Family Symmetries and multi Higgs Doublet Models

(Dark)(Matter To The Deepest 2019

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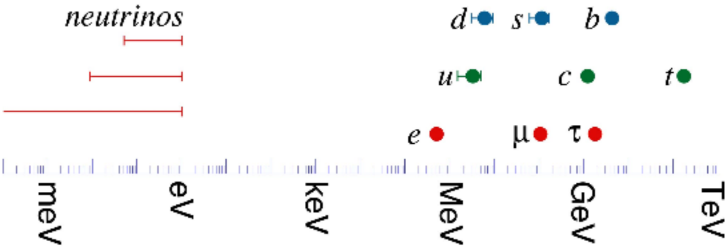
Motivation - Some open questions - Flavour Problem

Standard Model problems:

- lepton sector parameters (6 masses, PMNS matrix with 3 mixing angles, 1 or 3 CP phases)
- Origin and hierarchy of masses $\leftrightarrow$ Yukawa couplings
- Existence of exactly 3 families in the SM



Symmetry Realization

One of several proposals to solve this problems is:

FLAVOUR SYMMETRY

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \to G_{SM} \times \mathcal{G}$$

Flavour symmetry group $\mathcal{G}$:

- is not a gauge symmetry
- can be continuous or finite
- can reduce number of parameters in the SM

Two types of methods

"bottom-up" (experimental data $\to U_{PMNS} \to \mathcal{G}$)
"top-down" ($\mathcal{G} \to U_{PMNS}$)

2012 ($\theta_{13} \neq 0$) $\to$ Turning point $\to$ TriBiMaximal (A_4) excluded

Standard Model and Beyond

SM with one Higgs doublet

$$A_L^{i\dagger}(M_l M_l^\dagger)A_L^i = (M_l M_l^\dagger)$$

$$A_L^{i\dagger}(M_\nu M_\nu^\dagger)A_L^i = (M_\nu M_\nu^\dagger)$$

where $A_L^i = A_L(g_i), i = 1, 2, \ldots, N$ are 3-dim representations matrices for the left handed lepton doublets for some N-order flavour symmetry group $\mathcal{G}$.
The Schur's lemma implies that $M_l M_l^\dagger$ and $M_\nu M_\nu^\dagger$ are proportional to identity matrices, so one gets a trivial lepton mixing matrix.

Beyond SM

- Break family symmetry group by scalar singlet so called "flavon"
- add more Higgs multiplets

n-Higgs doublet models

To describe the coupling between lepton fields and the Higgs field we take the n-Higgs doublet Yukawa interaction term of the form:

$$L_Y = -(h_i^l)_{\alpha\beta}\bar{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R} - (h_i^\nu)_{\alpha\beta}\bar{L}_{\alpha L}\Phi_i \nu_{\beta R} + \text{H.c}$$

where $i = 1, 2, \ldots, n$ and $\alpha, \beta = e, \mu, \tau$.
The charged lepton states $l_{\beta R}$ and neutrinos $\nu_{\beta R}$ are right-handed $SU(2)$ singlets.

$$L_{\alpha L} = \begin{pmatrix} \nu_{\alpha L} \\ l_{\alpha L} \end{pmatrix}$$

for $\alpha = e, \mu, \tau$

$$\Phi_i = \begin{pmatrix} \varphi_i^0 \\ \varphi_i^- \end{pmatrix}, \quad \tilde{\Phi}_i = \begin{pmatrix} \varphi_i^{-*} \\ -\varphi_i^{0*} \end{pmatrix} = i\sigma_2\Phi^*$$

where $i = 1, 2, \ldots, n;$ and $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. φ_i^0 and φ_i^i are complex scalar fields in spacetime.

n-Higgs doublet models

The 3×3 Yukawa matrices h_i^l and h_i^ν each define the couplings of left-handed doublets with right-handed singlets via i-th Higgs doublet. Due to the form of the Higgs potentials, ground states occur at non-zero φ, with VeV:

$$< \Phi_i >= \frac{1}{\sqrt{2}} \begin{pmatrix} v_i \\ 0 \end{pmatrix} \quad \text{and} \quad < \tilde{\Phi}_i >= \frac{1}{\sqrt{2}} \begin{pmatrix} -v_i^* \\ 0 \end{pmatrix}$$

for some complex-valued v_i.

$$\sqrt{|v_1|^2 + |v_2|^2 + \cdots + |v_n|^2} = (\sqrt{2} G_F)^{-1/2} \sim 246 GeV$$

Mass matrices read as follows:

$$M^l = -\frac{1}{\sqrt{2}}(v_1^* h_1^{(l)} + \cdots + v_n^* h_n^{(l)})$$

$$M^\nu = \frac{1}{\sqrt{2}}(v_1 h_1^{(\nu)} + \cdots + v_n h_n^{(\nu)})$$

n-Higgs doublet models

For some discrete flavour group $\mathcal{G}$, Family Symmetry means that:

- after fields transformations Lagrangian doesn't change

$$\mathcal{L}(L_{\alpha L}, l_{\beta R}, \nu_{\beta R}, \Phi_i) = \mathcal{L}(L'_{\alpha L}, l'_{\beta R}, \nu'_{\beta R}, \Phi'_i)$$

- we need 3-dim representations for:

$$L_{\alpha L} \to L'_{\alpha L} = (A_L)_{\alpha,\chi} \, L_{\chi L}$$

$$l_{\beta R} \to l'_{\beta R} = \left(A_l^R\right)_{\beta,\delta} l_{\delta R}$$

$$\nu_{\beta R} \to \nu'_{\beta R} = \left(A_\nu^R\right)_{\beta,\delta} \nu_{\delta R}$$

- n-dim representations for:

$$\Phi_i \to \Phi'_i = (A_\Phi)_{ik} \, \Phi_k$$

n-Higgs doublet models

Symmetry conditions could be write as eigenequation problem for direct product of unitary group representations to the eigenvalue 1.
For any group elements (so also for generators) we have (Dirac case):

$$\left((A_\Phi)^\dagger \otimes (A_L)^\dagger \otimes (A_l^R)^T\right)_{k,\alpha,\delta;i,\beta,\gamma} (h_i^l)_{\beta,\gamma} = (h_k^l)_{\alpha,\delta}$$

$$\left((A_\Phi)^T \otimes (A_L)^\dagger \otimes (A_\nu^R)^T\right)_{k,\alpha,\delta;i,\beta,\gamma} (h_i^\nu)_{\beta,\gamma} = (h_k^\nu)_{\alpha,\delta}$$

In such a model, the invariance equations for the mass matrices are not trivial, so we avoid the consequences of Schur's Lemma

$$A_L M^{l(\nu)} \left(A^R_{l(\nu)}\right)^\dagger = \frac{1}{\sqrt{2}} \sum_{i,k=1}^{n} h_i^{l(\nu)} (A_\Phi)_{i,k} v_k \neq M^{l(\nu)}.$$

Model Verification

Algorithm

- find the groups which fulfil the requirements of our model
- impose flavour symmetry on the Yukawa Lagrangian
- calculate the Yukawa matrices
- create mass matrices and mixing matrix
- check agreement with experimental data

$\mathcal{G}$ candidates

DISCRETE, NON-ABELIAN, SUBGROUP of U(3)

GAP

In order to make calculations faster and easier we have used **GAP** CAS for computation.
We have used two libraries: SMALL GROUPS LIBRARY and REPSN

We have investigated discrete groups up to the order of 1025.

$[o, i]$	Structure description	2D	3D	$U(2)$	$U(3)$	L	DN	MN	L + DN	L + MN
[24, 3]	SL(2,3)	3	1	2	1 + 2			3		
[24, 12]	S4	1	2		3	4	4	2	8	4
[48, 28]	C2.S4=SL(2,3).C2	3	2	2	1 + 2	4	4	6	8	4
[48, 29]	GL(2,3)	3	2	2	1 + 2	4	4	6	8	4
[48, 30]	A4:C4	2	4		3	16	16	8	32	16
[48, 32]	C2xSL(2,3)	6	2		1 + 2			12		
[48, 33]	SL(2,3):C2	6	2	2	1 + 2					
[48, 48]	C2xS4	2	4		3	16	16	8	32	16
[54, 8]	((C3xC3):C3):C2	4	4		3	32	32		64	
[72, 3]	Q8:C9	9	3	2	1 + 2			9		
[72, 25]	C3xSL(2,3)	9	3	2	1 + 2			9		
[72, 42]	C3xS4	3	6		3	36	36	6	72	12
[96, 64]	((C4xC4):C3):C2	1	6		3	12	12	2	24	4
[96, 65]	A4:C8	4	8		3	64	64	16	128	32
[96, 66]	SL(2,3):C4	6	4		1 + 2	16	16	24	32	16
[96, 67]	SL(2,3):C4	6	4	2	1 + 2	16	16	8	32	16
[96, 69]	C4xSL(2,3)	12	4		1 + 2			24		
[96, 74]	((C8xC2):C2):C3	12	4	2	1 + 2					
[96, 186]	C4xS4	4	8		3	64	64	16	128	32
[96, 188]	C2x(C2.S4=SL(2,3).C2)	6	4		1 + 2	16	16	24	32	16
[96, 189]	C2xGL(2,3)	6	4		1 + 2	16	16	24	32	16
[96, 192]	(C2.S4=SL(2,3).C2):C2	6	4	2	1 + 2	16	16	8	32	16
[96, 200]	C2x(SL(2,3):C2)	12	4		1 + 2					

Chaber, et al. Physical Review D vol. 98 no. 55007 (06.09.2018)

2HDM results - Dirac case

There exist 267 groups that gave 748 672 different combinations of 2 and 3 dim irred. representations that gave 1-dim subspace for all generators. All possible solutions for Yukawa matrices can be expressed in 7 base forms ($\omega = e^{2\pi i/3}$):

$$h_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ \omega^2 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ \omega & 0 & 0 \\ 0 & \omega^2 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

$$h_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega^2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega^2 \\ \omega & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

All the rest can be obtained by interchange h_1 and h_2 for the first three solutions above.

