Radiative corrections to $e^+e^- \rightarrow hadrons + \gamma$ and $e^+e^- \rightarrow \mu^+\mu^-\gamma$

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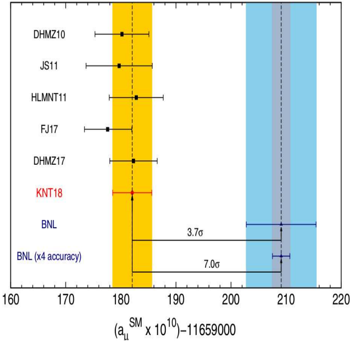
Outline

1. Motivation
2. Full NLO radiative corrections to the reaction $e^+e^- \to \pi^+\pi^-\gamma$
3. ISR NNLO radiative corrections
4. Conclusions

Anomalous magnetic moment of the muon a_μ

$$a_\mu^{SM} = 11659182.04 \pm 3.56$$

$$a_\mu^{exp} = 11659208.9 \pm 5.4 \pm 3.3$$



A.Keshavarzi, D.Nomura and T.Teubner, Phys. Rev. D 97, 114025 (2018)
Muon g-2 Collaboration (G.W. Bennett et al.), Phys. ReV. D 73, 072003 (2006) [hep-ex/0602035]

Theoretical value of the anomalous magnetic moment of the muon a_μ

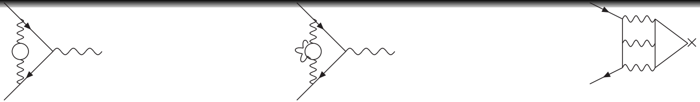
$$a_\mu^{exp} - a_\mu^{SM} = 27.06 \pm 7.26$$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$$

$$a_\mu^{QED} = 11658471.8971 \pm 0.007$$

$$a_\mu^{EW} = 15.36 \pm 0.10$$

A.Keshavarzi, D.Nomura and T.Teubner, Phys. Rev. D 97, 114025 (2018),
Muon g-2 Collaboration (G.W. Bennett et al.), Phys. ReV. D 73, 072003 (2006) [hep-ex/0602035].



$$a_\mu^{had} = a_\mu^{had,LO} + a_\mu^{had,NLO} + a_\mu^{had,LBL}$$

$$a_\mu^{had,NLO} = -9.82 \pm 0.04$$
$$a_\mu^{had,LO} = 684.68 \pm 2.42$$
$$a_\mu^{had,LBL} = 9.8 \pm 2.6$$

$$a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma_0}$$

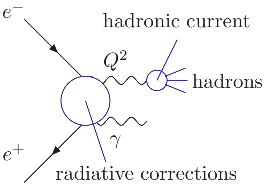
$a_\mu^{had,LBL}$ - effective models

Radiative corrections to the reaction $e^+e^- \to \pi^+\pi^-\gamma$

F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]

Radiative return method

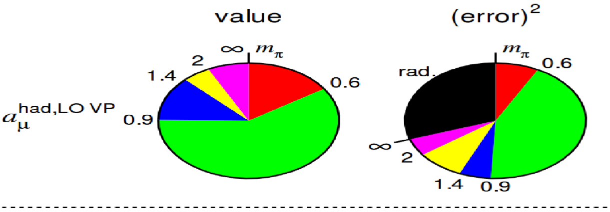
$$d\sigma(e^+e^- \to hadrons+\gamma_{isr}) = H(Q^2,\theta_\gamma)d\sigma(e^+e^- \to hadrons)(Q^2)$$



- measurement of $R(s)$ over the wide range of energies, from threshold up to $\sqrt{s}$
- large luminosity from factories compensate α/π from photon radiation
- precise measurement involves radiative corrections, precision of the simulation is a part of systematic error of the experiments.
- FSR contribution has to be subtracted
- Monte Carlo generators needed (Phokhara)
- Monte Carlo generators have to rely on reliable models

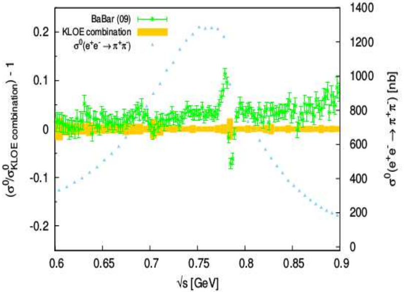
Contributions to HVP

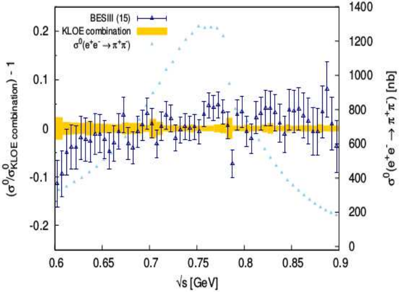
$$a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{ds}{s}K(s)R(s)$$



