

# Neutrino physics in gauged U(1) extensions of the standard model

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16 September 2021

Matter to the Deepest 2021

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arXiv:hep-ph/2104.14571 (accepted for publication in PRD),

arXiv:hep-ph/2105.13360

# Problems in the Standard Model

- > Why neutrinos are massive?
- > What is dark matter?
- > Why is the electroweak vacuum metastable?
- > What is the origin of cosmic inflation?
- > Why is the matter-antimatter asymmetry larger than expected?

# Problems in the Standard Model

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⇒ seesaw mechanism
- > What is dark matter?  
⇒ keV scale sterile neutrinos
- > Why is the electroweak vacuum metastable?  
⇒ it is stabilized by an extra scalar field
- > What is the origin of cosmic inflation?  
⇒ two-field curvaton inflation
- > Why is the matter-antimatter asymmetry larger than expected?  
⇒ resonant leptogenesis mechanism

# Super-weak model answers these

Main paper: Trócsányi, 1812.11189

> Why neutrinos are massive?

⇒ seesaw mechanism

Iwamoto, Kärkkäinen, Peli, Trócsányi, 2104.14571,

Kärkkäinen, Trócsányi, 2105.13360 ⇒ This talk

> What is dark matter?

⇒ keV/MeV scale sterile neutrinos

Iwamoto, Seller, Trócsányi, 2104.11248 ⇒ Károly Seller's talk

> Why is the electroweak vacuum metastable?

⇒ it is stabilized by an extra scalar field

Peli, Trócsányi, 1902.02791

> What is the origin of cosmic inflation?

⇒ two-field curvaton inflation

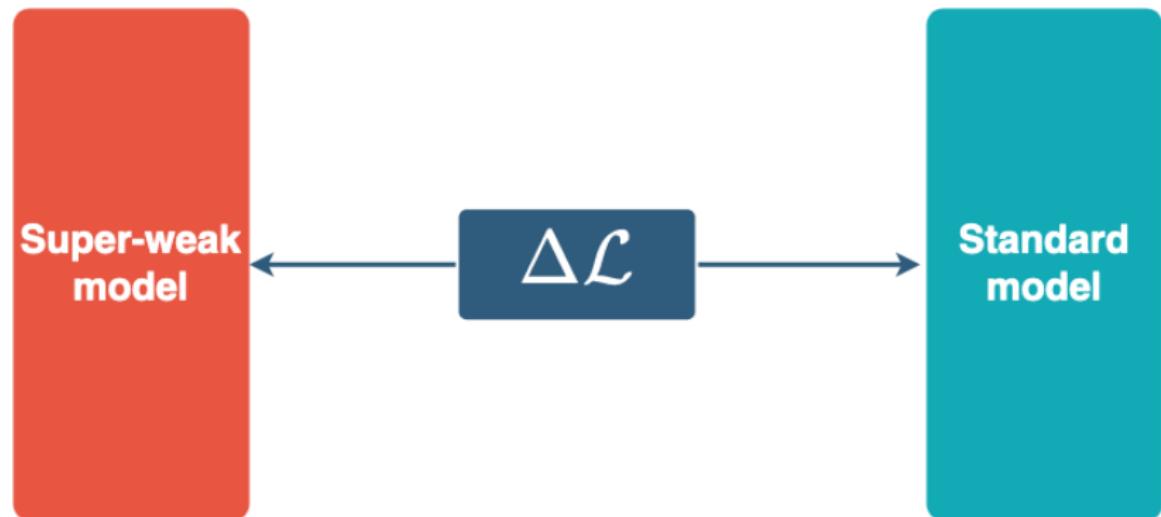
Nándori, Peli, Trócsányi, 1911.07082

> Why is the matter-antimatter asymmetry larger than expected?

⇒ resonant leptogenesis mechanism

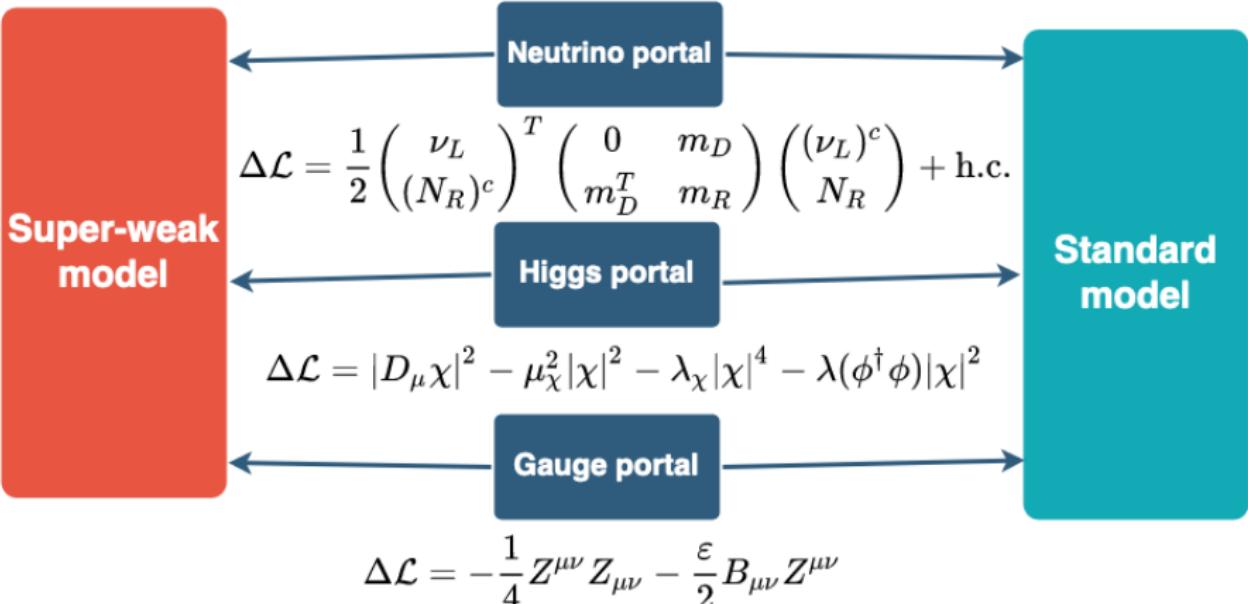
# How to access physics beyond the SM?

