Theory aspects of precision physics at the LHC: M_W and $\sin^2 \vartheta^\ell_{eff}$

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precision physics \Longrightarrow LEP/SLC legacy at the Z pole



• given precisely measured parameters, e.g. α , G_{μ} , M_Z , m_f , $(\Delta \alpha_h)$, $\alpha_s(M_Z)$ and, after LHC run I, m_H , all other quantities can be computed with high precision through perturbative calculations • in particular, two **SM parameters**: M_W and $\sin^2 \vartheta_{eff}^{\ell}$

M_W calculated in the Standard Model

$$M_W^2 = \frac{M_Z^2}{2} \left\{ 1 + \left[1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2} \left(1 + \Delta r \right) \right]^{1/2} \right\}$$
$$M_W^2 = 80.357 \pm 0.009 \pm 0.003 \text{GeV}$$

• one loop $\mathcal{O}(\alpha)$ calculation

A. Sirlin, PRD22 (1980) 971

• two loop $\mathcal{O}(\alpha \alpha_s)$

A. Djouadi, C. Verzegnassi, PLB195 (1987) 265

• three loop $\mathcal{O}(\alpha \alpha_s^2)$

L. Avdeev et al., PLB336 (1994) 560;

K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394

• $\mathcal{O}(\alpha^2)$ for large top / Higgs mass

R. Barbieri et al., PLB288 (1992) 95; NPB409 (1993) 105

G. Degrassi, P. Gambino, A. Vicini, PLB383 (1996) 219

A. Freitas et al., PLB495 (2000) 338; NPB632 (2002) 189 M. Awramik, M. Czakon, PLB568 (2003) 48; PRL89 (2002) 241801 A. Onishchenko, O. Veretin, PLB551 (2003) 111; M. Awramik et al., PRD68 (2003) 053004

G. Degrassi, P. Gambino, P.P. Giardino, JHEP 1505 (2015) 154

• exact $\mathcal{O}(\alpha^2)$

$$\sin^2 \vartheta_{eff}^l = \frac{1}{4} \left(1 - \operatorname{Re} \frac{g_v}{g_a} \right), \qquad \operatorname{Zl}\bar{l} \operatorname{vertex} \sim \bar{l} \gamma^{\mu} (g_v - g_a \gamma_5) l Z_{\mu}$$

- measured at Z peak: 0.23153 ± 0.00016
- uncertainty in the Standard Model calculations: $\sim 0.00005 \oplus 0.00004$
 - at one loop $\mathcal{O}(\alpha)$

A. Sirlin, PRD22, (1980) 971, W.J. Marciano, A. Sirlin, PRD22 (1980) 2695

G. Degrassi, A. Sirlin, NPB352 (1991) 352, P. Gambino and A. Sirlin, PRD49 (1994) 1160

at higher orders:

 $\star \mathcal{O}(\alpha \alpha_s)$

A. Djouadi, C. Verzegnassi, PLB195 (1987) 265 B. Kiehl, NPB353 (1991) 567; B. Kniehl, A. Sirlin, NPB371 (1992) 141, PRD47 (1993) 883

A. Djouadi, P. Gambino, PRD49 (1994) 3499

 $\star \mathcal{O}(\alpha \alpha_s^2)$

L. Avdeev et al., PLB336 (1994) 560;

Chetyrkin, Kühn, Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394; NPB482 (1996) 213

 $\star \mathcal{O}(\alpha \alpha_s^3)$

***** exact $\mathcal{O}(\alpha^2)$

Y. Schröder, M. Steinhauser, PLB622 (2005) 124;

K.G. Chetyrkin et al., hep=ph/0605201; R. Boughezal, M. Czakon, hep-ph/0606232

★ $\mathcal{O}(\alpha^2)$ for large Higgs / top mass

G. Degrassi, P. Gambino, A. Sirlin, PLB394 (1997) 188

M. Awramik, M. Czakon, A. Freitas, JHEP0611 (2006) 048

W. Hollik, U. Meier, S. Uccirati, NPB731 (2005) 213; I. Dubovik et al., arXiv:1906.08815

At hadron colliders



LHC luminosity



with L ~ 100 fb⁻¹ ~ 10¹⁰W!!
statistics is not an issue

• at hadronic colliders we have the opportunity to perform direct determination of both M_W and $\sin^2 \vartheta_{eff}^\ell$ through Drell-Yan processes, to compare with the calculated values for a stress test of the SM internal consistency





