SUPERWEAK FORCE

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OUTLINE

1. Status of particle physics
2. $U(1)_Z$ extension of SM
3. Constraints on the parameter space

Status of particle physics: energy frontier

- LEP, LHC: SM describes final states of particle collisions precisely
- SM is unstable
- No proven sign of new physics beyond SM at colliders*

*There are some indications below discovery significance (such as lepton flavor non-universality in meson decays)

Status of particle physics: cosmic and intensity frontiers

- Universe at large scale described precisely by cosmological SM: ΛCDM ($\Omega_m = 0.3$), without astrophysical explanation

- Neutrino flavours oscillate requiring neutrino masses

- Existing baryon asymmetry cannot be explained by CP asymmetry in SM

- Inflation of the early, accelerated expansion of the present Universe

Extension of SM

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- SM is highly efficient – let us stick to efficiency

the only exception of economical description is the relatively large number of Yukawa couplings

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- Neutrinos must play a key role

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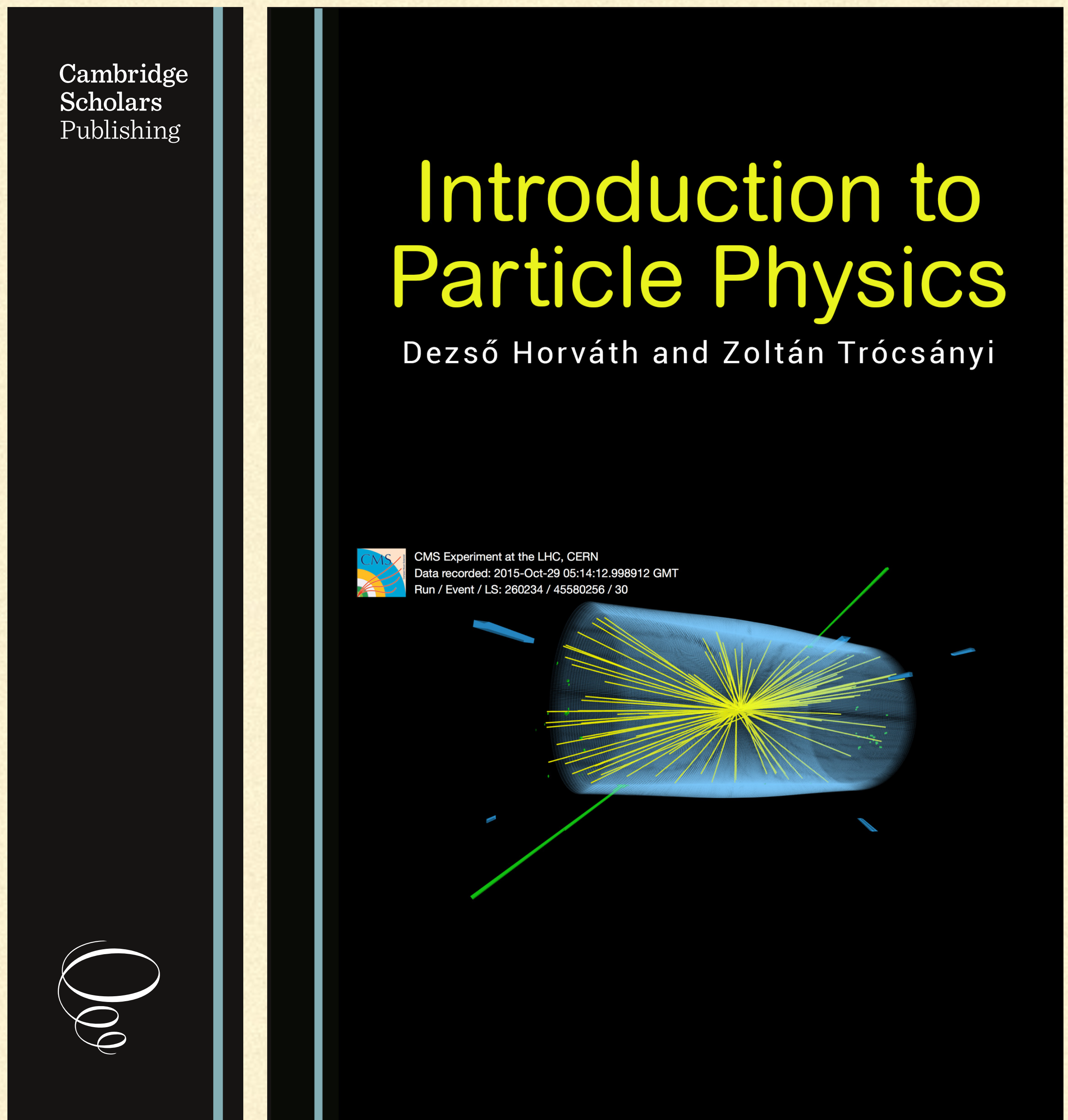
- Fix Z-charges by requirement of
  - gauge and gravity anomaly cancellation and
  - gauge invariant Yukawa terms for neutrino mass generation

Focus only on addition to the SM, find SM in this new book:



Fermions
(with new highlighted)

