Asymptotic expansions through the loop-tree duality

Judith Plenter

in collaboration with Germán Rodrigo

Instituto de Física Corpuscular - Universitat de València/CSIC

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Matter to the Deepest

Motivation

precision calculations:

- need hints where exactly the SM is failing
- one of the points of interest: the Higgs sector $\rightarrow$ necessarily includes calculation with various scales (at the very least m_t, m_H and Mandelstam variables)
- example: highly-boosted Higgs production. interesting in order to rule out an effective point-like ggH-coupling. first calculations at NLO (two loops) only last year (numerical or using expansions in the IBPs)

[Jonas, Kerner, Luisoni '18, Lindert et al. '18]

$\Rightarrow$ to obtain independent NLO result for this and similar processes: develop formalism for asymptotic expansions within new regularization technique

Motivation

Loop-tree duality:

- Loop-tree duality (LTD) is still a new method, need to explore possible applications
- formalism allows to conveniently identify and interpret divergences in the integrand
- integrands are functions of Euclidean and not Minkowski momenta $\rightarrow$ allows to determine hierarchies between the scales and scalar products and thus opens the possibility for well-defined asympotic expansions

Loop-tree duality theorem (LTD)

use Cauchy residue theorem: $\int_{\ell_0} \to \sum$ residues

LTD theorem

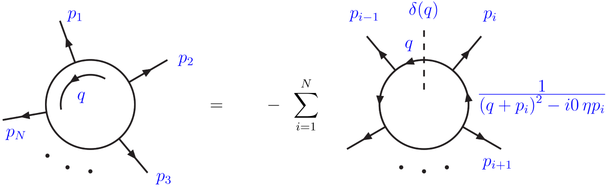
[Catani et al. '08]

$$\int_\ell N(\ell) \prod_i G_F(q_i) = -\sum_i \int_\ell N(\ell)\ \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i; q_j),$$

$$\tilde{\delta}(q_i) = 2\pi i\ \theta(q_{i,0})\ \delta(q_i^2 - m_i^2)$$

dual propagator:

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot \underbrace{(q_j - q_i)}_{k_{ji}}}$$



- η may be any future-like vector, typically choose $\eta = (1, \mathbf{0})$
- with the energy-component integrated out, the remaining loop three-momentum is Euclidean:
  $\tilde{\delta}(q_i) q_j^2 = \tilde{\delta}(q_i) \left(k_{ji}^2 + 2k_{ji,0} q_{i,0}^{(+)} - 2\mathbf{k_{ji}} \cdot \mathbf{q_i} + m_i^2 \right)$, where $q_{i,0}^{(+)} = \sqrt{m_i^2 + \mathbf{q_i}^2}$

Four-dimensional unsubtraction

[Sborlini, Driencourt-Mangin, Hernández-Pinto, Rodrigo '16]

- NLO corrections:

$$\sigma^{(1)} = \int_m \mathrm{d}\sigma^{(1,R)}_{\text{virtual}} + \int_{m+1} \mathrm{d}\sigma^{(1)}_{\text{real}}$$

- $\sigma^{(1,R)}_{\text{virtual}}$: LTD expresses loop integral through phase-space integral
- introduce suitable mapping between loop and external momenta

$\Rightarrow$ cancellation of soft and collinear divergences at *integrand level*

- can be performed entirely in four dimensions

succesful LTD applications to:

- physical cross-section for $\gamma^* \to q\bar{q}(g)$ [Sborlini et al. '16]
- extended to massive particles [Sborlini, Driencourt-Mangin, Rodrigo '16]
- amplitude for $gg \to H$ and $H \to \gamma\gamma$ [Driencourt-Mangin, Rodrigo, Sborlini '18]
- $H \to \gamma\gamma$ at two loops [Driencourt-Mangin et al. '19]

Expansions through LTD: plan

apply loop-tree duality theorem to amplitude

↓

expand dual integrand

↓

integrate

Objective for expansion

expansion shall be:

- well-defined, this we understand to mean that
  - expansion does not fundamentally change the analytic behaviour of the amplitude
  - convergence at integrand-level in all of the integration space, except for possibly in a limited region around the divergences
- simplify either the calculation or the result
- will have to hold up in comparison with *Expansion by Regions*

