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Multi-Field False Vacuum Decay

POLYGONAL BOUNCE

Victor Guada

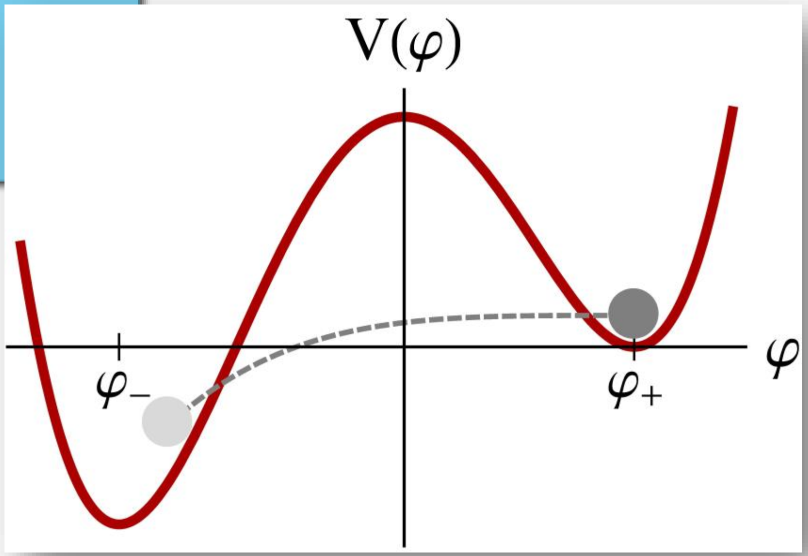
with A. Maiezza and M. Nemevšek.

Based on ArXiv: 1803.02227
PhysRevD. 99.056020

Katowice- Sep 2019

Introduction

- Bubble nucleation
- Forbidden at the classical level
- First-order phase transition
- Different context of physics



a. I. Y. Kobzarev, L. B. Okun and M. B. Voloshin, Sov. J. Nucl. Phys. 20 (1975) 644
b. S. R. Coleman, Phys. Rev. D 15 (1977) 2929
c. S. R. Coleman, V. Glaser and A. Martin, Commun. Math. Phys.
d. A. D. Linde, Phys. Lett. 100B (1981) 37. Nucl. Phys. B 216 (1983) 421

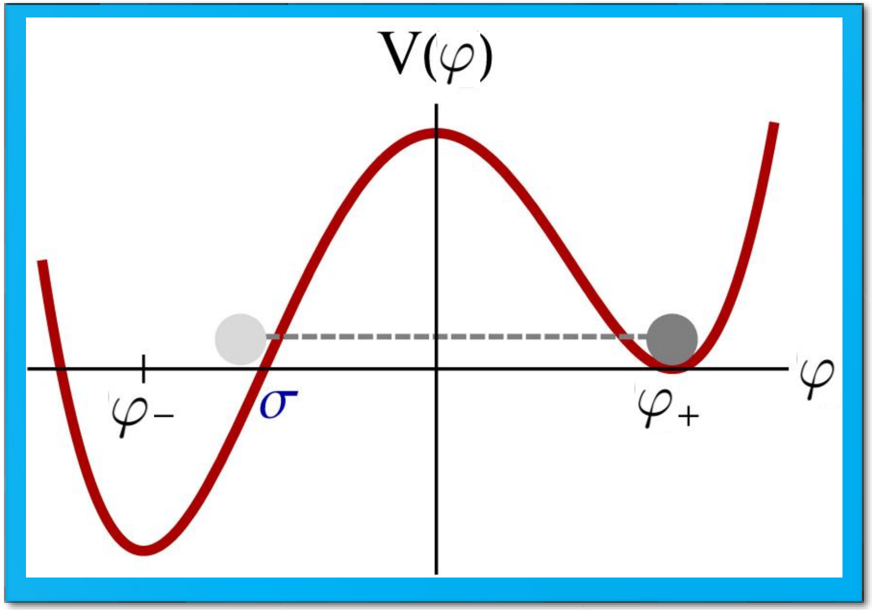
Decay Rate

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = \int_{-\infty}^{\infty} d\tau L_E \equiv S_E$$

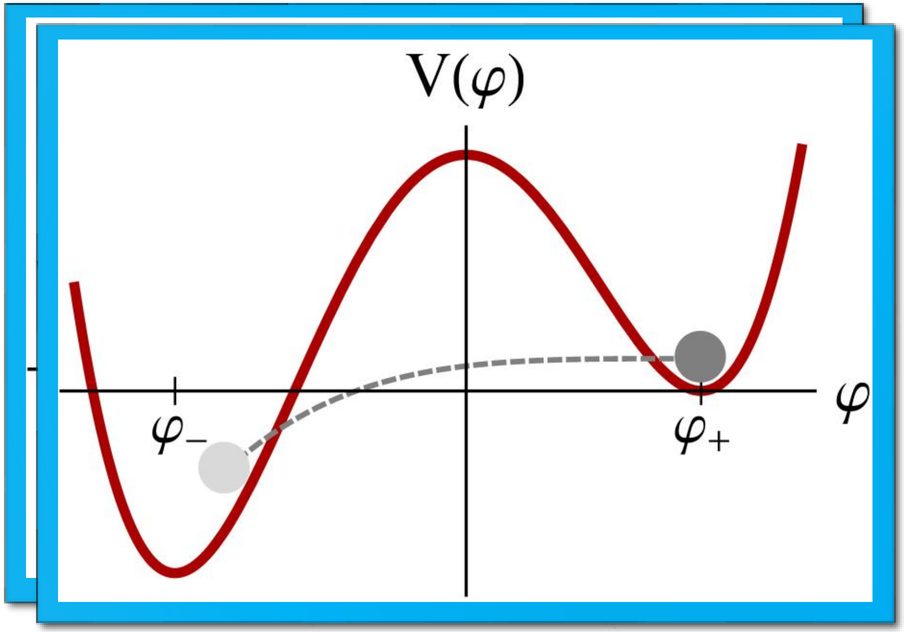
Single Field False Vacuum Decay

$$S_E = \int dx \left[\frac{1}{2} \left(\frac{\partial \varphi}{\partial \tau} \right)^2 + V \right]$$



Single Field False Vacuum Decay

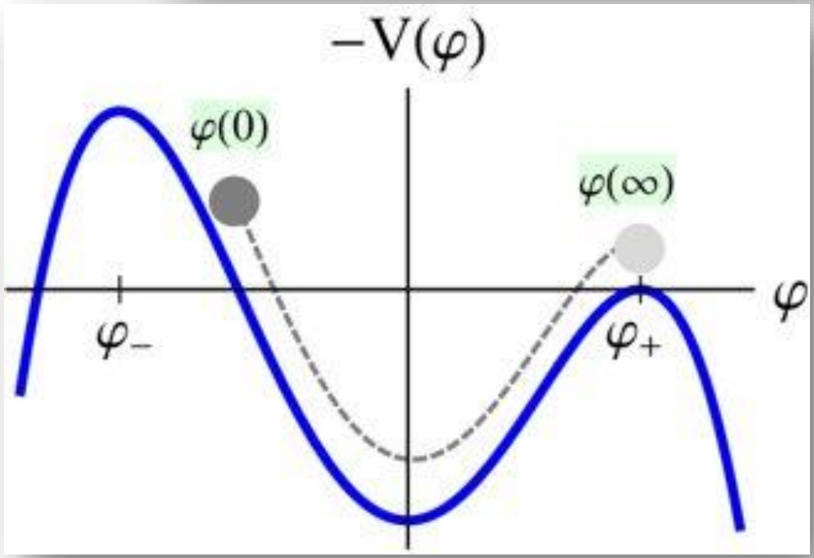
$$S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V \right]$$



Single Field False Vacuum Decay

[a] $\rho = \left(\tau^2 + |\vec{x}|^2\right)^{1/2}$

$$S_E = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty \rho^{D-1} d\rho \left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$



$$\frac{d^2\varphi}{d\rho^2} + \frac{D-1}{\rho}\frac{d\varphi}{d\rho} = V'(\varphi)$$

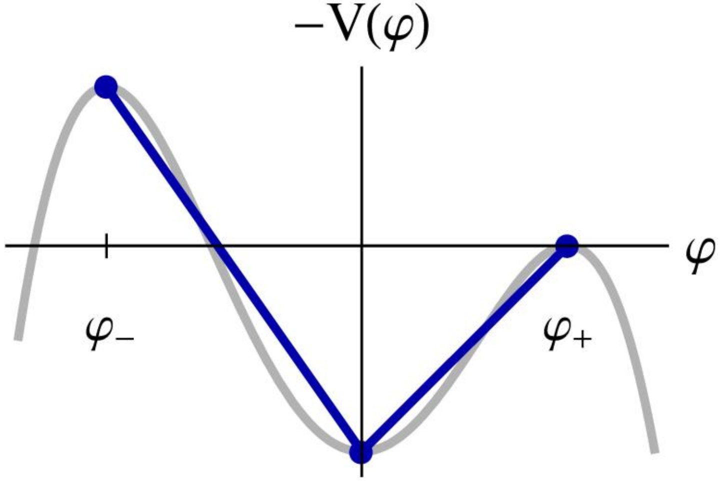
$$\varphi(0) = \varphi_0$$
$$\dot{\varphi}(0) = 0$$
$$\dot{\varphi}(\infty) = 0$$

a. S. R. Coleman, V. Glaser and A. Martin, Commun.Math. Phys. 58 (1978) 211

Piecewise Linear potential[a]

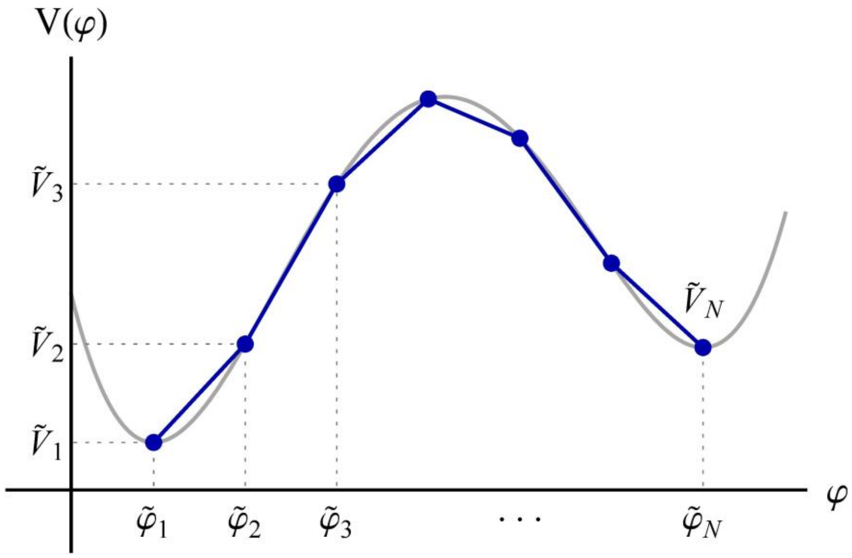
$$\frac{d^2\varphi}{d\rho^2} + \frac{D-1}{\rho}\frac{d\varphi}{d\rho} = V'(\varphi)$$

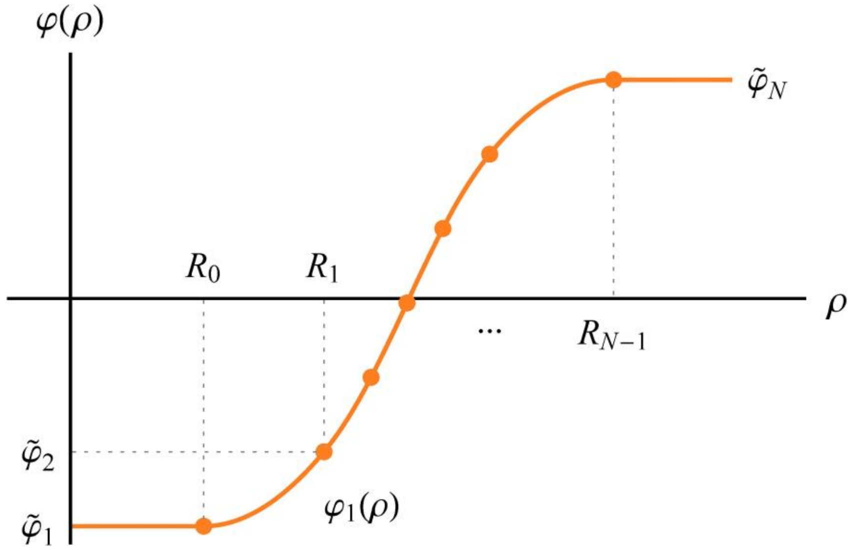
$$V(\varphi) = 8a\varphi$$
$$\varphi(\rho) = v + a\rho^2 + \frac{b}{\rho^2}$$



a. M. J. Duncan and L. G. Jensen, Phys. Lett. B 291 (1992) 109.