- fermion fields:

$$\psi^f_{q,1} = \begin{pmatrix} U^f \\ D^f \end{pmatrix}_{\rm L} \qquad \psi^f_{q,2} = U^f_{\rm R}\,, \qquad \psi^f_{q,3} = D^f_{\rm R}$$

$$\psi^f_{l,1} = \begin{pmatrix} \nu^f \\ \ell^f \end{pmatrix}_{\rm L} \qquad \psi^f_{l,2} = \nu^f_{\rm R}\,, \qquad \psi^f_{l,3} = \ell^f_{\rm R}$$

where

$$\psi_{\rm L/R} \equiv \psi_{\mp} = \frac{1}{2}\left(1 \mp \gamma_5\right)\psi \equiv P_{\rm L/R}\psi$$

($\nu_{\rm L}$ can $\nu_{\rm R}$ can also be Majorana neutrinos, embedded into *different* Dirac spinors)

- covariant derivative (includes kinetic mixing):

$$D^\mu_j = \partial^\mu + {\rm i}g_{\rm L}\,\boldsymbol{T}\cdot\boldsymbol{W}^\mu + {\rm i}g_Y\,y_j B'^\mu + {\rm i}(\underbrace{g'_Z\,z_j - g'_Y\,y_j}_{\equiv g'_Z r_j + (g'_Z - g'_Y)y_j})Z'^\mu$$

Scalars

- Standard Φ complex SU(2)$_L$ doublet and new χ complex singlet:

$$\mathcal{L}_{\phi,\chi} = [D_\mu^{(\phi)}\phi]^* D^{(\phi)\,\mu}\phi + [D_\mu^{(\chi)}\chi]^* D^{(\chi)\,\mu}\chi - V(\phi,\chi)$$

- with scalar potential

$$V(\phi,\chi) = V_0 - \mu_\phi^2|\phi|^2 - \mu_\chi^2|\chi|^2 + \left(|\phi|^2, |\chi|^2\right)\begin{pmatrix} \lambda_\phi & \frac{\lambda}{2} \\ \frac{\lambda}{2} & \lambda_\chi \end{pmatrix}\begin{pmatrix} |\phi|^2 \\ |\chi|^2 \end{pmatrix}$$

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- After SSB, G → SU(3)$_c$×U(1)$_{QED}$:

$$\phi = \frac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\boldsymbol{T}\cdot\boldsymbol{\xi}(x)/v}\begin{pmatrix}0\\ v+h'(x)\end{pmatrix} \;\&\; \chi(x) = \frac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\eta(x)/w}\left(w+s'(x)\right)$$

Anomaly free charge assignment

field	$SU(3)_c$	$SU(2)_L$	y_j	$z_j^{(a)}$	$z_j^{(b)}$	$r_j = z_j/z_\phi - y_j^{(c)}$
$U_L,\ D_L$	3	2	$\frac{1}{6}$	Z_1	$\frac{1}{6}$	0
U_R	3	1	$\frac{2}{3}$	Z_2	$\frac{7}{6}$	$\frac{1}{2}$
D_R	3	1	$-\frac{1}{3}$	$2Z_1 - Z_2$	$-\frac{5}{6}$	$-\frac{1}{2}$
$\nu_L,\ \ell_L$	1	2	$-\frac{1}{2}$	$-3Z_1$	$-\frac{1}{2}$	0
ν_R	1	1	0	$Z_2 - 4Z_1$	$\frac{1}{2}$	$\frac{1}{2}$
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(a) anomaly free charges (b) from neutrino-scalar interactions (c) from re-parametrization of couplings

Fermion-scalar interactions

- Standard Yukawa terms:

$$\mathcal{L}_{\rm Y} = -\left[c_D \left(\bar{U},\, \bar{D}\right)_{\rm L} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} D_{\rm R} + c_U \left(\bar{U},\, \bar{D}\right)_{\rm L} \begin{pmatrix} \phi^{(0)\,*} \\ -\phi^{(+)\,*} \end{pmatrix} U_{\rm R} + c_\ell \left(\bar{\nu}_\ell,\, \bar{\ell}\right)_{\rm L} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \ell_{\rm R}\right] + {\rm h.c.}$$

- lead to fermion masses after SSB:

$$\mathcal{L}_{\rm Y} = -\left(1 + \frac{h(x)}{v}\right)\left[\bar{D}_{\rm L}\, M_D\, D_{\rm R} + \bar{U}_{\rm L}\, M_U\, U_{\rm R} + \bar{\ell}_{\rm L}\, M_\ell\, \ell_{\rm R}\right] + {\rm h.c.}$$

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- Neutrino Yukawa terms $(z_\chi = -2z_{\nu_{\mathrm{R}}})$:

$$\mathcal{L}^{\nu}_{\mathrm{Y}} = -\sum_{i,j}\left((c_\nu)_{ij}\bar{L}_{i,\mathrm{L}}\cdot\tilde{\phi}\,\nu_{j,\mathrm{R}} + \frac{1}{2}(c_{\mathrm{R}})_{ij}\,\overline{\nu^c_{i,\mathrm{R}}}\nu_{j,\mathrm{R}}\,\chi\right) + \mathrm{h.c.}$$

(Dirac mass terms)

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(Majorana mass terms)

