Kinematics of t-channel and photon radiation processes in `carlomat`

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Outline

- Motivation
- Phase space integration in `carlomat`
- Phase space parameterizations for t-channel and photon/gluon radiation processes in `carlomat`
- Sample results
- Summary and outlook

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Higher energies and higher luminosity of the current and future colliders, such as LHC, ILC, CLIC, CEPC or FCC, pose a challenge not only for experimenters but for theoreticians, too.

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`carlomat_2.0` has built-in interfaces to parton density functions, includes ISR within the structure function approach, generates a single phase space parameterization for the Feynman diagrams of the same topology; improved color matrix computation; the Cabibbo-Kobayashi-Maskawa mixing in the quark sector included; the effective models including scalar electrodynamics, the *Wtb* interaction with operators of dimension up to 5 and a general top-higgs coupling implemented.

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The precise knowledge of the vacuum polarization is vital, e.g. for the SM predictions for the muon anomaly and for the electromagnetic coupling at the Z peak, $\alpha(M_Z)$, the latter is needed to predict, e.g. M_W and $\sin^2\theta_W$.

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To obtain reliable theoretical predictions for that many hadronic processes is a challenge again.

It is obvious that the correct description of the most relevant hadronic channels as, e.g., $\pi^+\pi^-$, requires the inclusion of radiative corrections.

However, it is probably enough to have the LO predictions for many sub-dominant channels, with 3 or more particles in the final state.

If those channels are measured with the method of radiative return, as is done by KLOE, BaBar and BES, the predictions must also include radiation of photons, both from the initial (ISR) and final (FSR) state.

Production of hadrons at low energies, as well as the photon radiation off them, is usually described in the framework of some effective model, as e.g. Hidden Local Symmetry (HLS) model, which includes quite a number of interaction vertices and mixing terms.

⇒ # Feynman diagrams of such processes may become quite big.
⇒ There is again a strong need for full automation of the MC code generation.

carlomat_3.0 is dedicated to the description of processes of electron-positron annihilation to hadrons at low centre of mass energies.

carlomat_3.1 is a version of the program which, among others, includes a possibility of taking into account

- either the initial or final state radiation separately, or both at a time,
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Multi particle reactions

Consider a multi particle reaction of the form

$$1 + 2 \to 3 + 4 + \ldots + n$$

with the maximum of $n = 12$.

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The phase space integration element

$$\mathrm{d}^{3n_f-4}Lips=(2\pi)^4\delta^{(4)}\left(p_1+p_2-\sum_{i=3}^{n}p_i\right)\prod_{i=3}^{n}\frac{\mathrm{d}p_i^3}{(2\pi)^3 2E_i},$$

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Final state particles $\{3,4,\ldots,n\}$ are divided into two subsets of four momenta q_{i_1} and q_{i_2}, defined in the relative c.m.s., dependent on a topology of the diagram.

Using consecutively the identity

$$\int \mathrm{d}s_i \int \frac{\mathrm{d}^3 q_i}{2E_i}\,\delta^{(4)}\left(q_i - q_{i_1} - q_{i_2}\right) = 1, \qquad E_i^2 = s_i + \vec{q}_i^{\,2}$$

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with $s_i = \left(q_{i_1}+q_{i_2}\right)^2$, $s_0 = s = (p_1+p_2)^2$, λ being the kinematical function, Ω_i the solid angle of momentum $\vec{q}_{i_1}$ in the relative c.m.s., $\vec{q}_{i_1} + \vec{q}_{i_2} = \vec{0}$.

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Denote i-th of N phase space parameterizations generated by

$$f_i(x) = \mathrm{d}^{3n_f-4} Lips_i(x) \qquad i = 1, \ldots, N,$$

where $x = (x_1, ..., x_{3n_f-4})$ are random arguments, $x_i \in [0,1]$.
It must satisfy the normalization condition

$$\int_0^1 \mathrm{d}x^{3n_f-4} f_i(x) = \mathrm{vol}(Lips).$$

All the parameterizations $f_i(x)$ are then automatically combined into a single multichannel probability distribution

$$f(x) = \sum_{i=1}^{N} a_i f_i(x),$$

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The actual MC integration is done with the random numbers generated according to probability distribution $f(x)$.

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Phase space integration - t-channel singularity

What if the t-channel Feynman diagrams become relevant, or if we have to do with a photon/gluon radiation with a soft photon, or collinear singularity?