Polygonal Bounce





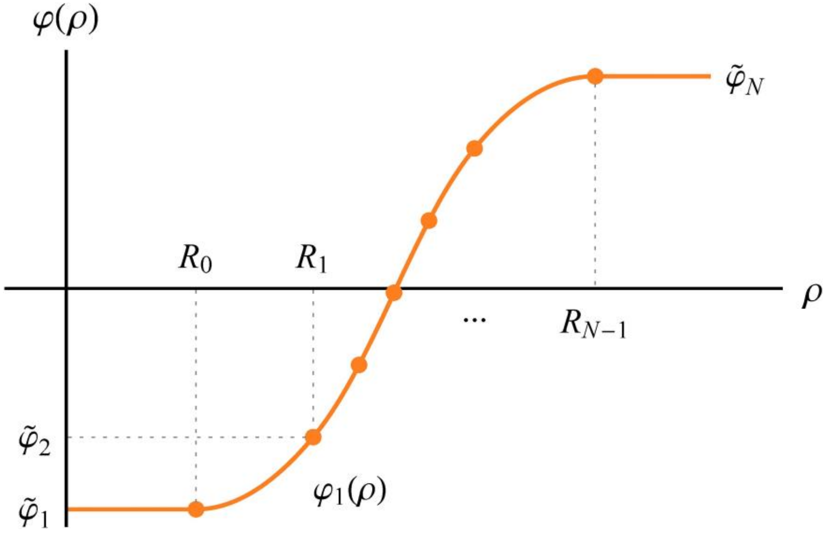
$$V_s(\varphi) = \underbrace{\left(\frac{\tilde{V}_{s+1} - \tilde{V}_s}{\tilde{\varphi}_{s+1} - \tilde{\varphi}_s}\right)}_{8\,a_s}(\varphi - \tilde{\varphi}_s) + \tilde{V}_s - \tilde{V}_N .$$

$$\varphi_s(\rho) = v_s + \frac{4}{D} a_s \rho^2 + \frac{2}{D-2}\frac{b_s}{\rho^{D-2}}$$

Boundary Conditions

$$\dot{\varphi}_s(R_s) = \dot{\varphi}_{s+1}(R_s)$$

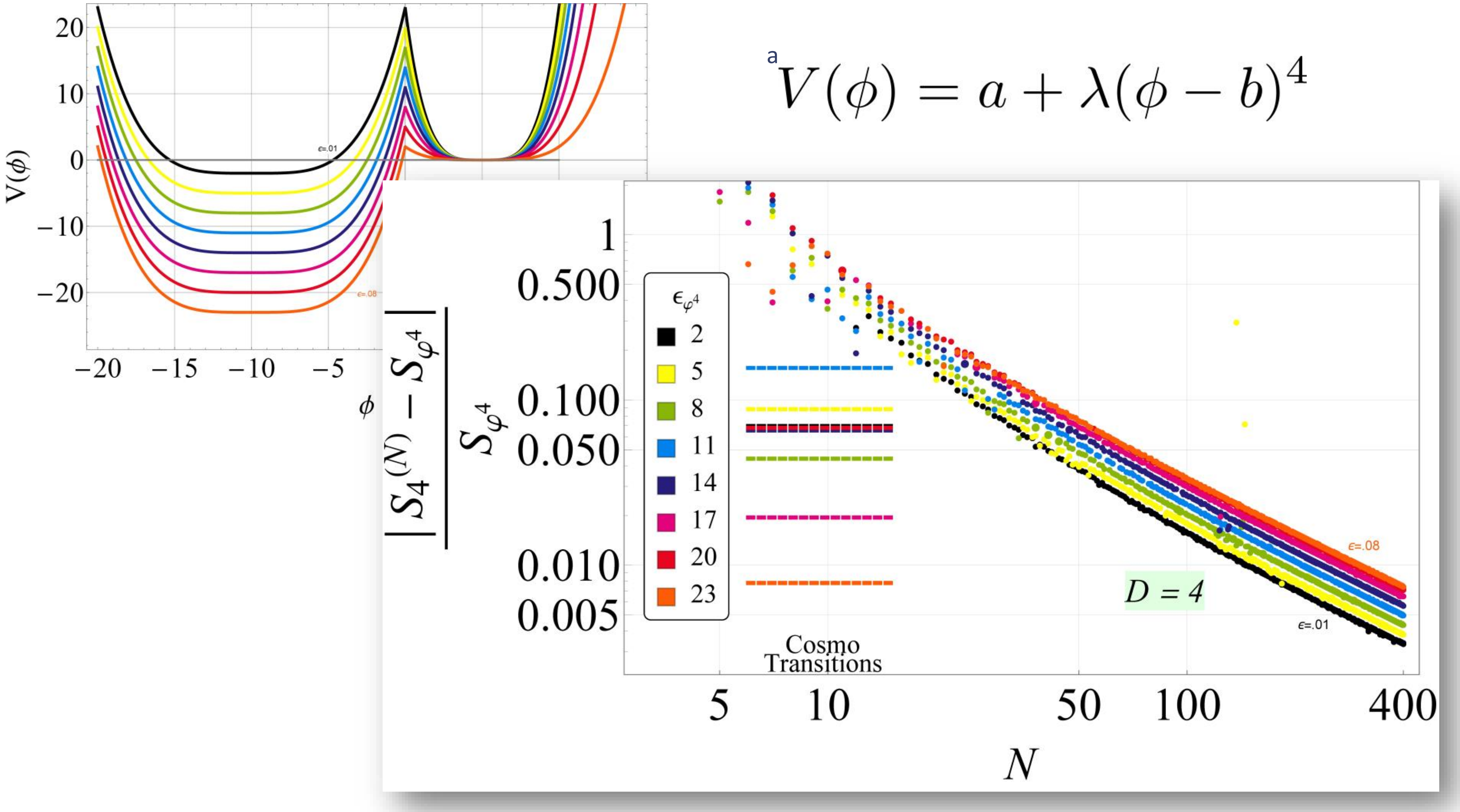
$$\varphi_s(R_s) = \tilde{\varphi}_{s+1} = \varphi_{s+1}(R_s)$$



$$a_s R_s^D - \frac{D}{4}(\tilde{\varphi}_{s+1} - v_s) R_s^{D-2} + \frac{D}{2(D-2)} b_s = 0$$

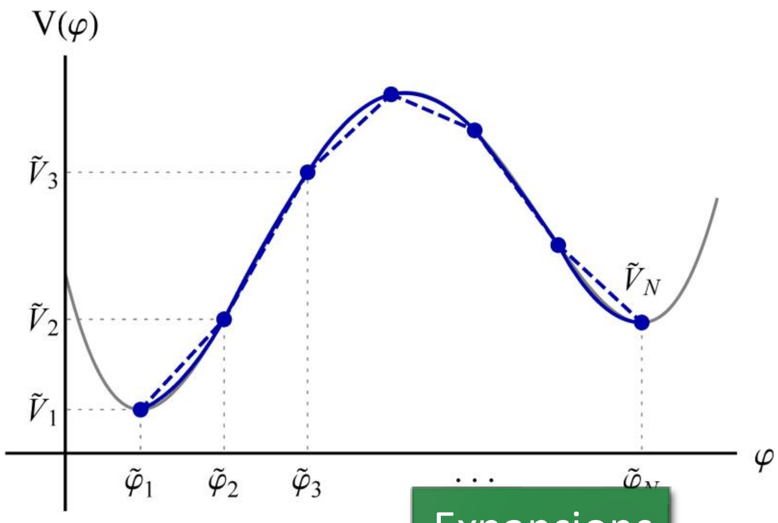
Bi-Quartic Potential: Exact Solution

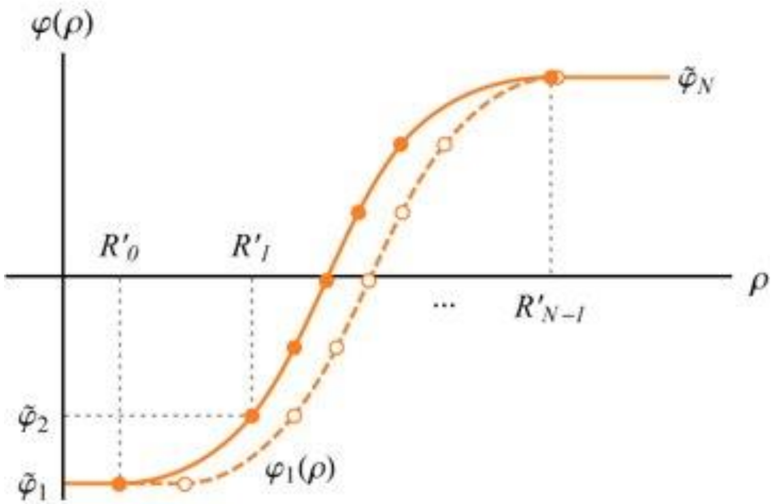
[a] $V(\phi) = a + \lambda(\phi - b)^4$



a. K. Dutta, C. Hector, P. M. Vaudrevange and A. West- phal, Phys. Lett. B 708 (2012) 309

