Production of Purely Gravitational Vector Dark Matter

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Matter To The Deepest, September 2019

Outline

Outline

Motivation

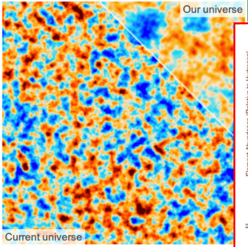
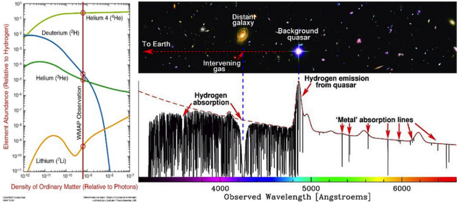


Image credit: CMB pattern for a universe with normal matter only compared do our own, which inc dark matter and dark energy. Generated by Amanda Yoho on the Planck CMB simulator at http://strudel.org.uk/planck/#.



Images credit: NASA / WMAP science team, Gary Steigman (L), of Big Bang Nucleosynthesis and the baryon-to-photon ratio; Michael Murphy, Swinburne U.; HUDF: NASA, ESA, S. Beckwith (STScI) et al. (R), of the Lyman-alpha forest from intervening intergalactic clumps of non-luminous matter.

labeled "disk", but observations (black squares) showed constant, rather than decreasing velocity. Adding a contribution from a dark matter halo (center line) makes the theory match predictions.

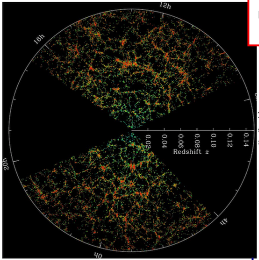
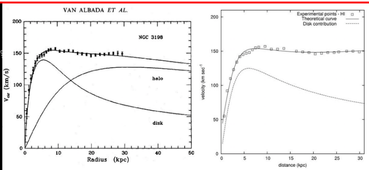


Image credit: "Sloan Digital Sky Survey 1.25 Declination Slice 2013 Data" by M. Blanton and the Sloan Digital Sky Survey.



Images credit: Van Albada et al. (L), A. Carati, via arXiv:1111.5793 (R). Observed velocities versus distance from the center of galaxy NGC 3198. The theoretical prediction before observations followed the trend labeled "disk", but observations (black squares) showed constant, rather than decreasing velocity. Adding a contribution from a dark matter halo (center line) makes the theory match predictions.

Spin-1 Dark Matter

The Einstein-Hilbert action with the matter field reads

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} + \mathcal{L}_{SM} + \mathcal{L}_{DM}\right\},$$

where $\kappa = M_{Pl}^{-1}$ is the effective gravitational constant.
After expanding in metric (the graviton field) fluctuations,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},$$

one can rewrite the above action as

$$S_{eff} = \int d^4x\mathcal{L}_{eff} = \int d^4x[\mathcal{L}^{(4)} + \mathcal{L}_{int}^{(5)} + \mathcal{L}_{int}^{(6)} + \mathcal{O}(\kappa^3, h^2)],$$

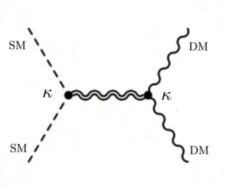
where $\mathcal{L}^{(4)} = \mathcal{L}_{SM}^{(4)} + \mathcal{L}_{DM}^{(4)}$ is dimension-4 renormalizable Lagrangian, whereas $\mathcal{L}_{int}^{(5)}$, $\mathcal{L}_{int}^{(6)}$ contain operators of dim-5 and 6 suppressed by M_{Pl}^{-1} and M_{Pl}^{-2}, respectively.

Spin-1 Dark Matter

In minimal scenario, communication between the SM and DM sectors is only mediated indirectly through gravity, which couples to their energy-momentum tensors:

$$\mathcal{L}_{int}^{(5)} = \frac{\kappa}{2} h^{\mu\nu} (T_{\mu\nu}^{DM} + T_{\mu\nu}^{SM}).$$

The annihilation of SM particles into DM that comes from this Lagrangian is through s-channel graviton exchange



Note, there is no dimension-5 direct SM and DM interaction.

Spin-1 Dark Matter

The lowest order direct interaction between the SM and DM appears at dim-6 operator.

