Low lying vector mesons - Aspects of pi-a1 mixing

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J. Morais*, B. Hiller*, A. A. Osipov†

*Centro de Física da Universidade de Coimbra, †Joint Institute for Nuclear Research

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Introduction - Basic Ideas

- **Effective Model of low energy QCD**
  [Osipov,Hiller,Blin 2013(1), Eur. Phys. J. A (2013) 49: 14]
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- Multi-quark effective interactions $\Rightarrow$ Effective Lagrangian for light mesons.

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- **Multiquark interactions:** 4q (NJL), 6q ('t Hooft), 8q (relevant at 4D)

$$\mathcal{L}_{int} = \frac{\bar{G}}{\Lambda^2}\mathrm{tr}\left(\Sigma^\dagger\Sigma\right) + \frac{\bar{\kappa}}{\Lambda^5}\left(\det\Sigma + \det\Sigma^\dagger\right) + \frac{\bar{g}_1}{\Lambda^8}\left[\mathrm{tr}\left(\Sigma^\dagger\Sigma\right)\right]^2 + \frac{\bar{g}_2}{\Lambda^8}\mathrm{tr}\left(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma\right)$$

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$$\mathcal{L}_2 = \frac{\bar{\kappa}_2}{\Lambda^3}\epsilon_{ijk}\epsilon_{lmn}\Sigma_{il}\Sigma_{jm}\chi_{kn} + \text{h.c.}$$

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$$\mathcal{L}_7 = \frac{\bar{g}_7}{\Lambda^4}\left[\mathrm{tr}\left(\Sigma^\dagger\chi\right) + \text{h.c.}\right]^2$$

$$\mathcal{L}_8 = \frac{\bar{g}_8}{\Lambda^4}\left[\mathrm{tr}\left(\Sigma^\dagger\chi\right) - \text{h.c.}\right]^2$$

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- Use $\chi \to m/2$ to make **Dirac mass term** $\mathcal{L}_0 = -\bar{q}mq$.

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- **Multiply by a functional unit:**

$$1 = \int \mathcal{D}s_a\mathcal{D}p_a\delta\left(s_a - \bar{q}\lambda_a q\right)\delta\left(p_a - i\bar{q}\lambda_a\gamma_5 q\right)$$
$$= \int \mathcal{D}\sigma_a\mathcal{D}\phi_a \int \mathcal{D}s_a\mathcal{D}p_a e^{i\int d^4x[\sigma_a(s_a-\bar{q}\lambda_a q)+\phi_a(p_a-i\bar{q}\lambda_a\gamma_5 q)]}$$

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- Quark determinant computed through a HK expansion.

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- **Inclusion of spin 1 states:** [Morais,Hiller,Osipov 2017, Phys. Rev. D 95, 074033 (2017)]
  - Study **decay processes of spin 1 mesons**.
  - Vector mesons have important impact in the **phase diagram and the EoS**.

Extension to Spin 1 - Multiquark Terms (I)

- Vector and axial vector bilinears: $v_a^\mu = \bar{q}\lambda_a\gamma^\mu q$, $a_a^\mu = \bar{q}\lambda_a\gamma^\mu\gamma_5 q$; $U(3)$-**valued fields:** $R^\mu, L^\mu = \frac{1}{2}\left(v_a^\mu \pm a_a^\mu\right)\lambda_a$.

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- **Source-independent** terms (vector):

$$\mathcal{L}'_{int} = \frac{\bar{w}_1}{\Lambda^2}\mathrm{tr}\left(R^\mu R_\mu + L^\mu L_\mu\right) + \frac{\bar{w}_2}{\Lambda^8}\left[\mathrm{tr}\left(R^\mu R_\mu + L^\mu L_\mu\right)\right]^2 + \frac{\bar{w}_3}{\Lambda^8}\left[\mathrm{tr}\left(R^\mu R_\mu - L^\mu L_\mu\right)\right]^2 + \frac{\bar{w}_4}{\Lambda^8}\mathrm{tr}\left(R^\mu R^\nu R_\mu R_\nu + L^\mu L^\nu L_\mu L_\nu\right) + \frac{\bar{w}_5}{\Lambda^8}\mathrm{tr}\left(R^\mu R_\mu R^\nu R_\nu + L^\mu L_\mu L^\nu L_\nu\right)$$

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$$\mathcal{L}'_1 = \frac{\bar{w}_9}{\Lambda^6} \mathrm{tr}\left(R^\mu R_\mu + L^\mu L_\mu\right) \mathrm{tr}\left(\Sigma^\dagger \chi + \Sigma \chi^\dagger\right)$$

$$\mathcal{L}'_2 = \frac{\bar{w}_{10}}{\Lambda^6} \mathrm{tr}\left(\chi^\dagger R^\mu \Sigma L_\mu + \Sigma^\dagger R^\mu \chi L_\mu\right)$$

$$\mathcal{L}'_3 = \frac{\bar{w}_{11}}{\Lambda^6} \mathrm{tr}\left(\left(\Sigma^\dagger \chi + \chi^\dagger \Sigma\right) L^\mu L_\mu + \left(\Sigma \chi^\dagger + \chi \Sigma^\dagger\right) R^\mu R_\mu\right)$$

$$\mathcal{L}'_4 = \frac{\bar{w}_{12}}{\Lambda^4} \mathrm{tr}\left(\chi^\dagger R^\mu \chi L_\mu\right)$$

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- **13 new parameters**, but not all will contribute to the vacuum properties of the model.