Extending Polygonal Bounce





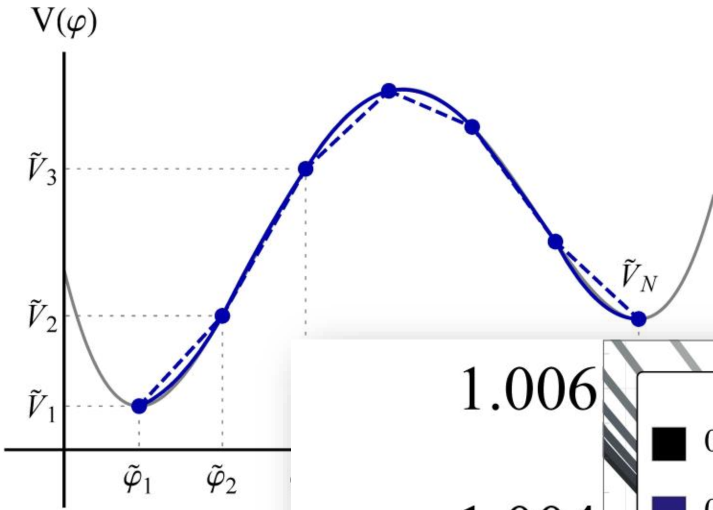
Expansions

$$\varphi = \varphi_{PB} + \xi$$

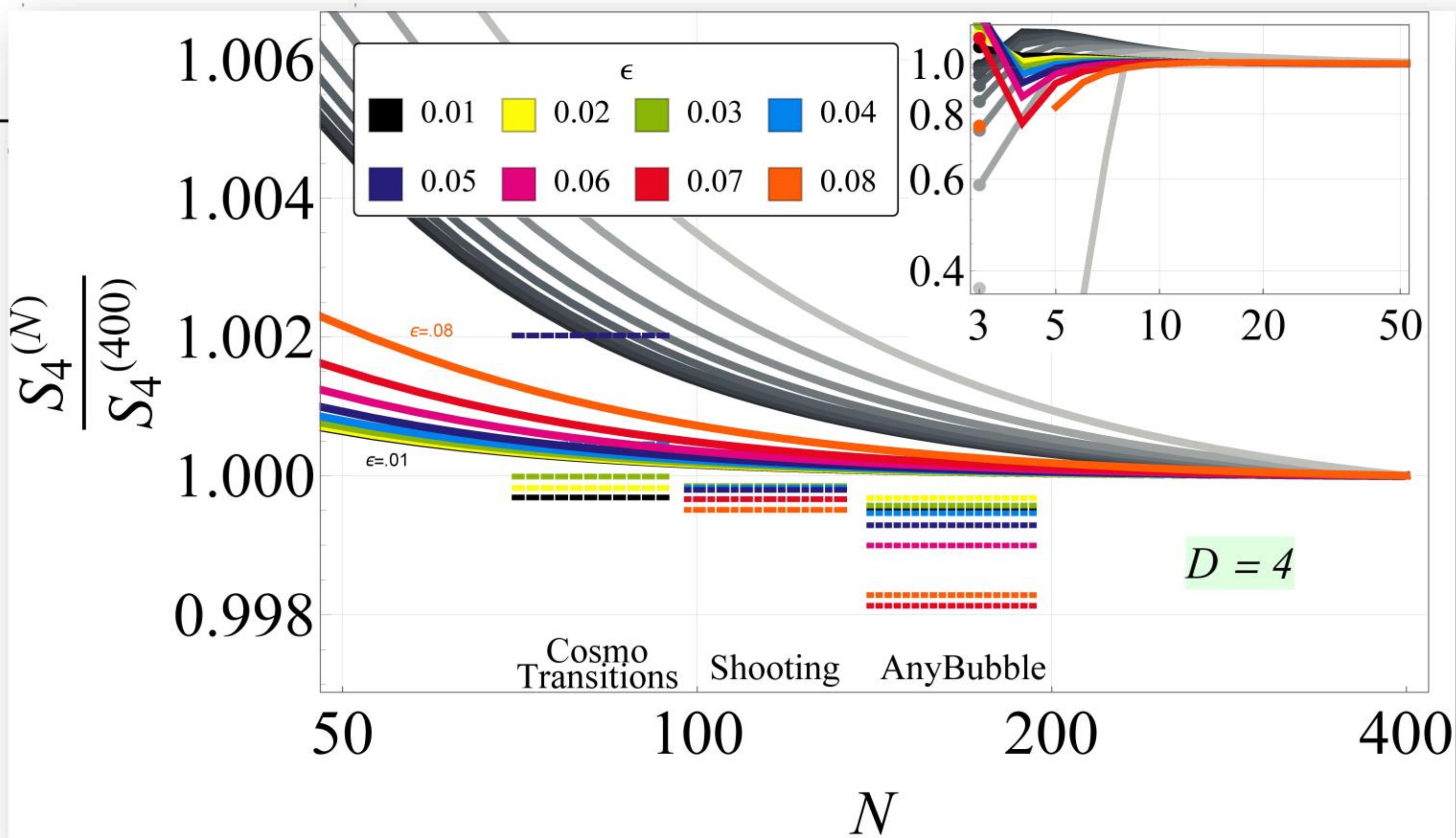
$$\tilde{V}_s - \tilde{V}_N + \partial\tilde{V}_s\left(\varphi_s - \tilde{\varphi}_s\right) + \frac{\partial^2\tilde{V}_s}{2}\left(\varphi_s - \tilde{\varphi}_s\right)^2 + \dots$$

$$R_s \to R_s\left(1 + r_s\right) \qquad r_s \ll 1$$

$$\xi = \nu + \frac{2}{D-2}\frac{\beta}{\rho^{D-2}} + \frac{4}{D}\alpha\rho^2 + \mathcal{I}(\rho)$$



V(φ)
$\tilde{V}_3$
$\tilde{V}_2$
$\tilde{V}_1$
$\tilde{V}_N$
$\tilde{\varphi}_1$
$\tilde{\varphi}_2$



ϵ
0.01 0.02 0.03 0.04
0.05 0.06 0.07 0.08
1.006
1.004
1.002
1.000
0.998
$\frac{S_4^{(N)}}{S_4^{(400)}}$
ϵ=.08
ϵ=.01
Cosmo Transitions
Shooting
AnyBubble
D = 4
50
100
200
400
N
1.0
0.8
0.6
0.4
3
5
10
20
50

Pre-factor A on Polygonal Bounce

$$\frac{\Gamma}{\mathcal{V}} = A e^{-S_E}\left[1 + \mathcal{O}(\hbar)\right]$$

[a]

$$\Gamma = \left(\frac{S_4}{2\pi}\right)^2 \left|\frac{\det'(-\partial^2 + V''(\varphi(\rho)))}{\det(-\partial^2 + V''(\varphi_+))}\right|^{-1/2} e^{-S_4-\delta_4}$$

[b,c]

$$\frac{\det \mathcal{O}_l}{\det \mathcal{O}_l^{\mathrm{free}}} = \mathcal{R}_l(\rho = \infty)^{(l+1)^2}$$

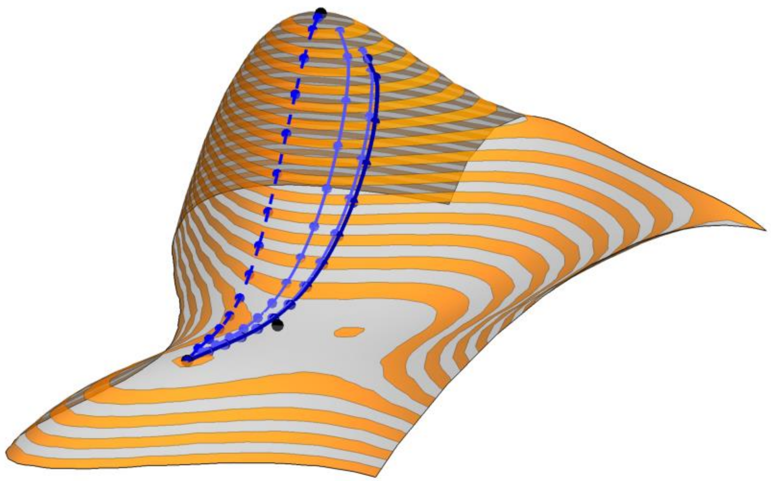
$$\mathcal{R}_l(\rho) = \frac{\psi_l(\rho)}{\psi_l^{\mathrm{free}}(\rho)}$$

a. C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D 16 (1977) 1762
b. G. V. Dunne and H. Min, Phys. Rev. D 72 (2005) 125004.
c. I. M. Gelfand and A. M. Yaglom, J. Math. Phys. 1 (1960) 48.

Multi-Field Vacuum Decay

$\mathcal{O}(D)$ [a]

$$S_D = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty \rho^{D-1}\, \mathrm{d}\rho \left(\frac{1}{2} \sum_i \dot{\varphi}_i^2 + V(\varphi_i) \right)$$



