluate loop functions for light neutrino masses at $p^2 = 0$.

1-Loop masses

$$Y_\nu^1 = (0,0,y)\,V^\dagger\,,\;\; Y_\nu^2 = \left(0,d,d'\right)\,V^\dagger$$



Low seesaw mass limit

- Finite, gauge invariant 2×2 mass matrix.
- for $m_3 = O\left(\sqrt{\Delta m^2_{\text{atm}}}\right)$, the approximation works well down to $m_4 > 10\,\text{eV}$ ($< 1\%$ difference between GL vs. full 1-loop masses).
- All loop corrections are

$$\delta m \sim m_4 \frac{m_i^2}{m_4^2 - m_i^2} \ln\left(\frac{m_4^2}{m_i^2}\right)$$

  where $i = h, H, A$ or SM Higgs, neutral Higgs, and pseudoscalar Higgs respectively.
- For $m_3 \ll m_4 < v \sim m_i$, we have

$$\delta m \sim m_4 \ln\left(\frac{m_4^2}{m_i^2}\right)$$

Low seesaw mass limit

- Consider the dependence of d^2 on m_4:
  - To reproduce fixed pole masses of $O\left(\Delta m^2_{\rm atm}\right)$ at 1-loop, we must have

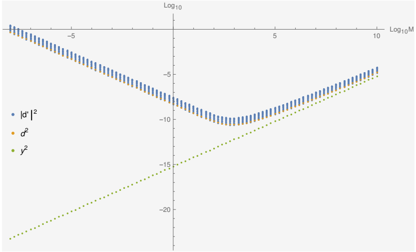
$$\delta m \sim d^2 m_4 \Rightarrow d^2 \sim \frac{1}{m_4}$$

  - Recalling tree-level seesaw relation

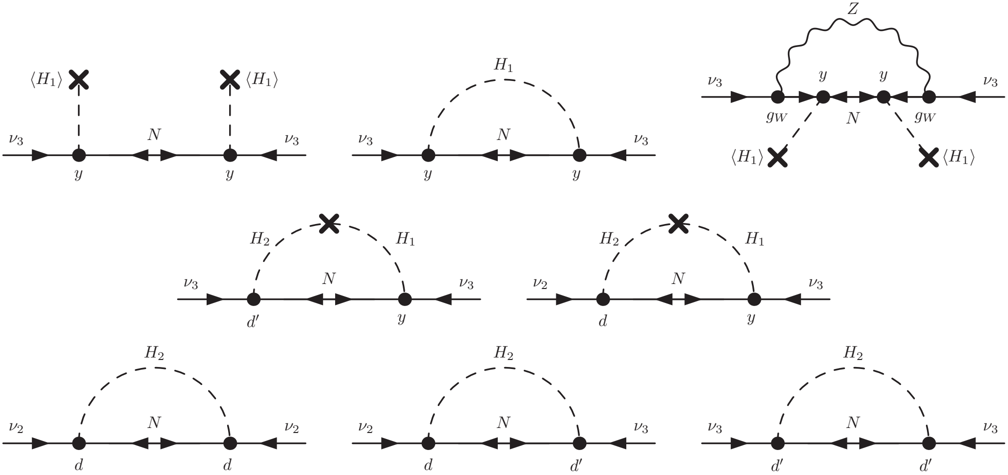
$$y^2 = m_4 \cdot \frac{2m_3}{v^2}$$

$\Rightarrow$for $m_3 = O\left(\Delta m^2_{\rm atm}\right)$ and $m_4 < v$ we get:

$$y^2 \ll d^2$$



Loop corrections



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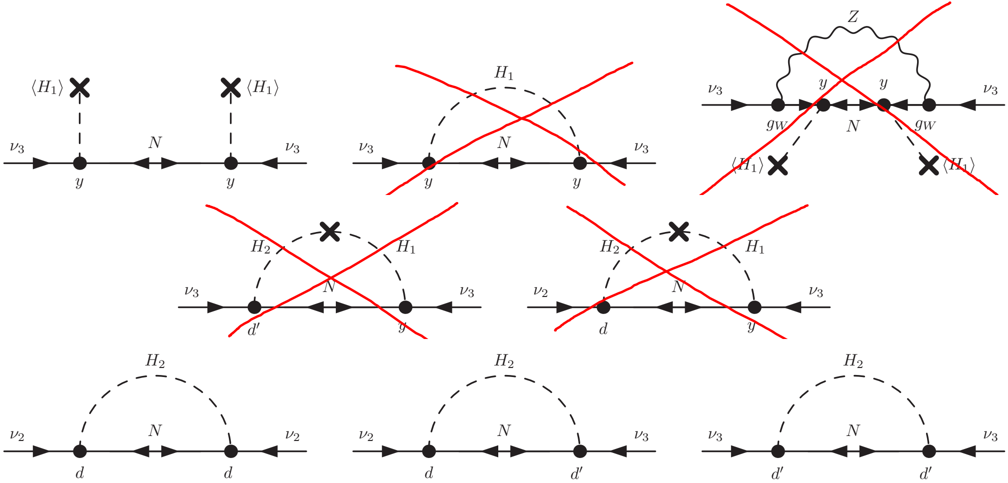


