Pole masses of Neutrinos in the Grimus–Neufeld model

Vytautas Dūdėnas

Vilniaus Universitetas

In collaboration with Thomas Gajdosik, Darius Jurčiukonis

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Outline

Motivation

- BSM physics already:
  - neutrinos mix and have mass...
    - but what is the exact mechanism?
- Unknown BSM physics: More scalars?

Motivation

- BSM physics already:
  - neutrinos mix and have mass...
    - but what is the exact mechanism?
- Unknown BSM physics: More scalars?
- Being general but minimal:
  - 2HDM + 1 Seesaw neutrino $\rightarrow$ Grimus–Neufeld model [GN '89].
    - Incorporates masses and mixings at one loop.

Yukawa couplings

- Singlet Weyl spinor neutrino N:

$$\mathscr{L}_M = -\frac{1}{2} M(NN + h.c.)$$

- Take 2HDM in the Higgs basis ($\langle H_1 \rangle = \frac{1}{\sqrt{2}} v$, $\langle H_2 \rangle = 0$) ,
- Leptons are in the Flavour basis ($Y^1_{\ell_i} = \frac{\sqrt{2} m_i}{v}$, $i = e, \mu, \tau$,)
- Neutrino Yukawa couplings in GN Y^1_ν and Y^2_ν are 2 general complex 3-vectors:

$$\mathscr{L}_{\text{Yuk}} = -Y^1_{\nu_i} \ell_i \tilde{H}_1 N - Y^2_{\nu_i} \ell_i \tilde{H}_2 N + H.c. , i = e, \mu, \tau ;$$
$$\tilde{H}_I \equiv \varepsilon_{ij} H_{Ij} , \; \varepsilon_{12} = -\varepsilon_{12} = 1 ,$$

- By a unitary transformation U we can pick a basis:

$$Y^1_\nu U = (0, 0, y) , \; Y^2_\nu U = \left(0, d, d'\right)$$

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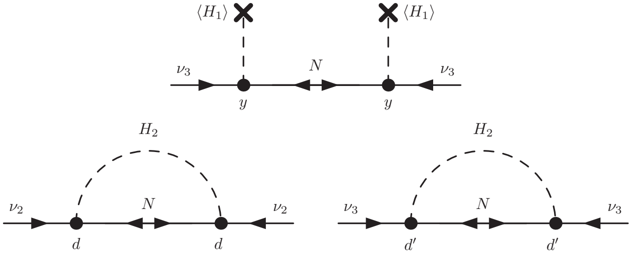
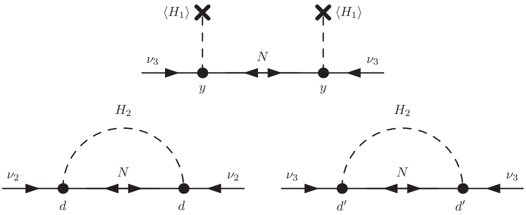


Figure: Seesaw and radiative mass generation in Weyl spinor notation. The arrow shows chirality propagation.

- Approximate mass eigenstate basis $\Rightarrow$ $U \approx V_{PMNS}$

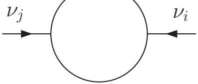
$$Y_\nu^1 V_{PMNS} = (0, 0, y) \, , \; Y_\nu^2 V_{PMNS} = (0, d, d')$$

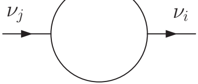


- **Task**: take PMNS, Δm^2_{12}, and Δm^2_{13} from experiment and relate them to d, d', y at one loop.
  - Also relates the scalar sector to Yukawa.
  - Implement in FlexibleSUSY (FS) spectrum generator.

Pole masses

- Effective two point functions for Majorana neutrinos:

$i\Gamma_{ij} =$ 

$i\sigma p\Sigma_{ij} =$ 

- We get poles:

$$\det(\mu^2 - \Gamma\Sigma^{-1}\Gamma\Sigma^{-1}) = 0 \Rightarrow \det(\mu \pm \Gamma\Sigma^{-1}) = 0$$

- We solve it perturbatively:

$$\mu = \sum_n \varepsilon^n \mu^{(n)}\,,\ \Sigma = \sum_n \varepsilon^n \Sigma^{(n)}\,,\ \Gamma = \sum_n \varepsilon^n \Gamma^{(n)}$$