$$\ddot{\varphi}_i + \frac{D-1}{\rho} \dot{\varphi}_i = d_i V$$

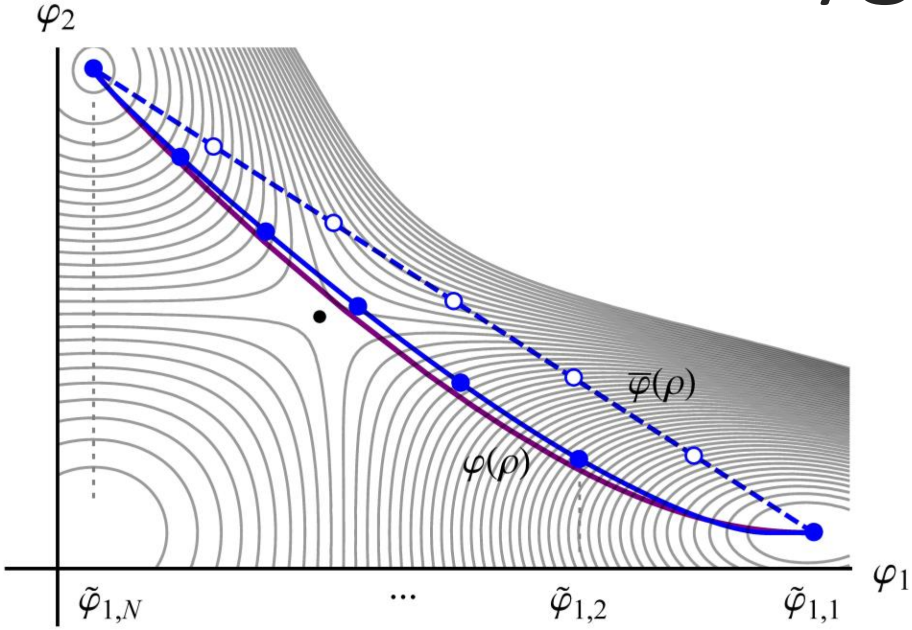
$$\varphi_i(0) = \varphi_{i0}$$

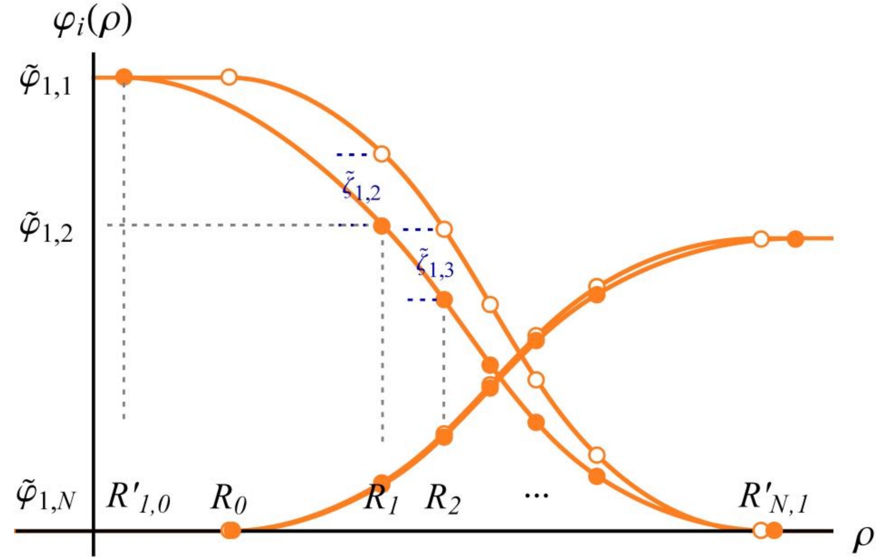
$$\dot{\varphi}_i(0) = 0$$

$$\dot{\varphi}_i(\infty) = 0$$

a. K. Blum, M. Honda, R. Sato, M. Takimoto and K. To-bioka, JHEP 1705 (2017) 109

Multi-Field Polygonal Bounce





$$\varphi_{is}(\rho) = \bar{\varphi}_{is} + \zeta_{is}$$

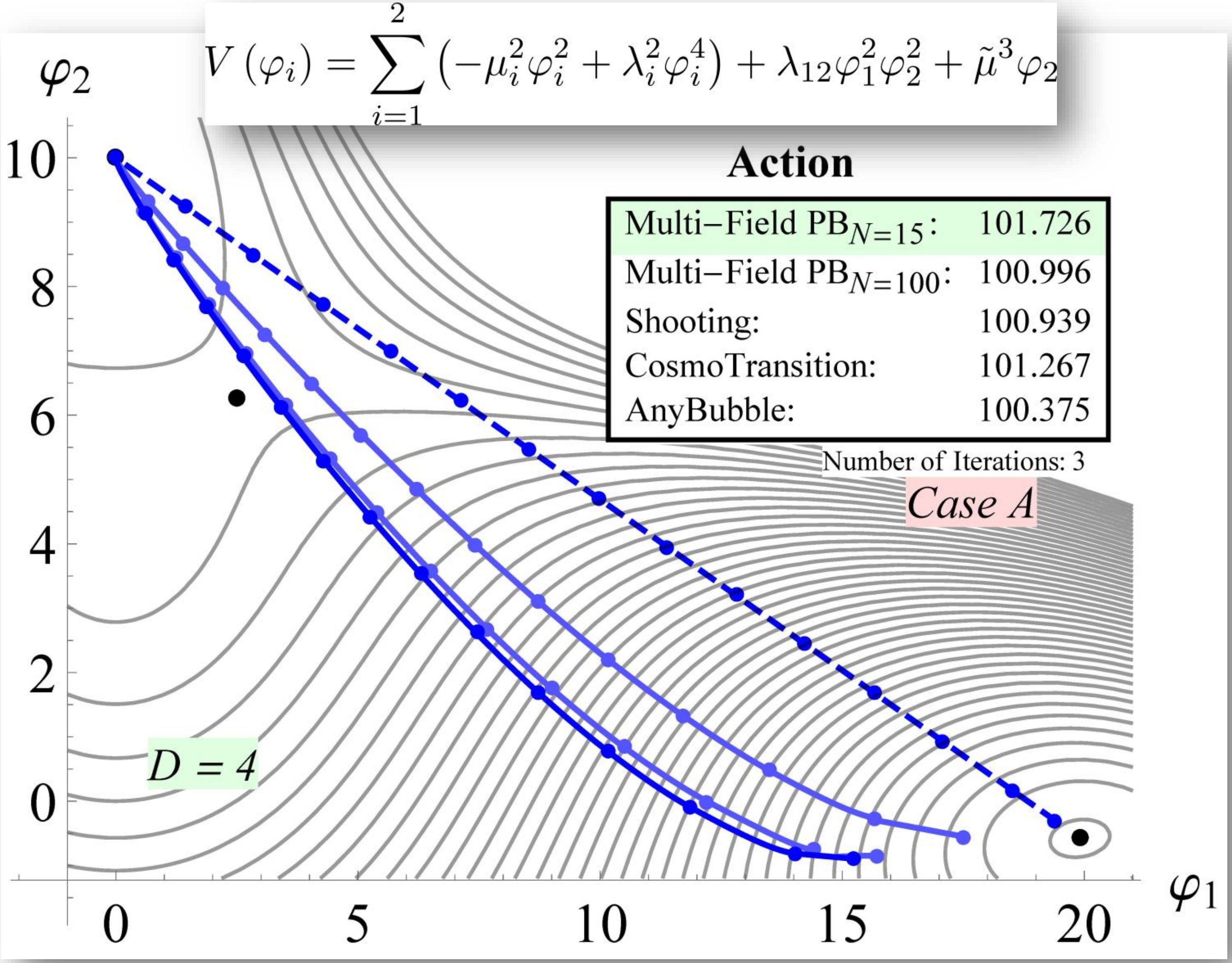
$$\underbrace{\ddot{\bar{\varphi}}_{is} + \frac{D-1}{\rho}\dot{\bar{\varphi}}_{is}}_{8\bar{a}_{is}} + \underbrace{\ddot{\zeta}_{is} + \frac{D-1}{\rho}\dot{\zeta}_{is}}_{8a_{is}} = \frac{dV}{d\varphi_i}\left(\bar{\varphi} + \zeta\right)$$

$$\underbrace{\ddot{\bar{\varphi}}_{is} + \frac{D-1}{\rho}\dot{\bar{\varphi}}_{is}}_{8\bar{a}_{is}} + \underbrace{\ddot{\zeta}_{is} + \frac{D-1}{\rho}\dot{\zeta}_{is}}_{8a_{is}} = \frac{dV}{d\varphi_i}(\bar{\varphi} + \zeta)$$

$$\frac{dV}{d\varphi_i} \simeq \frac{1}{2}\left(d_i\tilde{V}_s + d_i\tilde{V}_{s+1} + d^2_{ij}\tilde{V}_s\tilde{\zeta}_{js} + d^2_{ij}\tilde{V}_{s+1}\tilde{\zeta}_{js+1}\right)$$

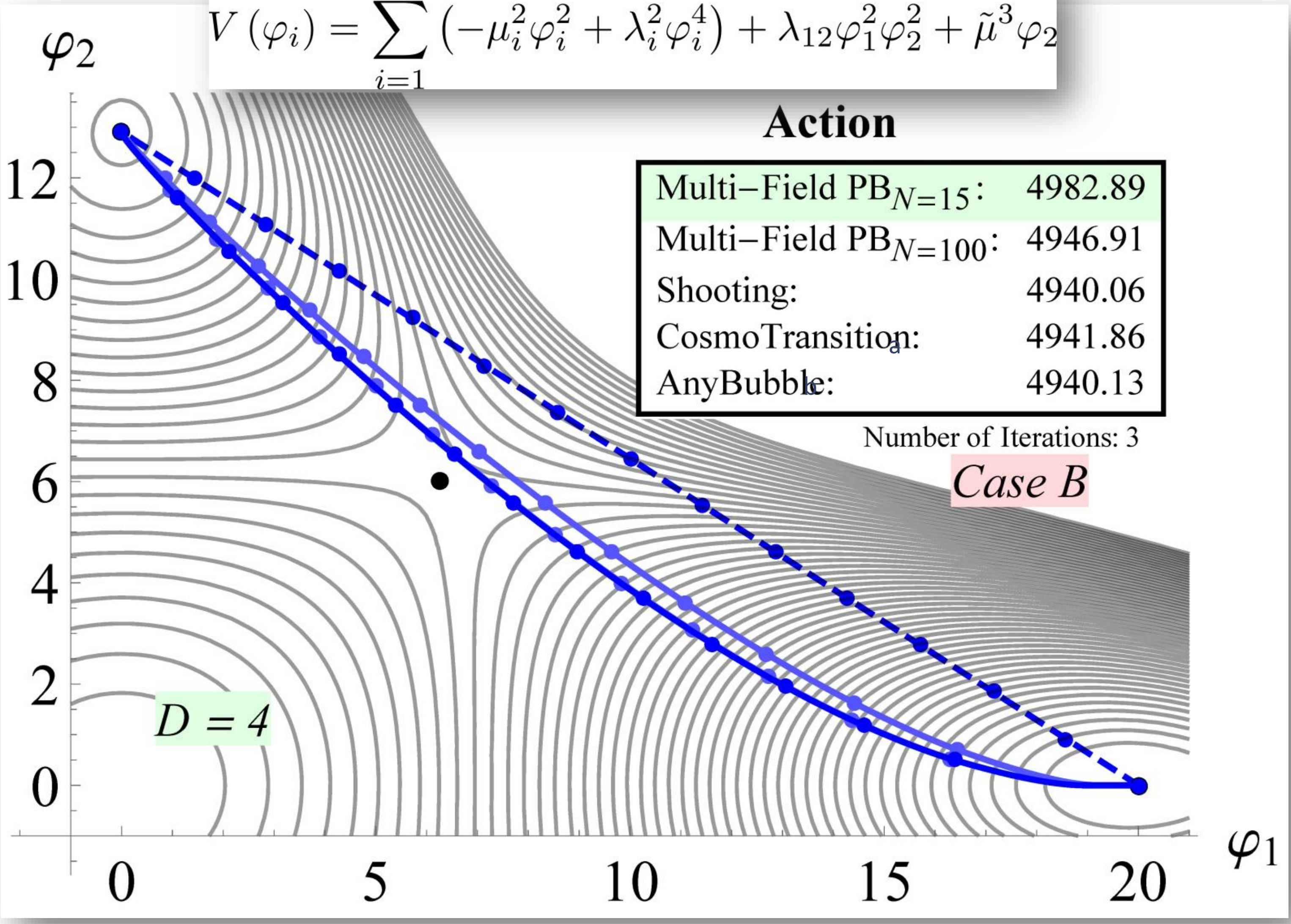
$$\zeta_{is} = v_{is} + \frac{2}{D-2}\frac{b_{is}}{\rho^{D-2}} + \frac{4}{D}a_{is}\rho^2$$

a. S. Akula, C. Balázs and G. A. White, Eur. Phys. J. C 76(2016) no.12, 681



a. J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D60 (1999) 105035.
   C. L. Wainwright, Comput. Phys. Commun. 183 (2012) 2006.
b. A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701(2017) no.01, 051
c. P. Athron, C. Balázs, M. Bardsley, A. Fowlie, D. Harries and G. White.

$$V(\varphi_i) = \sum_{i=1}^{2} \left(-\mu_i^2 \varphi_i^2 + \lambda_i^2 \varphi_i^4\right) + \lambda_{12} \varphi_1^2 \varphi_2^2 + \tilde{\mu}^3 \varphi_2$$



a. J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D60 (1999) 105035.
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b. A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701(2017) no.01, 051

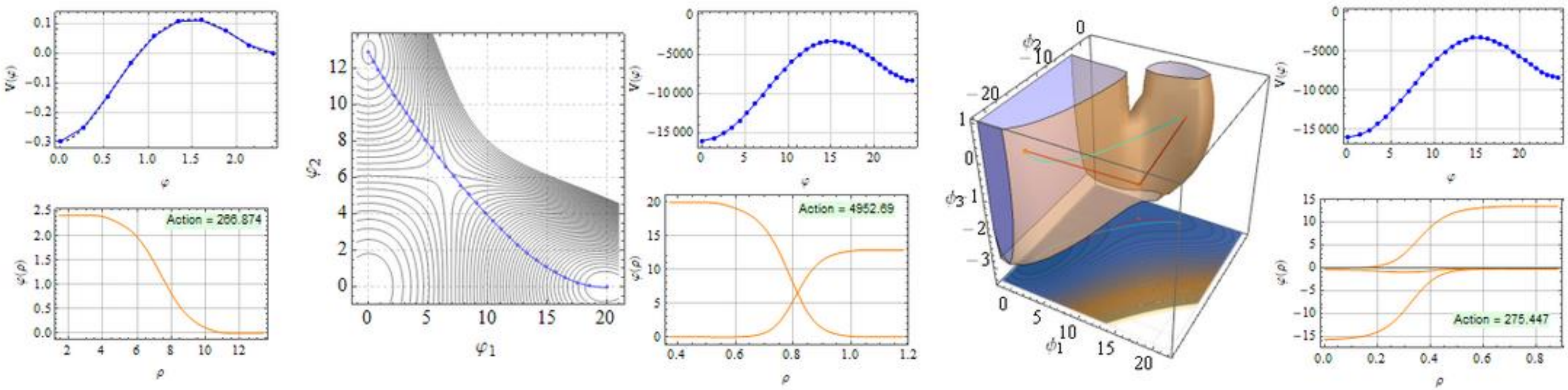
c. P. Athron, C. Balázs, M. Bardsley, A. Fowlie, D. Harries and G. White.