Charge assignment from gauge invariant neutrino interactions

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After SSB neutrino mass terms appear

$$\mathcal{L}_{\mathrm{Y}}^{\nu} = -\frac{1}{2}\sum_{i,j}\left[\left(\overline{\nu_{\mathrm{L}}},\, \overline{\nu_{\mathrm{R}}^{c}}\right)_i M(h,s)_{ij} \begin{pmatrix} \nu_{\mathrm{L}}^{c} \\ \nu_{\mathrm{R}} \end{pmatrix}_j + \mathrm{h.c.}\right]$$

where

$$M(h,s)_{ij} = \begin{pmatrix} 0 & m_{\mathrm{D}}\left(1+\frac{h}{v}\right) \\ m_{\mathrm{D}}\left(1+\frac{h}{v}\right) & M_{\mathrm{M}}\left(1+\frac{s}{w}\right) \end{pmatrix}_{ij}$$

6x6 symmetric matrix (m_{D} complex, M_{M} real)

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6x6 symmetric matrix ($m_{\rm D}$ complex, $M_{\rm M}$ real)

in diagonal: Majorana mass terms (so $\nu_{\rm L}$ massless!)

but $\nu_{\rm L}$ and $\nu_{\rm R}$ have the same q-numbers,
can mix, leading to type-I see-saw

Effective light neutrino masses

If $m_i << M_j$, can integrate out the heavy neutrinos

$$\mathcal{L}^{\nu}_{\mathrm{dim}-5} = -\frac{1}{2}\sum_i m_{\mathrm{M},i}\left(1+\frac{h}{v}\right)^2\left(\overline{\nu'^{c}_{i,\mathrm{L}}}\nu'_{i,\mathrm{L}} + \mathrm{h.c.}\right)$$

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if m_i ~ O(100keV) and M_j ~ O(100GeV), then

$$m_{M,i} \sim \mathrm{O(0.1eV)}$$

Mixing in the neutral gauge sector

$$\begin{pmatrix} W_\mu^3 \\ B'_\mu \\ Z'_\mu \end{pmatrix} = \underline{M}(\sin\theta_{\mathrm{W}}, \sin\theta_{\mathrm{T}}) \begin{pmatrix} Z_\mu^0 \\ T_\mu \\ A_\mu \end{pmatrix}$$

- QED current remains unchanged:

$$\mathcal{L}_{\mathrm{QED}} = -eA_\mu J^\mu_{\mathrm{em}}$$

$$J^\mu_{\mathrm{em}} = \sum_{f=1}^{3}\sum_{j=1}^{3} e_j \Big(\overline{\psi}^f_{q,j}(x)\gamma^\mu \psi^f_{q,j}(x) + \overline{\psi}^f_{l,j}(x)\gamma^\mu \psi^f_{l,j}(x) \Big)$$

Neutral current interactions

- current with Z^0 remains unchanged, but mixes with new current J_T of new couplings:

$$\mathcal{L}_Z = -eZ_\mu \Big(\cos\theta_T J_Z^\mu - \sin\theta_T J_T^\mu \Big) = -eZ_\mu J_Z^\mu + \mathrm{O}(\theta_T)$$

$$\mathcal{L}_T = -eT_\mu \Big(\sin\theta_T J_Z^\mu + \cos\theta_T J_T^\mu \Big) = -eT_\mu J_T^\mu + \mathrm{O}(\theta_T)$$

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both can be written as v-a interactions for non-chiral fields:

$$J_X^\mu = \sum_f \overline{\psi}_f(x)\gamma^\mu\big(v_f^{(X)} - a_f^{(X)}\gamma_5\big)\psi_f(x)$$

with $X = Z$ or T and summation over q and l flavors

Possible consequences with 5 new parameters

- The lightest massive new particle is a natural candidate for WIMP dark matter if it is sufficiently stable.
- Majorana neutrino mass terms are generated by the SSB of the scalar fields, providing the origin of neutrino masses and oscillations.
- Diagonalization of neutrino mass terms leads to the PMNS matrix, which in turn can be the source of lepto-baryogenesis.
- The vacuum of the χ scalar is charged ($z_j = -1$) that may be a source of accelerated expansion of the universe as seen now.
- The second scalar together with the established BEH field may be the source of hybrid inflation.

Credibility requirement

Is there any region of the parameter space of the model that is not excluded by experimental results, both established in standard model phenomenology and elsewhere?