The interaction of a graviton with the SM and DM energy-momentum tensors leads to an interaction between those two sectors proportional to κ^2 which is the same order as the interaction via the effective operator:

$$\mathcal{L}_{int}^{(6)} = \frac{\kappa^2}{2} \kappa_X m_X^2 |\mathcal{H}|^2 X_\mu X^\mu,$$

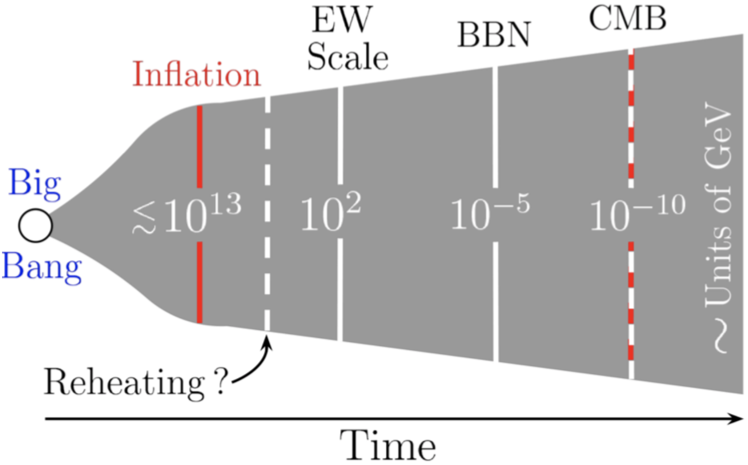
which arises through the following coupling

$$\frac{1}{\Lambda^2} D_\mu \Phi (D^\mu \Phi)^* \mathcal{H}^\dagger \mathcal{H} \supset \frac{1}{\Lambda^2} g^2 X_\mu X^\mu \Phi \Phi^* \mathcal{H}^\dagger \mathcal{H},$$

where

$$D_\mu \Phi \equiv \partial_\mu \Phi - i g X_\mu \Phi, \quad \frac{\kappa^2}{2} \kappa_X \equiv \Lambda^{-2}$$

Outline


Inflation
EW
Scale
BBN
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Big
Bang
≳10^13
10^2
10^−5
10^−10
∼ Units of GeV
Reheating ?
Time

Boltzmann equations

Let us consider a universe composed of:

- unstable, massive particles ϕ,
- stable, massive DM species X,
- ultra-relativistic SM particles $\mathcal{R}$.

The dynamics of such a system is governed by the following Boltzmann equations:

$$\dot{\rho}_\phi + 3(1+w)H\rho_\phi = -(\Gamma_R + \Gamma_X)\rho_\phi, \tag{1}$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_R\rho_\phi + \langle\sigma|v|\rangle 2\langle E_X\rangle[n_X^2 - (n_X^{eq})^2], \tag{2}$$

$$\dot{n}_X + 3Hn_X = \Gamma_X\frac{\rho_\phi}{m_\phi} - \langle\sigma|v|\rangle[n_X^2 - (n_X^{eq})^2]. \tag{3}$$

where Γ_X and Γ_R denote the inflaton decay widths into DM and SM, respectively, and $\langle\sigma v\rangle$ is a thermally-averaged cross-section.

T(a) and H(a) relations

To solve the above system of coupled equations we assume that:

- during reheating, the epoch between H_I^{-1} and $H_{RH}^{-1} = \Gamma_R^{-1}$, the total energy ρ density is dominated by the ϕ field,
- after reheating, during the RD period, the main contribution to the total energy density comes from SM radiation.

It has been shown [1] that the evolution of the Hubble rate:

$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_R + \rho_X),$$

and the temperature T with time (scale factor) follows different laws during and after reheating.

