Randomness and Quantum Mechanics

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Bit Sequence as a Result of An Experiment

PERFORM REPEATEDLY AT GIVEN TIMES $t_0, t_1, t_2, t_3, t_4, \ldots$:

- CARRY OUT THE STATE PREPARATION PROCEDURE (USING A GIVEN DENSITY OPERATOR).
- APPLY THE QUESTION PROCEDURE (WITH A GIVEN PROJECTION OPERATOR).
- OBSERVE OUTCOME AND DISCARD SYSTEM.

Quantum Mechanics: resulting $0-1$ sequence should be random.

LOCAL VIEW: revealing sequence bit by bit as the experiment proceeds in time (at finite time we have access to some initial segment of outcome sequence).

⋆ **We work in a separable Hilbert space.**

Mathematical Objects of Study

Infinite Sequences of Bits

$\Updownarrow$

Sets of Natural Numbers

$\Updownarrow$

Real Numbers

Cantor Space: $2^\omega \sim \{0,1\}^{\mathbb{N}}$

- Compact space of infinite bit sequences.
- Discrete topology on $\{0,1\}$.
- Product topology on $\{0,1\}^{\mathbb{N}} = \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \dots$

Randomness vs. Probability: Heuristics

Consider uniform probability distribution on space of 0-1 sequences with finite length - then all sequences are equally probable ($\mathbb{P} = \frac{1}{2^n}$).

0010010000111111011010101000100010000010110
1000110000100011010011000100100011001100010

10101010101010101010101010101010101010101010
10101010101010101010101010101010101010101010

Even worse for infinite dimensional sequences - then all have <u>zero</u> probability of appearance! Thus:

PROBABILITY CALCULUS CANNOT FORMALIZE OUR HEURISTIC IMAGE OF RANDOMNESS

Definitions of Randomness

⋆ **Some historical remarks**[1]

Intuitions on random bit sequences

- Should not have exceptional properties.
- Should be hard to describe.

Different approaches - randomness as ...

1. Unpredictiblity (von Mises-Wald-Church).
2. High algorithmic complexity (Kolmogorov, Solomonoff, Chaitin, Schnorr).
3. **TYPICALITY** (Laplace-Ville-Martin-Löf).

[1] Andrei Khrennikov. *Probability and randomness: quantum versus classical*. World Scientific, 2016.

Intuitions on 2 Main (Equivalent) Approaches

'Typicality' randomness

The set of regular sequences has a small measure, while the set of random sequences has an essentially larger measure. A random sequence should satisfy all properties of probability one (tests of randomness).

'Algorithmic' randomness

A random sequence is incompressible: no prefix can be produced by a program much shorter than the prefix. An algorithmically random sequence should appear random to any algorithm.

⋆ **Both approaches are equivalent. Moreover such random sequences are random also in the sense of unpredictibility.**[2]

[2] Claus P Schnorr. *Zufälligkeit und Wahrscheinlichkeit: eine algorithmische Begründung der Wahrscheinlichkeitstheorie*. Vol. 218. Springer-Verlag, 2007.

Models of Set Theory

- ZFC axioms - logical sentences determining which structures are sets (hence also sequences)
- Mathematical structure in which ZFC axioms hold true is a model of set theory

2^{ω} is relative to a model, i.e. in two different ZFC models there might be different sets of $0-1$ sequences!

Cohen Forcing can be seen as a formal technique to extend given model of ZFC

$$M \to M[G] \text{ - model extension, } M \subset M[G]$$

$$2^{\omega}_{M} \to 2^{\omega}_{M[G]} \text{ - reals extension, } 2^{\omega}_{M} \subset 2^{\omega}_{M[G]}$$

$\star$ **First used to prove ZFC independence of Continuum Hypothesis**

Gödel's Constructible Universe - L

Recursive Construction

- $L_0 = \varnothing$,
- $L_{\alpha+1} = def(L_\alpha)$,
- $L_\lambda = \bigcup_{\alpha<\lambda} L_\alpha$, λ - limit ordinal,
- $L = \bigcup_{\alpha \in Ord} L_\alpha$.

⋆ **Model that is a universe of all constructible sets.**

⋆ **No "esoteric" infinite cardinalities - Generalized Continuum Hypothesis holds.**

[3] **Quantum Mechanics CAN be formalized in L!**

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[3] Paul A Benioff. "Models of Zermelo Frankel set theory as carriers for the mathematics of physics. I". . In: *Journal of mathematical Physics* 17.5 (1976), pp. 618–628.

Assume that quantum mechanics is random according to the typicality definition (for bit sequences taken from experiment):

Random sequences lie on the intersection of sets with probability measure 1 (avoiding the null sets).

[4]

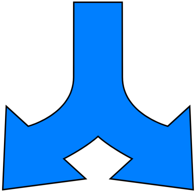
L <u>cannot</u> be the carrier for the mathematics of QM! (it is formalized but does not contain all its statistical predictions)

⋆ **Moreover, the same holds for all Cohen forcing extensions of L!**[5]

[4] Paul A Benioff. "Models of Zermelo Frankel set theory as carriers for the mathematics of physics. I". . In: *Journal of mathematical Physics* 17.5 (1976), pp. 618–628.

[5] Paul A Benioff. "Models of Zermelo Frankel set theory as carriers for the mathematics of physics. II". . In: *Journal of Mathematical Physics* 17.5 (1976), pp. 629–640.

QUANTUM MECHANICS



Random

- We need to go beyond L and its Cohen forcing extensions.
- Large cardinal ZFC axioms may shed some light on randomness of QM?
- Is there the weakest possible randomness definition needed for QM?

Not Random

- We cannot rule this out!
- Constructible QM inside L?
- Probably not since all predicitions of QM would be constructible - that could eventually contradict QM itself?

Connection with J. Król's talk from before an hour.

Assume that large scale spacetime structure of Universe is compatible with QM microscale structure.

⬇[6]

Bacause of the presence of random forcing in QM lattice of projections the large scale smooth structure has to be exotic and 4-dimensional.[7]

Is randomness essential also in cosmological theories?

[6] Jerzy Król and Torsten Asselmeyer-Maluga. "Quantum Mechanics, Formalization and the Cosmological Constant Problem". In: (revision).

[7] Jerzy Król et al. "From Quantum to Cosmological Regime. The Role of Forcing and Exotic 4-Smoothness". In: *Universe* 3.2 (2017), p. 31.

- Paul A Benioff. "Models of Zermelo Frankel set theory as carriers for the mathematics of physics. I". In: *Journal of mathematical Physics* 17.5 (1976), pp. 618–628.
- Paul A Benioff. "Models of Zermelo Frankel set theory as carriers for the mathematics of physics. II". In: *Journal of Mathematical Physics* 17.5 (1976), pp. 629–640.
- Andrei Khrennikov. *Probability and randomness: quantum versus classical*. World Scientific, 2016.
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- Claus P Schnorr. *Zufälligkeit und Wahrscheinlichkeit: eine algorithmische Begründung der Wahrscheinlichkeitstheorie*. Vol. 218. Springer-Verlag, 2007.

Thank you for your attention!