Multi-Field False Vacuum Decay

FindBounce

Computes the `Bounces` of a false vacuum decay with multiple scalar fields in QFT.

release | no releases or repo not found



Installation

The following description is for people who just want to use the package functionality and are not interested in package development. To use *FindBounce* package you need Mathematica version 10. or later.

Package

```
BounceFunction[ Action: 4.95×10^3
                Domain: {0.507, 1.05}
                Dimensions: 4
                InitialSegment: 1
                Iterations: 3
                Segments: 30 ]
```


a. J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D60 (1999) 105035.
   C. L. Wainwright, Comput. Phys. Commun. 183 (2012) 2006.
b. A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701(2017) no.01, 051
c. P. Athron, C. Balázs, M. Bardsley, A. Fowlie, D. Harries and G. White.

Conclusions

*We developed a **semi-analytical**, **simple** and **fast** approach to compute the false vacuum **decay rate** for arbitrary potentials with any number of fields and any space-time dimensions **up to the desired precision**.*

*Provides an **analytical insight** in describing the **vacuum structure** of the potential, thermodinamical **bubble nucleations** and it's related spectrum of **gravitational waves** at the early Universe.*

Thank you

Back up Slides

Pre-factor A on Polygonal Bounce

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-S_E}\left[1+\mathcal{O}(\hbar)\right]$$

[a]

$$\Gamma = \left(\frac{S_4}{2\pi}\right)^2 \left|\frac{\det'(-\partial^2 + V''(\varphi(\rho)))}{\det(-\partial^2 + V''(\varphi_+))}\right|^{-1/2} e^{-S_4-\delta_4}$$

$\mathcal{O}(4)$

$$\mathcal{O}_l = -\frac{d^2}{d\rho^2} - \frac{3}{\rho}\frac{d}{d\rho} + \frac{l(l+1)}{\rho^2} + V''(\rho) + 1$$

$$V''(\rho) = -3\varphi(\rho) + \frac{3\alpha}{2}\varphi^2(\rho)$$

[b,c]

$$\frac{\det \mathcal{O}_l}{\det \mathcal{O}_l^{\text{free}}} = \mathcal{R}_l(\rho=\infty)^{(l+1)^2} \qquad \mathcal{R}_l(\rho) = \frac{\psi_l(\rho)}{\psi_l^{\text{free}}(\rho)}$$

a. C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D 16 (1977) 1762
b. G. V. Dunne and H. Min, Phys. Rev. D 72 (2005) 125004.
c. I. M. Gelfand and A. M. Yaglom, J. Math. Phys. 1 (1960) 48.

Pre-factor A on Polygonal Bounce

$$\mathcal{R}_l(\rho) = \frac{\psi_l(\rho)}{\psi_l^{\text{free}}(\rho)}$$

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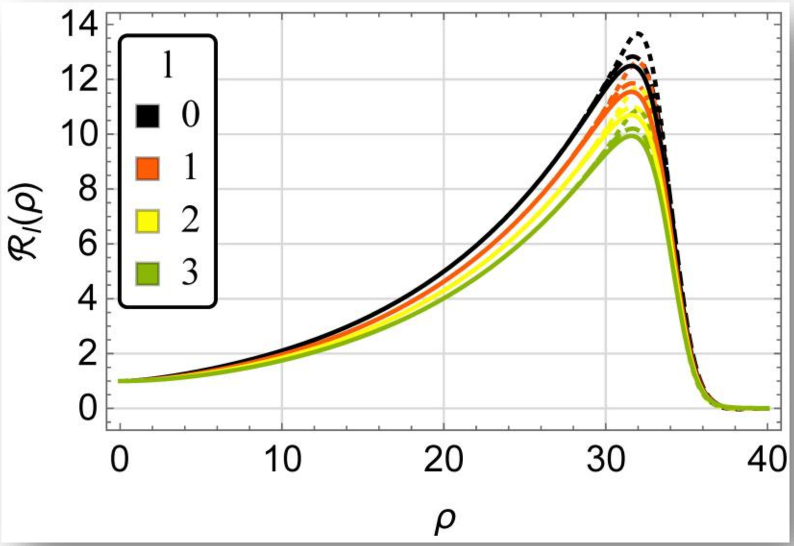
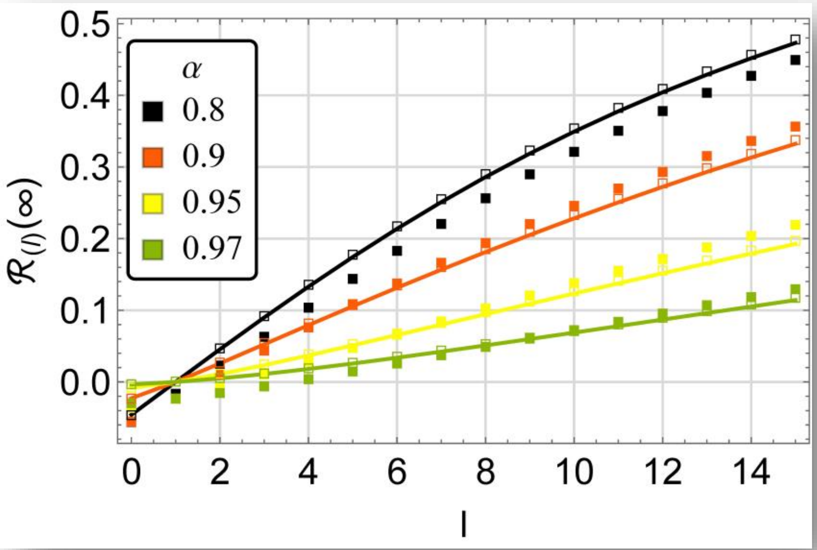




$$\frac{\Gamma}{\mathcal{V}} = Ae^{-S_E}\left[1 + \mathcal{O}(\hbar)\right]$$

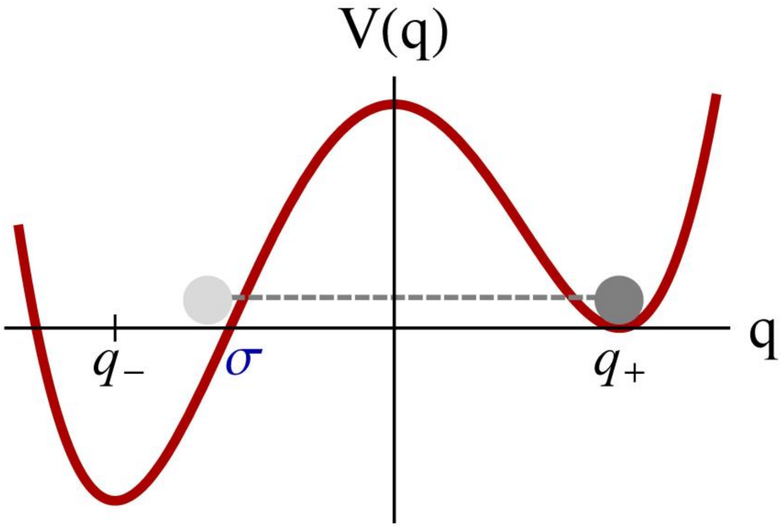
$\alpha \to 1$ Thin wall limit

Around 10% contribution

Tunneling in Quantum Mechanics

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$

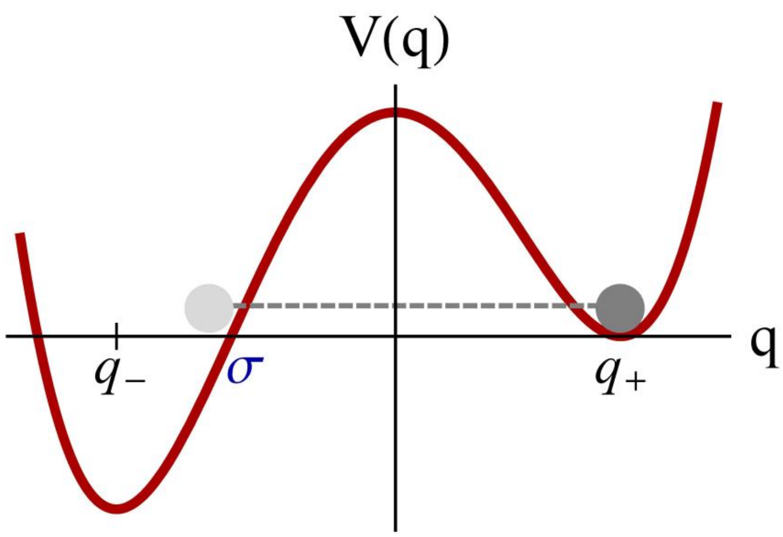


$$\frac{\Gamma}{\mathcal{V}} = A e^{-B} \left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2 \int_{q_+}^{\sigma} dq \left(2V\right)^{1/2}$$

Minimum

$$\delta \int_{q_+}^{\sigma} dq \left(2V\right)^{1/2} = 0$$



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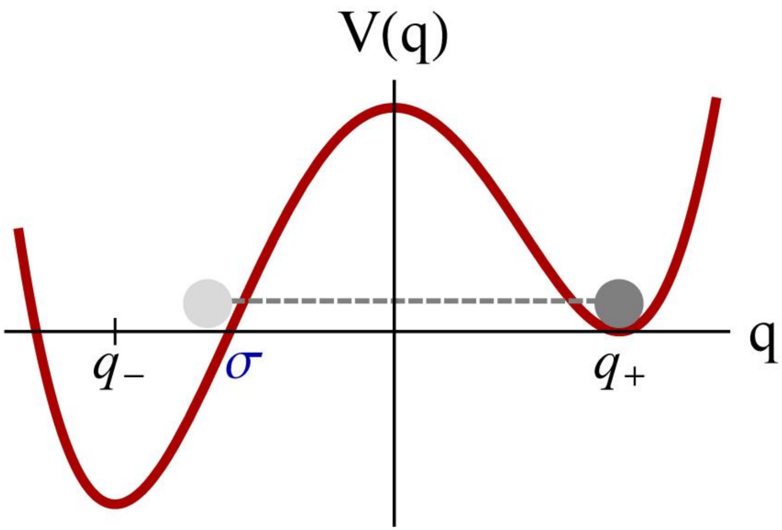