• experimentally clean signals

- extremely important that the direct determinations of M_W and $\sin^2\vartheta^\ell_{eff}$ be as much as possible independent of each other, also from a theoretical point of view
- remind that M_W and $\sin^2 \vartheta_{eff}^{\ell}$ are tightly intertwined with other electroweak parameters in the gauge sector (quite different w.r.t. m_{top} and m_H)

on the experimental side



ATLAS 2017

ATLAS 2016

• largest th. systematics: PDF's, QCD scale uncertainties

• main difficulty: protons are composite objects



$$\begin{split} \sigma^{\text{theory}} &\equiv \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a,H_1}(x_1,\mu_F^2,\mu_R^2) f_{b,H_2}(x_2,\mu_F^2,\mu_R^2) \times \\ &\times \int_{\Phi} d\hat{\sigma}_{a,b}(x_1,x_2,Q^2/\mu_F^2,Q^2/\mu_R^2) + \ \mathcal{O}\left(\frac{\Lambda_{QCD}^n}{Q^n}\right) \end{split}$$

Main relevant observables for M_W



• M_{\perp}

- theoretically stable vs radiative corrections
- determination of neutrino momentum exp. challenging
- p_{\perp}^{ℓ}
 - experimentally clean
 - theoretically very sensitive to ISR

sensitivity to M_W



Farry, Lupton, Pili, Vesterinen, arXiv:1902.04323

by A. Vicini

• control of shapes below 1% scale for $\Delta M_W \sim 10 - 20$ MeV

Main relevant observable for $\sin^2 \vartheta^{\ell}_{eff}$

• the matrix element for the production and decay of a spin one vector boson can be parameterized by an expansion on second order polinomials with nine coefficients (corresponding to nine polarization terms)

$$\begin{aligned} \frac{d\sigma}{dq_T^2 \, dy \, d\cos\vartheta \, d\phi} &= \frac{3}{16\pi} \frac{d\sigma^{\text{unpol.}}}{dq_T^2 \, dy \, d\cos\vartheta \, d\phi} \Big\{ 1 + \cos^2\vartheta \frac{1}{2} A_0 (1 - 3\cos^2\vartheta) + A_1 \sin 2\vartheta \cos\phi \\ &+ \frac{1}{2} A_2 \sin^2\vartheta \cos 2\phi + A_3 \sin\vartheta \cos\phi + A_4 \cos\vartheta + A_5 \sin^2\vartheta \sin 2\phi \\ &+ A_6 \sin 2\vartheta \sin\phi + A_7 \sin\vartheta \sin\phi \Big\} \end{aligned}$$

- ϑ and ϕ refer to a $\ell\bar\ell$ rest frame
- orientation given by the Collin-Soper frame



 integrating over the azimuthal angle the general parameterization of production and decay of a spin-one vector in terms of angular coefficients,



ATLAS 2017

•
$$\Delta A_{FB} \sim 10^{-4} \Longrightarrow \Delta \sin^2 \vartheta_{eff}^{\ell} \sim 2 \cdot 10^{-4}$$

\bullet crucial common ingredients to ${\bf W}$ and ${\bf Z}$

- ▶ p_{\perp}^{Z} , p_{\perp}^{W} (and their ratio), mainly sensitive to ISR QCD and different parton luminositites
- reliable PDF's determinations

lepton pair (Z/W) p_{\perp} : two regimes

large p⊥ (≥ 20 GeV), where pert. th. is reliable
 state of the art is NNLO QCD

• small p_{\perp} ($\lesssim 20$ GeV): ~90% of the cross section

- resummation of $\log\left(\frac{M_V}{q_\perp}\right)$ is needed
- sensitivity to the non-perturbative model of the MC Evt Gen

Large p_{\perp} region





- A. Gehrmann-De Ridder et al., arXiv:1605.04295
- A. Huss, $p_{T}^{Z} \ \mathrm{and} \ p_{T}^{W}$ theory meeting, CERN 2018
- R. Boughezal et al., 1512.01291, 1602.05612, 1602.08140

yellow dash: ew corrections

A. Gehrmann-De Ridder et al., arXiv:1712.07543 A. Huss at p_T^Z and p_T^W theory meeting, CERN 2018 R. Boughezal et al., arXiv:1602.06965

