The Grimus-Neufeld model

with FlexibleSUSY at 1-loop

(seesaw + radiative neutrino masses)

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The Grimus-Neufeld model:

Standard Model (SM) + one fermionic singlet + two Higgs doublets

- [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

Features of the Grimus-Neufeld model (GNM)

- an extension of the SM to describe neutrino masses
  - no change to the strong sector
  - only lepton- and Higgs-sector are modified
    - by adding only one fermionic singlet
    - and a second Higgs doublet

The particle spectrum contains additional to the SM

- four Majorana neutrinos
  - one heavy — the added fermionic singlet
  - and three light Majorana neutrinos
    - at tree-level two of them are massless
    - at loop-level one of them gains a radiative mass

- the second Higgs doublet gives
  - a charged scalar H^+ and a two neutral scalars H_k^0

The role of seesaw mechanism and of 2HDM in the GNM

- "normal" seesaw gives a small mass for each heavy mass

  $\Rightarrow$ one heavy fermionic singlet gives only one light neutrino

  – the other two SM-like light neutrinos stay massless

- in the SM fermion masses come from Yukawa couplings

  – massless light neutrinos have vanishing Yukawa couplings

  $\Rightarrow$ massless light neutrinos stay massless

- in the GNM the 2HDM allows different Yukawa couplings

  – the Yukawa couplings to the second Higgs doublet can generate radiative masses for the light neutrinos

  $\Rightarrow$ the 2HDM has to be general (i.e. not a type-I or type-II or . . .)

The Grimus-Lavoura approximation for calculating light neutrino masses

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

- taking the interaction eigenstates of the neutral leptons ν_j and N_R
  - coupled to first Higgs doublet and vev by the Yukawa coupling $(Y_N^{(1)})_j$
- calculating the $(3+1)\times(3+1)$ symmetric mass matrix

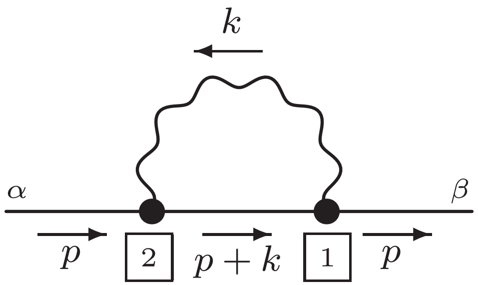
$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} \quad \text{with} \quad \begin{aligned} M_L &= 0_{3\times 3} \quad \ldots \text{ at tree level} \\ M_D &= (m_{De}, m_{D\mu}, m_{D\tau}) = \frac{v}{\sqrt{2}} Y_N^{(1)\top} \end{aligned}$$

  - at tree-level M_ν has rank 2 $\Rightarrow$ only two masses are non-zero
- considering the loop corrections to M_ν:

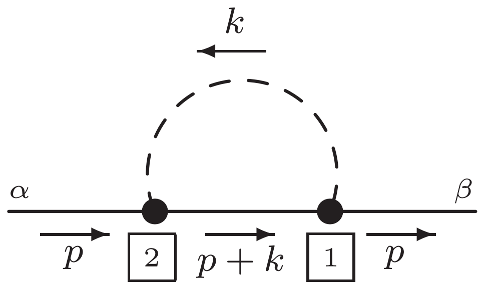
$$\delta M_\nu = \begin{pmatrix} \delta M_L & \delta M_D^\top \\ \delta M_D & \delta M_R \end{pmatrix} \quad \text{with} \quad \delta^{\text{ct}} M_L = 0_{3\times 3}\,, \quad \text{since} \quad M_L = 0_{3\times 3}$$

