Phenomena in non-trivial backgrounds

Outline:

- Decay of a muonic atom
  - daughter electron can remain bound
  - decay rate unexpectedly sensitive to components with $E<0$

- Ocean waves vs. de Broglie waves:
  - gravitational time dilation explains the free fall

Matter to the Deepest

Andrzej Czarnecki   University of Alberta

September 16, 2021

Part 1/2: Bound muon decay

PHYSICAL REVIEW D **102,** 073001 (2020)

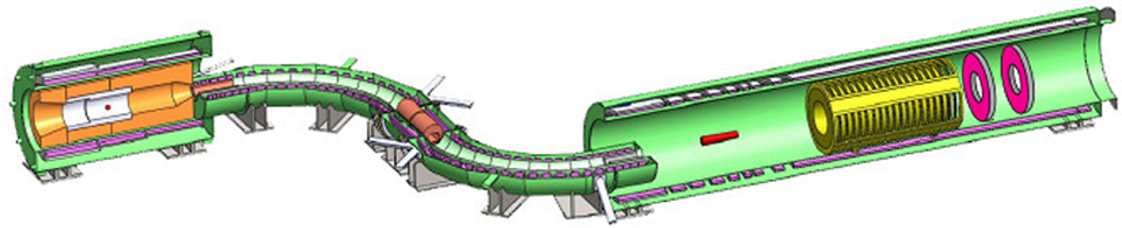
Decay of a bound muon into a bound electron

M. Jamil Aslam,[1,2] Andrzej Czarnecki,[1] Guangpeng Zhang,[1] and Anna Morozova[1]

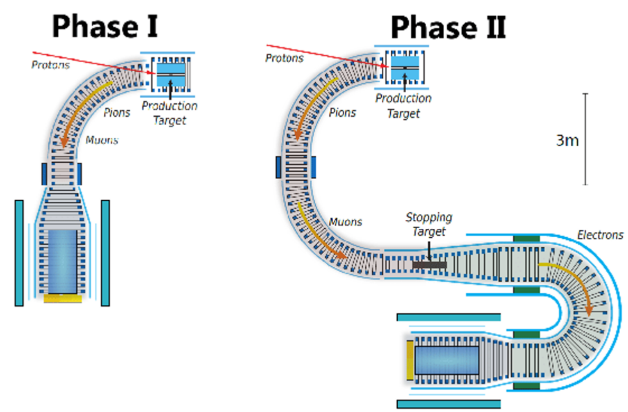
Bound muon decay: why do we care?

Muon-electron conversion searches

Mu2e
Fermilab



COMET
J-PARC



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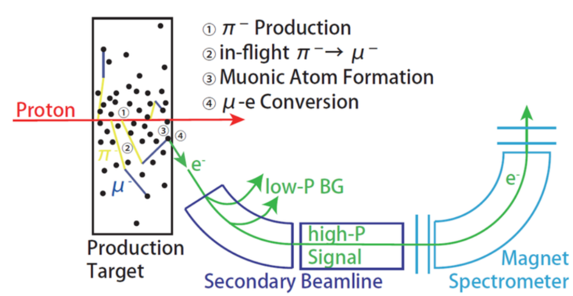

Bound muon decay

Studied in TRIUMF

PHYSICAL REVIEW D **80,** 052012 (2009)