2HDM results - Dirac case

Symmetric Yukawa matrices for any considered group and for any irreducible representation within the group can be expressed now by seven (i=1,2,..,7) basic matrix forms as follows:
a) for Dirac neutrinos:

$$\{h_1^{(\nu)}, h_2^{(\nu)}\} = \{h_1^{(i)}, e^{i\varphi} h_2^{(i)}\},$$

b) for charged leptons:

$$\{h_1^{(l)}, h_2^{(l)}\} = \{h_2^{(i)}, e^{-i(\delta_l+\varphi)} h_1^{(i)}\},$$

where φ is a phase distinctive for a group and for irreducible representations and $\delta_l = 0, \pi$.

2HDM results - Dirac case

In order to find the lepton masses and mixing matrix we constructed the hermitian matrices $M^l M^{l\dagger}$ and $M^\nu M^{\nu\dagger}$. For all the possible Yukawa matrices we have obtained only three different forms ($x = l, \nu$):

$$M_x M_x^\dagger = |c_x|^2 \begin{pmatrix} 1+\kappa^2 & \kappa e^{-i(\eta_x + 2k\pi/3)} & \kappa e^{i(\eta_x - 2k\pi/3)} \\ \kappa e^{i(\eta_x + 2k\pi/3)} & 1+\kappa^2 & \kappa e^{-i\eta_x} \\ \kappa e^{-i(\eta_x - 2k\pi/3)} & \kappa e^{i\eta_x} & 1+\kappa^2 \end{pmatrix},$$

with $k = -1, 0, +1$ and $\kappa = |v_2|/|v_1|$, the same for neutrinos and for charged leptons. The only difference lies in the phase η_x. For Dirac neutrinos,

$$\eta_\nu = \varphi + \varphi_2 - \varphi_1,$$

and for charged leptons,

$$\eta_l = \delta_l + \varphi + \varphi_2 - \varphi_1,$$

where $\varphi_i (i = 1, 2)$ are phases of the VEVs v_i.

2HDM results - Dirac case

After diagonalization

$$U^{\dagger}\left(M_x M_x^{\dagger}\right)U = diag\left(m_{x1}^2, m_{x2}^2, m_{x3}^2\right),$$

we obtain:

$$\begin{aligned} m_{x1}^2 &= |c_x|^2\left(1+\kappa^2+2\kappa cos\left(\eta_x\right)\right), \\ m_{x2}^2 &= |c_x|^2\left(1+\kappa^2+2\kappa sin\left(\eta_x-\frac{\pi}{6}\right)\right), \\ m_{x3}^2 &= |c_x|^2\left(1+\kappa^2-2\kappa sin\left(\eta_x+\frac{\pi}{6}\right)\right), \end{aligned}$$

and the diagonalization matrix U:

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} e^{-\frac{2}{3}\pi ik} & \omega e^{-\frac{2}{3}\pi ik} & \omega^2 e^{-\frac{2}{3}\pi ik} \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix}.$$

2HDM results - Dirac case

- We are not able to get correct masses for charged leptons.
- The U matrix does not depend on the phase η_x , so it is identical for charged leptons and for the neutrinos.
  Therefore, it is not possible to reconstruct the correct PMNS mixing matrix. One always gets a trivial one ($U_{PMNS} = U_l^\dagger U_\nu$).

2HDM results - Majorana case

- Out of all the groups under consideration, there exist 195 groups that gave in total 20888 solutions.
- Solutions for charged leptons were exactly the same as in the Dirac case (masses can not be reproduced).
- All found symmetries for the Majorana neutrinos gave a two-dimensional space which was always diagonal (it is also not possible to reconstruct the correct PMNS matrix).

3HDM - work in progress

Dirac case is in study. Up to now:

- analysis included all finite groups of order up to 1032 that have at least one 3-dimensional faithful irreducible representation.
- There are 939 such groups. For each of these groups we determined the set of irreducible three-dimensional representations.
- The total number of one-dimensional solution vectors for charged leptons and also neutrinos is 6 012 859 which gives 14 241 772 joint solutions.
- Up to now we have studied separately only mass matrices for charged leptons and neutrinos.
- There exist 15 types of such solutions. Among them we have found mass matrices which can reproduce experimental values separately.
- Still don't know about joint solutions.

Summary and Outlook

- 2HDM gave no solutions neither for Dirac or Majorana case (for the SMALL GROUPS LIBRARY up to order 1025).
- 3HDM preliminary results suggest that it is worth to investigate further (continue Dirac case and later also Majorana case).