- **Functional Integral:**

$$Z = \int \mathcal{D}\sigma_a \mathcal{D}\phi_a \mathcal{D}V_{a\mu} \mathcal{D}A_{a\mu} \int \mathcal{D}s_a \mathcal{D}p_a \mathcal{D}v_a^\mu \mathcal{D}a_a^\mu$$
$$\times\ e^{i\int d^4x \left[\mathcal{L}_{int} + s_a(\sigma_a + \Delta_a) + p_a\phi_a + v_a^\mu(V_{a\mu} + \eta_a\delta_{\mu 0}) + a_a^\mu A_{a\mu}\right]}$$
$$\times \int \mathcal{D}q\mathcal{D}\bar{q} \quad e^{i\int d^4x \bar{q}\left(i\gamma^\mu\partial_\mu - M - \gamma^0\eta - \sigma - i\gamma_5\phi - \gamma^\mu V_\mu - \gamma^\mu\gamma_5 A_\mu\right)q}$$

Extension to Spin 1 - Functional Bosonization

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$$Z = \int \mathcal{D}\sigma_a \mathcal{D}\phi_a \mathcal{D}V_{a\mu} \mathcal{D}A_{a\mu} \int \mathcal{D}s_a \mathcal{D}p_a \mathcal{D}v_a^\mu \mathcal{D}a_a^\mu$$
$$\times\, e^{i\int d^4x \left[\mathcal{L}_{int} + s_a(\sigma_a + \Delta_a) + p_a\phi_a + v_a^\mu(V_{a\mu} + \eta_a\delta_{\mu 0}) + a_a^\mu A_{a\mu}\right]}$$
$$\times \int \mathcal{D}q\mathcal{D}\bar{q} \quad e^{i\int d^4x \bar{q}\left(i\gamma^\mu\partial_\mu - M - \gamma^0\eta - \sigma - i\gamma_5\phi - \gamma^\mu V_\mu - \gamma^\mu\gamma_5 A_\mu\right)q}$$

- For SPA, use the expansions

$$s_a^{st} = h_a + h_{ab}^{(1)}\sigma_b + h_{abc}^{(1)}\sigma_b\sigma_c + h_{abc}^{(2)}\phi_b\phi_c + H_{abc}^{\mu\nu\,(1)} V_{b\mu}V_{c\nu}$$
$$+ H_{abc}^{\mu\nu\,(2)} A_{b\mu}A_{c\nu} + \ldots$$
$$p_a^{st} = h_{ab}^{(2)}\phi_b + h_{abc}^{(3)}\phi_b\sigma_c + H_{abc}^{\mu\nu\,(3)} V_{b\mu}A_{c\nu} + \ldots$$
$$v_a^{\mu\, st} = H_{ab}^{\mu\nu(1)} V_{b\nu} + H_{abc}^{\mu\nu\,(4)}\sigma_b V_{c\nu} + H_{abc}^{\mu\nu\,(5)}\phi_b A_{c\nu} + \ldots$$
$$a_a^{\mu\, st} = H_{ab}^{\mu\nu(2)} A_{b\nu} + H_{abc}^{\mu\nu\,(6)}\phi_b V_{c\nu} + H_{abc}^{\mu\nu\,(7)}\sigma_b A_{c\nu} + \ldots$$

Extension to Spin 1 - $V - \sigma$ and $A - \phi$ Mixing

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$$\mathrm{tr}_F(i\,[V^\mu, M]\,\partial_\mu\sigma - \{A^\mu, M\}\,\partial_\mu\phi).$$

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  - Diagonalization requires

$$k = \left(2\varrho^2 H^{(1)} - \Delta_M^{\circ 2}\right)^{\circ -1}, \qquad k' = \left(2\varrho^2 H^{(2)} + \Sigma_M^{\circ 2}\right)^{\circ -1},$$

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- The shifts provide new contributions to spin 0 kinetic terms $\Longrightarrow$ New (unsymmetric) field rescaling:

$$\sigma \to \varrho \left(1_H + k \circ \Delta_M^{\circ 2}\right)^{\circ -\frac{1}{2}} \circ \sigma \ , \ \phi \to \varrho \left(1_H - k' \circ \Sigma_M^{\circ 2}\right)^{\circ -\frac{1}{2}} \circ \phi$$

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- **Vector** and **axial vector** meson masses (ideal mixing):

$$M_\rho^2 = M_\omega^2 = \frac{3}{2}\varrho^2 H_{11}^{(1)} \quad , \quad M_{a_1}^2 = M_{f_1}^2 = \frac{3}{2}\varrho^2 H_{11}^{(2)} + 6M_u^2$$

$$M_{K^*}^2 = \frac{3}{2}\left[\varrho^2 H_{44}^{(1)} + (M_u - M_s)^2\right] \quad , \quad M_{K_1}^2 = \frac{3}{2}\left[\varrho^2 H_{44}^{(2)} + (M_u + M_s)^2\right]$$

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- **Isospin approximation** $(m_u = m_d \neq m_s) \Longrightarrow \sigma_0 - \sigma_8$ and $\phi_0 - \phi_8$ (or $\sigma_{ns} - \sigma_s$ and $\phi_{ns} - \phi_s$) mixing $\Rightarrow$ Introduce **mixing angles** θ_σ and θ_ϕ (or ψ_σ and ψ_ϕ).