Decay of negative muons bound in ^{27}Al

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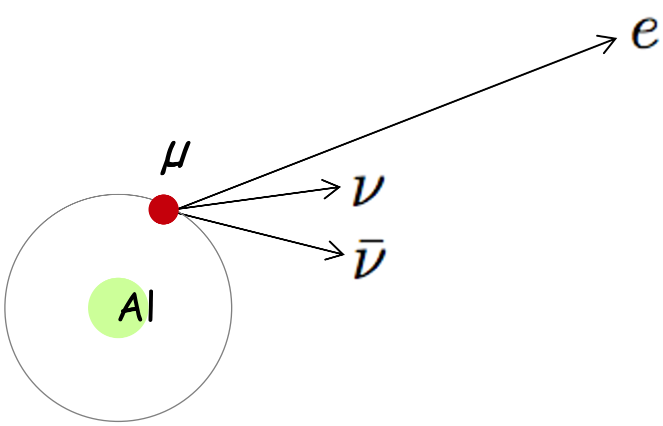
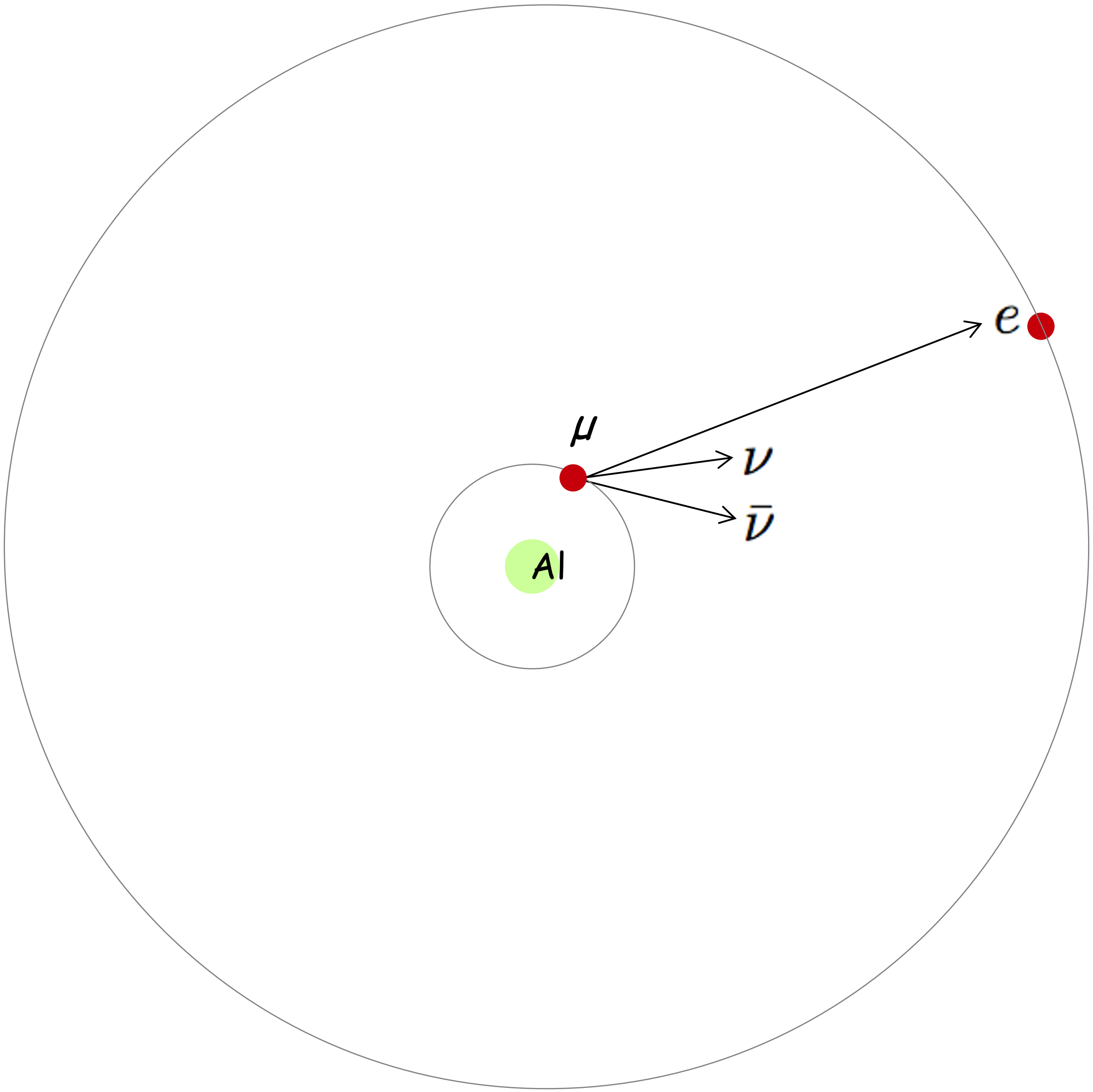


Bound muon decay into a bound electron



What is the probability of binding of the daughter electron?