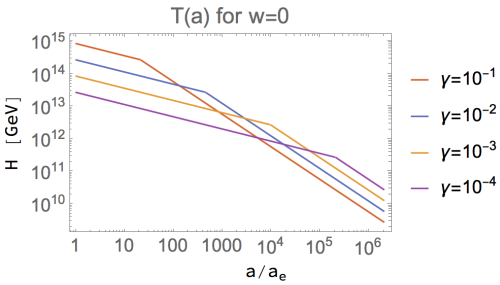
T(a) and H(a) relations

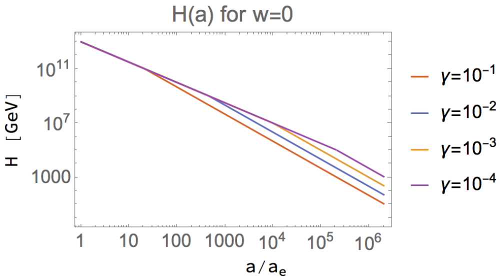
The temperature scales as

$$T(a) \simeq \begin{cases} T_{max}\, a^{-\frac{3}{8}(1+w)}, & \text{for} \quad a_e < a < a_{RH} \\ T_{RH}\, \frac{a_{RH}}{a}, & \text{for} \quad a \geq a_{RH}, \end{cases}$$

whereas the Hubble rate evolves as

$$H(a) = \begin{cases} H_I a^{-\frac{3}{2}(1+w)}, & \text{for} \quad a < a_{RH} \\ H_{RH} \left(\frac{a_{RH}}{a}\right)^2, & \text{for} \quad a > a_{RH} \end{cases}$$





Boltzmann equation for DM

To simplify the system of Boltzmann equations it is convenient to introduce dimensionless variables:

$$\tilde{\Phi} = \rho_\phi \frac{a^{3(1+w)}}{T_{RH}^4}, \quad R = \rho_R \frac{a^4}{T_{RH}^4}, \quad X = n_X \frac{a^3}{T_{RH}^3},$$

which allows us to rewrite Eq.(3) as

$$\frac{dX}{da} = \frac{\Gamma_X}{H} \frac{T_{RH}}{m_\phi} \tilde{\Phi} a^{-1-3w} - \frac{\langle \sigma |v| \rangle}{H T_{RH}^3} a^2 \left(n_X^2 - (n_X^{eq})^2 \right). \tag{4}$$

Inflaton decay into DM

Now let us consider the coupling

$$\mathcal{L}_{int} = -\frac{\kappa_V}{2}\kappa m_X^2 g^{\mu\nu}\phi X_\mu X_\nu,$$

which originate from the term

$$\mathcal{L}_{int} = -\frac{1}{\Lambda} g^{\mu\nu}(D_\mu \Phi)(D_\nu \Phi)^* \phi,$$

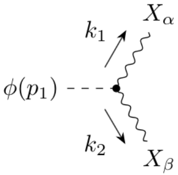
where

$$D_\mu \Phi \equiv (\partial_\mu - igX_\mu)\Phi,$$

and

$$\Lambda^{-1} \equiv \frac{\kappa_V}{2}\kappa.$$

The corresponding vertex:



$$i\gamma^{\phi XX}_{\alpha\beta} = -i\kappa_V \frac{m_X^2}{m_{pl}}\eta_{\alpha\beta}$$

and the decay width:

$$\Gamma_X = \frac{\kappa_V^2 \kappa^2 \sqrt{m_\phi^4 - 4m_X^2 m_\phi^2}}{128\pi m_\phi^3} \times \times \left(12m_X^4 - 4m_X^2 m_\phi^2 + m_\phi^4\right)$$

Inflaton decay into DM

Assuming that initially $X(a_e) = 0$ we get:

$$X(a_{RH}) = \int_{a_e}^{a_{RH}} \frac{\Gamma_X}{H(a)} \frac{T_{RH}}{m_\phi} \tilde{\Phi} a^{-1-3w} da. \tag{5}$$