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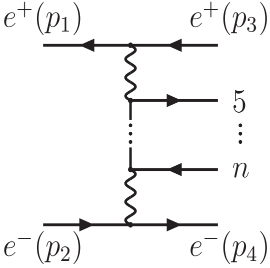

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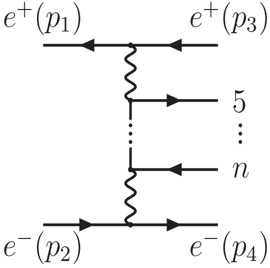

Phase space integration - t-channel singularity

We parameterize the phase space integration element in the following way

$$\begin{aligned} \mathrm{d}^{3n_f-4}Lips &= (2\pi)^{4-3n_f}\mathrm{d}s'\delta^{(4)}\left(p_1+p_2-p_3-p_4-p'\right)\frac{\mathrm{d}p_3^3}{2E_3}\frac{\mathrm{d}p_4^3}{2E_4}\frac{\mathrm{d}p'^3}{2E'} \\ &\times \delta^{(4)}\left(p'-\sum_{i=3}^{n}p_i\right)\prod_{i=5}^{n}\frac{\mathrm{d}p_i^3}{2E_i} \\ &= (2\pi)^{4-3n_f}\mathrm{d}s'\mathrm{d}PS_3(s,m_3^2,m_4^2,s')\mathrm{d}PS_{n_f-3}(s',m_5^2,...,m_n^2), \end{aligned}$$

with $E' = \sqrt{s'+\vec{p}'^2} = \sqrt{s'+(\vec{p}_3+\vec{p}_4)^2}$.

The 3-particle phase space element can be brought to the form

$$\mathrm{d}PS_3(s,m_3^2,m_4^2,s') = \frac{1}{8}\delta(\sqrt{s}-E_3-E_4-E')\frac{|\vec{p}_3||\vec{p}_4|}{E'}\mathrm{d}E_3\mathrm{d}E_4\mathrm{d}\Omega_3\mathrm{d}\Omega_4.$$

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Phase space integration - t-channel singularity

Introducing dimensionless variables $x = 2E_3/\sqrt{s}$ and $y = 2E_4/\sqrt{s}$ we get

$$\mathrm{d}PS_3 = \frac{1}{8}\frac{|\vec{p}_3||\vec{p}_4|}{2 - x + y\frac{|\vec{p}_3|}{|\vec{p}_4|}\cos\theta_{34}}\mathrm{d}x\left[\delta(y - y_+) + \delta(y - y_-)\right]\mathrm{d}y\mathrm{d}\Omega_3\mathrm{d}\Omega_4,$$

where $y_\pm$ are the solutions of the energy conservation equation

$$\sqrt{s} - E_3 - E_4 - E' = \sqrt{s} - \frac{\sqrt{s}}{2}x - \frac{\sqrt{s}}{2}y - \Bigg[s' + \frac{s}{4}x^2 - m_3^2 + \frac{s}{4}y^2 - m_4^2 + 2\sqrt{\frac{s}{4}x^2 - m_3^2}\sqrt{\frac{s}{4}y^2 - m_4^2}\cos\theta_{34}\Bigg]^{1/2} = 0.$$

KK, M. Zrałek, Phys. Rev. D43 (1991) 3619;
NEXTCALIBUR: F.A. Berends, et.al,. Comput. Phys. Commun. 136 (2001) 148.

Phase space integration - t-channel singularity

Random variables are generated in the following sequence.

- s' is generated according to $\sim 1/s$ distribution.
- x is generated according to $\sim 1/(1-x)$ distribution.
- φ_3 and φ_4 are generated according to the uniform distribution.
- $\cos\theta_3 \in [-\cos\theta_{\rm cut}, \cos\theta_{\rm cut}]$ is generated according to $\sim 1/(1-\beta_3\cos\theta_3)$, with $\beta_3 = 2|\vec{p}_1||\vec{p}_3|/(2E_1E_3 - m_3^2)$.
- $\cos\theta_4 \in [-\cos\theta_{\rm cut}, \cos\theta_{\rm cut}]$ is generated according to $\sim 1/(a_4 + \cos\theta_4)$, where $a_4 = (2E_2E_{4-} - m_4^2)/(2|\vec{p}_2|\sqrt{E_{4-}^2 - m_4^2})$ and E_{4-} being the solution of the energy conservation eq. for $\cos\theta_{34} = -1$.
- If two solutions $y_\pm$ exist, then one of them is chosen randomly and the phase space normalization is multiplied by a factor 2.
- The remaining $n_f - 2$-particle phase space element is generated in a way similar to the s-channel phase space generation of `carlomat`.

Phase space integration - photon emission

Consider the process of the photon or gluon radiation

$$1+2 \to 3+4+...+n+\gamma(p_\gamma).$$

Discarding powers of 2π, the phase space element for the initial state radiation is parameterized by

$$\mathrm{d}^{3(n_f+1)-4}Lips = \frac{1}{2}E_\gamma \mathrm{d}E_\gamma \mathrm{d}\Omega_\gamma \mathrm{d}PS_{n_f}(s', m_3^2, ..., m_n^2),$$

where $s' = s - 2\sqrt{s}E_\gamma$ is the reduced c.m.s. energy squared.

- E_γ is generated according to $\sim 1/E_\gamma$ with the minimum photon energy $E_\gamma^{\rm cut}$.
- $\cos\theta_\gamma \in [-\cos\theta_\gamma^{\rm cut}, \cos\theta_\gamma^{\rm cut}]$ is generated according to $\sim 1/(1-\beta^2\cos^2\theta_\gamma)$, with $\beta = \sqrt{1-4m_1^2/s}$ and $m_1 = m_2$.
- φ_γ is generated according to a uniform distribution.
- $\mathrm{d}PS_{n_f}(s', m_3^2, ..., m_n^2)$ is again generated in a way that takes into account the s-channel Feynman propagators.