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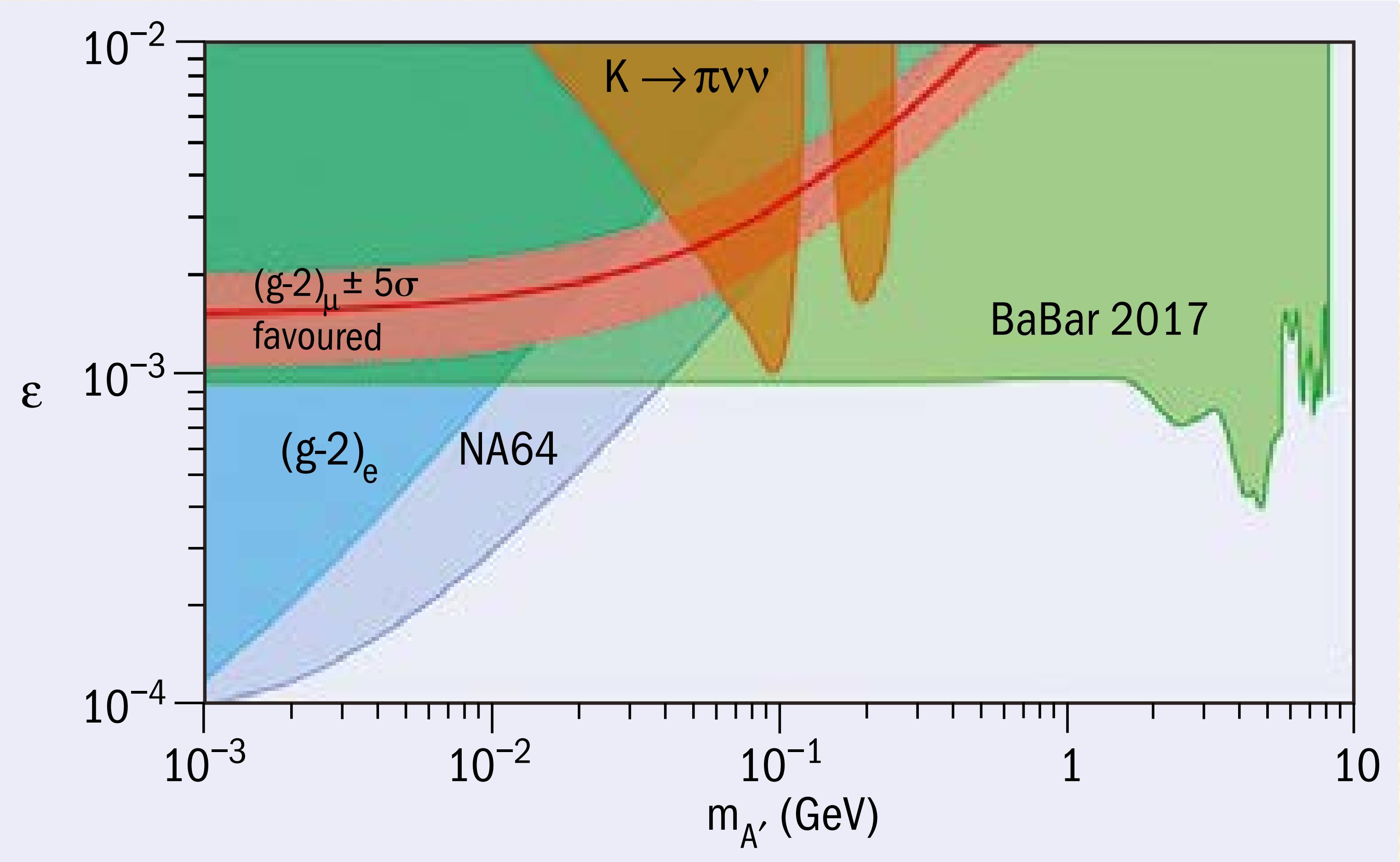
Answer is not immediate, extensive studies are needed

A′ explanation of the muon magnetic moment anomaly ruled out?

CERN Courier **April 2017**



Further reading

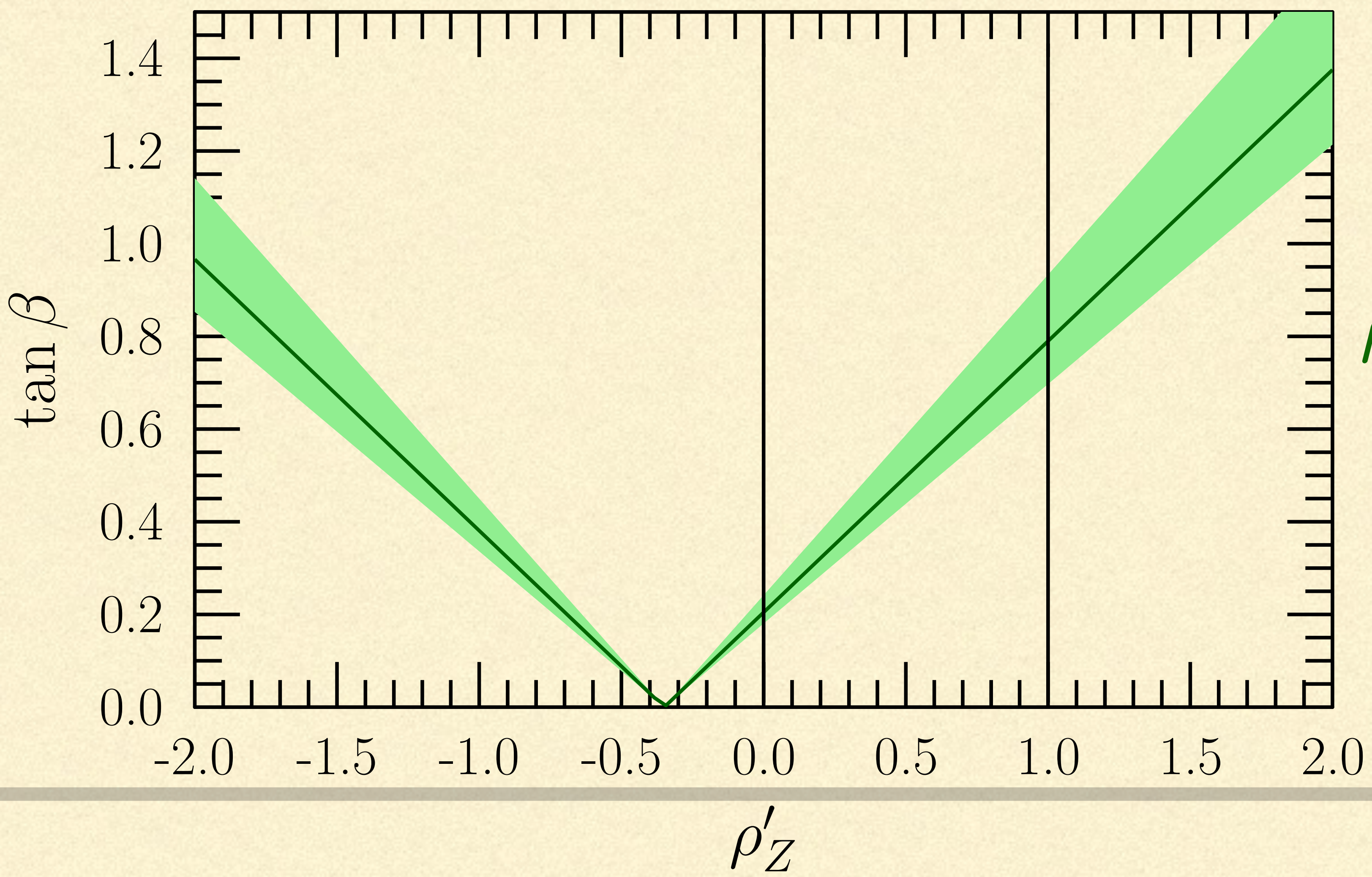
BaBar Collaboration 2017 arXiv:1702.03327.
NA64 Collaboration 2017 *Phys. Rev. Lett.* **118** 011802.