A. Denner et al., arXiv:1103.0914

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Small p_{\perp} region: resummation techniques

- recent progress by different groups on resummation
 - q_{\perp} resummation in impact parameter space
 - DYRES, DYTURBO
 S. Catani et al., arXiv:1507.06937; G. Ferrera, S. Camarda
 - ★ ReSolve T. Cridge and F. Coradeschi
 - ★ Resbos2, CSS formalism J. Isaacson
 - SCET based resummation
 - * GENEVA, SCETIb S. Alioli et al., arXiv:1211.7049, arXiv:1508.01475; F. Tackmann et al
 - CuTe
 T. Becher et al., arXiv:1109.6027, arXiv:1212.2621

T. Becher, Hager, arXiv:1904.08325

Höche, Li, Prestel, arXiv:1405.3607

- resummation in direct space (RadISH) W. Bizon et al., arXiv:1705.09127; 1604.02191
- resummation throuh TMD factorisation (NangaParbat) V. Bertone and G. Bozzi
- recent progress in Monte Carlo generators
 - inclusion of NLO splitting kernels (DIRE)

S. Höche, F. Krauss, S. Prestel, 1705.00982

S. Höche, S. Prestel, arXiv:1705.00742

- DY at NNLOPS accuracy with different methods
 - * MiNLO with POWHEG Karlberg, Re, Zanderighi, arXiv:1407.2940
 - UNNLOPS with SHERPA
 - ★ GENEVA Alioli, Bauer, Berggren, Guns, Tackmann, Walsh, '15-'16
- first investigations of possible flavour dependence of non-perturb

partonic intrinsic k_{\perp} A. Bacchetta et al., arXiv:1807.02101; G. Bozzi, A. Signori, arXiv:1901.01162

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 M_W and $\sin^2 \vartheta_{off}^\ell$ at the l

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- different approaches, even if with the same nominal accuracy, can differ
 - subleading terms
 - different matching effects in the transition region
 - matching schemes (additive vs multiplicative)
 - non-perturbative corrections/MC tune
 - order of PDF evolution
 - thresholds and treatment of heavy quarks
- within the LHC EWWG, important benchmarking activity among different codes
 - ► this is the first time such an exercise is beeing performed, after the studies of S. Alioli, arXiv:1606.02330

latest results from RadISH+NNLOjet: N³LL+NNLO

W. Bizon et al., arXiv:1905.05171







ATLAS coll., arXiv:1512.02192

$$p_{\perp}^{W} = (p_{\perp}^{Z})_{\text{measured}} \left(\frac{p_{\perp}^{W}}{p_{\perp}^{Z}}\right)_{\text{th}}$$

• How to treat th uncertainties in numerator and denominator?



W. Bizon et al., arXiv:1905.05171

• stability of the best predictions vs (un)correlation of scales • remaining $\mathcal{O}(\%)$ th. uncertainty

Higher-order corrections (for M_W fit)

$$d\sigma = d\sigma_0 + d\sigma_{\alpha_s} + d\sigma_\alpha + d\sigma_{\alpha_s^2} + d\sigma_{\alpha\alpha_s} + d\sigma_{\alpha^2} + \dots$$

• multi-photon emission from the final state $\rightarrow \delta M_W \simeq 10$ MeV for $\mu \nu_{\mu}$ final state

Carloni Calame et al., PRD 69 (2004) 037301, JHEP 0710 (2007)

mixed QCD-EWK corrections

Dittmaier, Huss, Schwinn, NPB 885 (2014) 318, NPB 904 (2016) 216

NNLO EWK effects

C.M. Carloni Calame et al., Phys.Rev. D96 (2017) 093005

- EWK input scheme
- lepton pair emission

QCD-EWK interference

- \bullet the $\mathcal{O}(\alpha\alpha_s)$ calculation involves as building blocks
 - ▶ NNLO virtual corrections at $\mathcal{O}(\alpha \alpha_s)$ (not yet available)
 - ★ necessary two-loop master integrals (with m = 0 external particles and $M_W = M_Z$, or with one massive internal line)