Bound muon decay into a bound electron

PHYSICAL REVIEW D VOLUME 52, NUMBER 7 1 OCTOBER 1995

Atomic alchemy: Weak decays of muonic and pionic atoms into other atoms

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Bound particle wave function can be expanded in plane waves

$$\tilde{\Phi}(\vec{k}) = \sum_r \left[A_r(\vec{k}) \frac{u_r(\vec{k})}{\sqrt{2k^0}} + B_r^*(-\vec{k}) \frac{v_r(-\vec{k})}{\sqrt{2k^0}} \right]$$

negative energy states:
neglected in the "alchemy" paper:

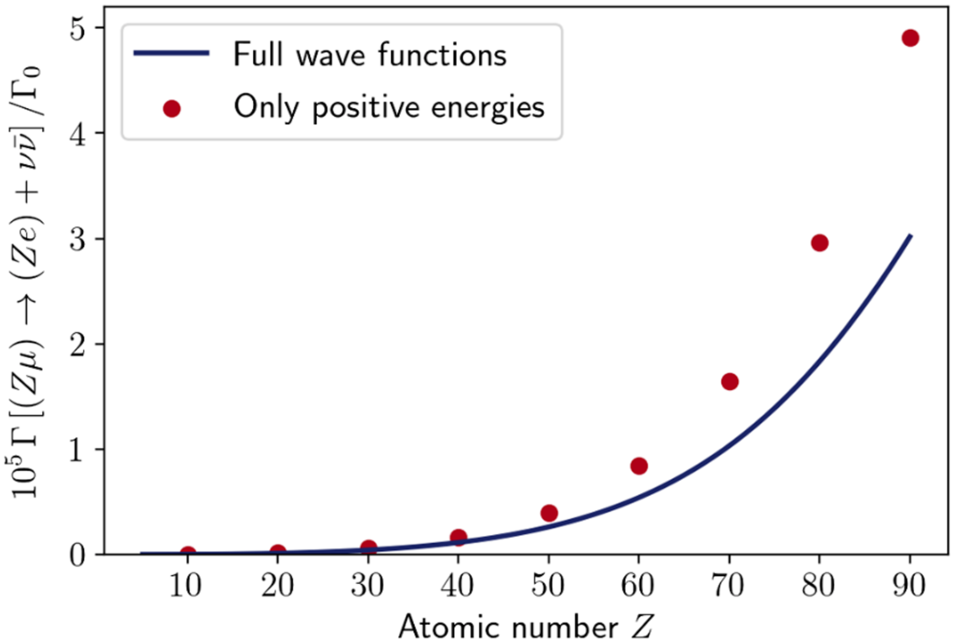
The integral $\int [d^3k/(2\pi)^3] \sum_r |B_r(\vec{k})|^2$ gives the probability to find a three particle Fock state $(e^+e^-e^-)$ in the atom. Even for $Z = 80$ this fraction is tiny ($\approx 0.2\%$), so we only consider the one-Fock contribution characterized by $A_r(\vec{k})$.

Bound muon decay: our study

PHYSICAL REVIEW D **102,** 073001 (2020)

Decay of a bound muon into a bound electron

M. Jamil Aslam,[1,2] Andrzej Czarnecki,[1] Guangpeng Zhang,[1] and Anna Morozova[1]



Why such large effect of negative energies?

Tentative explanation:

The decay happens where the muon and the electron wave functions overlap.
This is a tiny fraction of the electron's range, close to the nucleus.
In that region, $E < 0$ is relatively likely.

Check: position space calculation

$$\mathcal{M} = \frac{g}{\sqrt{2}} \int \mathrm{d}^3 \boldsymbol{r} \exp\left(i\boldsymbol{q} \cdot \boldsymbol{r}\right) \bar{\Phi}_\mu(\boldsymbol{r}) \not{\epsilon}^{\lambda_A \star} L \Phi_e(\boldsymbol{r})$$

$$\frac{1}{\Gamma_0} \Gamma[(Z\mu^-) \to (Ze^-) + \nu\bar{\nu}] = 128 \int_0^{z_{\max}} (N_a^2 + N_b^2 + F_a^2 + F_b^2) k_A z^3 \mathrm{d}z.$$

Our position space and momentum space evaluations agree (if both A and B included).

Part 2/2: Why does everything fall?