- The 2x2 mass matrix becomes:

$$M = \frac{1}{32\pi^2} m_4 F_1 \times \begin{pmatrix} -d^2 & -i\,dd' \\ -i\,dd' & \left(16\pi^2 \frac{1}{F_1}\frac{v^2}{m_4^2}\right) y^2 - (d')^2 \end{pmatrix},$$

$$F_1 = \ln\left(\frac{m_H^2}{m_A^2}\right) - \ln\left(\frac{m_H^2}{m_h^2}\right)\cos(\beta-\alpha)\,.$$

- Takagi decomposition with Unitary matrix R:

$$R^T M R = \mathrm{diag}\left(m_2^{\mathrm{pole}}, m_3^{\mathrm{pole}}\right); \quad 0 < m_2^{\mathrm{pole}} < m_3^{\mathrm{pole}}$$

- If Yukawa couplings are Z_2-symmetric:

$$y^2 \to 0 \Rightarrow \det M = 0$$

$$\Rightarrow m_2^{\mathrm{pole}} = 0$$

$\Rightarrow$ruled out by 2 non-vanishing Δm_{21}^2 and Δm_{31}^2

$\Rightarrow y$ is small ($O\left(10^{-9}\right)$ for $m_4 = O(\mathrm{MeV})$), but not vanishing.

Relating to experiments

- We order $m_2^{\text{pole}} < m_3^{\text{pole}}$, so

$$\begin{aligned}
\text{NH:} \quad & m_2^{\text{pole}} = \sqrt{\Delta m_{21}^2}\,,\ m_3^{\text{pole}} = \sqrt{\Delta m_{31}^2}\,, \\
\text{IH:} \quad & m_2^{\text{pole}} = \sqrt{\Delta m_{31}^2}\,,\ m_3^{\text{pole}} = \sqrt{\Delta m_{21}^2 + \Delta m_{31}^2}\,.
\end{aligned}$$

Summary of parameters

- Parameters, that we use as a direct "Input" from neutrino mixing data:

$$U_{\mathrm{PMNS}}, m_2^{\mathrm{pole}}, m_3^{\mathrm{pole}}$$

- Free parameters:

$$Z_{m3}\,,\ m_4\,,\ \alpha = \arg\left(d'\right)\,, F_1\,(\text{scalar sector})\,,$$

where we traded y for Z_{m3} via:

$$y^2 = \frac{2m_3 m_4}{v^2}\,,\ m_3 = Z_{m3} m_3^{\mathrm{pole}}\,.$$

- "Internal" parameters, that we get from solving $R^T M R = \mathrm{diag}\left(m_2^{\mathrm{pole}}, m_3^{\mathrm{pole}}\right)$:

$$d, \left|d'\right|\,,\ V \approx U^{\mathrm{Input}} R^\dagger\,,\ U^{\mathrm{Input}} = \begin{cases} U_{\mathrm{PMNS}} & \text{for NH} \\ U_{\mathrm{PMNS}} O_{IH} & \text{for IH to match the conventions} \end{cases}$$

all Yukawa couplings are then completely determined by:

$$Y_\nu^1 = (0,0,y)\,V^\dagger\,,\ Y_\nu^2 = \left(0,d,d'\right)\,V^\dagger$$

Determining d and $|d'|$

- $\det\left(M^\dagger M\right) = \left(m_3^{\text{pole}} m_2^{\text{pole}}\right)^2$ gives

$$d^2 = \frac{32\pi^2\, m_2^{\text{pole}}}{Z_{m3} m_4 \left|F_1\right|}\,,\; F_1 = F_1\left(\text{scalar sector}\right),\, Z_{m3} = \frac{m_3}{m_3^{\text{pole}}}$$

- The description breaks down only if $F_1 \approx 0$ (we can estimate the validity from perturbativity $d^2 < 4\pi$)
- $\mathrm{Tr}\left(M^\dagger M\right) = \left(m_2^{\text{pole}}\right)^2 + \left(m_3^{\text{pole}}\right)^2$ gives:

$$(1+x)^2 \frac{1}{Z_{m3}^2} + \mathrm{sign}(F_1)\left(2\cos^2(\alpha) - 1\right) k_{32} x + k_{32}^2\left(Z_{m3}^2 - 1\right) - 1 = 0\,.$$

$$\alpha = \arg\left(d'\right)\,,\; k_{32} = \frac{m_3^{\text{pole}}}{m_2^{\text{pole}}}\,,\; x = \left|\frac{d'}{d}\right|^2\,.$$