  - but $\delta M_L \neq 0$, from

$\Sigma_V^{[2]}(p^2) =$



$\Sigma_S^{[2]}(p^2) =$



The Grimus-Lavoura approximation

- reducing the problem to the "light" neutrinos
- leads to an effective 3×3 mass matrix $\mathcal{M}_\nu$
  - at tree level $\mathcal{M}_\nu^{\text{tree}} = -M_D^\top\, M_R^{-1}\, M_D$ has rank 1 ,
  - at one-loop level $\mathcal{M}_\nu^{\text{1-loop}} = \mathcal{M}_\nu^{\text{tree}} + \delta\mathcal{M}_\nu$ can have rank > 1 ,

    with $\delta\mathcal{M}_\nu = \delta M_L - \delta M_D^\top\, M_R^{-1}\, M_D - M_D^\top\, M_R^{-1}\, \delta M_D + M_D^\top\, M_R^{-1}\, \delta M_R\, M_R^{-1}\, M_D$
- the approximation assumes
  - δM_R is irrelevant (as M_R is a free unmeasured parameter, set $\delta M_R = 0$)
  - corrections with δM_D are subdominant
    - suppressed by $Y^2 m_{\ell^\pm}/m_D$ or $g^2 m_{\ell^\pm}/m_D$ compared to $\mathcal{M}_\nu^{\text{tree}}$
  - the correction δM_L is of the same order as $\mathcal{M}_\nu^{\text{tree}}$

⇒ at 1-loop only neutral bosons contribute to δM_L

- calculated from $\Sigma^{[2]}_{V=Z^0}(p^2)$ and $\Sigma^{[2]}_{S=G^0,h,H,A}(p^2)$
  - Z^0 and G^0 sum up to a gauge invariant contribution

Using the Grimus-Lavoura approximation

- parametrize the Yukawa couplings as

$$(Y_N^{(1)})_k = \frac{\sqrt{2} m_D}{v} u_{3k} \qquad (Y_N^{(2)})_k := d\, u_{2k} + d' u_{3k}$$

  - with three orthonormal vectors $\vec{u}_\alpha = u_{\alpha k}$
    - that mix the three neutrinos $\nu_k = u_{\alpha k} P_L \zeta_\alpha^M$ at tree-level

- calculate the effective 1-loop 3×3 mass matrix

$$(\mathcal{M}_\nu^{\text{1-loop}})_{jk} = u_{2j} u_{2k} A + (u_{2j} u_{3k} + u_{3j} u_{2k}) B + u_{3j} u_{3k} C$$

  - which is obviously only rank 2 :

$$u_{\alpha j}^* (\mathcal{M}_\nu^{\text{1-loop}})_{jk} u_{\beta k}^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B & C \end{pmatrix}_{\alpha\beta} ,$$

  - with $A = d^2 f_1, \quad B = d' d f_1 + i d \frac{m_D}{v} f_2, \quad C = d'^2 f_1 + 2 i d' \frac{m_D}{v} f_2 + \frac{m_D^2}{v^2} f_3,$

    and the f_i depending on the parameters of the Higgs sector (and the SM)

- diagonalize $\mathcal{M}_\nu^{\text{1-loop}}$ by the Takagi decomposition :

$$V_{\text{PMNS}}^\top \mathcal{M}_\nu^{\text{1-loop}} V_{\text{PMNS}} = \text{diag}(\widehat{m}_o = 0, \widehat{m}_r, \widehat{m}_s)$$

Using the Grimus-Lavoura approximation part II

- since we get $\hat{m}_r$ and $\hat{m}_s$ as analytic functions
  - we can invert these functions to determine parameters
- we choose to detemine d and $|d'|$
- since $\hat{m}_o = 0$, the measured neutrino mass differences
  - determine $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{sol}}}$ and $\tilde{m}_3 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}}$
    - * attention: there are several possibilities of ordering the neutrinos
- the Takagi decomposition also determines the mixing matrix V_{PMNS}
  - which determines the vectors $u_{\alpha k}$ that define the Yukawa couplings

⇒ we determine the Yukawa couplings $(Y_N^{(1)})_k$ and $(Y_N^{(2)})_k$ (12 real parameters)

- in terms of SM parameters
- the parameters of the neutrino sector: $\Delta m^2_{\text{atm/sol}}$ and V_{PMNS}
- parameters of the Higgs sector: m_h^2, m_H^2, m_A^2, mixing angle $\beta - \alpha$
- and two free parameters: m_D^2 and $\phi' = \arg[d']$