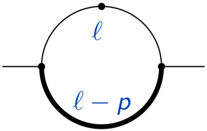
[Beneke, Smirnov '98]

toy amplitude

- whole range of problems in a physically sensible amplitude: many scales, ≥ 2 loops, multiple legs (angular dependence of the integrand), above threshold

$\Rightarrow$ study benchmark toy amplitudes

$$\mathcal{A}_n^{(1)} = \int_\ell (G_F(\ell; m))^n \, G_F(\ell - p; M)$$



n=2 finite amplitude

n=1 singular in the UV

- renormalization trough **local UV counter-term**:

$$\mathcal{A}_1^{(1,R)} = \mathcal{A}_1^{(1)} - \mathcal{A}_{1,\mathrm{UV}}^{(1,\mathrm{cnt})},$$

$$\mathcal{A}_{1,\mathrm{UV}}^{(1,\mathrm{cnt})} = \int_\ell (G_F(\ell; \mu))^2$$

- various limits to be studied:
  - one large mass: $M^2 \gg m^2, p^2$
  - large external momentum: $p^2 \gg M^2, m^2$
  - threshold: $\beta = 1 - \frac{p^2}{(m+M)^2} \to 0$

identifiying divergences

- most important property of amplitudes are their divergences
- LTD amplitude allows to easily identify divergences and their meaning:

$$-\sum_i \int_\ell N(\ell)\ \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

- consider the on-shell dual propagator:

$$\tilde{\delta}(q_i) G_D(q_i;q_j) \sim \frac{\theta(q_{i,0})\ \delta(q_i^2-m_i^2)}{\left(q_{j,0}-q_{j,0}^{(+)}\right)\left(q_{j,0}+q_{j,0}^{(+)}\right)-i0\eta\cdot k_{ji}}, \quad k_{ji}=q_j-q_i$$

$$= \frac{\delta\left(q_{i,0}-q_{i,0}^{(+)}\right)}{2q_{i,0}^{(+)}\lambda_{ij}^{+-}\lambda_{ij}^{++}}, \quad \lambda_{ij}^{\pm\pm}=\pm q_{i,0}^{(+)}\pm q_{j,0}^{(+)}+k_{ji,0}$$

- causal unitarity threshold represented by $\lambda_{ij}^{++}\to 0$
- unphysical thresholds appear at $\lambda_{ij}^{+-}\to 0$ in two cuts at once and cancel
- conditions for these limits to occur are easily derived using this notation
  [Aguilera-Verdugo, Driencourt-Mangin, JP, Ramírez-Uribe, Rodrigo, Sborlini, Torres Bobadilla, Tracz arXiv:1904.08389]

the dual propagator and general expansion

- first identify divergences in propagator (unphysical ones may be eliminated by a different choice of loop momentum)
- dual propagator may be written as

$$G_D(q_i; q_j) = \frac{1}{2q_i \cdot k_{ji} + \Gamma_{ji} + \Delta_{ji} - i0\eta \cdot k_{ji}}, \quad \Gamma_{ji} + \Delta_{ji} = m_i^2 - m_j^2 + k_{ji}^2$$

- choose parameters s.t.: $|2q_i \cdot k_{ji} + \Gamma_{ji}| > |\Delta_{ji}|$

$$= \sum_{n=0}^{\infty} \frac{(-\Delta_{ji})^n}{(2q_i \cdot k_{ji} + \Gamma_{ji} - i0\eta \cdot k_{ji})^{n+1}}$$

- additional condition on parameters: want that there are Q_i^2 and r_{ij} s.t.