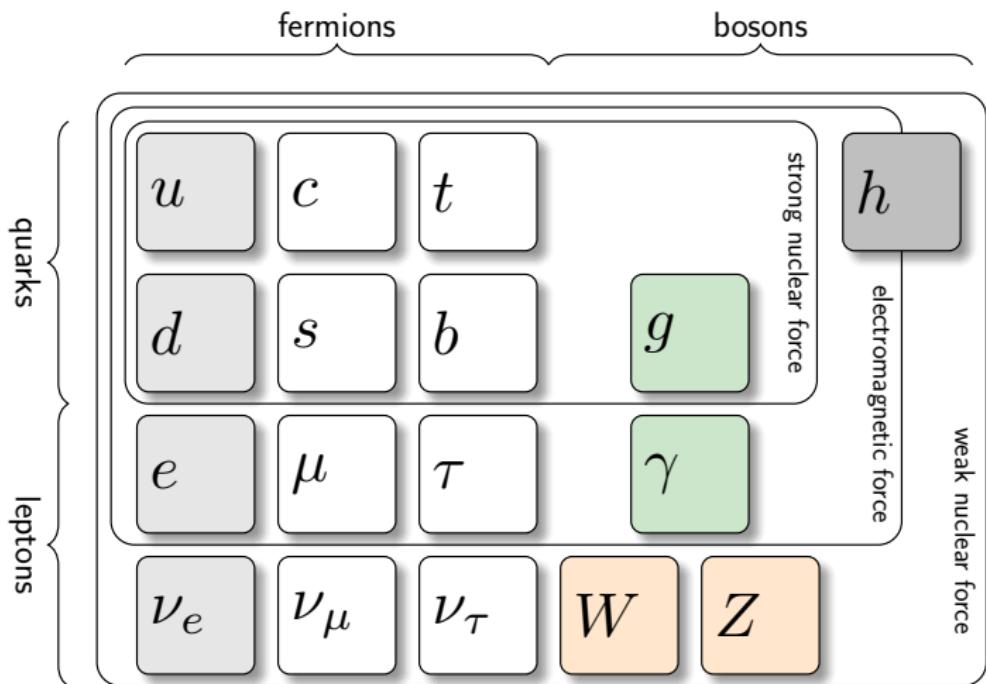
Existence of dark matter, baryonic asymmetry of the universe, metastability of vacuum *et cetera* suggests that SM is incomplete.

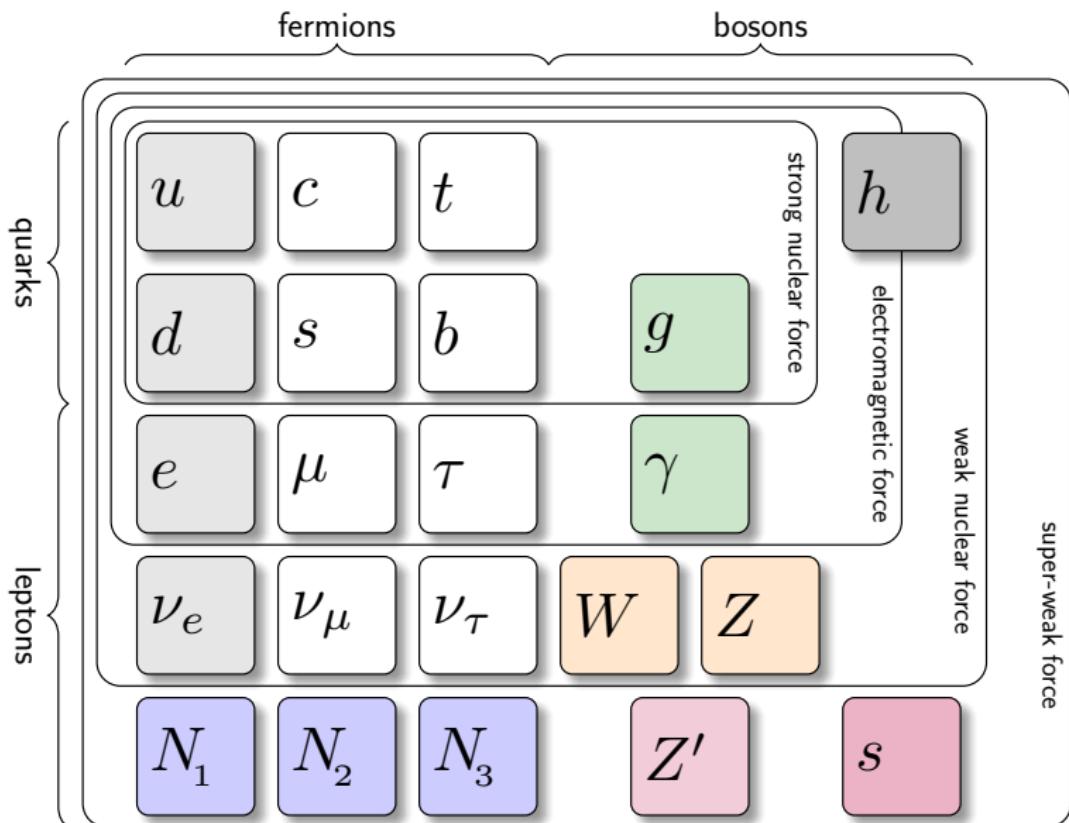


New interactions should exist! Possible new particles must be either very heavy or must interact super-weakly, otherwise we would have seen notable deviations from SM predictions.

# Pick your favourite?







# We choose all three simple portal extensions

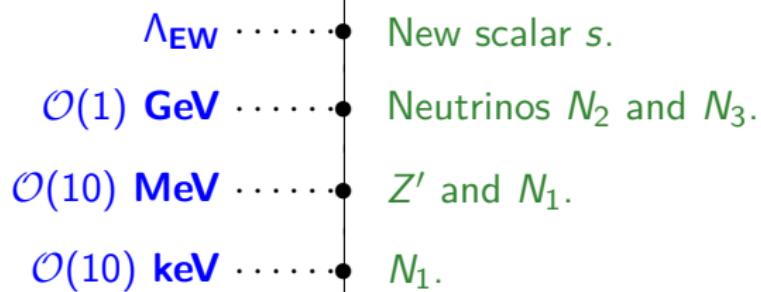
$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_Z$$

Field	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_Z$
$Q_L$	2	$\frac{1}{6}$	$\frac{1}{6}$
$u_R$	1	$\frac{2}{3}$	$\frac{7}{6}$
$d_R$	1	$-\frac{1}{3}$	$-\frac{5}{6}$
$L_L$	2	$-\frac{1}{2}$	$-\frac{1}{2}$
$\ell_R$	1	-1	$-\frac{3}{2}$
$\phi$	2	$\frac{1}{2}$	1
$N_R$	1	0	$\frac{1}{2}$
$\chi$	1	0	-1

SM  
+  
three sterile right-handed Majorana neutrinos  
+  
complex scalar singlet,  
which breaks the  $\mathrm{U}(1)_Z$   
symmetry and produces a massive gauge boson,  $Z'$ .