Contribution of the new gauge boson to a_μ

$$a_\mu^{(\mathrm{T+SM})} - a_\mu^{(\mathrm{SM})} = \frac{G_\mathrm{F} m_\mu^2}{6\sqrt{2}\pi^2}\left(\frac{(1+\rho'_Z)\cos^2\theta_\mathrm{W} - \frac{1}{2}}{\tan\beta} + \mathrm{O}(\theta_T,\ \gamma'_Z)\right)^2$$

experimentally: $a_\mu^{(\mathrm{exp})} - a_\mu^{(\mathrm{SM})} = 268(76)\cdot 10^{-11}$

$\tan\beta = \frac{w}{v}$

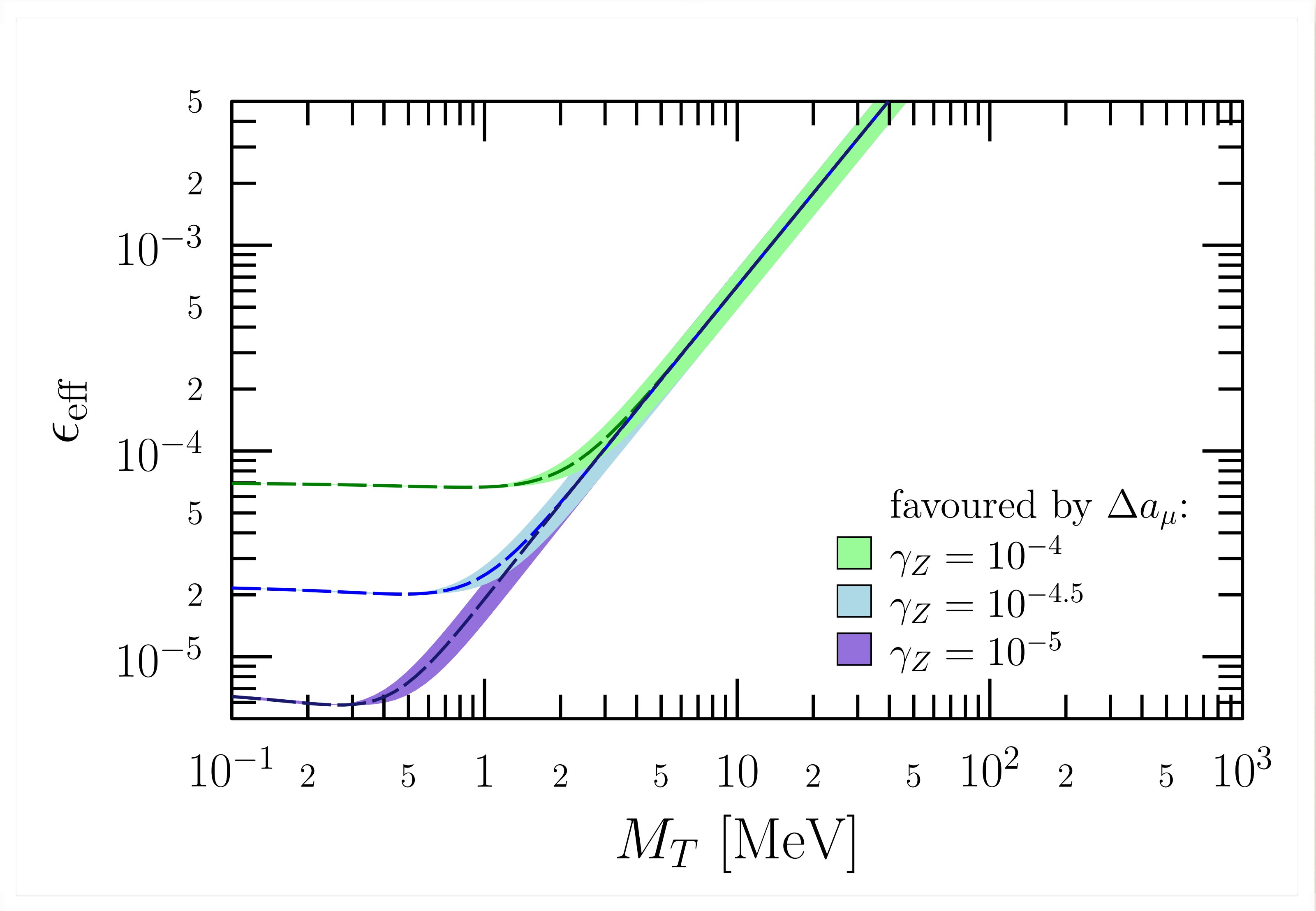