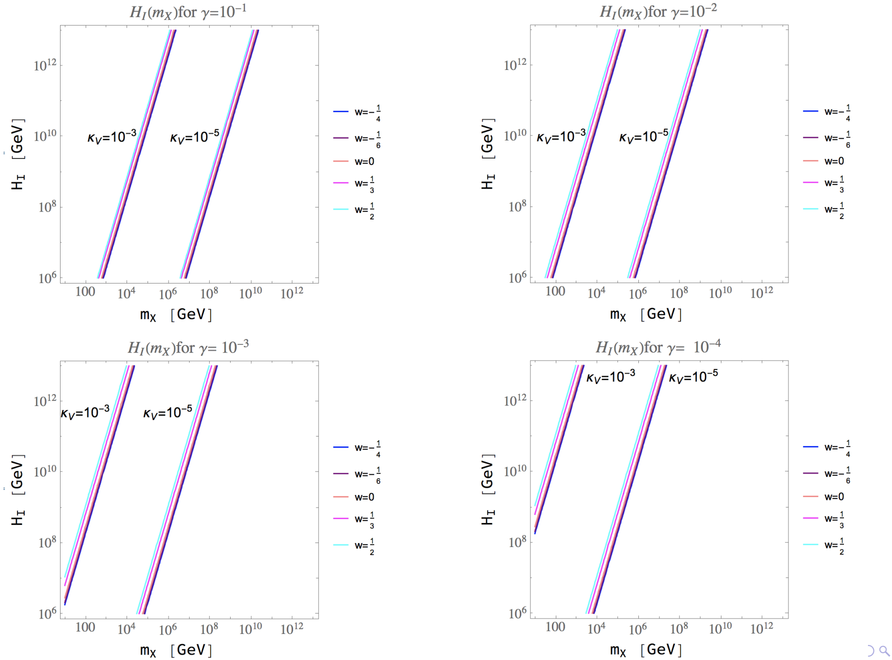
The present day relic abundance of DM particles is

$$\Omega_X h^2 = \frac{\rho_X}{\rho_c} h^2 = \frac{m_X n_X(T_0)}{\rho_c} h^2, \tag{6}$$

where

$$n_X(T_0) = X(T_0) T_{RH}^3 \gamma^{\frac{4}{1+w}} \frac{s_0}{s_{RH}},$$

with $X(T_0) \simeq X(a_{RH})$ and $\gamma^2 \equiv \frac{H_{RH}}{H_I}$.



H_I(m_X) for γ=10^-1
H_I(m_X) for γ=10^-2
H_I(m_X) for γ= 10^-3
H_I(m_X) for γ= 10^-4
κ_V=10^-3
κ_V=10^-5
H_I [GeV]
m_X [GeV]
10^6
10^8
10^10
10^12
100
10^4
w=-1/4
w=-1/6
w=0
w=1/3
w=1/2

Freeze-in from SM particles annihilation

Let us now focus on the second term in (4), which takes the form:

$$\frac{dX}{da} = \frac{\langle\sigma|v|\rangle}{T_{RH}^3 H} a^2 (n_X^{eq})^2, \tag{7}$$

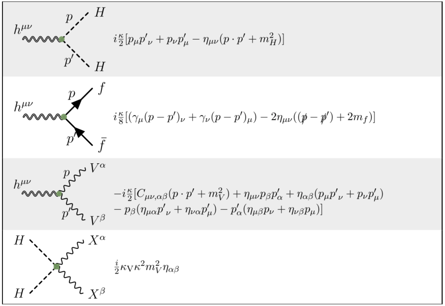
where we have assumed that $n_X \ll n_X^{eq}$.
The Gondolo-Gelmini formula for the thermally- averaged cross sections is given by

$$\langle\sigma|v|\rangle = \frac{1}{8m^4 T K_2(m/T)^2} \int_{4m^2}^{\infty} ds \sqrt{s}(s-4m^2)\sigma(s) K_1\left(\frac{\sqrt{s}}{T}\right),$$

where

$$\sigma(s) = \frac{-1}{16\pi s(s-4m^2)} \int_{t_+}^{t_-} dt |\mathcal{M}|^2,$$

with $t_\pm = -(\sqrt{s/4 - m^2} \mp \sqrt{s/4})^2$.



Interaction vertices for the graviton SM(DM) interaction. Here H, f, V represents scalar, fermion, vector degrees of freedom, respectively.

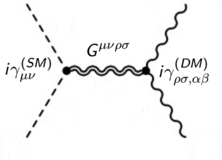
The annihilation of SM particles into DM through s-channel graviton exchange results in the following amplitude