$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$

Minimum

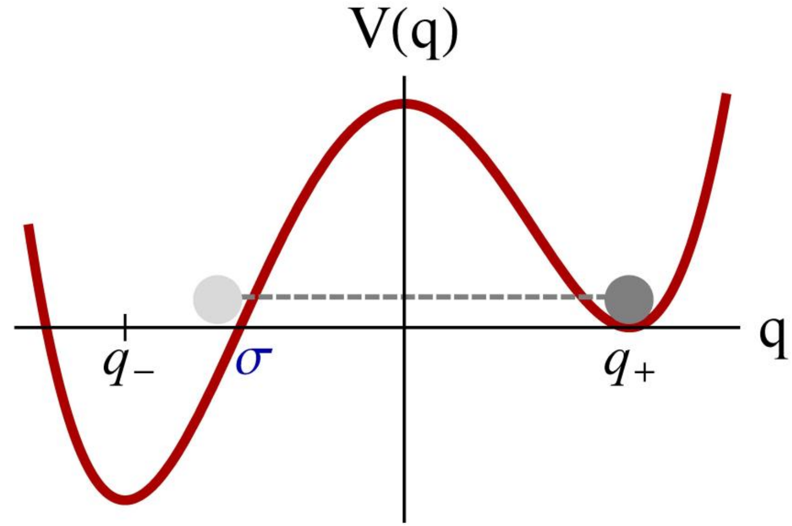
$$\delta\int_{q_+}^{\sigma} dq\,(2V)^{1/2} = 0$$

$$\delta\int_{q_+}^{\sigma} dq\left[2\left(E - V\right)\right]^{1/2} = 0$$

$$\frac{1}{2}\frac{dq}{dt}\cdot\frac{dq}{dt} + V = E$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\left(2V\right)^{1/2}$$



$$\delta\int_{q_+}^{\sigma} dq\left(2V\right)^{1/2} = 0$$

$$\delta\int_{q_+}^{\sigma} dq\left[2\left(E - V\right)\right]^{1/2} = 0$$

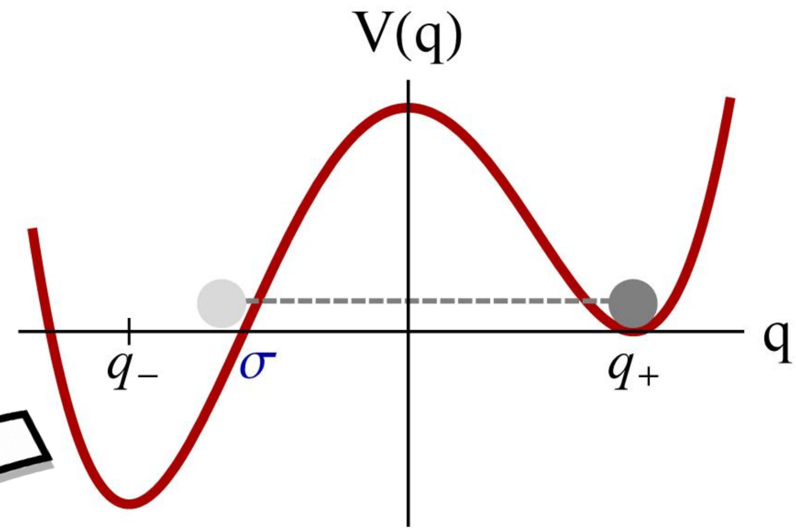
$$t \rightarrow i\tau$$

$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

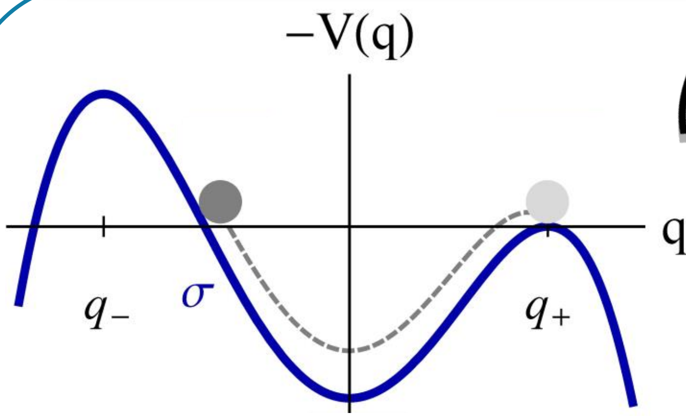
$$\frac{1}{2}\frac{dq}{dt}\cdot\frac{dq}{dt} + V = E$$

$$\frac{\Gamma}{\mathcal{V}} = A e^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,\left(2V\right)^{1/2}$$



$$t \rightarrow i\tau$$



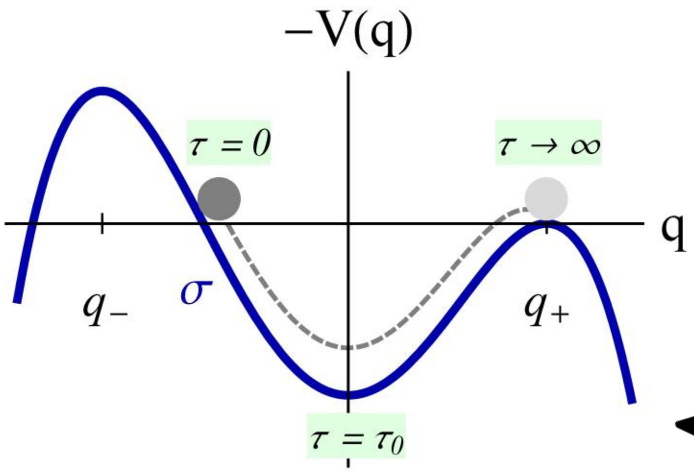
$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

$$\delta\int_{q_+}^{\sigma} dq\,\left[2\left(E - V\right)\right]^{1/2} = 0$$

$$\frac{1}{2}\frac{dq}{dt}\cdot\frac{dq}{dt} + V = E$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}(\hbar)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$



$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

$$\left.\frac{dq}{d\tau}\right|_{\tau_\sigma=0} = 0$$

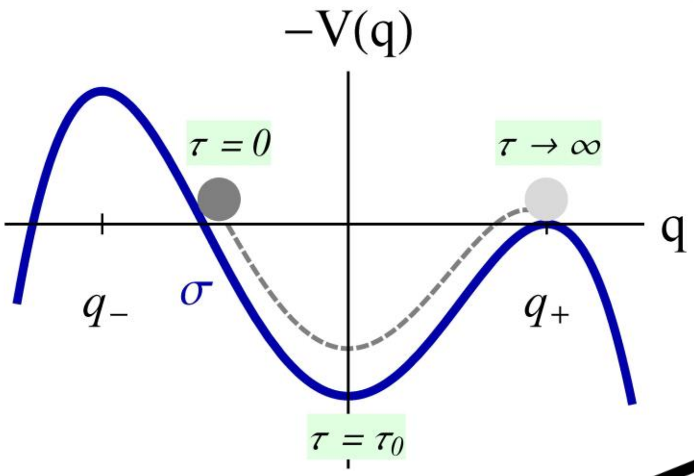
$$\lim_{\tau\to\pm\infty} q = q_+$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}(\hbar)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$

$$\left.\frac{dq}{d\tau}\right|_{\tau_\sigma = 0} = 0$$

$$\lim_{\tau \to \pm\infty} q = q_+$$



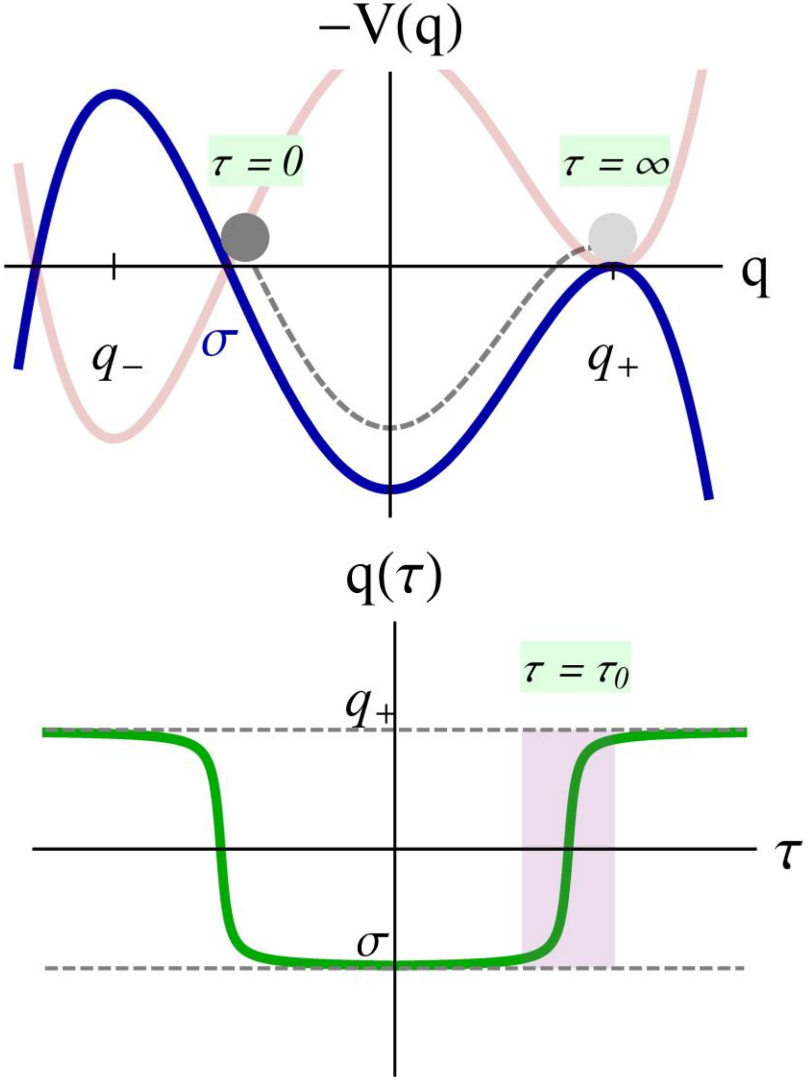
$$\int_{q_+}^{\sigma} dq\,(2V)^{1/2} = \int_{-\infty}^{0} d\tau L_E$$