ε – perturbation series ordering parameter

Pole mass 1 loop

- When ε is a loop order parameter, then GN model has:

$$\Gamma^{[0]} = -\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \; \Gamma^{[1]} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix},$$

$$m_3 \approx \frac{y^2 v^2}{2M}, \; m_4 \approx M$$

- Solving $\det(\mu + \Gamma\Sigma^{-1}) = 0$ for $\varepsilon = 1$:

$$\begin{aligned} \mu_1 &= 0 & \mu_3 &= m_3 - \Gamma^{[1]}_{33} - m_3 \Sigma^{[1]}_{33} \\ \mu_2 &= -\Gamma^{[1]}_{22} & \mu_4 &= m_4 - \Gamma^{[1]}_{44} - m_4 \Sigma^{[1]}_{44} \end{aligned}$$

- Off diagonal 2pt functions, e.g. Γ_{23} enter only at $\varepsilon = 2$.

Pole mass 1 loop

$$\mu_1 = 0 \qquad \mu_3 = m_3 - \Gamma^{[1]}_{33} - m_3\Sigma^{[1]}_{33}$$
$$\mu_2 = -\Gamma^{[1]}_{22} \qquad \mu_4 = m_4 - \Gamma^{[1]}_{44} - m_4\Sigma^{[1]}_{44}$$

- μ_2 is finite without any UV subtractions.
- μ_3 and μ_4 needs a subtraction scheme.
- Interested only in μ_2 and μ_3
  - Grimus and Lavoura (GL) appoximation [GL '02]: they are finite, gauge invariant, no UV subtraction. How?

Getting μ_3 without subtraction scheme ?

- We need all **three** masses, μ_1, μ_2 and μ_3, to be zero at $O\left(\varepsilon^0\right)$:
  - There is no multiplicative counterterm possible for $O\left(\varepsilon^1\right)$
  - Then $O\left(\varepsilon^1\right)$ result is finite and gauge invariant for μ_2 and μ_3
  - Counterterms should appear only at $O\left(\varepsilon^2\right)$.

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  - Then $O(\varepsilon^1)$ result is finite and gauge invariant for μ_2 and μ_3
  - Counterterms should appear only at $O(\varepsilon^2)$.
- Ordering series in terms of couplings?

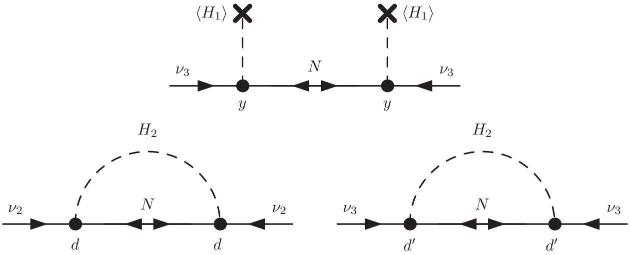

Figure: These diagrams have the same number of Yukawa couplings

- Assign ordering parameter to all Yukawa couplings, i.e. $Y \to \varepsilon^{\frac{1}{2}} Y$, then:

$$\Gamma^{\mathrm{GL}[0]} = -\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \; \Gamma^{\mathrm{GL}[1]} = \begin{pmatrix} 0 & 0 & 0 & * \\ 0 & * & * & * \\ 0 & * & * & * \\ * & * & * & * \end{pmatrix},$$

$$m_3 \approx \frac{y^2 v^2}{2M} = O\left(\varepsilon^1\right), \; m_4 \approx M$$

- Solving $\det(\mu + \Gamma \Sigma^{-1}) = 0$ for $\varepsilon = 1$:

$$\mu_2 \mu_3 = \Gamma^{\mathrm{GL}[1]}_{22} \Gamma^{\mathrm{GL}[1]}_{33} - \left(\Gamma^{\mathrm{GL}[1]}_{23}\right)^2, \; \mu_2 + \mu_3 = -\Gamma^{\mathrm{GL}[1]}_{22} - \Gamma^{\mathrm{GL}[1]}_{33}$$

- Checked: we get the same $\Gamma^{\mathrm{GL}[1]}_{ij}$ as in [GL '02]

Determining Yukawas in GL

- In GL we have:

$$(\mu_2, \mu_3)_i^{\rm GL} = f_i\left(m_S, U_S, U_{PMNS}, \mu_4, Z_3, d, d'\right)\,,$$

$$m_3 \equiv Z_3 \mu_3 \approx \frac{y^2 v^2}{2\mu_4}\,,\ m_S, U_S\text{-masses and mixings of scalars}$$

