Lepton flavor symmetry in a three-Higgs doublet model

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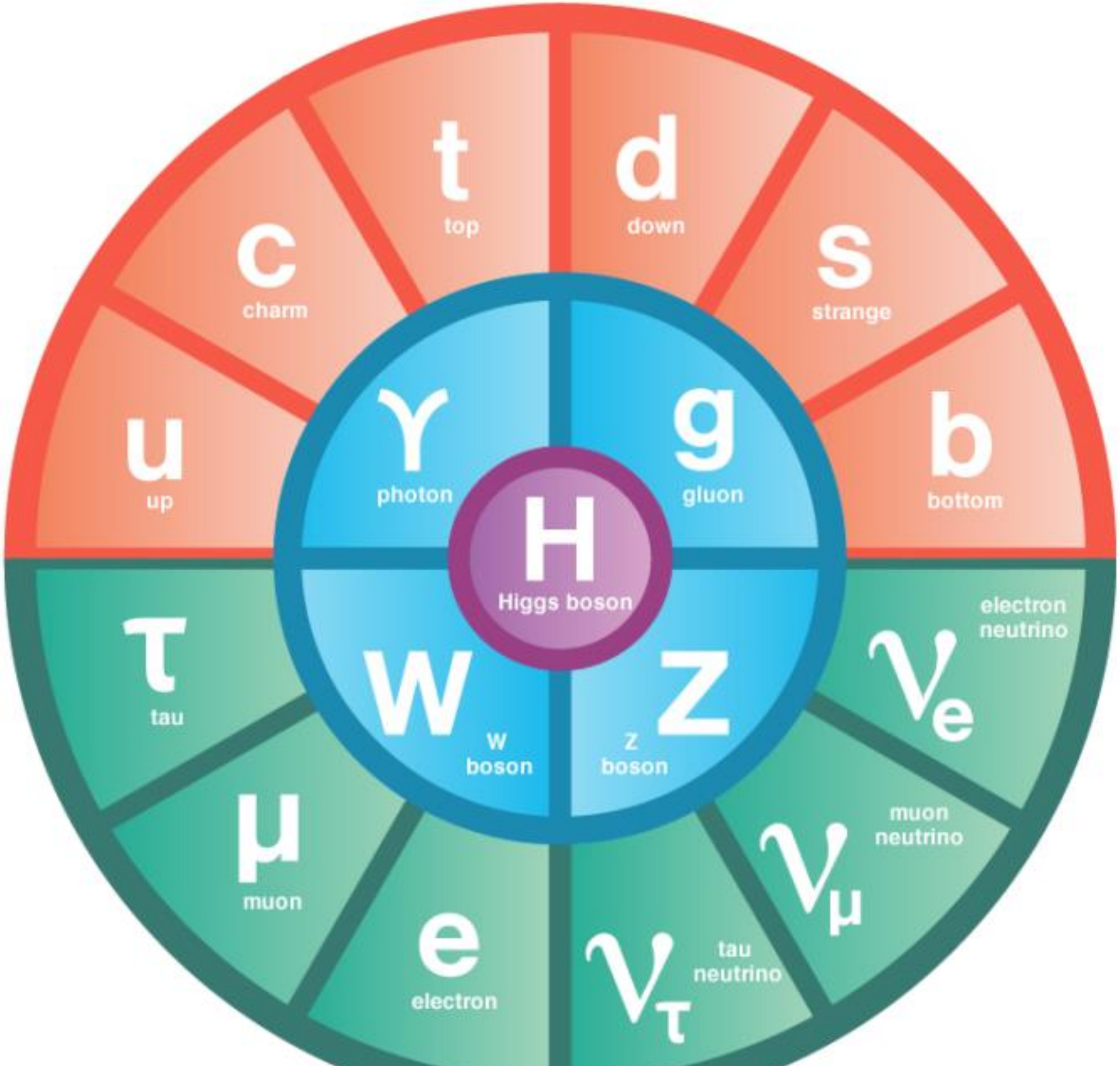
What is the 3HDM?



Picture by Michael S. Gold

What is the 3HDM?