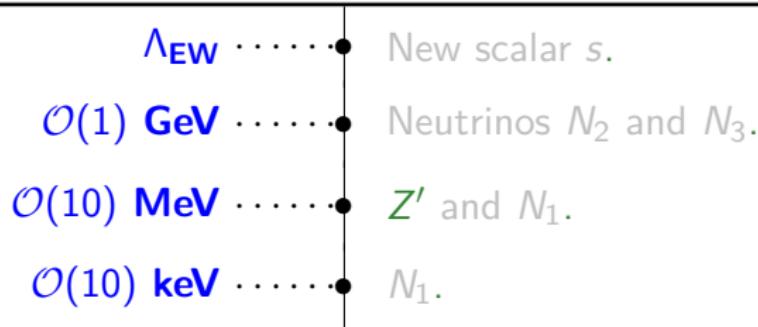
## Chosen mass scales for the new particles

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_Z$$



## Gauge sector

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_Z$$



$$\mathcal{L}_{\text{gauge}} = \underbrace{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}}_{\mathrm{U}(1)_Y} - \underbrace{-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu}}_{\mathrm{U}(1)_Z} - \underbrace{-\frac{\varepsilon}{2}B_{\mu\nu}Z^{\mu\nu}}_{\text{kinetic mixing}} - \underbrace{-\frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}}_{\dots}$$

## Gauge sector

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{\varepsilon}{2}B_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \dots$$

The model exhibits kinetic mixing between the U(1) gauge fields:

$$\Delta\mathcal{L} = -\frac{\varepsilon}{2}B_{\mu\nu}Z^{\mu\nu},$$

$$D'^\mu = \partial^\mu + ig_L \boldsymbol{\tau} \cdot \mathbf{W}^\mu + i(y g_y B_\mu + z g_z B'_\mu)$$

## Gauge sector

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{\varepsilon}{2}B_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \dots$$

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$$\Delta\mathcal{L} = -\frac{\varepsilon}{2}B_{\mu\nu}Z^{\mu\nu},$$

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We can choose a basis where the mixing is absent, corresponding to covariant derivative

$$D^\mu = \partial^\mu + ig_L \boldsymbol{\tau} \cdot \mathbf{W}^\mu + i \left( y g_y - (z - y\eta) g'_z \right) \times (\dots)$$

$$\eta = \frac{\varepsilon g_y}{g_z}, \quad g'_z = \frac{g_z}{\sqrt{1 - \varepsilon^2}}$$

# The neutral gauge bosons $Z$ and $Z'$ mix weakly

$$\begin{pmatrix} \hat{B}^\mu \\ W^{3\mu} \\ \hat{B}'^\mu \end{pmatrix} = \mathbf{R}(\theta_W, \theta_Z) \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix},$$

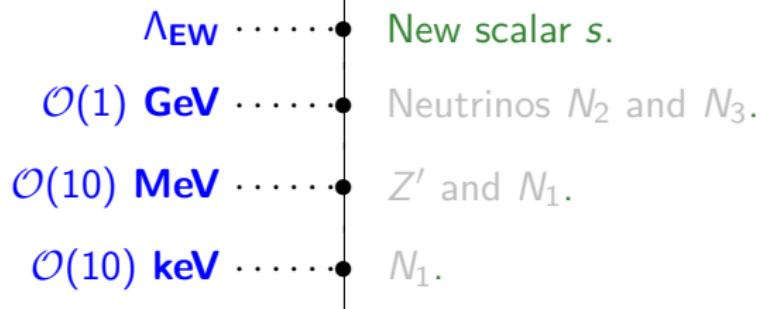
where the  $Z - Z'$  mixing angle is

$$\theta_Z = \theta_Z \left( g_y, g'_z, \frac{\langle \chi \rangle}{\langle h \rangle} \right) \ll 1,$$

suppressed due to small couplings.

## Scalar sector

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_Z$$



$$\mathcal{L}_{\text{scalar}} = |D_\mu \phi|^2 + |\not{D}_\mu \chi|^2 - \mu_\phi^2 |\phi|^2 - \mu_\chi^2 |\chi|^2 - \lambda_\phi |\phi|^4 - \lambda_\chi |\chi|^4 - \lambda |\phi|^2 |\chi|^2,$$

Lagrangian:

$$\mathcal{L}_{\text{scalar}} = |D_\mu \phi|^2 + |D_\mu \chi|^2 - \mu_\phi^2 |\phi|^2 - \mu_\chi^2 |\chi|^2 - \lambda_\phi |\phi|^4 - \lambda_\chi |\chi|^4 - \lambda |\phi|^2 |\chi|^2,$$

Vacuum expectation values:

$$\langle \phi \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \chi \rangle \equiv \frac{w}{\sqrt{2}},$$

Lagrangian:

$$\mathcal{L}_{\text{scalar}} = |D_\mu \phi|^2 + |D_\mu \chi|^2 - \mu_\phi^2 |\phi|^2 - \mu_\chi^2 |\chi|^2 - \lambda_\phi |\phi|^4 - \lambda_\chi |\chi|^4 - \lambda |\phi|^2 |\chi|^2,$$

Vacuum expectation values:

$$\langle \phi \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \chi \rangle \equiv \frac{w}{\sqrt{2}},$$

Eigenvalues of the scalar mass matrix are given by

$$m_{s,h} = \lambda_\phi v^2 + \lambda_\chi w^2 \pm \sqrt{(\lambda_\phi v^2 - \lambda_\chi w^2)^2 + \lambda^2 v^2 w^2}$$

RGE analysis and vacuum stability constrain  $m_s \in [144, 598]$  GeV.

# Goldstone and scalar mixing

After spontaneous symmetry breaking, in  $R_\xi$  gauge

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sqrt{2}\sigma^+ \\ v + h' + i\sigma_\phi \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}}(w + s' + i\sigma_\chi)$$

Mass eigenstates are  $h, s, \sigma_h$  and  $\sigma_s$ :

$$\begin{pmatrix} h \\ s \end{pmatrix} = \mathbf{Z}_S \begin{pmatrix} h' \\ s' \end{pmatrix}, \quad \begin{pmatrix} \sigma_Z \\ \sigma_{Z'} \end{pmatrix} = \mathbf{Z}_G \begin{pmatrix} \sigma_\phi \\ \sigma_\chi \end{pmatrix}$$

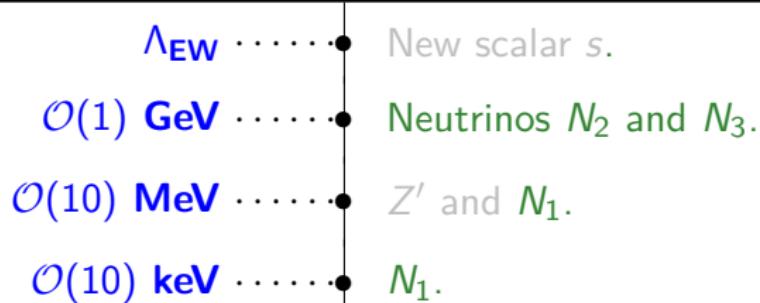
Goldstone mixing between  $\sigma_Z$  and  $\sigma_{Z'}$  is given by a particularly simple form,

$$\boxed{\tan \theta_G = \frac{M_{Z'}}{M_Z} \tan \theta_Z},$$

double-suppressed by  $Z - Z'$  mass ratio and mixing.