Gravitational time dilation, free fall, and matter waves

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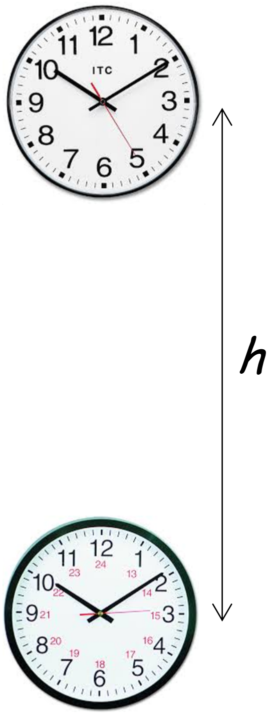
Am. J. Phys. **89** (6), June 2021

Free fall and time dilation

Gravitational time dilation:



$$\Delta t = \frac{gh}{c^2} t$$

Example: a weekend trip from Seattle to Mount Rainier, t = 40 hours, elevation h = 1340 metres,

$$\Delta t = \frac{gh}{c^2} t = \frac{9.8 \cdot 1340}{9 \cdot 10^{16}} 40 \cdot 3600 \text{ s} = 21 \text{ ns.}$$

How does this tiny difference cause falling?

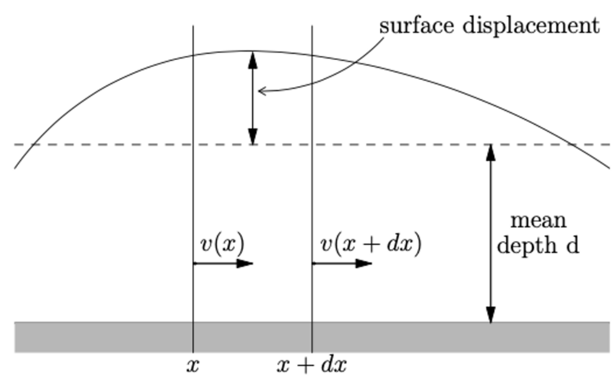
Matter waves evolve faster at a higher elevation.

Intuitive example: ocean waves; why do they always approach the beach?

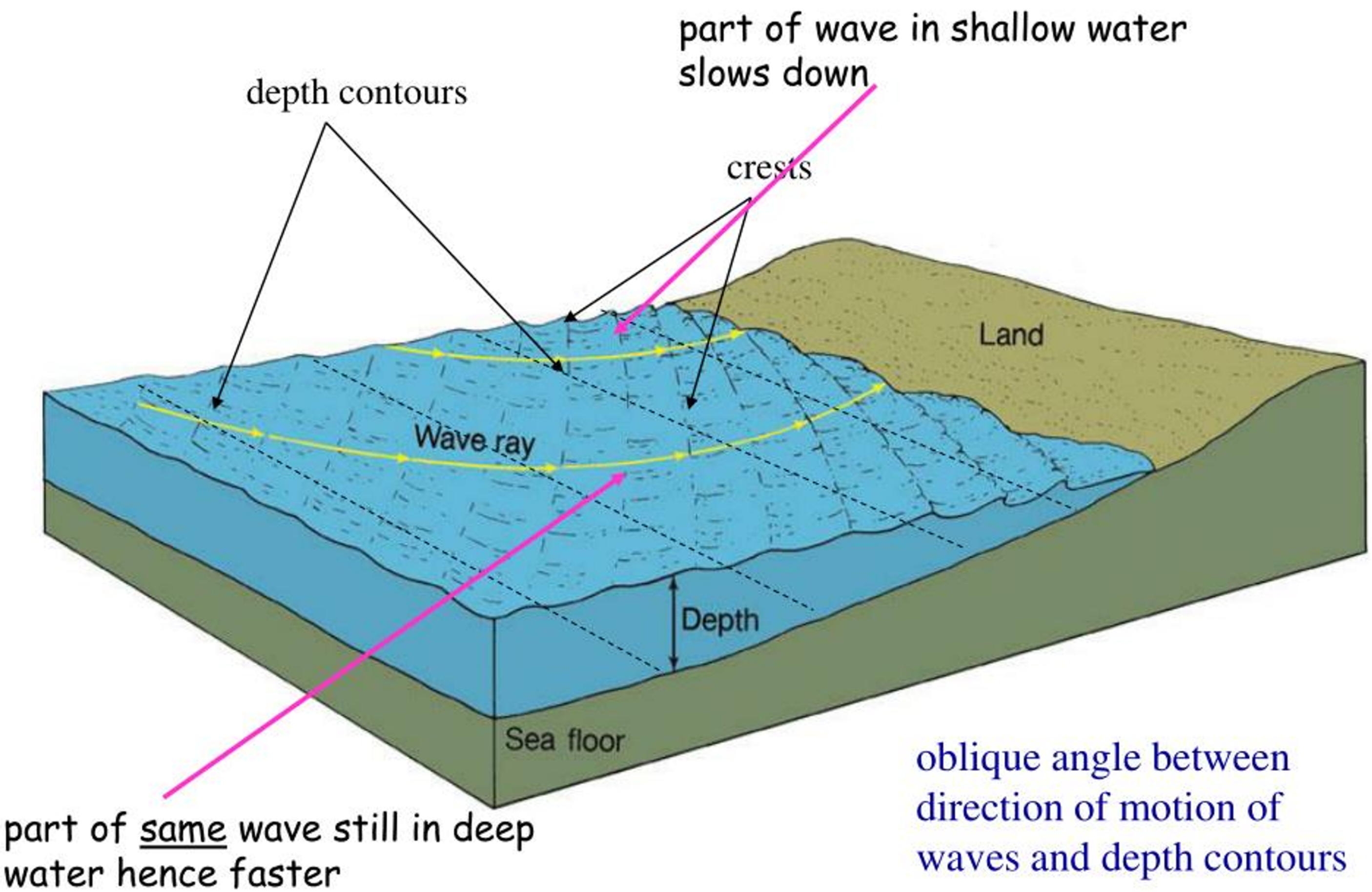
Reason: waves are slower in shallower water,