$$2q_i \cdot k_{ji} + \Gamma_{ji} = \frac{Q_i^2}{x}(r_{ij} + x_i)(r_{ij} x_i + 1), \quad x_i = \frac{|\ell| + \sqrt{\ell^2 + m_i^2}}{m_i}$$

the dual propagator and general expansion

- if this parametrization is possible the toy amplitudes introduced simplify:

$$\mathcal{A}_1^{(1,R)} = \frac{1}{16\pi^2}\left[2 - \frac{M^2 - m^2}{2p^2}\log\left(\frac{M^2}{m^2}\right) - \log\left(\frac{M \cdot m}{\mu_{\mathrm{UV}}^2}\right)\right.$$
$$\left. - \frac{m^2}{Q_1^2}\sum_{n=0}^{\infty}\left(c_{r_{12}}^{(n)}\log(r_{12}) + c_1^{(n)}\right) - \frac{M^2}{Q_2^2}\sum_{n=0}^{\infty}\left(c_{r_{21}}^{(n)}\log(r_{21}) + c_1^{(n)}\right)\right]$$

- the remaining integrals are of the type

$$\int_1^{\infty} \frac{\mathrm{d}x}{(r+x)(rx+1) \pm i0} = \frac{\log(r \pm i0)}{r^2 - 1}, \quad \forall\ |r| < 1$$

so the coefficients are proportional to $(r^2 - 1)^{-1}$

the dual propagator and general expansion

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- local renormalization required only for the first two terms of the expansion
- further orders only modify the sums of coefficients (i.e. higher-order terms can be calculated numerically in 4 dimensions)
- solution is general without need to specify a limit
- divergences in a propagator correspond to $r_{ij} < 0$

expansions for scale hierarchies

- if the chosen process and limit contains a hierarchy between scales a set of simple rules can be followed to determine the parameters

  1. with $Q_i^2 = \pm Q^2$ and Q the large scale

  $$\Gamma_{ji} = Q_i^2 \left(1 + {r_{ij}}^2\right), \; \Delta_{ji} = m_i^2 - m_j^2 + k_{ji}^2 - \Gamma_{ji}, \quad r_{ij} = \frac{m_i k_{ji,0}}{Q_i^2}$$

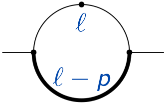
  2. fix the sign of Q_i^2: it directly determines the sign of r_{ij} (always reproduce the analytic structure of the original amplitude)

- no integrations necessary, just insert the determined parameters r_{ij}, Q_i in the solution

one mass large

- consider explicitly the large mass limit for the toy amplitude $\mathcal{A}^{(1)}_{1/2}$:

$$M^2 \gg m^2, p^2$$

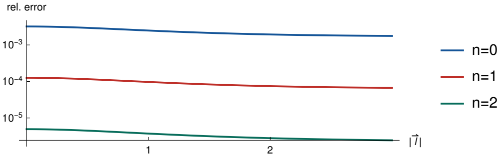


- the two appearing dual propagators are:

$$G_D(\ell; q_1) = \frac{1}{-2\ell \cdot p + p^2 + m^2 - M^2} = \sum_{n=0}^{\infty} \frac{\left(-m^2 - p^2 - M^2 r_1^2\right)^n}{\left(-2\ell \cdot p - M^2\left(1 + r_1^2\right)\right)^{n+1}}$$

$$G_D(q_1; \ell) = \frac{1}{2q_1 \cdot p + p^2 - m^2 + M^2} = \sum_{n=0}^{\infty} \frac{\left(m^2\right)^n}{\left(2q_1 \cdot p + M^2\left(1 + r_2^2\right)\right)^{n+1}}$$

$$r_1 = \frac{m^2 p^2}{M^4}$$



$$r_2 = \frac{p^2}{M^2}$$

- no need to integrate, just use the formula provided before
- relative error of result for $\mathcal{A}^{(1,R)}_1$ at order $n = 1, 2$: $2.7\%, 0.03\%$

alternative: one mass large through Taylor expansions

- loop energy in the LTD integrand is fixed, remaining loop three-momentum is Euclidean

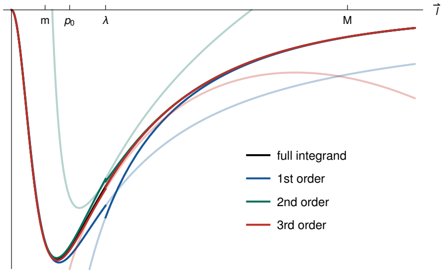
$\Rightarrow$ no cancellations within scalar products