# Neutrino sector

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_Z$$



$$-\mathcal{L}_Y^\nu = \underbrace{\frac{1}{2} \bar{\nu}_R \mathbf{Y}_N (\nu_R)^c \chi}_{\Rightarrow \text{Majorana mass matrix}} + \underbrace{\bar{\nu}_R \mathbf{Y}_\nu \varepsilon_{\alpha\beta} L_{L\alpha} \phi_\beta}_{\Rightarrow \text{Dirac mass matrix}} + \text{h.c.}$$

# Neutrino sector - seesaw mechanism

Yukawa terms:

$$-\mathcal{L}_Y^\nu = \frac{1}{2} \overline{\nu_R} \mathbf{Y}_N (\nu_R)^c \chi + \overline{\nu_R} \mathbf{Y}_\nu \varepsilon_{\alpha\beta} L_{L\alpha} \phi_\beta + \text{h.c.}$$

$$-\mathcal{L}_m^\nu = \frac{1}{2} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}^T C \begin{pmatrix} \mathbf{0}_3 & \mathbf{M}_D^T \\ \mathbf{M}_D & \mathbf{M}_N \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}$$

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R = \begin{pmatrix} N_{R1} \\ N_{R2} \\ N_{R3} \end{pmatrix}, \quad \mathbf{M}_D = \frac{v}{\sqrt{2}} \mathbf{Y}_\nu, \quad \mathbf{M}_N = \frac{w}{\sqrt{2}} \mathbf{Y}_N$$

$$\mathbf{M}_L = -\mathbf{M}_D \mathbf{M}_N^{-1} \mathbf{M}_D^\dagger + (\text{subleading terms}),$$

## Neutrino sector - mixing

$$-\mathcal{L}_m^\nu = \frac{1}{2} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}^T C \begin{pmatrix} \mathbf{0}_3 & \mathbf{M}_D^T \\ \mathbf{M}_D & \mathbf{M}_N \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}$$

$$\mathbf{U}^T \begin{pmatrix} \mathbf{0}_3 & \mathbf{M}_D^T \\ \mathbf{M}_D & \mathbf{M}_N \end{pmatrix} \mathbf{U} = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)$$

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_L \\ \mathbf{U}_R^* \end{pmatrix}$$

$\mathbf{U}$  is unitary, but  $3 \times 6$ -matrices  $\mathbf{U}_L$  and  $\mathbf{U}_R$  are semiunitary:

$$\mathbf{U}_L \mathbf{U}_L^\dagger = \mathbf{1}_3, \quad \mathbf{U}_R \mathbf{U}_R^\dagger = \mathbf{1}_3$$

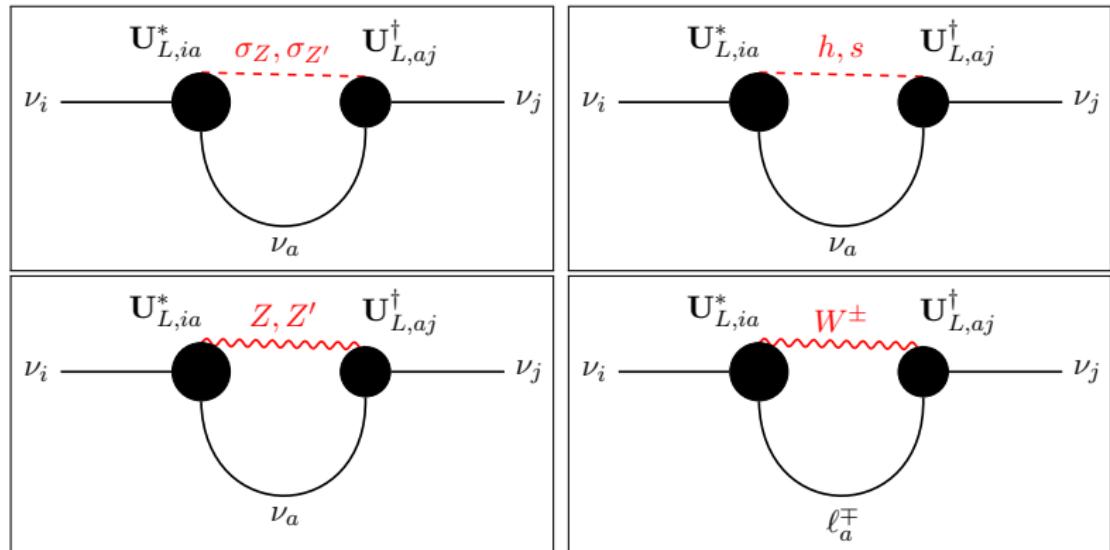
$$\mathbf{U}_L^\dagger \mathbf{U}_L \neq \mathbf{1}_6, \quad \mathbf{U}_R^\dagger \mathbf{U}_R \neq \mathbf{1}_6$$

$$\mathbf{U}^\dagger \mathbf{U} = \mathbf{U}_L^\dagger \mathbf{U}_L + \mathbf{U}_R^\dagger \mathbf{U}_R^* = \mathbf{1}_6$$

# The new fields may enlarge the radiative corrections to $\mathbf{M}_L$

Neutrino self-energy is a  $6 \times 6$  matrix,

$$i\boldsymbol{\Sigma}(p) = \mathbf{A}_L(p^2)\not{p}P_L + \mathbf{A}_R(p^2)\not{p}P_R + \mathbf{B}_L(p^2)P_L + \mathbf{B}_R(p^2)P_R$$



$$\boldsymbol{\Sigma} \equiv \mathbf{U}_L^T (\boldsymbol{\Sigma}^Z + \boldsymbol{\Sigma}^{Z'} + \boldsymbol{\Sigma}^h + \boldsymbol{\Sigma}^s + \boldsymbol{\Sigma}^{\sigma_h} + \boldsymbol{\Sigma}^{\sigma_s} + \boldsymbol{\Sigma}^W) \mathbf{U}_L$$

$$\delta \mathbf{M}_L = \mathbf{U}_L^* \mathbf{B}_L(0) \mathbf{U}_L^\dagger$$

# We generated neutrino interaction vertices via SARAH

Model file available online at arXiv and on PRD!

```
26 Model`Name = "U1XSM";
27 Model`NameLaTeX ="U(1)X extended Standard Model";
28 Model`Authors = "Sho Iwamoto, T. J. K\[\ADoubleDot]rkk\[\ADoubleDot]inen";
29 Model`Date = "2021-03-02";
30
31
32 (*-----*)
33 (*  Particle Content*)
34 (*-----*)
35
36 Gauge[[1]] = {B,    U[1], hypercharge, gYY,  False};
37 Gauge[[2]] = {WB,   SU[2], left,        g2,   True};
38 Gauge[[3]] = {G,    SU[3], color,       g3,   False};
39 Gauge[[4]] = {Bp,   U[1], superweak,  gXX,  False};
```

Mathematica extension SARAH generates Feynman rules, beta functions, mass matrices *et cetera*.