- Assuming normal hierarchy for neutrinos, we solve for d and d':

$$\mu_3^{\rm GL} = \sqrt{\Delta m^2_{31}}\,,\ \mu_2^{\rm GL} = \sqrt{\Delta m^2_{21}}$$
$$\Rightarrow \left(d_{GL}, d'_{GL}\right)_i = \tilde{f}_i\left(m_S, U_S, U_{PMNS}, \mu_4, Z_3\right)$$

GL vs MSbar vs FlexibleSUSY poles

- Put solutions for d_{GL} and d'_{GL} in calculation X (MSbar or FlexibleSUSY):

$$(\mu_2, \mu_3)_i^X = F_i \left(m_S, U_S, U_{PMNS}, \mu_4, Z_3, d_{GL}, d'_{GL} \right)$$

- Define an "error" function:

$$\frac{\Delta \mu_i}{\mu_i} \left(m_S, U_S, \mu_4, Z_3, U_{PMNS} \right) \equiv \frac{\mu_i^X - \sqrt{\Delta m_{i1}^2}}{\sqrt{\Delta m_{i1}^2}}, \, i = 2, 3$$

- We take a benchmark 2HDM point: $m_H = 300$, $m_A = 411$, $m_{H^\pm} = 442$, GeV, $\sin\theta_{H-A} \approx 0.07$
- Look how it depends on $Z_3 = \frac{m_3}{\mu_3}$ and μ_4
- Set charged lepton Yukawa couplings to the H_2 to zero for simplicity.

GL vs MSbar vs FlexibleSUSY poles

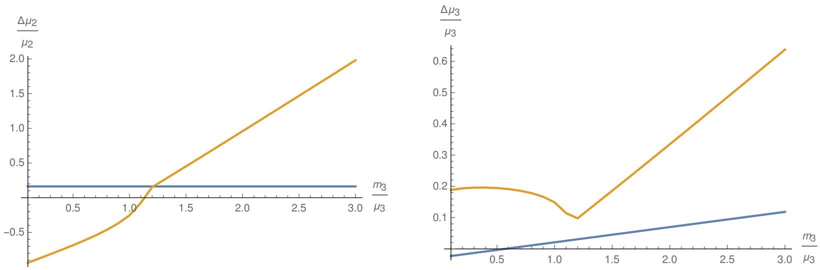

Figure: $\mu_4 = 10^5,\ m_H = 300,\ m_A = 411,\ m_{H^\pm} = 442$ GeV; $\sin\theta_{H-A} \approx 0.07$. MSbar - blue, FS - orange

GL vs MSbar vs FlexibleSUSY poles

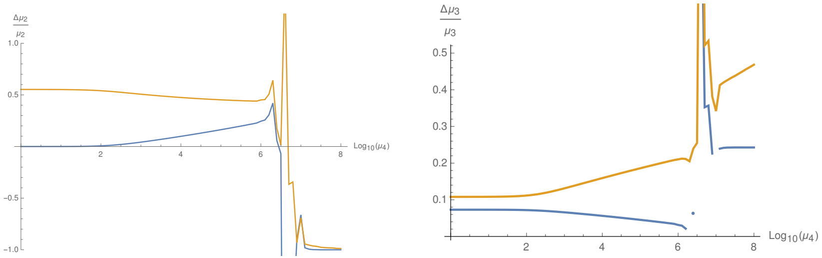

Figure: $\frac{\mu_3}{m_3} = 1.5,\ m_H = 300,\ m_A = 411,\ m_{H^\pm} = 442$ GeV; $\sin\theta_{H-A} \approx 0.07$. MSbar - blue, FS - orange

- Numerical effects start at $\mu_4 = 10^6$ and kills the calculation at 10^8 (MSbar with LoopTools).
- Radiative mass is more off for FS.

Conclusions

- GL approximation – coupling ordering instead of loop (they differ for a seesaw)
  - first order in GL is close to 1 loop approximation
  - But how close, depends on parameters
  - Which perturbation ordering is more accurate?
    - Can we formulate some condition and/or estimate error?
- it is possible to use FS to reproduce neutrino data
  - but up to $M < 10^6$GeV (Maybe 10^8 in some cases)
  - and not too big loop corrections for $\frac{m_3}{\mu_3}$.
  - The dependence on scalar potential is not yet fully researched...
- Further step: relate scalar potential with neutrino Yukawa couplings and make it work with FS.