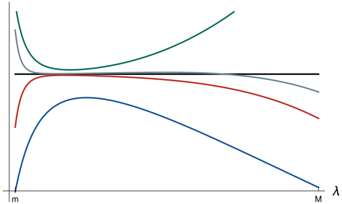
- any Taylor expansion still makes implicit assumptions about the size of the loop three-momentum

$\Rightarrow$ combine various sections, s.t. the expansion converges at integrand-level for any value of loop three-momentum $|\boldsymbol{\ell}|$

$$\mathcal{A}_n^{(1)} = \int_0^\infty \mathrm{d}|\boldsymbol{\ell}| \; A_n^{(1)}(|\boldsymbol{\ell}|)$$

$$= \int_0^\lambda \mathrm{d}|\boldsymbol{\ell}| \; TA_n^{(1)}(|\boldsymbol{\ell}|, 0) + \int_\lambda^\infty \mathrm{d}|\boldsymbol{\ell}| \; TA_n^{(1)}(|\boldsymbol{\ell}|, \infty)$$





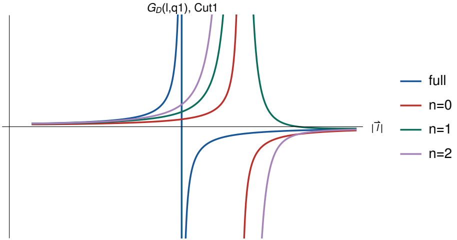
determine the switch point λ through extrema in the result

large external momentum

- in the limit of large external momentum the amplitude is considered above threshold

$$p^2 \gg M^2, m^2$$

- following the set of rules the pole is reproduced in the expanded amplitude *but not its position*



- obtain result from general formula again, no need for integration
- relative error of result for $\mathcal{A}_1^{(1,R)}$ at order $n = 1, 2, 3$: $0.24\%, 0.06\%, 0.02\%$

threshold expansion

- there are other types of limits that do not consist of having a large difference between the sizes of scales

$\rightarrow$ one such limit is the threshold expansion

$$\beta = 1 - \frac{p^2}{(m+M)^2} \rightarrow O$$

- actually two distinct limits: approaching the threshold from below or from above
- small parameter of expansion is essentially

$$\frac{|-\Delta_{ji}|}{|2q_i \cdot k_{ji} + \Gamma_{ji} - i0\eta \cdot k_{ji}|}$$

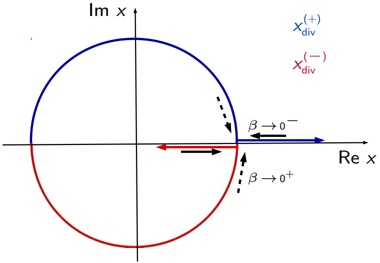
$\rightarrow$ minimize this? $\rightarrow$ leads to $|r_1| = 1$. coefficients of expansion diverge

$\Rightarrow$ more important to consider behaviour of the physical threshold!

threshold expansion

$$x_{\text{div}}^{(\pm)} = \frac{M+m}{2m\sqrt{1-\beta}}\left[\frac{2m}{m+M} - \beta \pm \sqrt{-\beta\left(\frac{4mM}{(m+M)^2} - \beta\right)}\right]$$

- for $\beta \to 0$ the pole in the integrand moves towards $x = 1$ (integration goes from 1 to ∞)
- also in the case below threshold ($\beta \to 0^+$) the pole is the most important property of the amplitude $\Rightarrow$ complex r_1
- expanding x_{div} gives



$$r_1 = -1 + \sqrt{\frac{M}{m}}\sqrt{-\beta} + \mathcal{O}(\beta)$$

threshold expansion

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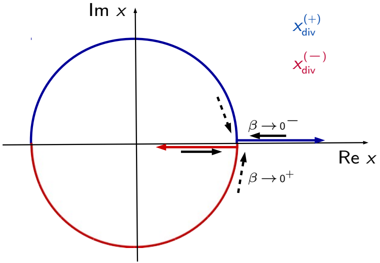