A.Keshavarzi, D.Nomura and T.Teubner, Phys. Rev. D 97, 114025 (2018)

BaBar, BES and KLOE data

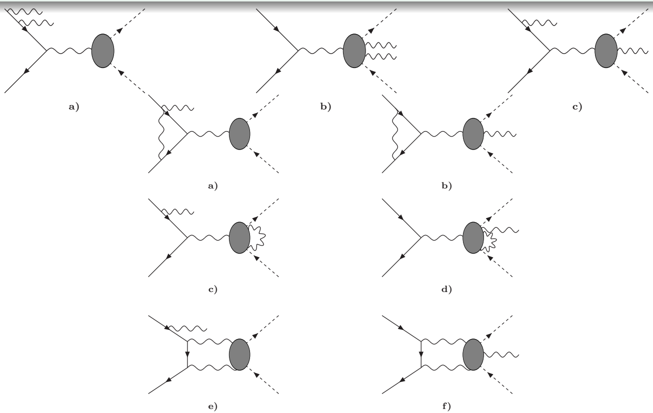




Data analysis used approximate radiative corrections from MC PHOKHARA.
Full NLO radiative corrections were added

A. Anastasi *et al.* [KLOE-2 Collaboration], JHEP **1803** (2018) 173, F. Campanario, H. Czyż, J. Gluza, M. Gunia, T. Riemann, G. Rodrigo and V. Yundin, JHEP **1402** (2014) 114

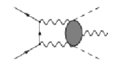
NLO radiative corrections for $e^+e^- \to \pi^+\pi^-\gamma$



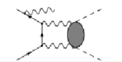
A. Denner and S. Dittmaier, Nucl. Phys. B **734** (2006) 62, T. Binoth, J. P. Guillet, G. Heinrich, E. Pilon and C. Schubert, JHEP **0510** (2005) 015, F. Campanario, JHEP **1110** (2011) 070

Modeling pion-photon interaction

-Real emission proportional to the form factor
-Factorization of the form factor:



$= \quad \mathbf{F}_\pi(\mathbf{s}) \times \mathbf{sQED}$



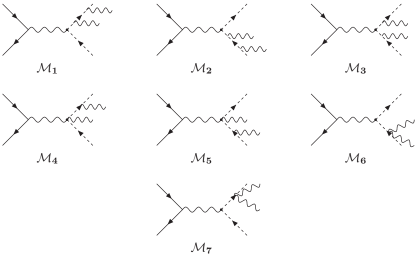
$= \quad \mathbf{F}_\pi(\mathbf{q}^2) \times \mathbf{sQED}$

-Form factor:

$$F_\pi(q^2) = \sum_n c^\pi_{\rho_n} BW_{\rho_n}(q^2)$$

F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]
H. Czyz, A. Grzelinska and J. H. Kuhn, Phys. Rev. D **81** (2010) 094014

Infrared divergences



$$\sigma_{1h,1s} = \sigma_{1h} \frac{-\alpha}{4\pi} \int_{0 \le |k| \le E_{max}} \frac{d^3k}{E_k} \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} + \frac{q_1}{q_1 \cdot k} - \frac{q_2}{q_2 \cdot k} \right)^2,$$

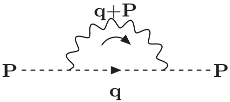
Dimensional regularization vs photon mass regulator scheme:

$$\log\left(\frac{\lambda^2}{s}\right) \to \Delta = \frac{(4\pi)^\epsilon}{\epsilon\Gamma(1-\epsilon)} \left(\frac{\mu^2}{s}\right)^\epsilon$$

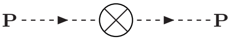
G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **153** (1979) 365

Counter-terms for virtual FSR corrections

self energy correction:



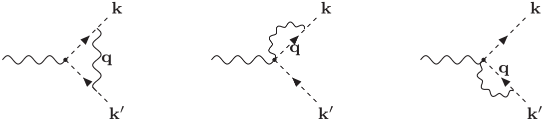
counter-term:



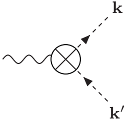
$$\bar{\mathcal{A}}_{1PI}|_{P^2=m_\pi^2} = 0,$$

$$\frac{d\bar{\mathcal{A}}_{1PI}}{dP^2}|_{P^2=m_\pi^2} = 0.$$

vertex corrections:

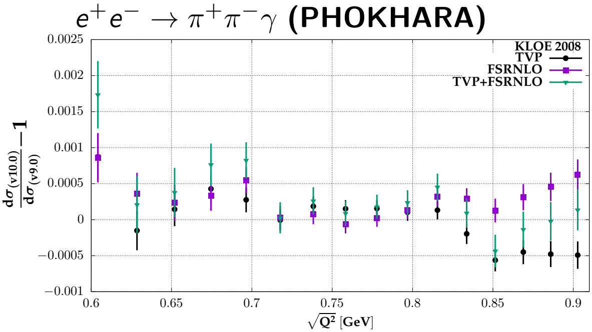


counter-term:



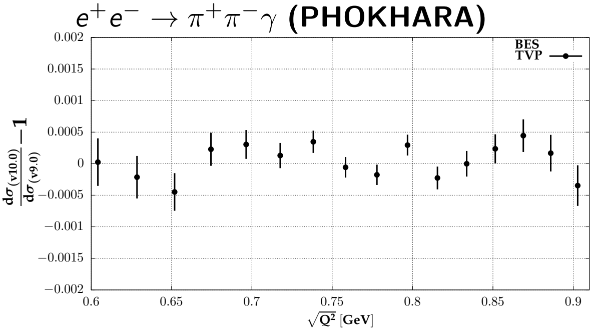
$$\bar{\mathcal{A}}_v|_{s=0} = 0.$$

- $\sqrt{s} = 1.02$ GeV
- Pion tracks: $50^o < \theta_{\pi^\pm} < 130^o$, $|p_{z_{\pi^\pm}}| > 90$ MeV
- Missing photon angle: $|\cos\theta_\gamma| > \cos 15^o$
- Track mass: $m_{trk} > 130$ MeV
- $q^2 \in (0.35, 0.95)$



F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]

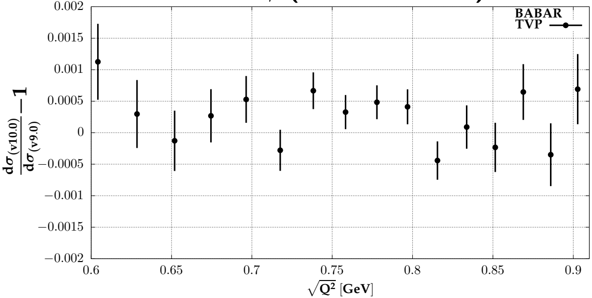
- $\sqrt{s} = 3.773$ GeV
- Pion tracks: $22.9^o < \theta_{\pi^\pm} < 157.1^o$, $|p_{T_{\pi^\pm}}| > 300$ MeV
- Minimal photon energy: $E_\gamma > 400$ MeV
- Missing photon angle: $|\cos\theta_\gamma| < 0.8$ or $0.86 < |\cos\theta_\gamma| < 0.92$
- $q^2 \in (0.35, 0.95)$



F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]

- $\sqrt{s} = 10.56$ GeV
- Pion tracks: $20^o < \theta_{\pi^\pm} < 160^o$, $|p_{T_{\pi^\pm}}| > 300$ MeV
- Minimal photon energy: $E_\gamma > 3$ GeV
- Missing photon angle: $20^o < \theta_\gamma < 160^o$
- $q^2 \in (0.35, 0.95)$, $|q_1| > 1$ GeV (π^-) and $|q_2| > 1$ GeV (π^+)

$e^+e^- \to \pi^+\pi^-\gamma$ **(PHOKHARA)**

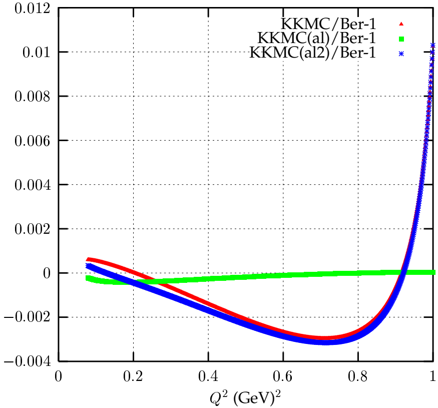


F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]

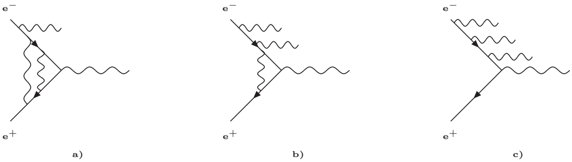
ISR NNLO radiative corrections for radiative return

- $e^+e^- \to \mu^+\mu^-\gamma$
- $e^+e^- \to \pi^+\pi^-\gamma$
- $e^+e^- \to \pi^0\pi^0\pi^+\pi^-\gamma$
- $e^+e^- \to \pi^+\pi^+\pi^-\pi^-\gamma$
- $e^+e^- \to \bar{p}p\gamma$
- $e^+e^- \to \bar{n}n\gamma$
- $e^+e^- \to K^+K^-\gamma$
- $e^+e^- \to K^0\bar{K}^0\gamma$
- $e^+e^- \to \pi^0(\eta)\ \pi^+\pi^-\gamma$

- **Declared precision of Phokhara for ISR: 0.5 %**
- **Fermilab and JPARC will improve precision by a factor 4**