$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

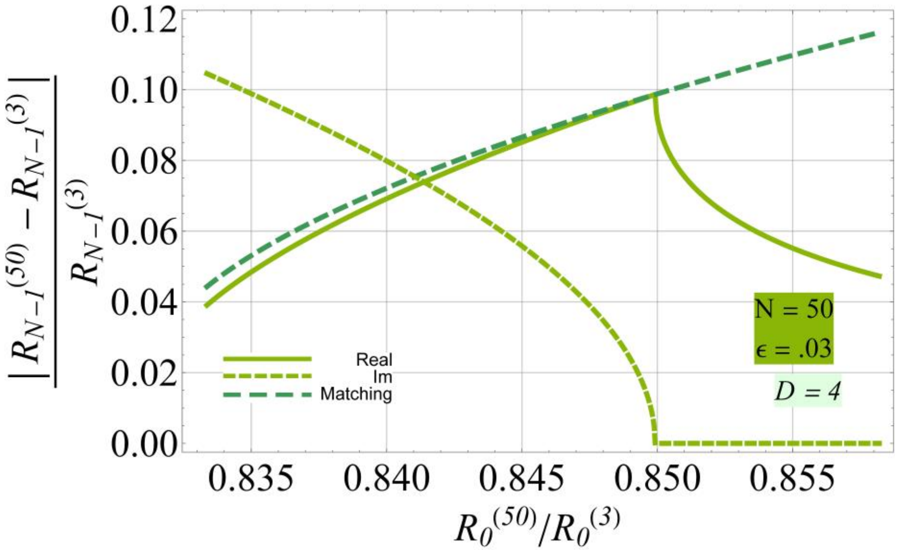
False Vacuum Decay

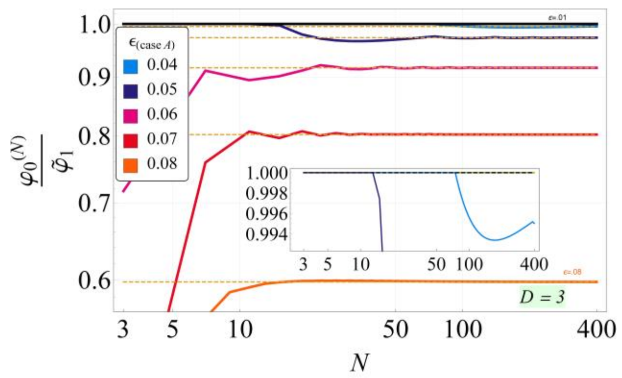
$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

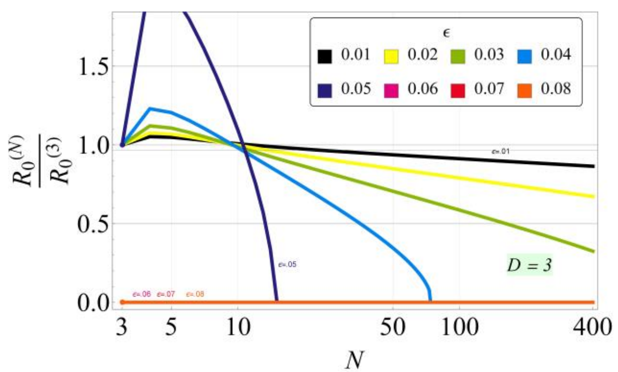
$$B = \int_{-\infty}^{\infty} d\tau L_E \equiv S_E$$



Parameters: Polygonal Bounce







Some Bounce Properties

Rescaling

$$V \to g^a V$$

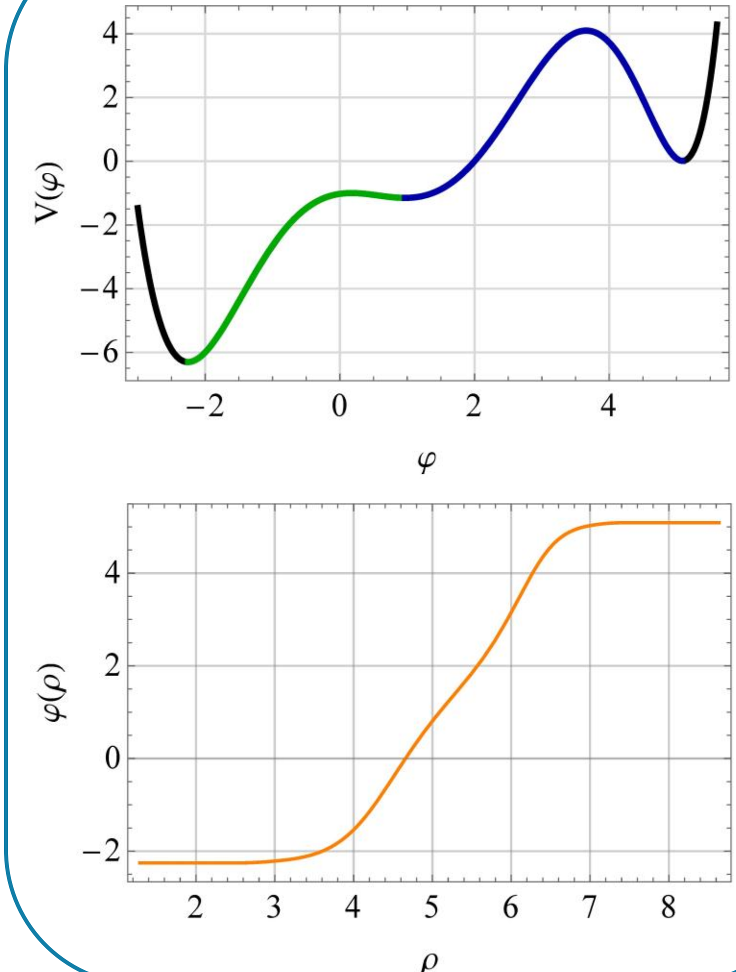
$$\phi \to g^b \phi$$

$$\rho \to g^c \rho$$

$$S_E \to g^{4b-a} S_E$$

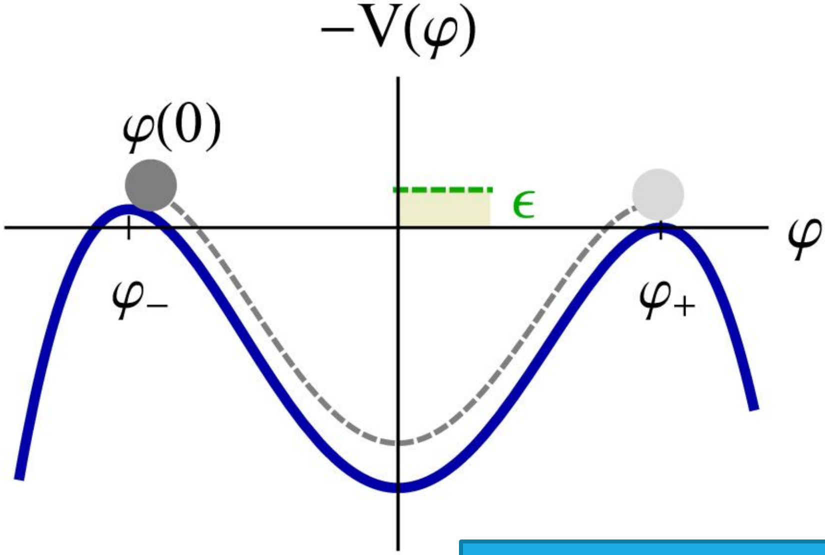
$$2c = 2b - a$$

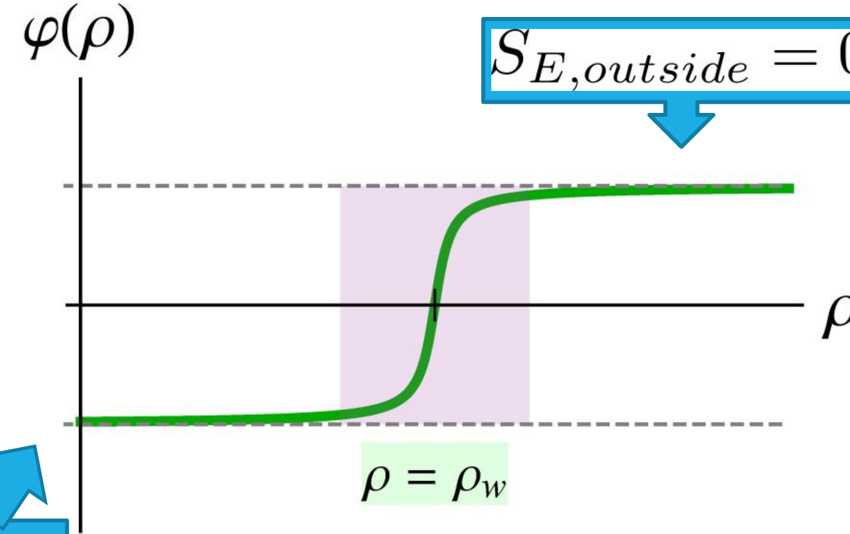
Multiple-Solutions



Thin wall approximation

$D = 4 \Rightarrow \frac{d^2\varphi}{d\rho^2} + \boxed{\frac{3}{\rho}\frac{d\varphi}{d\rho}} = V'(\varphi)$





$$S_{E,outside} = 0$$

$$S_{E,inside} = -\frac{\pi^2}{2}\rho_w^4 \epsilon$$

$$S_{E,wall} = 2\pi^2 \rho_w^3 S_1$$

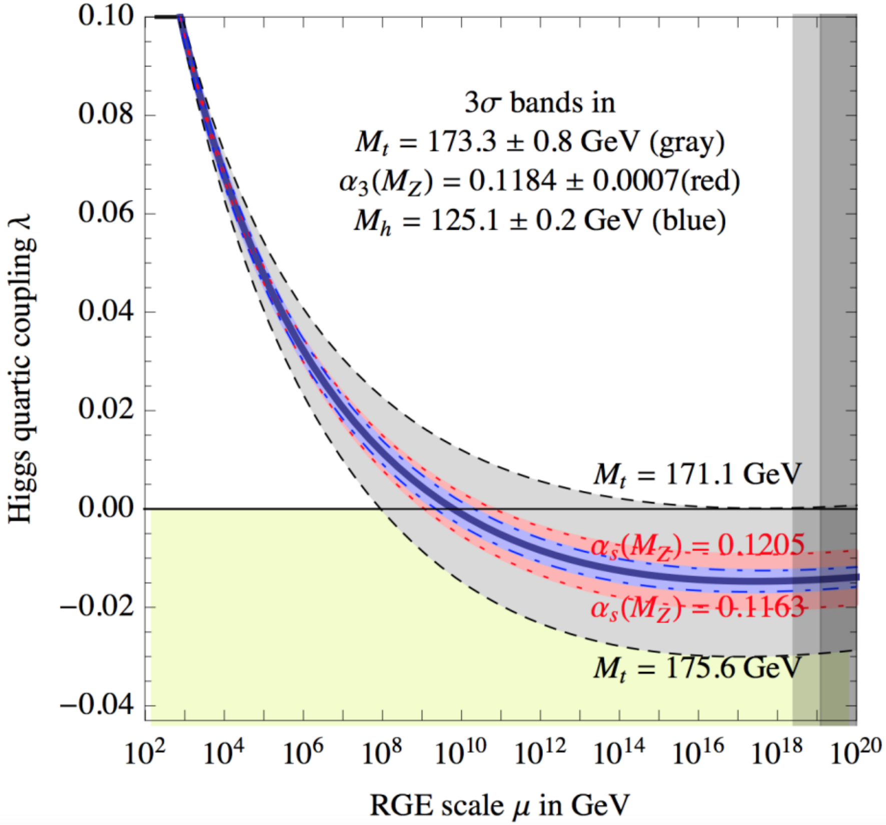
$$\frac{dS_E}{dR} = 0 \Rightarrow R = \frac{3S_1}{\epsilon}$$