- relative error for $\mathcal{A}_1^{(1,R)}$: $M/m = 3$

β	$n=1$	$n=2$	$n=3$
-0.1	0.06%	$4 \cdot 10^{-6}$	$5 \cdot 10^{-8}$
$+0.1$	0.2%	$2 \cdot 10^{-5}$	$2 \cdot 10^{-7}$

scalar three-point function

- consider amplitudes with more external legs
- three-point function $\mathcal{A}_3^{(1)} = \int_\ell G_F(q_1, q_{12}, q_3)$ ($\mathbf{p_1} = -\mathbf{p_2}$, equal masses M)

$$\mathcal{A}_3^{(1)} = -\int_\ell \left\{ \frac{\tilde{\delta}\,(q_1; M)}{(-2q_1 \cdot p_1)(2q_1 \cdot p_2)} + \frac{\tilde{\delta}\,(\ell; M)}{(-2\ell \cdot p_2)(-2\ell \cdot p_{12} + s_{12} + \imath 0)} + \frac{\tilde{\delta}\,(\ell; M)}{(2\ell \cdot p_1)(2\ell \cdot p_{12} + s_{12})} \right\}$$

- angular dependence not in the propagators to be expanded
- example: large mass expansion

$$\mathcal{A}_3^{(1)} = \int_\ell \frac{\tilde{\delta}\,(\ell; M)}{(2\ell \cdot p_{12})(\ell \cdot p_1)} \sum_{n=1} \left(\frac{s_{12}}{2\ell \cdot p_{12}} \right)^{2n} = \frac{1}{16\pi^2} \frac{1}{4M^2} \left(1 + \frac{r}{12} + \frac{r^2}{90} \right) + \mathcal{O}(r^3), \quad r = \frac{s_{12}}{M^2}$$

- additional strategies will be necessary for more legs, different internal masses, more loops, etc.

Conclusion

- development of a formalism for well-defined asymptotic expansions of amplitude integrands
- loop-tree duality allows to rewrite loop integrals in terms of a sum of phase-space integrals over the cut amplitude

$\Rightarrow$ the resulting expression can be expanded more straight-forwardly

- general result obtained for toy amplitude, applicable for different types of limits
- first steps towards more realistic amplitudes show promising results

Outlook:

- consider amplitudes with more legs
- toy amplitude at two loops: sunrise diagram
- reproduce LO result for $\bar{q}q \to Hg$ using LTD + direct expansions
- repeat for the other two LO contributions to large-p_T Higgs production: $gg \to Hg$ and $qg \to qH$

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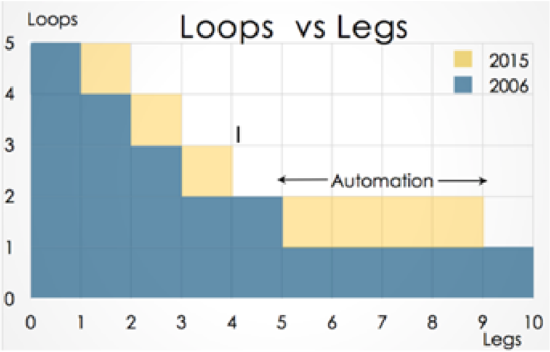
backup slides

difficulty of multi-loop or multi-leg calculations

amplitudes that contain up to one mass scale:



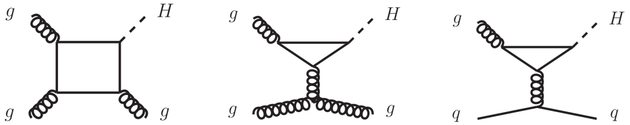
highly-boosted Higgs boson production

- effective point-like ggH-coupling not ruled out [Grojean et al. '14]

$$\frac{m_t}{v}\bar{t}tH \to -\kappa_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v}\bar{t}tH$$

- need to consider Higgs + jet production at p_T sufficiently high for resolving the top loop in order to disentangle possible BSM effects

LO result known for decades



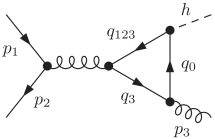
[Ellis et al. '87]
[Baur, Glover '89]