What we achieved is

- a one-loop improved parametrization for the GNM
  - this parametrization reproduces neutrino data exactly
    * the masses at one loop
    * the mixing matrix within the approximation
  - but not a full renormalization
    * . . . this is still a goal for the (far) future

- We can avoid doing the renormalization ourselves :
  - by using a generic tool: a spectrum calculator
  - a spectrum calculator does the renormalization numerically
  - the problem in our case:
    * implementing the seesaw numerically

- with limiting the seesaw scale to $\sim 10^8$ GeV
  - FlexibleSUSY provides qualitativly correct neutrino masses

  $\Rightarrow$ there is hope to make it work at least in this regime part II

FlexibleSUSY

How FlexibleSUSY works

1. fields and Lagrangian of the model are implemented in SARAH
   - this model file is then checked for consistency and completeness
     - done and works
   - if ok, FlexibleSUSY uses SARAH to produce the model (code)
     - done and works
2. then FlexibleSUSY needs a Mathematica steering file: FlexibleSUSY.m.in
   - that tells the boundary conditions for the RGE running
     - ... input and output are also boundary conditions ...
   - the first attempt is working
     - but the one-loop improved parametrization is not yet fully implemented
3. the model has to be compiled and the program then runs
   - from the terminal with SLHA input and output files
   - or with a Mathematica interface ... which is more convenient

FlexibleSUSY input for the GNM

- the SM masses and couplings
  - excluding the Higgs mass and Higgs self coupling
  - have their predefined input
    - but can be changed, too

- the "heavy" Majorana mass

- the parameters of the Higgs potential, λ_i,
  - the basis choice is done at the level of the model file, not here
  - some λ_is can be replaced by the masses of the Higgs bosons and their mixing
    - how to implement them is a matter of convenience

- the Yukawa couplings of the model
  - couplings of the first Higgs doublet to charged fermions are covered by the SM
  - couplings of the second Higgs doublet to charged fermions are free parameters
  - Yukawa couplings to the heavy Majorana fermion
    - either as 12 real parameters ... done and works
    - or with our approach ... implementation still not fully finished

Plans for the FlexibleSUSY implementation of the GNM

- checking the validity of the implementation
  - that includes the correctness of tree- and loop-level
  - and the parameter-range of validity for the seesaw and the GL-approximation
- exploring the parameter space of the GNM
  - seesaw scale / Majorana mass parameter
  - neutrino sector / Yukawa couplings $(Y_N^{(i)})_k$
    - * the other Yukawa couplings, $(Y_{E,U,D}^{(2)})_{jk}$, are not specific for the GNM
  - Higgs potential
- looking / searching for genuine predictions of the GNM
  - low energy observables like $(g-2)_\mu$ or $\mu \to 3e$ or $\mu \to e\gamma$ ...
  - ? limits on Higgs masses / potential parameters
  - ? restrictions on Higgs branching ratios
  - ? implications for cosmology

Activities with the FlexibleSUSY implementation of the GNM

- ✔ checking the validity of the tree-level
  - ✘ but not for all possible implementations of the input
    - $*$ entering Higgs masses as the FS input is not yet done

- ✔ estimating the range of validity for the seesaw
  - ✔ hints that it could work up to 10^8 GeV
    - ✘ but also hints, that the running of Higgs masses limits $m_4 < 10^4$ GeV

- searching for suitable points in the Higgs parameter space
  - to start the exploration of the neutrino parameter space
    - ✔ we have a benchmark point for the 2HDM
    - ✘ but it has near non-perturbative potential parameters for FS
  - ⇒ searching for better points

⇒ a lot of work to be done

- ☞ good for students (thesis themes, etc.)

Summary / Conclusions

✔ We could implement the Grimus-Lavoura approximation for the GNM

– arXiv:1909.00752

☞ The FlexibleSUSY implementation of the GNM is promising

- the full renormalization of the model is still missing
  - but work in progress ... with increased manpower ! ☺

Thank you

for discussion

and comments

and of course for the conference! ☺