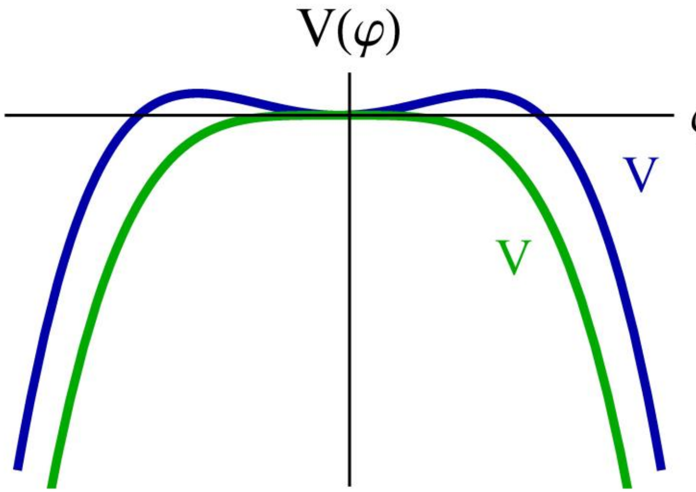
a. S. R. Coleman, Phys. Rev. D 15 (1977) 2929

Higgs Potential

$$V = -\mu^2|\Phi^2| + \lambda|\Phi^4| = \frac{1}{2}m^2h^2 + \sqrt{m\lambda}\,h^3 + \frac{1}{4}\lambda h^4 + \ldots$$

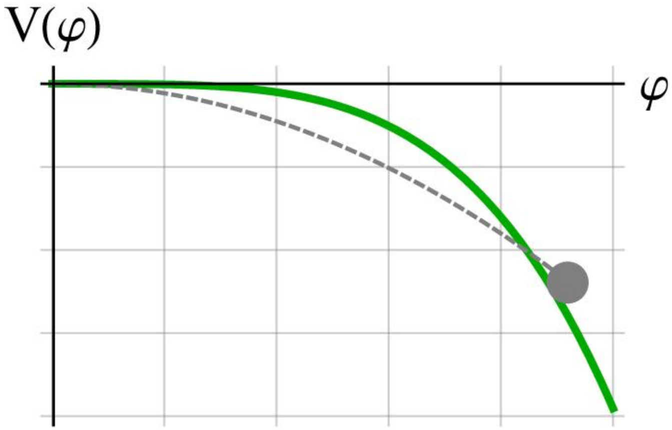
$$V\left(h \gg v\right) \cong \frac{1}{4}\lambda_{eff}\left(h\right)h^4$$





a. G. Isidori, V. S. Rychkov, A. Strumia and N. Tetradis, Phys. Rev. D 77 (2008) 025034.
b. A. Salvio, A. Strumia, N. Tetradis and A. Urbano, JHEP 1609 (2016) 054

Decay Rate: Higgs Potential



$$\lambda < 0$$

$$V(\phi) = \frac{\lambda}{4}\phi^4$$

$$\phi \to \frac{1}{\alpha}\phi \qquad \rho \to \alpha\rho$$

$$\Gamma \simeq \frac{V_U}{R^4} e^{-\frac{8\pi^2}{3|\lambda|}}$$

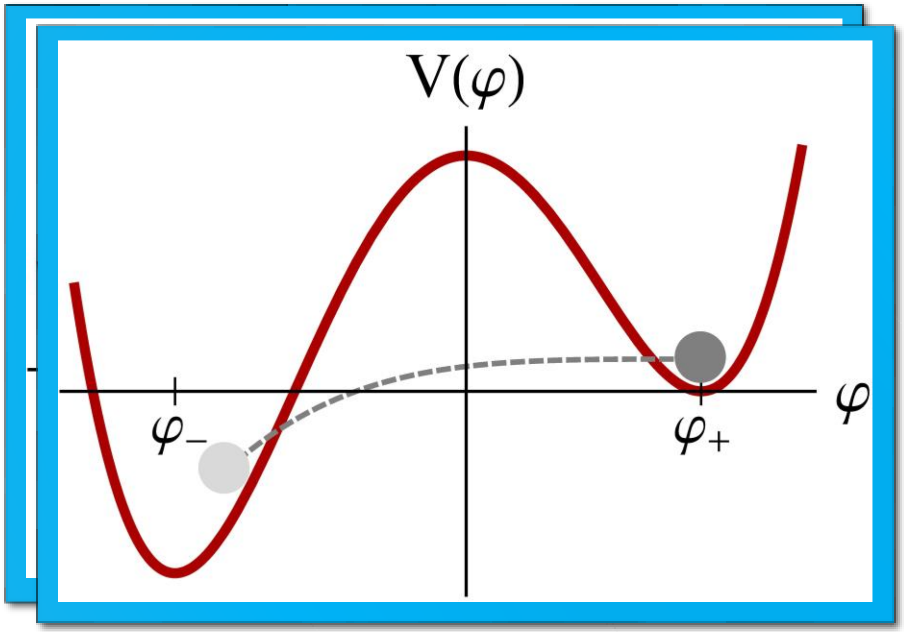
$$\Gamma \simeq \left(10^{25} eV\right)^4 \left(8.10^{101} eV^{-3}\right) e^{-\frac{8\pi^2}{3|-0.02|}} \simeq 10^{-370} eV$$

$$t \equiv \frac{1}{\Gamma} \simeq 10^{347} \text{ years}$$

a. K. M. Lee and E. J. Weinberg, Nucl. Phys. B 267, 181 (1986).

Single Field False Vacuum Decay

$$S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V \right]$$



$$\frac{\partial\varphi}{\partial\tau}(0,\vec{x}) = 0$$

$$\lim_{\tau\to\pm\infty} \varphi(\tau,\vec{x}) = \varphi_+$$

$$\lim_{\vec{x}\to\pm\infty} \varphi(\tau,\vec{x}) = \varphi_+$$

Solving Boundary Conditions

$$\sum_{\sigma=0}^{N-1} \left(a_{\sigma+1} - a_{\sigma}\right) R_{\sigma}^{D} = 0$$

$$S_D^{(\lambda)} = \lambda^{D-2}\mathcal{T} + \lambda^{D}\mathcal{V}$$

$$\left.\frac{dS_D^{(\lambda)}}{d\lambda}\right|_{\lambda=1} = 0$$

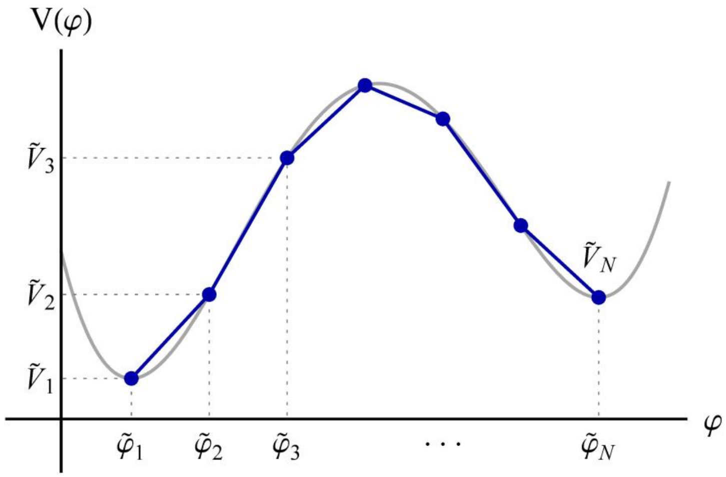
$$(D-2)\mathcal{T} + D\mathcal{V} = 0$$

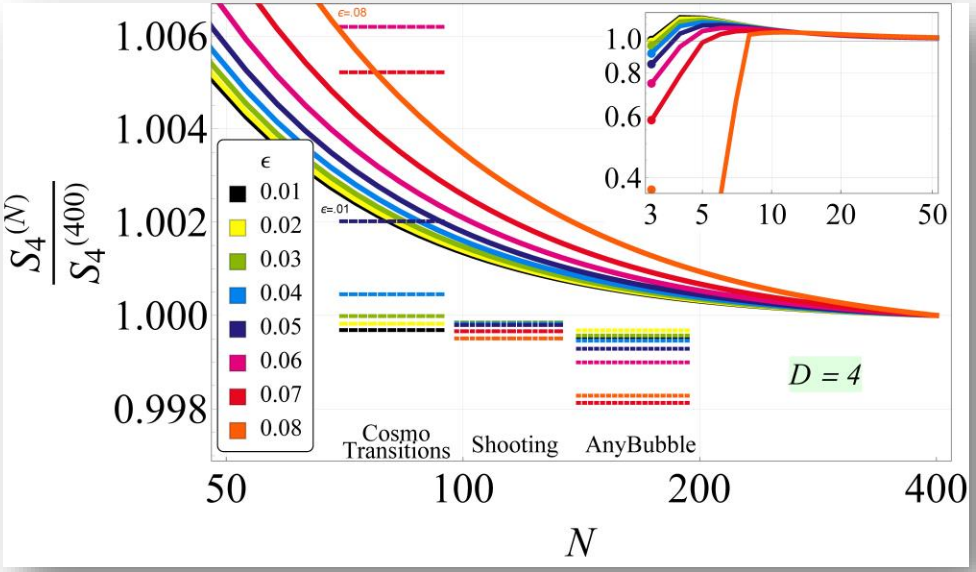
$$\lambda = \sqrt{\frac{(2-D)\mathcal{T}}{D\mathcal{V}}} = 1$$

a. G. H. Derrick, J. Math. Phys. 5 (1964) 1252.

Coleman Potential

$$V(\varphi) = \frac{\lambda}{8}\left(\varphi^2 - v^2\right)^2 + \varepsilon\left(\frac{\varphi - v}{2v}\right)$$





$$\mathcal{I}(\rho) = \int_{\rho_0}^{\rho} \mathrm{d}y\, y^{1-D} \int_{\rho_1}^{y} \mathrm{d}x\, x^{D-1} \delta dV(x)$$

$$\delta dV = dV(\varphi_{PB}(\rho)) - 8\,(a+\alpha)$$

Expansions

$$\tilde{V}_s - \tilde{V}_N + \partial\tilde{V}_s\,(\varphi_s - \tilde{\varphi}_s) + \frac{\partial^2\tilde{V}_s}{2}\,(\varphi_s - \tilde{\varphi}_s)^2 + \ldots$$

$$R_s \to R_s\,(1+r_s) \qquad r_s \ll 1$$

$$\dot{\varphi}_s(R_s) = \dot{\varphi}_{s+1}(R_s)$$

$$\varphi_s(R_s) = \tilde{\varphi}_{s+1} = \varphi_{s+1}(R_s)$$

Boundary Conditions

$$\zeta_{is}(R_s) = \tilde{\zeta}_{is+1}$$

$$R_{iN-1} = R_{N-1}\left(1 + r_{iN-1}\right)$$

$$R_{i0} = R_0\left(1 + r_{i0}\right)$$

$$\zeta_{is} = v_{is} + \frac{2}{D-2}\frac{b_{is}}{\rho^{D-2}} + \frac{4}{D}a_{is}\rho^2$$