M. Skrzypek and B. F. L. Ward, Phys. Lett. B **257** (1991) 173 F. A. Berends, W. L. van Neerven and G. J. H. Burgers, Nucl. Phys. B **297** (1988) 429, S. Jadach, Acta Phys. Polon. B **36** (2005) 2387



$$\sigma_{NNLO} = \Big(\Delta_{soft,2ph} + \Delta_{virt,soft,1ph} + \Delta_{virt,2ph}\Big)\sigma_{1\gamma} + \Big(\Delta_{soft,1ph} + \Delta_{virt,1ph}\Big)\sigma_{2\gamma} + \sigma_{3\gamma},$$

Leading logarithmic approximation:

$$\Delta_{soft,1ph} = \frac{\alpha}{\pi}\Big(\log(s/m_e^2) - 1\Big)\log(2w), \qquad \Delta_{virt,1ph} = \frac{3\alpha}{2\pi}\log(Q^2/m_e^2),$$

$$\Delta_{soft,2ph} = \frac{\Delta^2_{soft,1ph}}{2}, \qquad \Delta_{virt,soft,1ph} = \Delta_{soft,1ph}\Delta_{virt,1ph},$$

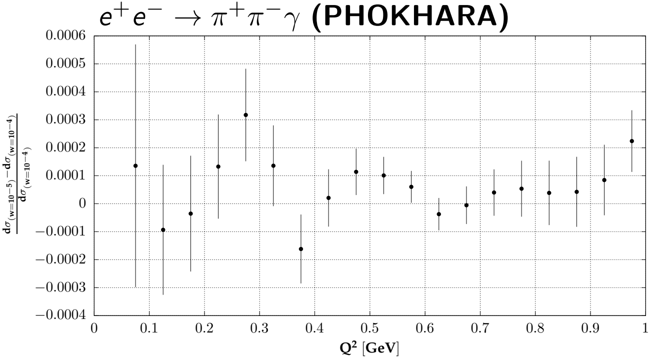
$$\Delta_{virt,2ph} = \frac{9\alpha}{8\pi}\log^2(Q^2/m_e^2),$$

$w = E_\gamma^{min}/\sqrt{s}$.

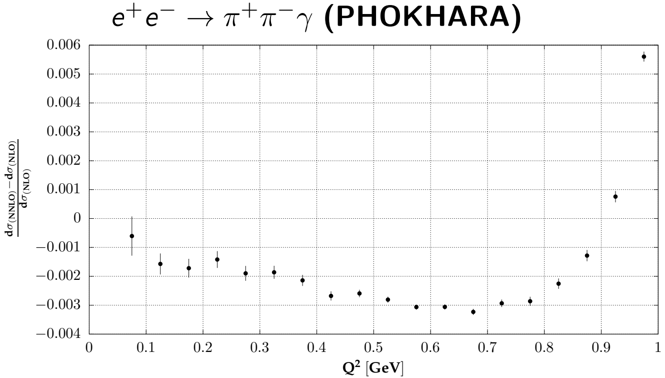
F. A. Berends, W. L. van Neerven and G. J. H. Burgers, Nucl. Phys. B **297** (1988) 429

Soft photon cutoff independence

- $\sqrt{s} = 1.02$ GeV
- $q^2 \in (0.35, 0.95)$



- $\sqrt{s} = 1.02$ GeV
- $q^2 \in (0.35, 0.95)$



More results soon...

Conclusions

- Discrepancy between BABAR and KLOE data for $e^+e^- \to \pi^+\pi^-\gamma$ cannot be explained by missing radiative corrections. The source of the difference can be only of the experimental origin.
- As a consequence, these corrections cannot be the origin of the discrepancy between the experimental measurement and the SM prediction of the muon anomalous magnetic moment.
- The size of the dominant NNLO corrections has been confirmed and its impact for experimental event selection will be investigated.

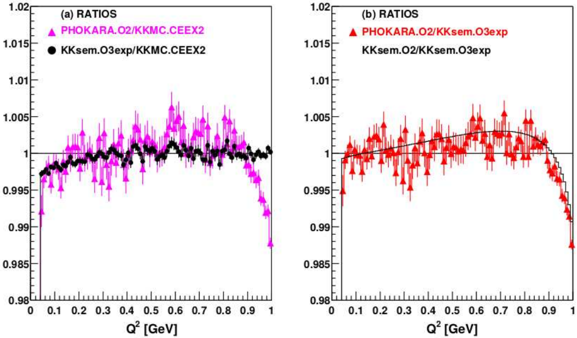


Figure 1: Mu-pair mass (square) spectrum in case of ISR only. $\sqrt{s} = 1.01942$GeV.

S. Jadach, Acta Phys. Polon. B **36** (2005) 2387