$$gd\frac{\partial^2 \zeta}{\partial x^2} = \frac{\partial^2 \zeta}{\partial t^2}$$

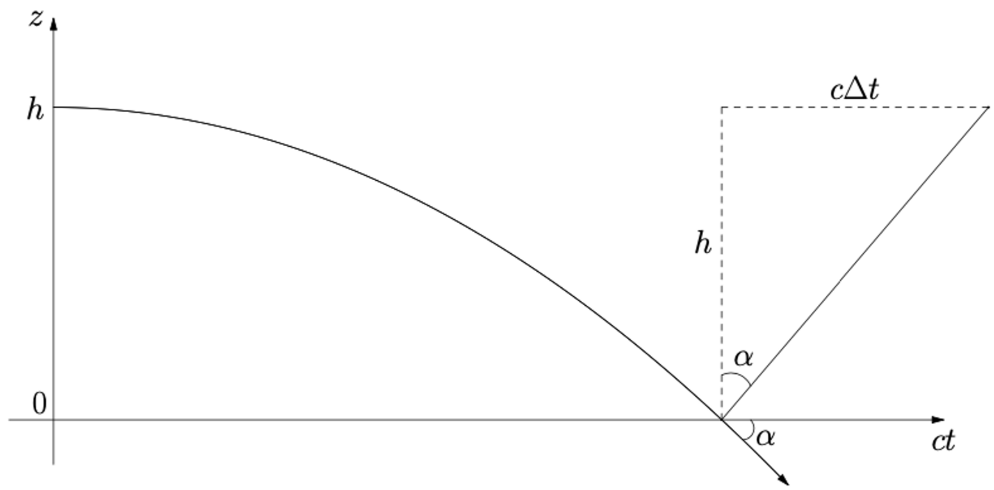
$$u = \sqrt{gd}$$



Wave Refraction – slowing and bending of waves as they approach shore at an angle



What does this mean for matter waves?



Note: this plot shows a one-dimensional motion (only vertical), not a two-dimensional projectile motion.

Velocity from the slope:

$$\frac{v}{c} = -\tan\alpha$$

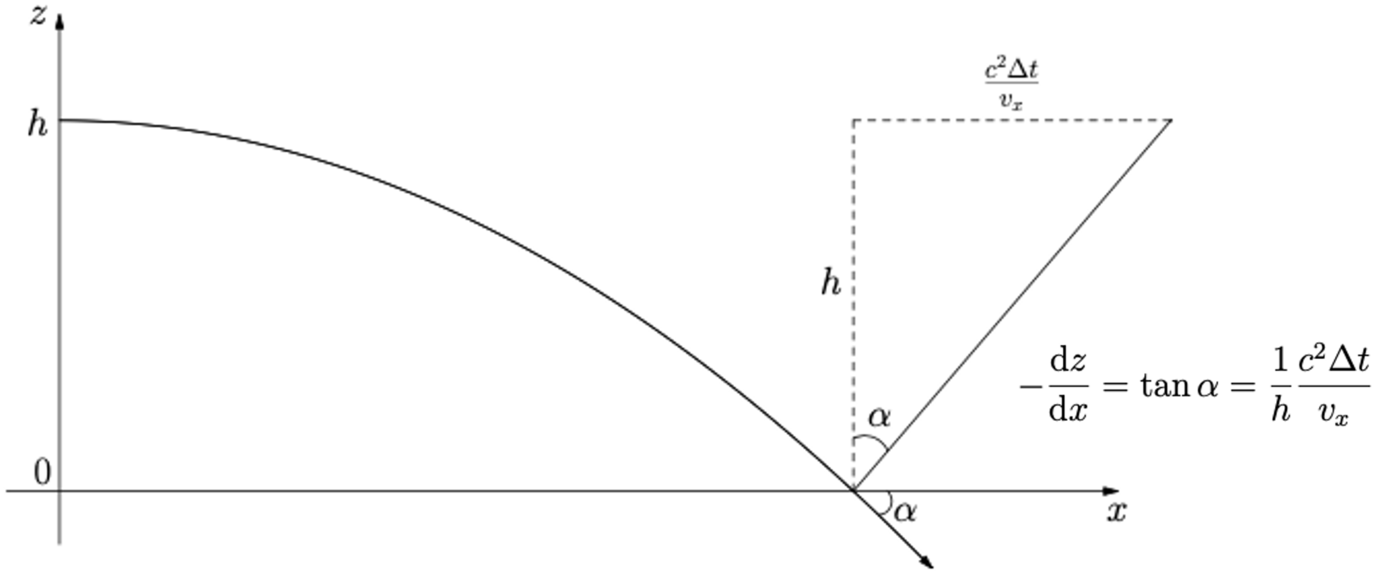
$$\frac{v}{c} = -\frac{c\Delta t}{h}$$

h cancels with $\Delta t = \frac{gh}{c^2}t$

We reproduce the familiar result,

$$\boxed{v = -gt}$$

Another point of view: projectile motion



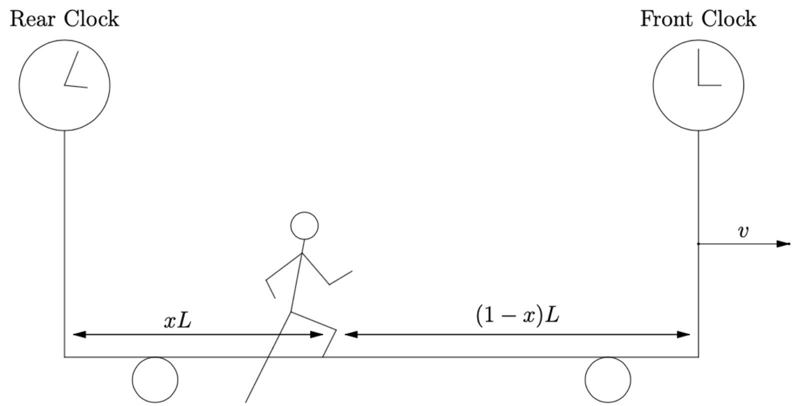
But $\mathrm{d}x = v_x \mathrm{d}t$

$$-\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{c^2 \Delta t}{h} = gt$$

Application: twin paradox (local version)

Rear clock is ahead by

$$\Delta t = \frac{Lv}{c^2}$$



duration of the spurt is $t = v/g$ (here "g" is the average acceleration)

$$\Delta t_R = g\,xL\,t/c^2 \qquad \Delta t_F = g(1-x)Lt/c^2$$

$$\Delta t = \Delta t_F + \Delta t_R = \frac{gLt}{c^2} = \frac{Lv}{c^2}$$

When the observer settles on the train, both clocks show the same time.

Summary

Decay of a bound muon into a bound electron:
- unexpected lesson in the Dirac equation
- negative energy components are important

Free fall explained by the evolution rate of matter waves at various altitudes. Analogy to ocean waves near beaches.