$\rho'_Z = 1 - \frac{g'_Y}{g'_Z}$

Favoured region by Δa_μ differs from that in kinetic mixing model

Favoured region of $\Delta a_\mu = a_\mu^{(exp)} - a_\mu^{(SM)} = (268 \pm 76)\, 10^{-11}$ in the kinetic mixing–vector boson mass plane with the existence of a T^0 boson

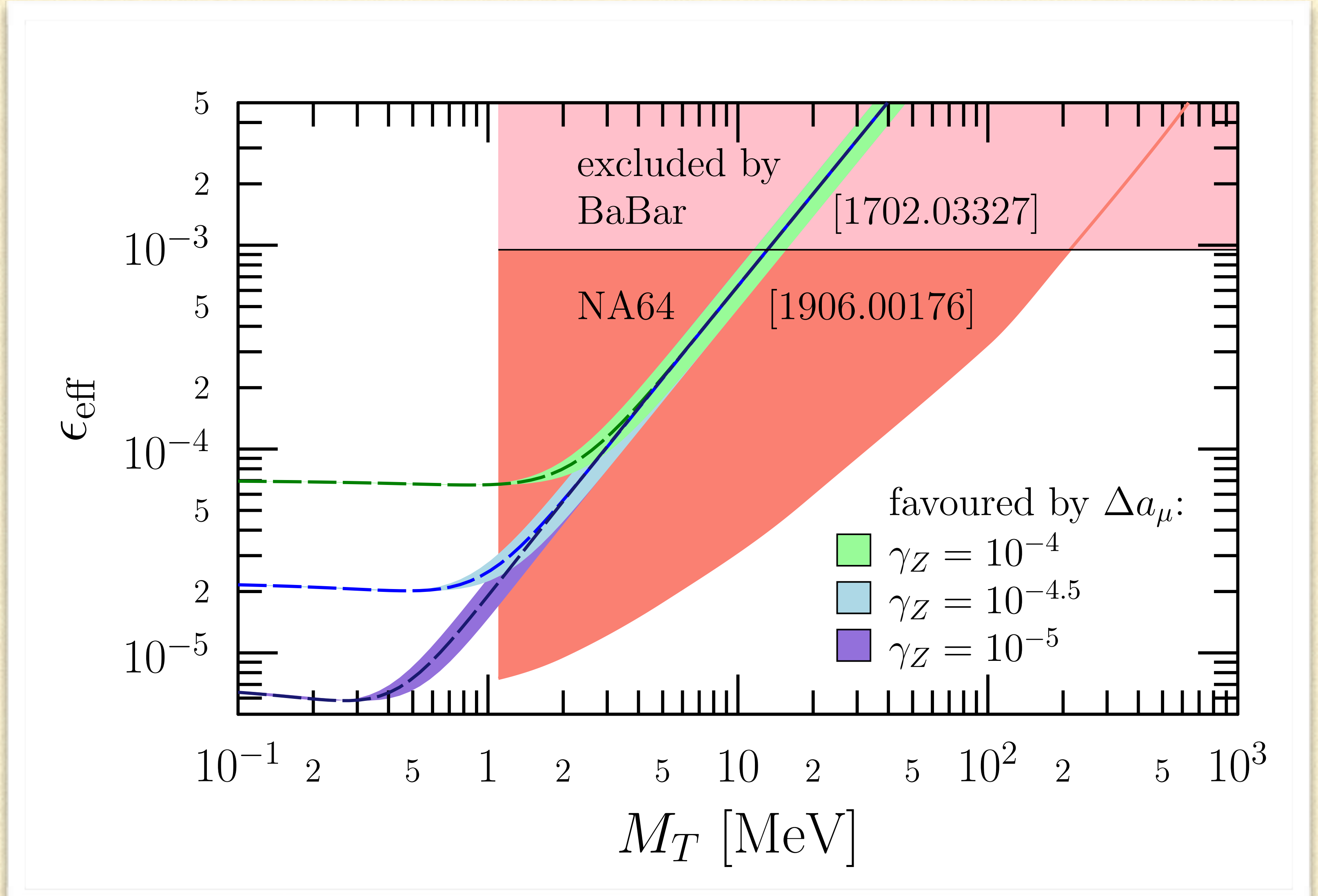
$$\epsilon_{\text{eff}} = \sqrt{\frac{\sigma(e^+e^- \to \gamma \mathrm{T}^0)}{\sigma(e^+e^- \to \gamma A')/\epsilon^2}}$$