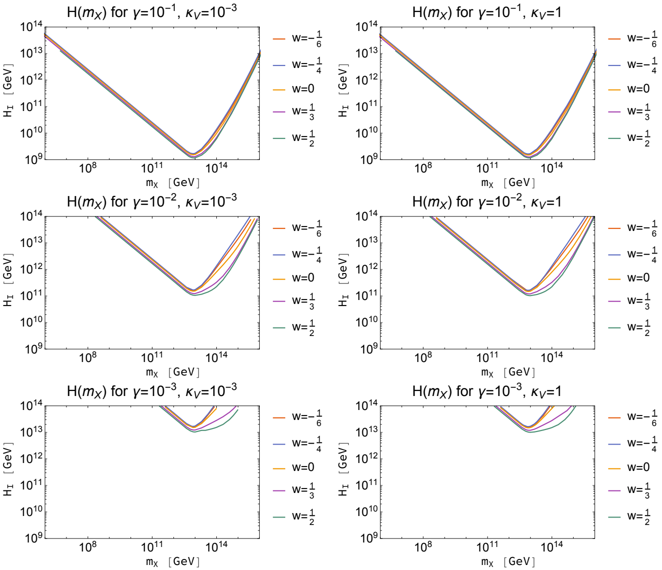
$$\mathcal{M} = \sum_{\alpha,\beta} i\gamma^{(SM)}_{\mu\nu}\, G^{\mu\nu\rho\sigma}\, i\gamma^{(DM)}_{\rho\sigma,\alpha\beta}$$

where

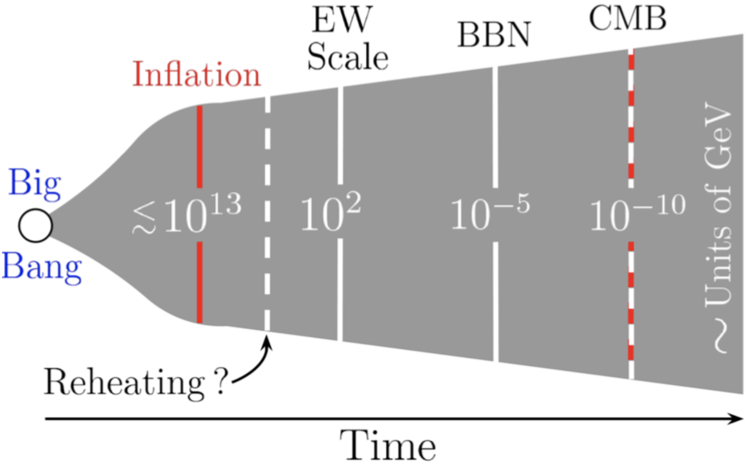
$$G^{\mu\nu\rho\sigma} = \frac{1}{2}\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}}{q^2 + i\epsilon},$$

denotes the graviton propagator in the de Donder gauge.




H(m_X) for γ=10^{-1}, κ_V=10^{-3}
H(m_X) for γ=10^{-1}, κ_V=1
H(m_X) for γ=10^{-2}, κ_V=10^{-3}
H(m_X) for γ=10^{-2}, κ_V=1
H(m_X) for γ=10^{-3}, κ_V=10^{-3}
H(m_X) for γ=10^{-3}, κ_V=1
H_I [GeV]
m_X [GeV]
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^8
10^{11}
10^{14}
w=$-\frac{1}{6}$
w=$-\frac{1}{4}$
w=0
w=$\frac{1}{3}$
w=$\frac{1}{2}$

Outline



Inflation
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$\gtrsim 10^{13}$
10^{2}
10^{-5}
10^{-10}
$\sim$ Units of GeV
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Gravitational production during inflation

Let us now consider the action for the Abelian massive DM spin-1 field:

$$S_{DM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} - \frac{1}{2}m_X^2 g^{\mu\nu}X_\mu X_\nu\right),$$

where $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ and the background metric is in the FLRW form, i.e. $ds^2 = a^2(d\tau^2 - d\mathbf{x}^2)$.
Extremizing the above action with respect to the X_μ field one obtains,

$$\mathcal{X}_\pm'' + \left(k^2 + a^2 m_X^2\right)\mathcal{X}_\pm = 0, \quad (8)$$

$$\tilde{\mathcal{X}}_L'' + \underbrace{\left(k^2 + m_X^2 a^2 - \frac{k^2}{k^2 + m_X^2 a^2}\frac{a''}{a} + 3\frac{k^2 m_X^2 a'^2}{(k^2 + m_X^2 a^2)^2}\right)}_{\omega_L^2}\tilde{\mathcal{X}}_L = 0, \quad (9)$$

where we have used the Fourier decomposition,

$$X_\mu(t,\mathbf{x}) = \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}}\epsilon_\mu^\lambda(\mathbf{k})\mathcal{X}_\mu^\lambda(t,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}.$$