Phase space integration - photon emission

Consider the process of the photon or gluon radiation

$$1+2 \to 3+4+...+n+\gamma(p_\gamma).$$

Discarding powers of 2π, the phase space element for the initial state radiation is parameterized by

$$\mathrm{d}^{3(n_f+1)-4}Lips = \frac{1}{2}E_\gamma \mathrm{d}E_\gamma \mathrm{d}\Omega_\gamma \mathrm{d}PS_{n_f}(s', m_3^2, ..., m_n^2),$$

where $s' = s - 2\sqrt{s}E_\gamma$ is the reduced c.m.s. energy squared.

- E_γ is generated according to $\sim 1/E_\gamma$ with the minimum photon energy E_γ^{cut}.
- $\cos\theta_\gamma \in [-\cos\theta_\gamma^{\mathrm{cut}}, \cos\theta_\gamma^{\mathrm{cut}}]$ is generated according to $\sim 1/(1-\beta^2\cos^2\theta_\gamma)$, with $\beta = \sqrt{1-4m_1^2/s}$ and $m_1 = m_2$.
- φ_γ is generated according to a uniform distribution.
- $\mathrm{d}PS_{n_f}(s', m_3^2, ..., m_n^2)$ is again generated in a way that takes into account the s-channel Feynman propagators.

Phase space integration - photon emission

The phase space element for the final state radiation from either particle 3 or 4 is parameterized by

$$\begin{aligned} \mathrm{d}^{3(n_f+1)-4} Lips(s) &= \mathrm{d}s'\mathrm{d}s''\mathrm{d}PS_2(s,s',s'')\mathrm{d}PS_3(s',m_3^2,m_4^2,0) \\ &\times \mathrm{d}PS_{n_f-3}(s'',m_5^2,...,m_n^2). \end{aligned}$$

The 3-particle phase space element corresponding to the photon radiation off particle 3 is parameterized by

$$\mathrm{d}PS_3(s',m_3^2,m_4^2,0) = \frac{1}{8}\mathrm{d}E_\gamma \mathrm{d}E_3 \mathrm{d}\cos\theta_3 \mathrm{d}\varphi_3 \mathrm{d}\varphi_{37},$$

where

- E_γ is generated according to $\sim 1/E_\gamma$ with the minimum photon energy E_γ^{cut} boosted to the c.m.s. of particles 3, 4 and γ,
- E_3 is generated according to $\sim 1/(c_3 - E_3) \sim 1/(p_4 \cdot p_\gamma)$,
- $\cos\theta_3$, φ_3 and φ_{37} are generated according to the uniform distribution.

ee4fγ: KK, F.Jegerlehner, Comput.Phys.Commun. 159(2004)106.

Phase space integration - combined

The multichannel probability distribution

$$f(x) = \sum_{i=1}^{N} a_i f_i(x)$$

is automatically supplemented with the parameterizations which map away the t-channel, soft and collinear photon/gluon emission.

If the set of final state four momenta $\{p_3, p_4, ..., p_n\}$ is randomly generated according to the probability distribution $f_j(x)$

$\Rightarrow$ all other distributions $f_i(x), i = 1, ..., j, ..., N$ must be calculated for that particular set of four momenta in order to obtain proper phase space normalization.

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Sample results: $e^+e^- \to 4f(\gamma)$

Cuts : $\cos\theta(l, \text{beam}) \le 0.985, \quad \theta(\gamma, l) > 5^\circ, \quad E_\gamma > 1$ GeV,
$\cos\theta(\gamma, \text{beam}) \le 0.985, \quad \theta(\gamma, q) > 5^\circ, \quad E_l > 5$ GeV.

Final state	$\sqrt{s} = 200$ GeV σ [fb]	$\sqrt{s} = 200$ GeV σ_γ [fb]	$\sqrt{s} = 500$ GeV σ [fb]	$\sqrt{s} = 500$ GeV σ_γ [fb]
$\mu^+\mu^-\tau^+\tau^-$	10.267(14) 10.250(36) 10.246(8)	2.1787(91) 2.1958(28) 2.1979(30)	2.5117(44) 2.4866(31) 2.4093(29)	0.6495(40) 0.6514(13) 0.6274(12)
$e^+e^-\mu^+\mu^-$	137.18(90) 134.6(1.8) 135.4(1.8)	12.93(31) 13.02(14) 13.28(15)	43.80(38) 40.93(74) 41.26(73)	4.58(12) 4.83(8) 4.82(7)

ee4fγ (KK, F.Jegerlehner) vs. `carlomat`: double, quartic precision

Summary and outlook

- The automatic generation of multichannel MC phase space integration routine of `carlomat`, which up to now took into account only mappings of $\sim 1/s$ or Breit-Wigner behaviour of the s-channel diagrams, is being supplemented with the parameterizations which map away the t-channel, soft and collinear photon/gluon emission.
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