R. Bonciani et al., arXiv:1604.08581; A. von Manteuffel and R.M. Schabinger, arXiv:1701.06583

- NLO EW corrections to $l\bar{l}^{(')}$ + jet
- NLO QCD corrections to $l\bar{l}^{(')} + \gamma$
- double real contributions $l\bar{l}^{(')} + \gamma + jet$
- ▶ PDF's with NNLO accuracy at $\mathcal{O}(\alpha \alpha_s)$ (not yet available)
- very recent progress on NNLO mixed QCD-QED ISR corrections

Cieri, Ferrera, Sborlini arXiv:1805.11948; De Florian, Der, Fabre, arXiv:1805.12214

what is available:

- ► fixed order dominant $O(\alpha_s \alpha)$ corrections to DY in pole approximation Dittmaier, Huss, Schwinn, NPB 885 (2014) 318, NPB 904 (2016) 216
- Monte Carlo estimates through NLO QCD
 NLO EW (with higher orders)
 L. Barzè et al., JHEP 1204 (2012) 037, Eur. Phys. J. C73 (2013) 2474

C.M. Carloni Calame et al., arXiv:1612.02841

fixed order $\mathcal{O}(\alpha_s \alpha)$ in pole approximation

- two main classes of contributions:
 - factorizable
 - non-factorizable



(a) Factorizable initial-initial corrections



(c) Factorizable final-final corrections



(b) Factorizable initial-final corrections



(d) Non-factorizable corrections S. Dittmaier, A. Huss and C. Schwinn, arXiv:1601.02027

a) small contribution (~ 0.1% level) (for on-shell Z) De Florian, Der, Fabre, arXiv:1805.12214 ($\mathcal{O}(\alpha)$ corrections in PA \Longrightarrow M_{\perp} and $M(l^+l^-)$ insensitive to QED ISR in addition M_{\perp} and $M(l^+l^-)$ mildly affected by NLO QCD corrections)

- b) this gives the bulk of the contribution
- c) no real contributions \implies no impact on shape of M_{\perp} and $M(l^+l^-)$
- d) numerical impact below 0.1%

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 $\mathcal{O}(\alpha_s \alpha)$ corrections through Monte Carlo

- (e.g.) the POWHEG-BOX includes NLO QCD & EW corrections interfaced to QCD/QED shower, i.e. NLOPS EW ⊕ QCD accuracy
 - POWHEG_W_ew_BMNNP, CC DY
 - POWHEG_W_ew_BW, CC DY
 - POWHEG_Z_ew_BMNNPV, NC DY
 - independent implementation

Mück and Oymanns, JHEP 1705 (2017) 090

Bernaciak and Wackeroth, PRD 85 (2012) 093003

Barzè et al. JHEP 1204 (2012) 037

Barzè et al, EPJC 73 (2013) 6, 2474

 correctly taken into account the NLO contribution with one additional radiation in the soft/collinear limit



comparison POWHEG-BOX-V2 vs NNLO in pole approx

C.M. Carloni Calame et al., Phys.Rev. D96 (2017) 093005

$$d\sigma_{\text{POWHEG}} = d\sigma_0 \left[1 + \delta_{\alpha_s} + \delta_\alpha + \sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right],$$

 $\Delta M_W^{\alpha_s \alpha}(\mu^+ \nu_\mu) = -16.0 \pm 3.0 \text{ MeV} \qquad \text{vs} \qquad \delta_{\text{NNLO}} = -14 \text{ MeV}$

Dittmaier, Huss, Schwinn, NPB 885 (2014) 318, NPB 904 (2016) 216

 \bullet summary of residual effects present in (QCD \oplus EW)_{\rm NLOPS} but missing in QCD_{\rm NLOPS} \otimes QEDPS

		$\Delta M_W ({ m MeV})$		
	QED FSR model	M_T	p_T^ℓ	
Tevatron	Pythia Photos	$^{+5}_{-2}{}^{\pm}{}^{2}_{\pm}{}^{1}_{1}$	$+17 \pm 5 \\ -8 \pm 5$	
LHC	Pythia Photos	$+6.2 \pm 0.8 \\ -0.6 \pm 0.8$	$+29 \pm 4$ -2 ± 4	

 differences in shifts induced by PYTHIA QEDPS and PHOTOS disappear when used on top of QCD⊕EW NLO

Lepton pair corrections: virtual and real contributions

• emission of a photon converting to a lepton pair $\sim \mathcal{O}(\alpha^2 L^2) \sim$ two-photon contribution