Favoured region by Δa_μ differs from that in kinetic mixing model

BaBar and NA64 together allow for the interpretation of $\Delta a_\mu = a_\mu^{(exp)} - a_\mu^{(SM)} = (268 \pm 76)\, 10^{-11}$ with the existence of a T^0 boson only if $M_T < 1.1$ MeV

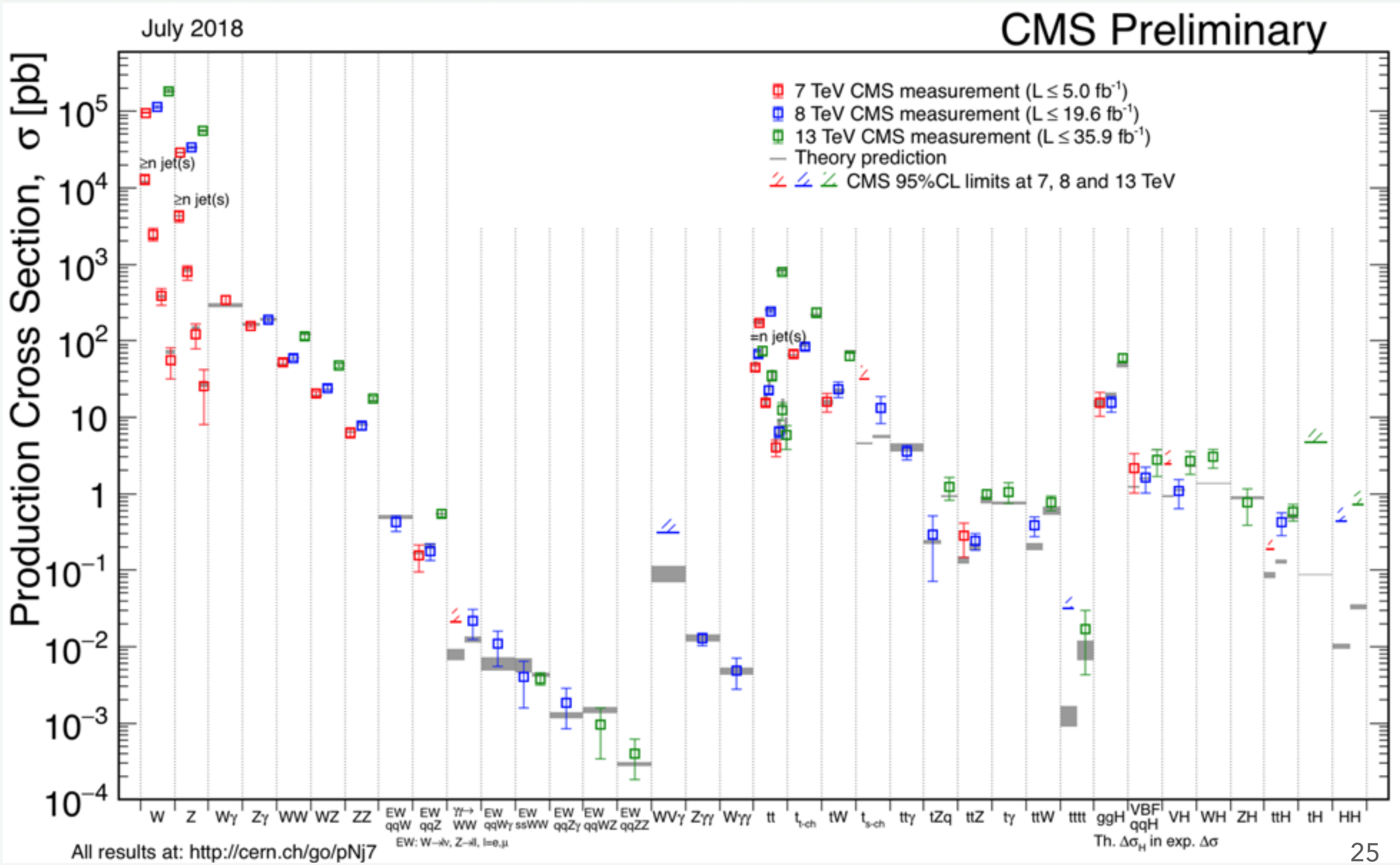
$$\epsilon_{\text{eff}} = \sqrt{\frac{\sigma(e^+e^- \to \gamma T^0)}{\sigma(e^+e^- \to \gamma A')/\epsilon^2}}$$