# We generated neutrino interaction vertices via SARAH

$$\Gamma_{V\nu_i\nu_j}^\mu = -ie\gamma^\mu \left( \Gamma_{V\nu\nu}^L P_L + \Gamma_{V\nu\nu}^R P_R \right)_{ij}$$

$$\Gamma_{V\nu\nu}^L = C_{V\nu\nu}^L \mathbf{U}_L^\dagger \mathbf{U}_L - C_{V\nu\nu}^R \mathbf{U}_R^T \mathbf{U}_R^*$$

$$\Gamma_{V\nu\nu}^R = -C_{V\nu\nu}^L \mathbf{U}_L^T \mathbf{U}_L^* + C_{V\nu\nu}^R \mathbf{U}_R^\dagger \mathbf{U}_R = -\left(\Gamma_{V\nu\nu}^L\right)^*$$

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$$\Gamma_{S_k/\sigma_k \nu_i \nu_j} = \left( \Gamma_{S_k/\sigma_k \nu\nu}^L P_L + \Gamma_{S_k/\sigma_k \nu\nu}^R P_R \right)_{ij}, \Gamma^R = -(\Gamma^L)^*$$

$$\Gamma_{S_k \nu\nu}^L = -i \left[ \left( \mathbf{M} \mathbf{U}_L^\dagger \mathbf{U}_L + \mathbf{U}_L^T \mathbf{U}_L^* \mathbf{M} \right) \frac{(\mathbf{Z}_S)_{k1}}{\nu} + \mathbf{U}_R^\dagger \mathbf{M}_N \mathbf{U}_R^* \frac{(\mathbf{Z}_S)_{k2}}{w} \right]$$

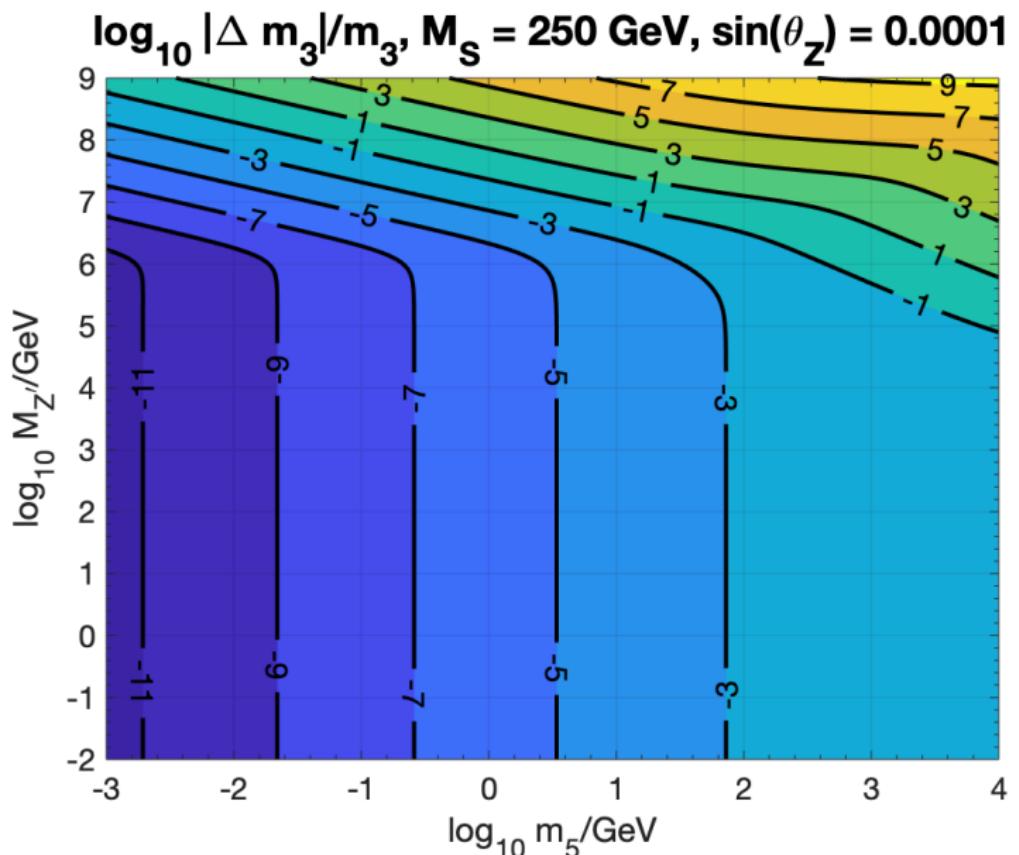
$$\Gamma_{\sigma_k \nu\nu}^L = - \left[ \left( \mathbf{M} \mathbf{U}_L^\dagger \mathbf{U}_L + \mathbf{U}_L^T \mathbf{U}_L^* \mathbf{M} \right) \frac{(\mathbf{Z}_G)_{k1}}{\nu} + \mathbf{U}_R^\dagger \mathbf{M}_N \mathbf{U}_R^* \frac{(\mathbf{Z}_G)_{k2}}{w} \right]$$

$$V \text{ (red wavy line)} = -ie\gamma^\mu \left( \Gamma_{V\nu\nu}^L P_L + \Gamma_{V\nu\nu}^R P_R \right)_{ij}, \quad V = Z, Z'$$

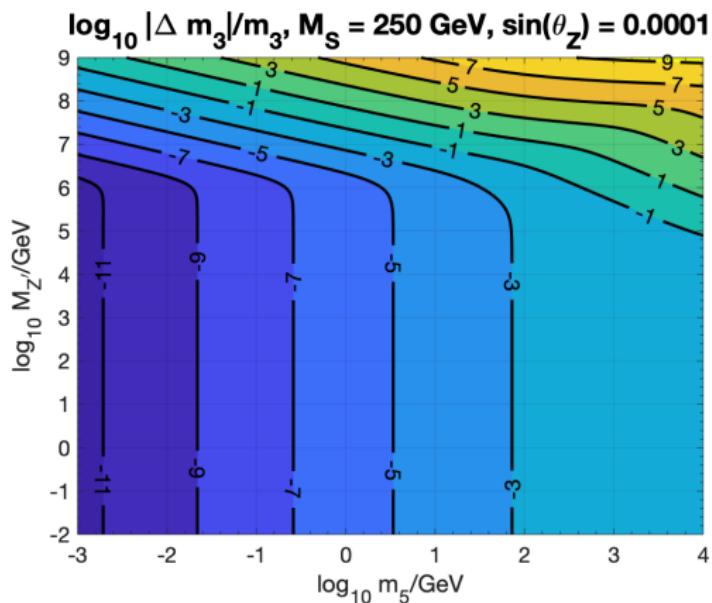
$$S \text{ (red dashed line)} = \left( \Gamma_{S\nu\nu}^L P_L + \Gamma_{S\nu\nu}^R P_R \right)_{ij}, \quad S = h, s$$

$$\sigma \text{ (red dotted line)} = \left( \Gamma_{\sigma\nu\nu}^L P_L + \Gamma_{\sigma\nu\nu}^R P_R \right)_{ij}, \quad \sigma = \sigma_Z, \sigma_{Z'}$$