$pp \rightarrow W^+$, $\sqrt{s} = 14 \text{ TeV}$ Templates accuracy: LO		M_W shifts (MeV) $W^+ \rightarrow \mu^+ \nu$ $W^+ \rightarrow e^+ \nu$			
	Pseudo-data accuracy	M_T	p_T^ℓ	M_T	p_T^ℓ
1	HORACE only FSR-LL at $\mathcal{O}(lpha)$	-94±1	-104±1	-204±1	-230±2
2	Horace FSR-LL	-89±1	-97±1	-179±1	-195±1
3	HORACE NLO-EW with QED shower	-90 ± 1	-94±1	-177 ± 1	-190 ± 2
4	HORACE FSR-LL + Pairs	-94±1	-102 ± 1	-182±2	-199 ± 1
5	PHOTOS FSR-LL	-92 ± 1	-100 ± 2	-182 ± 1	-199 ± 2

C.M. Carloni Calame et al., arXiv:1612.02841

 $\Delta M_W(\mu^+
u) \sim 5 \pm 1$ MeV (from M_\perp) and $\sim 3 \pm 2$ MeV (from p_\perp^ℓ)

NNLO uncertainty: input parameter scheme

- pert. EW calculations require a coherent set of input param. in the gauge sector, e.g.
 - $\alpha(0)$, M_W and M_Z
 - G_{μ} , M_W and M_Z to be preferred in the CC DY
 - we can define

$$\begin{aligned} \alpha_{\mu}^{tree} &\equiv \frac{\sqrt{2}}{\pi} G_{\mu} M_{W}^{2} \sin^{2} \vartheta \\ \alpha_{\mu}^{1l} &\equiv \frac{\sqrt{2}}{\pi} G_{\mu} M_{W}^{2} \sin^{2} \vartheta \left(1 - \Delta r\right) \end{aligned}$$

 \blacktriangleright three possible different expression for the cross section, starting to differ at $\mathcal{O}(\alpha^2)$

$$\begin{aligned} \alpha_0 &: \qquad \sigma = \alpha_0^2 \sigma_0 + \alpha_0^3 (\sigma_{SV} + \sigma_H) \,, \\ G_\mu I &: \qquad \sigma = (\alpha_\mu^{tree})^2 \sigma_0 + (\alpha_\mu^{tree})^2 \alpha_0 (\sigma_{SV} + \sigma_H) - 2\Delta r (\alpha_\mu^{tree})^2 \sigma_0 \,, \\ G_\mu II &: \qquad \sigma = (\alpha_\mu^{1l})^2 \sigma_0 + (\alpha_\mu^{1l})^2 \alpha_0 (\sigma_{SV} + \sigma_H) \end{aligned}$$

• potentially effects on M_W because of the different sharing among different photon multiplicities

	$p\bar{p} \rightarrow W^+$, $\sqrt{s} = 1.96$ Templates accuracy: LC	$ \begin{array}{c c} M_W \text{ shifts (MeV)} \\ W^+ \to \mu^+ \nu \end{array} $		
	Pseudodata accuracy	Input scheme	M_T	p_T^ℓ
1 2 3 4 5 6	Horace NLO-EW Horace NLO-EW+QED-PS	$\begin{array}{c} \alpha_0 \\ G_\mu - I \\ G_\mu - II \\ \alpha_0 \\ G_\mu - I \\ G_\mu - II \end{array}$	-101 ± 1 -112 ± 1 -101 ± 1 -70 ± 1 -72 ± 2 -72 ± 1	-117 ± 2 -130 ± 1 -117 ± 1 -81 ± 1 -83 ± 1 -82 ± 2

 differences present at NLO, after matching with higher orders, become much smaller

 $\Delta M_W \sim 2 \text{ MeV} \pm 1 - 2 \text{ MeV}$

Considerations on input parameter schemes

• independent quantities in the SM:

- 3 in the electroweak gauge sector (to be specified)
- $\alpha_s(Q^2)$, for a given Q^2 , e.g. $Q^2 = M_Z^2$
- lepton masses, m_t , m_H
- ▶ light quark masses (including c and b) crucial for the running of $\alpha \implies$ circumvented by using low-energy data and dispersion relations
- possible triplets of input (Lagrangian) parameters (in the gauge sector)
 - (e, M_W, M_Z) , (g, M_W, M_Z) , $(g, \sin \vartheta, M_Z)$, ...
- **renormalization scheme**: input parameters need to be defined with reference to three data points
- ullet \Longrightarrow everything else is calculated in terms of input parameters
- conceptually, independently of any simplified assumption (e.g. factorization properties at the Z peak), every parameter can be directly determined through a template fit procedure to any observable, provided our theoretical prediction of the observable allows us to freely move the measured parameter without spoiling the accuracy of the calculation (e.g. M_W with G_{μ} , M_W , M_Z)

requirement for an input/renormal. scheme

- minimize the parametric uncertainty of the reference observables defining the scheme
 - e.g. the LEP (G_{μ}, α, M_Z) scheme
- minimize the effects of higher order corrections, in order to have stable predictions in perturb. theory
 - ▶ e.g. the (G_{μ}, M_W, M_Z) scheme used for DY at Tevatron/LHC
- minimize the parametric uncertainty due to imperfect knowledge of other Lagrangian parameters (e.g. m_t , $\Delta \alpha_h$)
- for a direct determination of $\sin^2 \vartheta^\ell_{eff}$ at the LHC, based on simulations includig higher order EW corrections, investigation of a scheme with $\sin^2 \vartheta^\ell_{eff}$ as an input parameter

M. Chiesa, F.P., A. Vicini, arXiv:1906.11569

$lpha/G_{\mu}$, M_Z , $\sin^2 artheta_{eff}^\ell$ as reference data

M. Chiesa, F.P., A. Vicini, arXiv:1906.11569

- our Lagrangian parameters: e, $\sin^2 \vartheta$, M_Z
- e and M_Z renormalized parameters fixed as usual through α/G_μ and M_Z value measured at LEP
- $\sin^2 \vartheta$ renormalized parameter fixed at $\sin^2 \vartheta^{\ell}_{eff}$
 - ► this can be achieved by requiring that the ratio gⁱ_V does not get radiative corrections
 - \blacktriangleright procedure independent of QED corrections because g_L and g_R receive the same corrections
 - Δr has to be changed accordingly w.r.t. to the usual expression
- the scheme can be used for making predictions
 - "drawback": parametric uncertainty inherited by the LEP measurement of $\sin^2 \vartheta^\ell_{eff}$
- most important: the scheme can be used in a fitting procedure to measure $\sin^2 \vartheta^\ell_{eff}$ with a template fit

convergence of pert. th. on $d\sigma/dM_{\ell\ell}$



M. Chiesa, F.P., A. Vicini, arXiv:1906.11569



M. Chiesa, F.P., A. Vicini, arXiv:1906.11569

parametric dependence on $m_{ m top}$



M. Chiesa, F.P., A. Vicini, arXiv:1906.11569

parametric dependence on $m_{\rm top}$



Summary

- aiming at a precision $\Delta M_W \leq 10$ MeV as well as $\Delta \sin^2 \vartheta_{eff}^\ell \leq 30 \cdot 10^{-5}$, the details of simulating radiation in MC's become relevant
- QCD for p_T^Z , p_T^W : impressive recent progress in resummation matched to full fixed order results calculation
 - benchmarking activity started within LHC EWWG at Cern
- *M*_W
 - mixed QCD× EW: comparison with fixed order in pole approximation nicely compatible, at the MeV scale
 - the pragmatic recipe QCD NLOPS QEDLL (with PHOTOS) agrees at the MeV level with the factorized prescription QCD NLOPS EWNLOPS
 - * the above prescription inherits an uncertainty of \sim 5 MeV if QED FSR is simulated with PYTHIA (M_{\perp}) and of \sim 29 MeV (p_{\perp}^{ℓ})
 - \star the differences between PYTHIA and PHOTOS disappear if used on top of EW NLO precision
 - leptonic pair corrections at the level of 5 MeV
 - $\blacktriangleright~\mathcal{O}(\alpha^2)$ uncertainties through input param schemes of $\mathcal{O}(1-2)~{\rm MeV}$
- $\sin^2 \vartheta^\ell_{eff}$
 - \blacktriangleright proposal of a new input param. scheme with $\sin^2 \vartheta^\ell_{eff}$ as input
 - (not discussed) ongoing studies and estimates of QED radiative effects which break factorization
 - \blacktriangleright (not discussed) ongoing studies of the effects of γ induced processes and their dependence on PDF set