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Focus on the lepton sector

Masses, and neutrino mixing among flavors

$$\overline{\Psi}_L \, \Phi \, \Psi_R + \text{H.c.}$$

mass term of a fermion

in the Lagrangian

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$$\overline{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R}$$

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in the Lagrangian

α=1,2,3

β=1,2,3

i=1,2,3

$$-(h_i^l)_{\alpha\beta}\overline{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R}$$

mass term of the three charged leptons

$$\mathcal{L}^l = -(h_i^l)_{\alpha\beta}\overline{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R} + \text{H.c.}, \tag{1}$$

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3 left-handed fields (3 flavors)

3 right-handed fields (3 flavors)

3 Higgs fields

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h^l_i is a 3x3 matrix of coupling coefficients,

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$$\mathcal{L}^\nu = -(h_i^\nu)_{\alpha\beta}\overline{L}_{\alpha L}\Phi_i \nu_{\beta R} + \text{H.c.}, \tag{2}$$

$$\mathcal{L}^M = -\frac{g}{M}(h_{ij}^M)_{\alpha\beta}(\overline{L}_{\alpha L}\Phi_i)(\Phi_j^T L_{\beta R}^c) + \text{H.c.}, \tag{3}$$

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mass term of three neutrinos

mass term of three Majorana neutrinos

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In general L^l is not invariant after altering any of the flavor vectors.

But in *some* situations L^l remains invariant!

If so, we have found a flavor symmetry.

$$\cdot (h_i^l)_{\alpha\beta} \overline{L}_{\alpha L} \tilde{\Phi}_i l_{\beta R} \cdot$$

$$\cdot\,(h_i^l)_{\alpha\beta}\,\overline{L}_{\alpha L}\,\tilde{\Phi}_i\,l_{\beta R}\;\cdot$$

Simplifying the notation

$$\cdot (h_i^l)_{\alpha\beta} \overline{L}_{\alpha L} \tilde{\Phi}_i l_{\beta R} \cdot$$

$$\downarrow$$

$$\bar{l}_L \, (\tilde{\Phi}_i h_i^l) \, l_{R:}$$

Simplifying the notation

$$\bar{l}_L \, (\tilde{\Phi}_i h_i^l) \; l_R$$

3x3 matrix (for each *i*)

$$\bar{l}_L \, \underbrace{(\tilde{\Phi}_i h_i^l)} \, l_R$$

sum (over *i*) of three 3x3 matrices

=

one 3x3 matrix

$$\bar{l}_L \left(\tilde{\Phi}_i h_i^l\right) l_R$$

row vector

vector

sum of three 3x3 matrices

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Still the sum of 27 terms.

$\Phi = (\Phi_1, \Phi_2, \Phi_3)^T$ is a vector

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Still the sum of 27 terms.

Alltogether we have three vectors.

Question:

Can we alter the three vectors without changing L?

That is:

Can we impose a symmetry on (a part of) L?

Relevance:

Imposing a flavor symmetry restricts masses and mixing
(in case of leptons: neutrino mixing angles)

Insight in whether and how the SM could be extended

Why is the mixing between quarks and between leptons
so drastically different

For overview and other work:

- F. Feruglio *et al* (2021)
- M. Holthausen *et al* (2013)
- C.S. Lam (2011)
- P.O. Ludl (2010)
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$$\bar{l}_L \left(\tilde{\Phi}_i h_i^l\right) l_R:$$

$$\bar{l}_L\,(\tilde{\Phi}_i h_i^l)\ l_R:$$

$$\bar{l}_L A_L^\dagger\,((A_\Phi^* \tilde{\Phi})_i h_i^l)\,A_{lR}\,l_R$$

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$$\bar{l}_L A_L^\dagger \, ((A_\Phi^* \tilde{\Phi})_i h_i^l) \, A_{lR} \, l_R$$

All three vectors are being altered by some linear operator *A*

Each A is a linear operator in a 3D flavor space.

Lagrangian will in general change.

$$\bar{l}_L A_L^\dagger \left((A_\Phi^* \tilde{\Phi})_i h_i^l \right) A_{lR}\, l_R$$

Each *A* is an operator in a 3D flavor space.

Lagrangian will in general change.

UNLESS:

- *A* is unitary
- *A* is a group action
- h^l_i is well chosen (three 3x3 matrices)

$$\bar{l}_L A_L^\dagger \left((A_\Phi^* \tilde{\Phi})_i h_i^l \right) A_{lR} \, l_R$$

$$\bar{l}_L A_L^\dagger \left((A_\Phi^* \tilde{\Phi})_i h_i^l \right) A_{lR}\, l_R$$ is constant?