Extension to Spin 1 - Scalar Meson Masses

- **Scalar** meson masses:

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$$M_{a_0}^2 = \frac{2}{3} M_\rho^2 \frac{1}{H_{11}^{(1)}}\left(\frac{h_u}{M_u} - h_{11}^{(1)}\right) + 4M_u^2$$

$$M_\kappa^2 = \frac{2}{3} M_{K^*}^2 \frac{1}{H_{44}^{(1)}}\left(\frac{h_u - h_s}{M_u - M_s} - h_{44}^{(1)}\right)$$

$$M_{f_0}^2 = \frac{1}{1-\tan^2\psi_\sigma}\left[\frac{M_\omega^2}{3H_{uu}^{(1)}}\left(\frac{h_u}{M_u} - 2h_{uu}^{(1)} - 2h_{ud}^{(1)}\right) + 4M_u^2\right] + \frac{1}{1-\cot^2\psi_\sigma}\left[\frac{M_\varphi^2}{3H_{ss}^{(1)}}\left(\frac{h_s}{M_s} - 2h_{ss}^{(1)}\right) + 4M_s^2\right]$$

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- **Pseudoscalar** meson masses:

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$$M_K^2 = \frac{2}{3} M_{K_1}^2 \frac{1}{H_{44}^{(2)}} \left(\frac{h_u + h_s}{M_u + M_s} - h_{44}^{(2)} \right)$$

$$M_\eta^2 = \frac{1}{1 - \tan^2 \psi_\phi} \frac{M_{f_1}^2}{3 H_{uu}^{(2)}} \left(\frac{h_u}{M_u} - 2 h_{uu}^{(2)} - 2 h_{ud}^{(2)} \right) + \frac{1}{1 - \cot^2 \psi_\phi} \frac{M_{f_1'}^2}{3 H_{ss}^{(2)}} \left(\frac{h_s}{M_s} - 2 h_{ss}^{(2)} \right)$$

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$$M_{a_1}^2 = M_{f_1}^2 = \frac{3}{2} \varrho^2 H_{11}^{(2)} + 6 M_u^2$$

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Extension to Spin 1 - Weak Decay Constants

- Using the **PCAC hypothesis**, we may compute the π and K weak decay constants:

$$f_\pi = \frac{M_u}{\sqrt{\varrho^2 + \frac{4M_u^2}{H_{11}^{(2)}}}} \equiv \frac{M_u}{g_\pi}$$

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- These express the well-known **Goldberger-Treiman relations**.

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- These three parameters are **tightly constrained** by the spin 1 spectra $\Rightarrow$ Crucial for **vector - axial vector mass differences**.

Model Fitting - Analytic Results (II)

- The model predicts a relation between M_u and M_s depending solely on the spin-1 meson masses:

$$\frac{2}{M_{K^*}^2 - \frac{3}{2}(M_u - M_s)^2} + \frac{2}{M_{K_1}^2 - \frac{3}{2}(M_u + M_s)^2} =$$

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- These 3 relations **completely determine** M_u, M_s **and** Λ **from empirical data**.

Model Fitting - Results

- **Empirical Input:**

M_π	M_K	M_η	$M_{\eta'}$	M_σ	M_κ	M_{a_0}	M_{f_0}	M_ρ	M_{K^*}
138	496	548	958	500	850	980	980	778	893

M_φ	M_{a_1}	M_{K_1}	M_{f_1}	m_u	m_s	f_π	f_K	θ_ϕ
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- **Results of the Fit:** (boldface = externally fixed)

G	κ	g_1	g_2	κ_2	g_3	g_4	g_5	g_6
2.54	-2.66	15.3	-35.2	0.143	-148	36.1	-21.9	-115
g_7	g_8	θ_σ	Λ	M_u	M_s	$\mathbf{w_1}$	$\mathbf{w_6}$	w_7
-32.6	-21.8	25.1°	1633	244	508	-10	0	-1903
w_8	$\mathbf{w_9}$	w_{10}	w_{11}	w_{12}	$\mathbf{w_{13}}$			
2505	0	-2540	1425	-1523	0			

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  - Include a new class of terms which involve products of $tr\,(v^\mu)$ and $tr\,(a^\mu)$ $\Rightarrow$ Complicates the $A-\phi$ mixing scheme and subsequent mass diagonalization, and introduces more parameters.

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