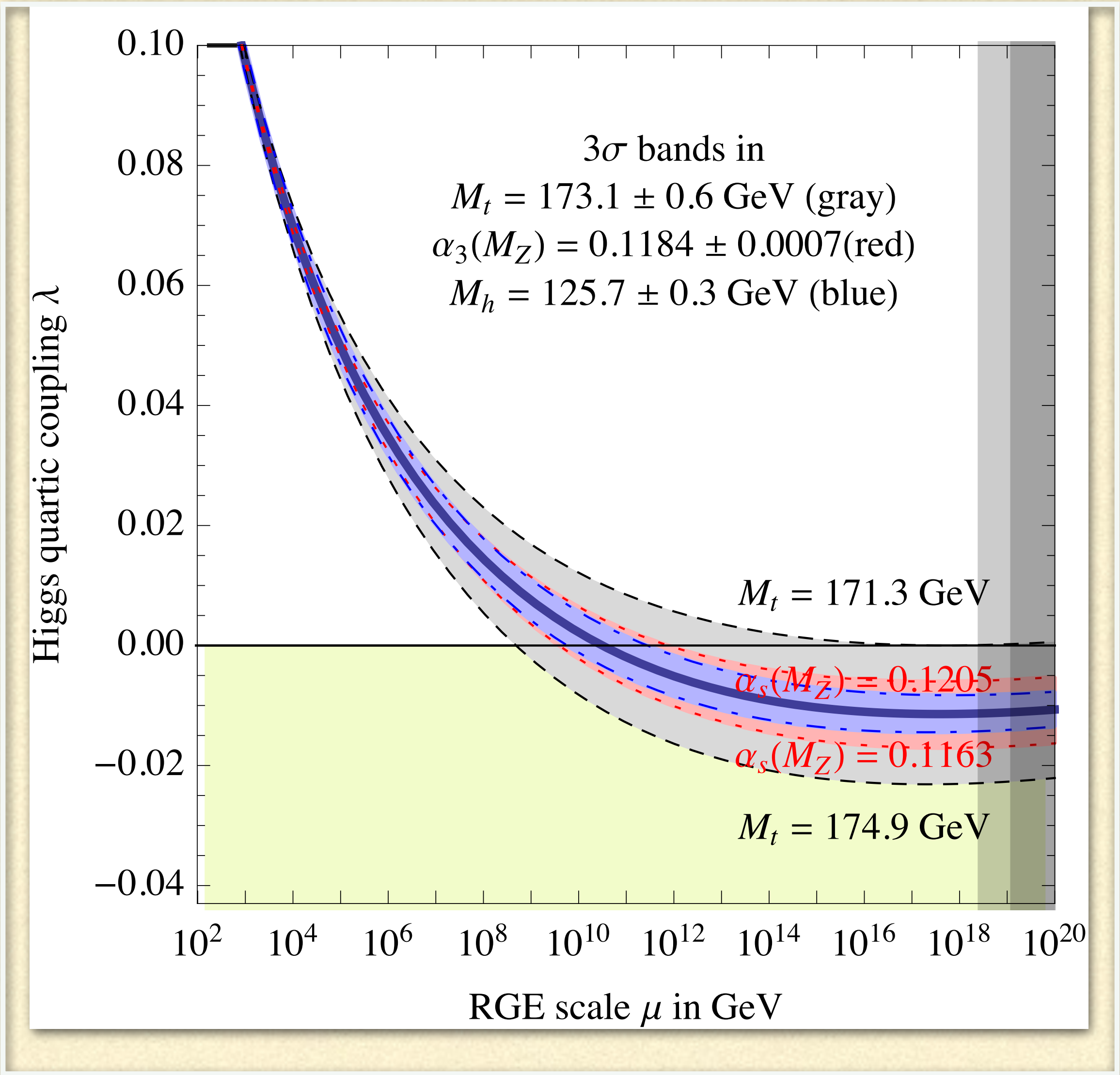
Conclusions

- Established observations require physics beyond SM, but do not suggest a rich BSM physics
- $U(1)_Z$ extension has the potential of explaining all known results
- Anomaly cancellation and neutrino mass generation mechanism are used to fix the Z-charges up to reasonable assumptions
- Parameter space can and need be constrained from existing experimental results (like searches in missing energy events)

SM@LHC: theory vs. 36 measurements at CMS



SM is unstable



Degrassi et al., arXiv:1205.6497

Neutrino masses

First diagonalize $m_{\rm D}$ and $M_{\rm M}$ by defining

$$\nu'_{{\rm L},i} = \sum_j (U_{\rm L})_{ij} \nu_{{\rm L},j} \quad \text{and} \quad \nu'_{{\rm R},i} = \sum_j (O_{\rm R})_{ij} \nu_{{\rm R},j}$$

so

$$\mathcal{L}_{\rm Y}^{\nu} = -\frac{1}{2} \sum_{i,j} \left[\left(\overline{\nu'_{\rm L}}, \overline{\nu'^{c}_{\rm R}} \right)_i M'(h,s)_{ij} \begin{pmatrix} \nu'^{c}_{\rm L} \\ \nu'_{\rm R} \end{pmatrix}_j + \text{h.c.} \right]$$

where

$$M'(h,s) = \begin{pmatrix} 0 & mV\left(1+\frac{h}{v}\right) \\ V^{\dagger}m\left(1+\frac{h}{v}\right) & M\left(1+\frac{s}{w}\right) \end{pmatrix}$$

with m and M diagonal, $V = U_{\rm L}^T O_{\rm R}$ unitary matrix