Can be solved algebraically by letting *A* be 3D representations of some group *G*.

That means: the *A*'s will be 3x3 matrices

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Trivial solution for *G* being the trivial group.

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Can be solved algebraically by letting A be 3D representations of some group G.

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Trivial solution for G being the trivial group.

We require:
- A's are 3D irreducible representations of G
- Not all reps are unfaithful
- h^l_i is a unique solution

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after symmetry-breaking

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Mass matrix

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VEV of Higgs *i*

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\begin{aligned}
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3x3 matrix

member of group *G*

vector of lenght 27

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For which groups *G* do we find a solution?

Scope: G finite, order of $G < 1032$

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There are about 50,000,000,000 groups of order < 1032

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6,012,859 vectors h^l (27D) were found, using GAP and Mathematica (same for h^ν)

2130 of them are linearly independent

They imply masses of the leptons, and the neutrino mixing matrix

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Consistent with experimental data?

Mixing angle example:
group S_4 (order 24), assuming ν are Dirac particles.

Implied mixing angles are functions of the v_i
(since the mass matrices are):

$$M^l = -\frac{1}{\sqrt{2}} v_i^* h_i^l$$

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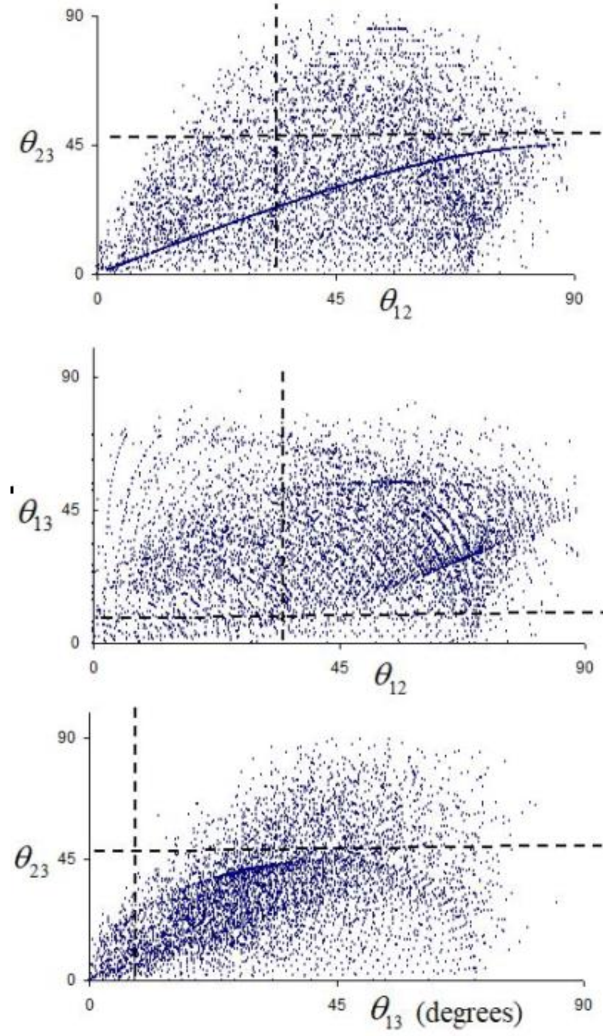