Solutions

- Meaningful solutions are $x = \left|\frac{d'}{d}\right|^2 \geq 0$, which give:

$$Z_{m3} \geq \frac{m_2^{\text{pole}}}{m_3^{\text{pole}}} \Rightarrow m_3 \geq m_2^{\text{pole}}$$

  - The phase α of $d' = |d'|\, e^{i\alpha}$ is restricted for $Z_{m3} \geq 1$:

$$F_1 > 0 : \cos^2\alpha < \frac{1}{2} \cdot \left(1 - \frac{m_2^{\text{pole}}}{m_3^{\text{pole}}}\right), \qquad F_1 < 0 : \cos\alpha \to \sin\alpha .$$

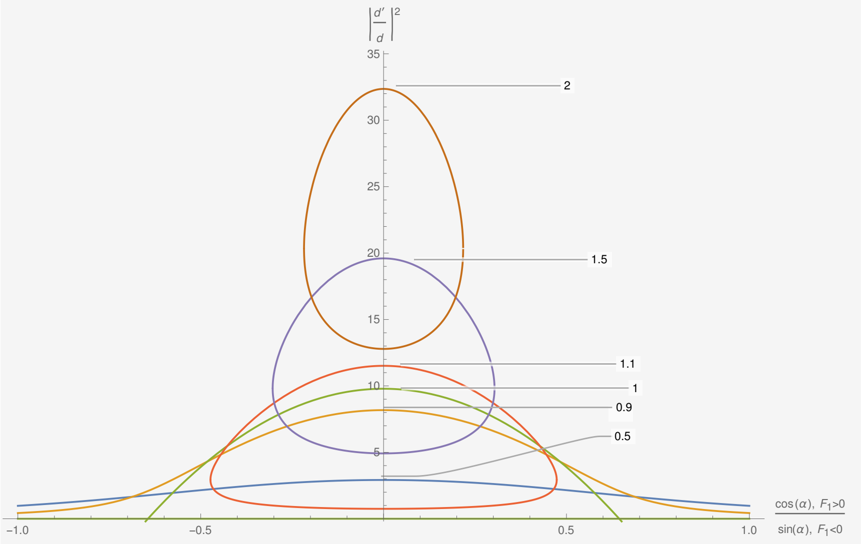
- Squeezed parameter space for IH, since $\frac{m_2^{\text{pole}}}{m_3^{\text{pole}}} \approx 0.986$:

$$Z_{m3} \geq 0.986\,; \text{ for } Z_{m3} > 1 : \begin{cases} \alpha \approx \frac{\pi}{2}\,, & F_1 > 0, \\ \alpha \approx 0\,, & F_1 < 0\,. \end{cases}$$

- Special case possible:

$$d' = 0 : \begin{cases} m_3 & = m_2^{\text{pole}} \\ m_3 & = m_3^{\text{pole}} \end{cases}$$

Solutions in NH



How large can d, d' be?

- If we restrict $\delta m < m_3^{\text{pole}}$

$$\Rightarrow \frac{1}{2} < Z_{m3} < \frac{3}{2} \Rightarrow \left|\frac{d'}{d}\right|^2 < 20$$

$\Rightarrow$ typically, we can say that $|d'| \sim d$ (except for the special case of $|d'| = 0$)

- Couplings:

$$\text{NH: } d^2 = \frac{2.7\text{ eV}}{Z_{m3} m_4} \cdot \frac{1}{|F_1|}, \qquad \text{IH: } d^2 = \frac{16\text{ eV}}{Z_{m3} m_4} \cdot \frac{1}{|F_1|}$$

- Example:

$$m_4 = O(\text{MeV}) \text{ and } d^2 = O\left(10^{-2}\right) : \begin{cases} \text{NH:} & |F_1| = O\left(10^{-4}\right), \\ \text{IH:} & |F_1| = O\left(10^{-3}\right) \end{cases}$$

Conclusions

- In GN model, the Yukawa sector is almost completely determined by neutrino oscillations
  - only 2 dofs. are free (Z_{m3} and $\arg(d')$) which have allowed ranges, **not** dependent on scalar sector.
- GN model can accomodate experimental neutrino values with almost any scalar potential (only for $F_1 \approx 0$, this description breaks down.)
- d, d' can be significant for some scalar potential points.
- TODO:
  - d and d' enter LFV processes, such as $\ell_i \to \ell_j \gamma \Rightarrow$ possible to restrict F_1 (scalars) from below.
  - DM, Higgs decays?
  - employ FlexibleSUSY for more elaborate scans.

Thank you!