As usual, we quantize our theory imposing equal-time commutation relations,

$$[\hat{X}_L(\tau, \mathbf{x}), \hat{\Pi}_L(\tau, \mathbf{y})] = i\delta^{(3)}(\mathbf{x} - \mathbf{y}),$$

where,

$$\hat{X}_L = \int \frac{d^3k}{(2\pi)^{3/2}} \left\{ \epsilon_L(\vec{k}) \hat{a}^-_{\mathbf{k}} \mathcal{X}_L(\tau, \vec{k}) e^{ik\cdot x} + \epsilon^*_L(\vec{k}) \hat{a}^\dagger_{\mathbf{k}} \mathcal{X}^*_L(\tau, \vec{k}) e^{-ik\cdot x} \right\}, \quad (10)$$

and $\hat{\Pi}_L = \hat{X}'_L$.

The quantum Hamiltonian:

$$\hat{H} = \int d^3x d\tau \left(\hat{\Pi}^2_L + (\nabla \hat{X}_L)^2 + m^2_{eff}(\tau) \hat{X}^2_L \right)$$

explicitly depends on time and thus does not have time-independent eigenstates that could serve as the vacuum.
This phenomenon is interpreted as a particle production.

Energy density

Using the classical definition of the energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{DM})}{\delta g_{\mu\nu}},$$

one can find the energy density of the spin-1 DM vector field

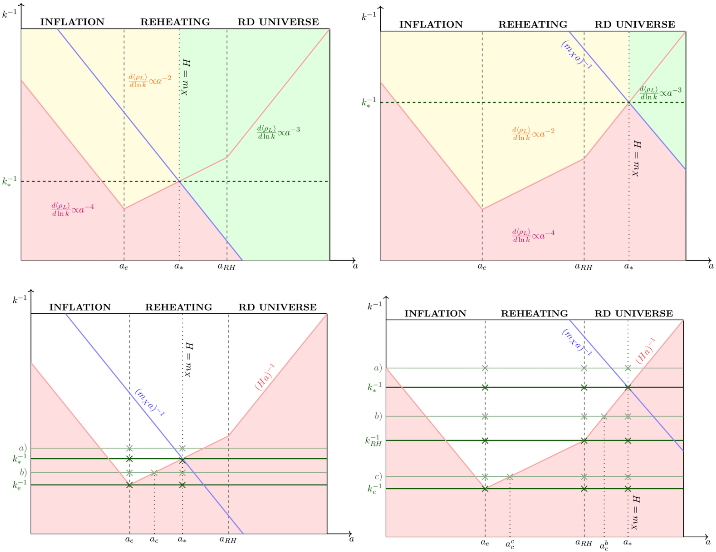
$$T_{00} \equiv \rho_X = \frac{1}{2a^2}(|\dot{\vec{X}} - \nabla X_0|^2 + \frac{1}{a^2}|\vec{\nabla} \times \vec{X}|^2 + m_X^2 a^2 X_0^2 + m_X^2 \vec{X}^2),$$

inserting mode decomposition (10) we the compute vacuum expectation value of the energy density

$$\frac{d\langle\rho_L\rangle}{d\ln k} = \frac{k^3}{2\pi^2 a^4}\left\{|\mathcal{X}_L'|^2 - \left(\mathcal{X}_L'\mathcal{X}_L^* + \mathcal{X}'^*_L\mathcal{X}_L\right)\frac{k^2}{k^2 + a^2 m_X^2}\frac{a'}{a} + \left(\frac{k^4}{(k^2 + a^2 m_X^2)^2}\left(\frac{a'}{a}\right)^2 + k^2 + a^2 m_X^2\right)|\mathcal{X}_L|^2\right\}$$

where we have used the Bunch-Davies vacuum.

Energy density scaling



- For DM vector bosons with mass $H_{RH} < m_X < H_I$

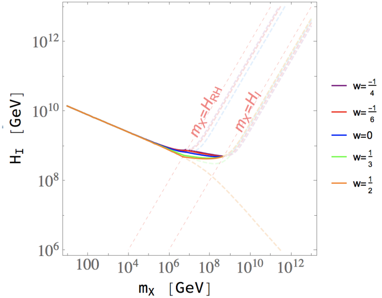
$$\frac{d\langle n_L(\tau : H = m_X)\rangle}{d\ln k} = \begin{cases} \frac{1}{8\pi^2} H_I^{\frac{2(1+3w)}{3(1+w)}} m_X^{\frac{1-3w}{3(1+w)}} \left(\frac{k}{a_e}\right)^2, & \text{for } k < k_* \\ \frac{1}{8\pi^2} H_I^{\frac{2\left(3w^2+3w+2\right)}{(w+1)(3w+1)}} m_X^{\frac{2}{1+w}} \left(\frac{a_e}{k}\right)^{\frac{3(1-w)}{(1+3w)}}, & \text{for } k_e > k > k_* \end{cases}$$