Angles of PMNS matrix

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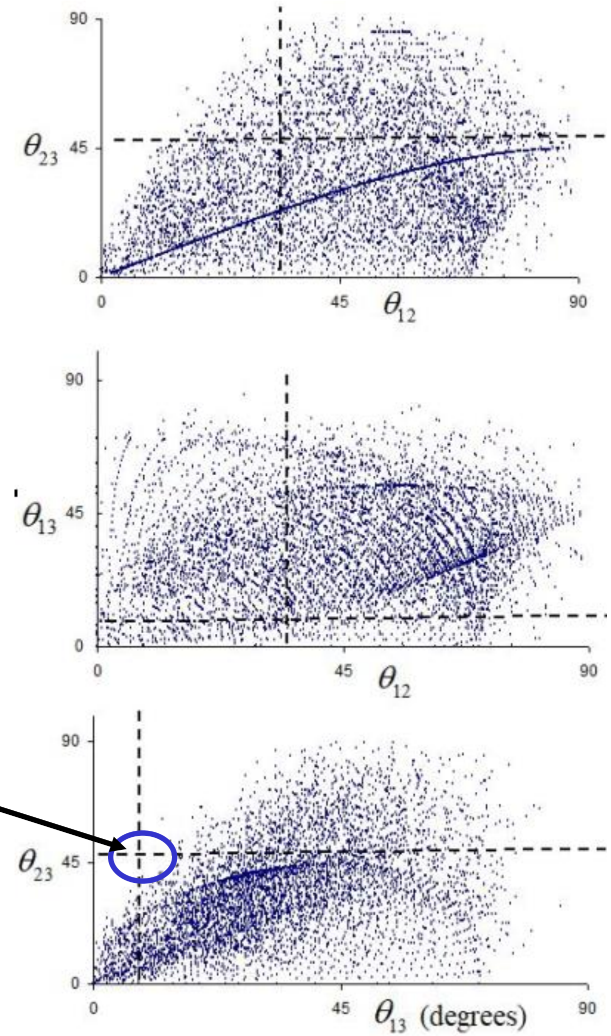

For a group and choice of 3D irreps we can calculate:

-Implied mass ratios of charged leptons
-Implied mass ratios of neutrinos
-Implied mixing angles

We check neutrinos Dirac or Majorana

We find groups providing <u>one or more</u> of:

A: Consistent mass ratios of charged leptons
B: Consistent mass ratios of neutrinos
C: Consistent neutrino mixing angles

for neutrinos being either Dirac or Majorana.

GAP-ID	Structure	3D Irreps (Faithful)	Dirac Mass	Maj. Mass	Dirac Mixing χ^2	Maj. Mixing χ^2
$[12, 3]$	A_4	1 (1)	-	-	-	-
$[21, 1]$	T_7	2 (2)	a	-	d	-
$[24, 12]$	S_4	2 (2)	b,c	-	21.1	-

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Group [24,12]: Dirac mixing $\chi^2 = 21.1$

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$[39,1]$	T_13	4 (4)	a	-	d	-
$[48,3]$	$\Delta(48)$	5 (4)	a	a	d	-
$[48,30]$	$A4:C4$	4 (2)	b	-	d	-
$[48,48]$	$C2\times S4$	4 (2)	b,c	-	21.1	-
$[60,5]$	$A5$	2 (2)	b	-	d	-
$[72,42]$	$C3\times S4$	6 (4)	b,c	-	21.1	-
$[84,11]$		9 (6)	a	a	d	-
$[96,64]$	$\Delta(96)$	6 (4)	a,b,c	a,c	d	0.0
$[96,68]$		10(4)	a	a	d	d
$[96,186]$	$C4\times A4$	8 (4)	b.c	-	21.1	-
$[108,15]$	$\Sigma(36\times 3)$	8 (8)	b,c	-	-	-
$[120,35]$	$C2\times A5$	4 (2)	b	-	d	-
$[120,37]$	$C5\times S4$	10 (8)	b,c	-	21.1	-
$[150,5]$	$\Delta(150)$	8 (8)	a,b,c	a,b,c	d	0.0
$[192,182]$	$\Delta(96,2)$	12 (4)	a,b,c	a,b	d	0.0
$[192,944]$	$C2\times\Delta(96)$	12 (4)	a,b,c	a,c	d	0.0
$[243,19]$	$Z''(3,3)$	24 (18)	a,c	-	600	-
$[432,239]$	$\Pi(1,2)$	16 (8)	b	a	d	d
$[729,63]$	$Z''(3,4)$	72 (54)	a,c	-	600	-

Conclusions:

$\Delta(96)$ is the smallest group defining a flavor symmetry of neutrinos (masses and mixing angles)

Conclusions for the 3HDM

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There are no groups of order < 1032 that can predict all masses and mixing angles of charged leptons and neutrinos simultaneously

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There are no groups of order < 1032 that can predict all masses and mixing angles of charged leptons and neutrinos simultaneously, *for the present criteria of the invariant eigenvectors*

Issues / improvements

Reconsider criteria, such as faithfulness, uniqueness of h, irreducibility

Some issues of non-unitary representations

Reliance on numerical methods

Computational efficiency

Thank you