# Final result for one-loop correction



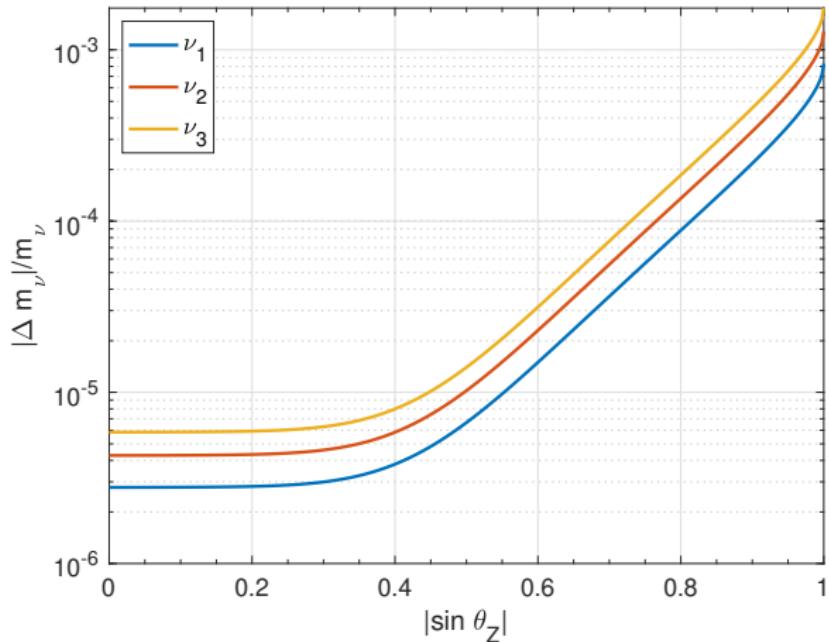
# Final result for one-loop correction



$$\delta \mathbf{M}_L = \frac{1}{16\pi^2} \sum_{k=1,2} \left[ 3(\mathbf{Z}_G)_{k1}^2 \frac{M_{V_k}^2}{v^2} \mathbf{F}(M_{V_k}^2) + (\mathbf{Z}_S)_{k1}^2 \frac{M_{S_k}^2}{v^2} \mathbf{F}(M_{S_k}^2) \right]$$

$$\mathbf{F}_{ij}(M^2) = \sum_{a=1}^6 (\mathbf{U}_L^*)_{ia} (\mathbf{U}_L^\dagger)_{aj} \frac{m(\nu_a)^3}{M^2} \frac{\ln \frac{m(\nu_a)^2}{M^2}}{\frac{m(\nu_a)^2}{M^2} - 1}$$

# Relative correction to $m_\nu$ , with $M_{Z'} = 50$ MeV



## Parameterization

We use the Casas-Ibarra parameterization, which allows us to input neutrino masses and PMNS matrix consistently

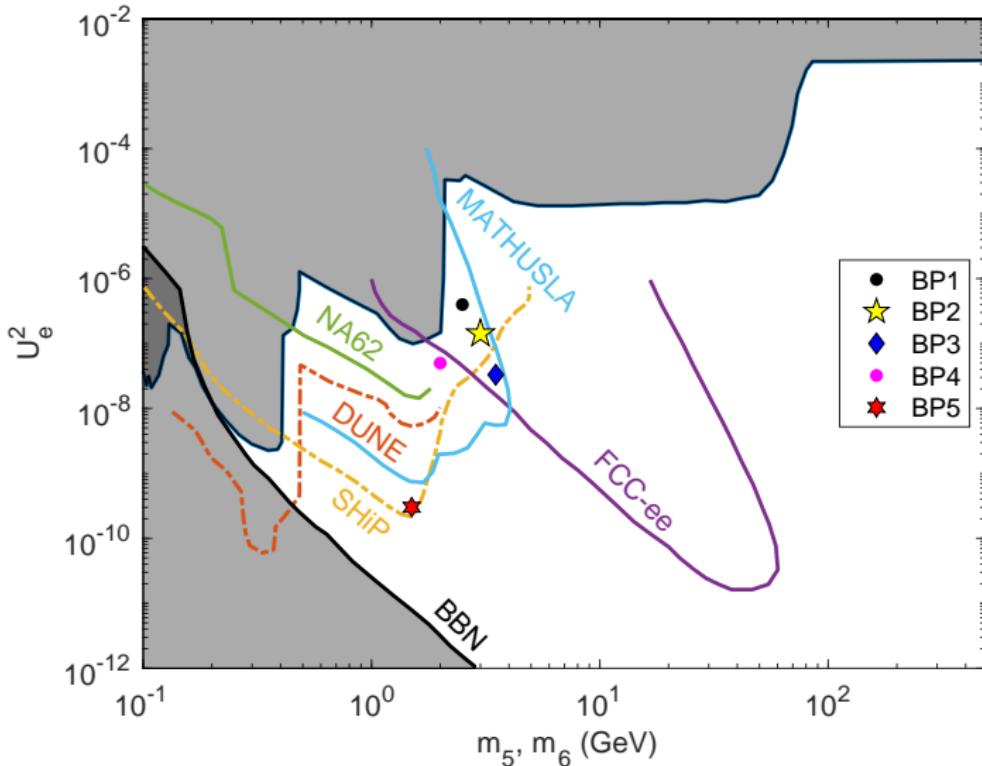
$$Y_\nu = \frac{\sqrt{2}}{\nu} U_{\text{PMNS}}^* \sqrt{m_\nu^{\text{diag}}} (-iR^T) \sqrt{m_R}, \quad R \in O(3)$$

$$U_{\nu \leftrightarrow N} = U_{\text{PMNS}} \sqrt{m_\nu^{\text{diag}}} iR^T \sqrt{m_R^{-1}}$$

The standard case  $R = \mathbf{1}_3$  produces the expected active-sterile mixing,  $|U_{\nu \rightarrow N}^{ij}| \approx \sqrt{m_\nu/m_R}$ .