- For DM vector bosons with mass $H_{r.m.e} < m_X < H_{RH}$

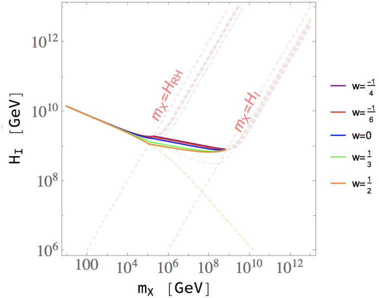
$$\frac{d\langle n_L(\tau : H = m_X)\rangle}{d\ln k} = \begin{cases} \frac{1}{8\pi^2} H_I \gamma^{\frac{2(1-3w)}{3(1+w)}} \left(\frac{k}{a_e}\right)^2, & \text{for } k < k_* \\ \frac{1}{8\pi^2} m_X^{3/2} H_I^{5/2} \gamma^{\frac{-1+3w}{3(1+w)}} \left(\frac{a_e}{k}\right), & \text{for } k_* < k < k_{RH} \\ \frac{1}{8\pi^2} m_X^{3/2} \gamma^{\frac{1-3w}{1+w}} H_I^{\frac{3(w+3)}{2(3w+1)}} \left(\frac{a_e}{k}\right)^{\frac{3(1-w)}{1+3w}}, & \text{for } k_e > k > k_{RH} \end{cases}$$

The number density per log frequency has a peak structure if and only if $w \in (-\frac{1}{3}, 1)$. Then, $\frac{d\langle n_L(H=m_X)\rangle}{d\ln k}$ is dominated by modes with $k = k_*$.

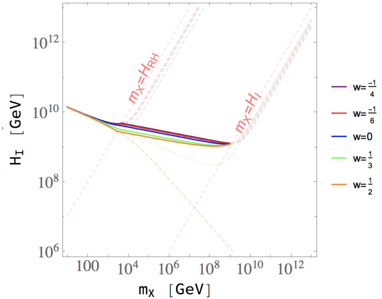
$H_I(m_X)$ for $\gamma=10^{-1}$



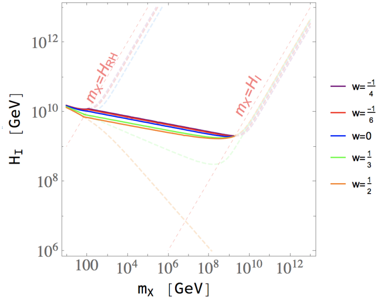
$H_I(m_X)$ for $\gamma=10^{-2}$



$H_I(m_X)$ for $\gamma=10^{-3}$



$H_I(m_X)$ for $\gamma=10^{-4}$



Outline

Summary

- Spin-1 particles that communicate with SM sector only through gravity can serve as a viable DM candidate.
- Even in the minimal scenario there exist several mechanisms, including:
    - the so-called pure gravitational production during and after inflation,
    - the inflaton decay,
    - freeze-in from SM particles,

  that can produce DM species in right abundance.

References

1. G. F. Giudice, E. W. Kolb and A. Riotto, *Largest temperature of the radiation era and its cosmological implications* , Phys. Rev. D64 (2001) 023508 , [hep-ph/0005123]
2. M. Garny, A. Palessandro, M. Sandora and M. S. Sloth, *Theory and Phenomenology of Planckian Interacting Massive Particles as Dark Matter*, JCAP 1802 (2018) 027, [1709.09688]
3. Edward W. Kolb and Andrew J. Long. *Superheavy dark matter through Higgs portal operators.*,Phys. Rev., D96(10):103540, 2017
4. Y. Ema, K. Nakayama, Y. Tang, *Production of Purely Gravitational Dark Matter: The Case of Fermion and Vector Boson*, JHEP 1907 (2019) 060,[1903.10973]