NLO
- fix Higgs-top mass ratio, numerical integration [Jones, Kerner, Luisoni '18]
- integration-by-parts identities + expansion in $\frac{m_H^2}{4m_t^2}$, $\frac{m_t^2}{s}$ [Lindert et al. '18]

goal: obtain independent NLO result using new regularization technique: loop-tree duality (LTD) [Catani et al. '08]

$q\bar{q} \to Hg$ amplitude

- **LO amplitude**: $\mathcal{M}_{\text{LO}} \sim$
  $$\varepsilon^*_\mu(p_3)\ \bar{v}(p_2)\gamma_\nu u(p_1)\ F_{12}\left(g^{\mu\nu} - \frac{p_{12}^\mu p_3^\nu}{p_3 \cdot p_{12}}\right)$$

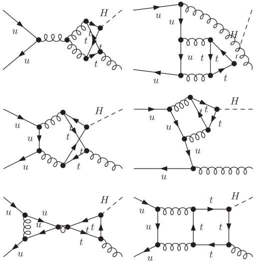


- straight-forward with dim. reg.: solutions for master integrals available

$$F_{12} = \int_\ell \frac{N\left(\ell \cdot p_3,\ \ell \cdot p_{12},\ \ell^2,\ s,\ m_H^2\right)}{[\ell^2 - m_t^2 + i0][(\ell + p_3)^2 - m_t^2 + i0][(\ell + p_{123})^2 - m_t^2 + i0]}$$

- while at LO only one kinematic variable s,
  NLO amplitude depends on: s, t

$$\mathcal{M}_{\text{NLO}} \sim\ F_{12}^{(1)}\left(g^{\mu\nu} - \frac{p_{12}^\mu p_3^\nu}{p_3 \cdot p_{12}}\right) + \bar{F}_{12}^{(1)}\left(g^{\mu\nu} - \frac{\bar{p}_{12}^\mu p_3^\nu}{p_3 \cdot \bar{p}_{12}}\right)$$



$\Rightarrow$ four scales + two loops: simplification through asymptotic expansion necessary

expansion by regions of $q\bar{q} \to Hg$

expansion by regions

- Divide space of loop momentum into various regions and expand the integrand into a Taylor series w.r.t. the parameters considered small there.
- Integrate expanded integrand *over the whole integration domain* of the loop momenta.
- Set to zero any scaleless integral. [Beneke, Smirnov '98]

- 'region': $\ell \sim m$ instead of $0 \leq \ell \leq \Lambda$.
- let's try for one of the integrals needed for $\bar{q}q \to Hg$ at LO:

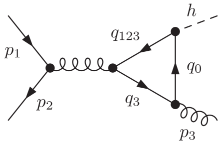
$$\int_\ell \frac{1}{(\ell^2 - m_t^2 + i0)\,(\ell^2 + 2\ell \cdot p_{12} + 2\ell \cdot p_3 + m_H^2 - m_t^2 + i0)}$$

- How many regions? At least $\ell \sim m$ (soft) and $\ell \sim p_\mathrm{T}$ (hard)
  - How about $\ell \sim \frac{m}{p_\mathrm{T}}$ (ultrasoft)? Or $\ell_0 \sim \frac{m}{p_\mathrm{T}}, |\boldsymbol{\ell}| \sim m$ (potential)?
  - How to treat the scalar products?

application of LTD on $q\bar{q} \to Hg$

- test applicability: need to extend usage of LTD to further processes
- asymptotic expansions (needed in the NLO calculation) expected to be easier in a phase-space integral

$$F_{12} \overset{LTD}{=} -\int_\ell \left[\frac{\tilde{\delta}(q_0)\, N(q_0)}{(2p_3 \cdot q_0 - i0)\,(2p_{123} \cdot q_0 + m_H^2 + i0)} + \frac{\tilde{\delta}(q_3)\, N(q_3 - p_3)}{(-2p_3 \cdot q_3 + i0)\,(2p_{12} \cdot q_3 + s + i0)} + \frac{\tilde{\delta}(q_{123})\, N(-p_{123} + q_{123})}{(-2p_{12} \cdot q_{123} + s - i0)\,(-2p_{123} \cdot q_{123} + m_H^2 - i0)} \right]$$



note: integrand now only depends on loop three-momenta