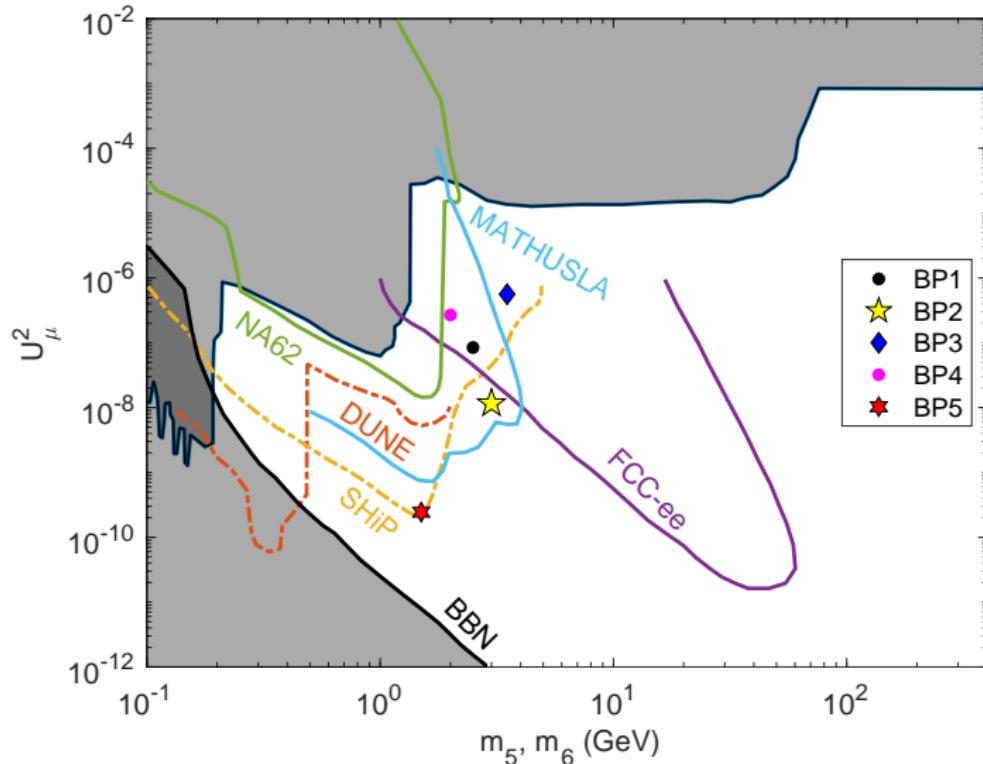
However since  $R$  is a general orthogonal matrix, **benchmarks corresponding to large mixing can be generated** by parameterizing  $R$  as Euler rotation matrix and scanning over the possible values.

# Experimental constraints for GeV-scale sterile neutrinos



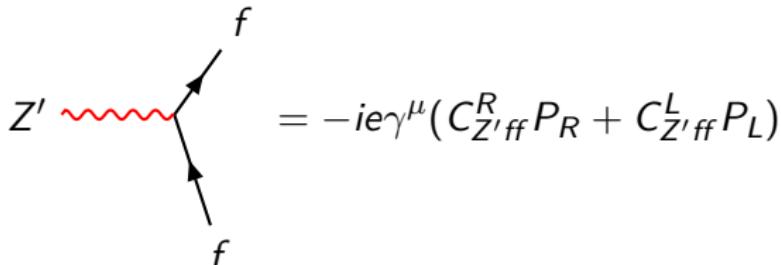
$$U_e^2 = \sum_{i=4}^6 |U_{ei}|^2$$

# Experimental constraints for GeV-scale sterile neutrinos



$$U_{\mu}^2 = \sum_{i=4}^6 |U_{\mu i}|^2$$

## $Z'$ interactions

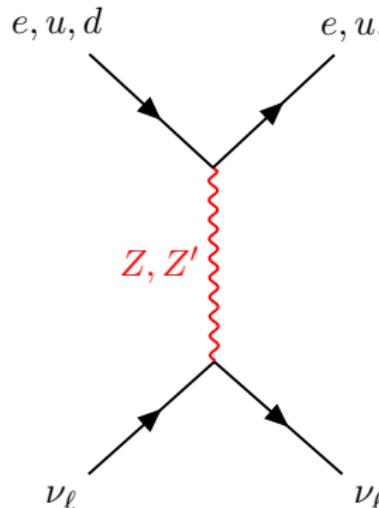


The couplings have the form

$$eC_{Z'ff}^{L,R} = (Q_1 g'_y + Q_2 g'_z) \cos \theta_Z + (Q_3 + Q_4 \sin^2 \theta_W) g_L \frac{\sin \theta_Z}{\cos \theta_W}$$

where  $Q_1, Q_2, Q_3, Q_4 \in \mathbb{Q}$  are different for different fermions and chiralities.

# Nonstandard interactions



We integrate out the  $Z'$  boson to obtain effective nonrenormalizable dimension-6 operator.

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \varepsilon_{\ell\ell'}^{ff} (\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)(\bar{f} \gamma_\mu P_L f)$$

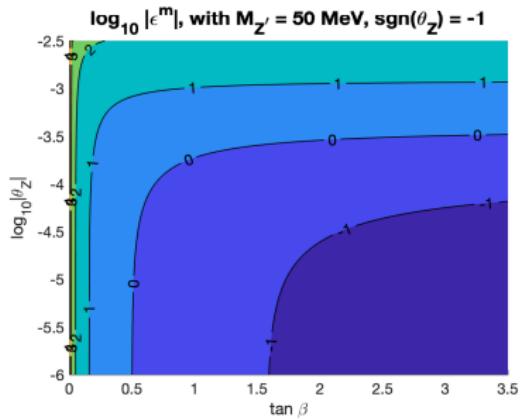
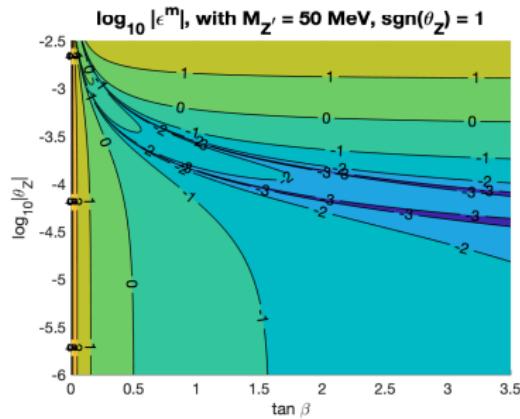
$$\varepsilon_{\ell\ell}^{m,X} = \frac{v^2}{2M_{Z'}^2} e C_{Z'\nu\nu}^L e C_{Z'ff}^X$$

$$\varepsilon_{\ell\ell}^m = \underbrace{\varepsilon_{\ell\ell}^e + 2\varepsilon_{\ell\ell}^u + \varepsilon_{\ell\ell}^d}_{=0} + \frac{N_n}{N_e} (\varepsilon_{\ell\ell}^u + 2\varepsilon_{\ell\ell}^d)$$

Sum over the fermions  $f = e, u, d$  and chiralities  $X = L, R$ .

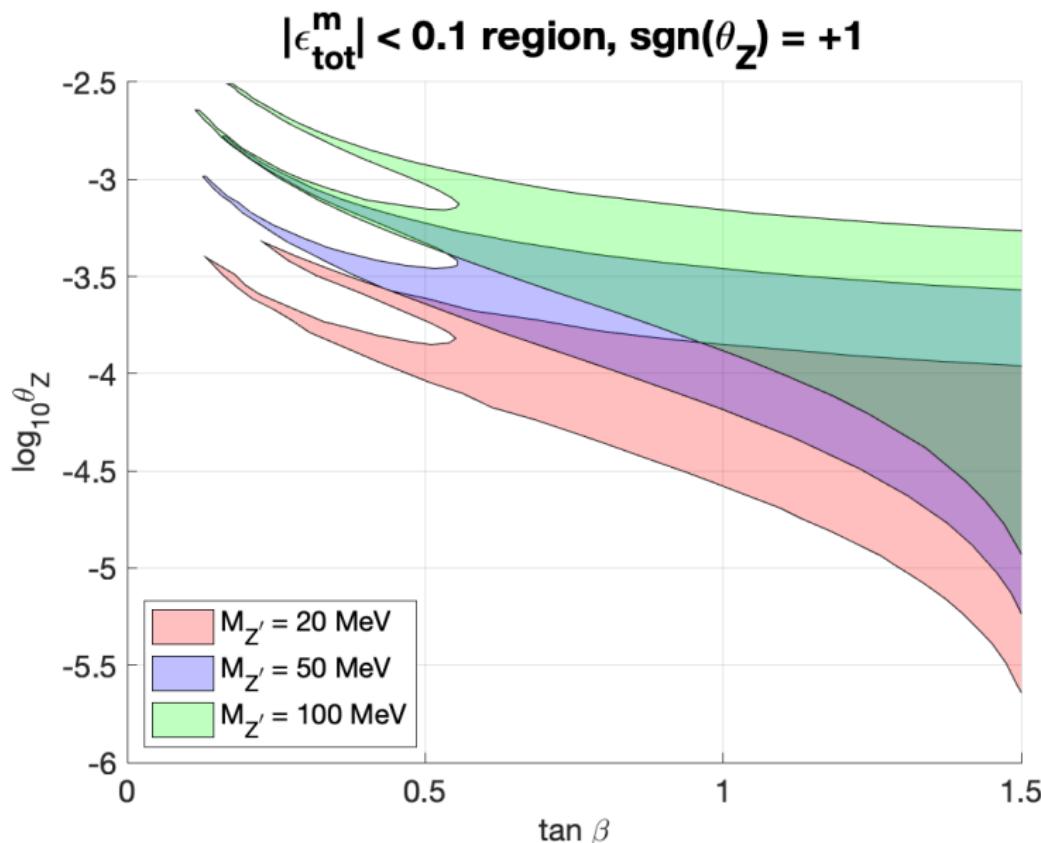
$$\varepsilon_{\ell\ell}^m = -\frac{v^2}{8M_{Z'}^2} \frac{N_n}{N_e} \left( g'_y \cos \theta_Z - \frac{g_L \sin \theta_Z}{\cos \theta_W} \right) \left( (g'_y - g'_z) \cos \theta_Z - \frac{g_L \sin \theta_Z}{\cos \theta_W} \right)$$

# NSI constraints for the gauge sector



Effective NSI for neutrino oscillation is suppressed by active-sterile mixing:  $|\varepsilon^{\text{eff}}| \sim |\varepsilon U_{\nu \leftrightarrow N}|$ .

# MeV scale $Z'$ boson constrains neutral gauge boson mixing



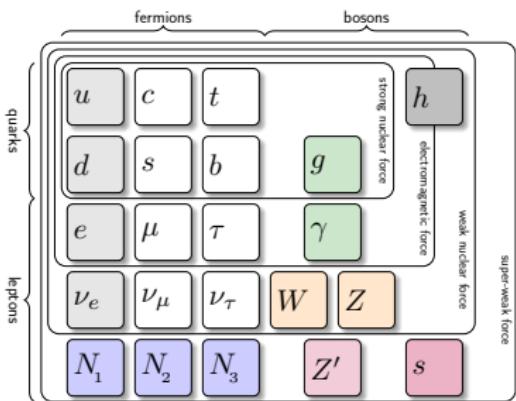
## Conclusions

- Super-weak model is an economical U(1) extension of the standard model, adding only sterile neutrinos, complex singlet scalar and a massive neutral vector boson.
- New neutrino interactions are manifested via nonstandard interactions and active-sterile mixing.
- Higher order corrections to neutrino masses are small.

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- Super-weak model is an economical  $U(1)$  extension of the standard model, adding only sterile neutrinos, complex singlet scalar and a massive neutral vector boson.
- New neutrino interactions are manifested via nonstandard interactions and active-sterile mixing.
- Higher order corrections to neutrino masses are small.

Thank you!



# Backup slides

$\varepsilon, g'_z \ll 1$  translate to light  $M'_Z \ll M_Z$  and  $\theta_Z \ll 1$

$$\begin{pmatrix} \hat{B}^\mu \\ W^{3\mu} \\ \hat{B}'^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\cos \theta_Z \sin \theta_W & -\sin \theta_Z \sin \theta_W \\ \sin \theta_W & \cos \theta_Z \cos \theta_W & \cos \theta_W \sin \theta_Z \\ 0 & -\sin \theta_Z & \cos \theta_Z \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix},$$

$$\tan(2\theta_Z) = \frac{2\kappa}{1 - \kappa^2 - \tau^2}$$

$$\kappa = \cos \theta_W (\gamma'_y - 2\gamma'_z), \quad \tau = 2 \cos \theta_W \gamma'_z \tan \beta \ll 1$$

$$\gamma'_y = \frac{\varepsilon g_y}{g_L \sqrt{1 - \varepsilon^2}}, \quad \gamma'_z = \frac{g'_z}{g_L} \ll 1$$

$$M_Z = \frac{M_W}{\cos \theta_W} (1 + \mathcal{O}(\kappa^2)) \quad M_{Z'} = \frac{M_W}{\cos \theta_W} \tau (1 + \mathcal{O}(\kappa^2)) \ll M_Z$$

$$\Rightarrow \tau \approx M_{Z'}/M_Z$$

# Goldstone and scalar mixing

After spontaneous symmetry breaking, in  $R_\xi$  gauge

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sqrt{2}\sigma^+ \\ v + h' + i\sigma_\phi \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}}(w + s' + i\sigma_\chi)$$

Mass eigenstates are  $h, s, \sigma_h$  and  $\sigma_s$ :

$$\begin{pmatrix} h \\ s \end{pmatrix} = \mathbf{Z}_S \begin{pmatrix} h' \\ s' \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_S & -\sin \theta_S \\ \sin \theta_S & \cos \theta_S \end{pmatrix} \begin{pmatrix} h' \\ s' \end{pmatrix}$$

$$\begin{pmatrix} \sigma_Z \\ \sigma_{Z'} \end{pmatrix} = \mathbf{Z}_G \begin{pmatrix} \sigma_\phi \\ \sigma_\chi \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_G & -\sin \theta_G \\ \sin \theta_G & \cos \theta_G \end{pmatrix} \begin{pmatrix} \sigma_\phi \\ \sigma_\chi \end{pmatrix}$$

Goldstone mixing between  $\sigma_Z$  and  $\sigma_{Z'}$  is given by

$$\sin \theta_G = \frac{\tau \sin \theta_Z}{\sqrt{(\cos \theta_Z - \kappa \sin \theta_Z)^2 + (\tau \sin \theta_Z)^2}}$$