

Electroweak bosonic 2-loop corrections to Z-pole precision observables

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I. Dubovsky, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, arXiv:1607.08375, arXiv:17mm.nnnnn

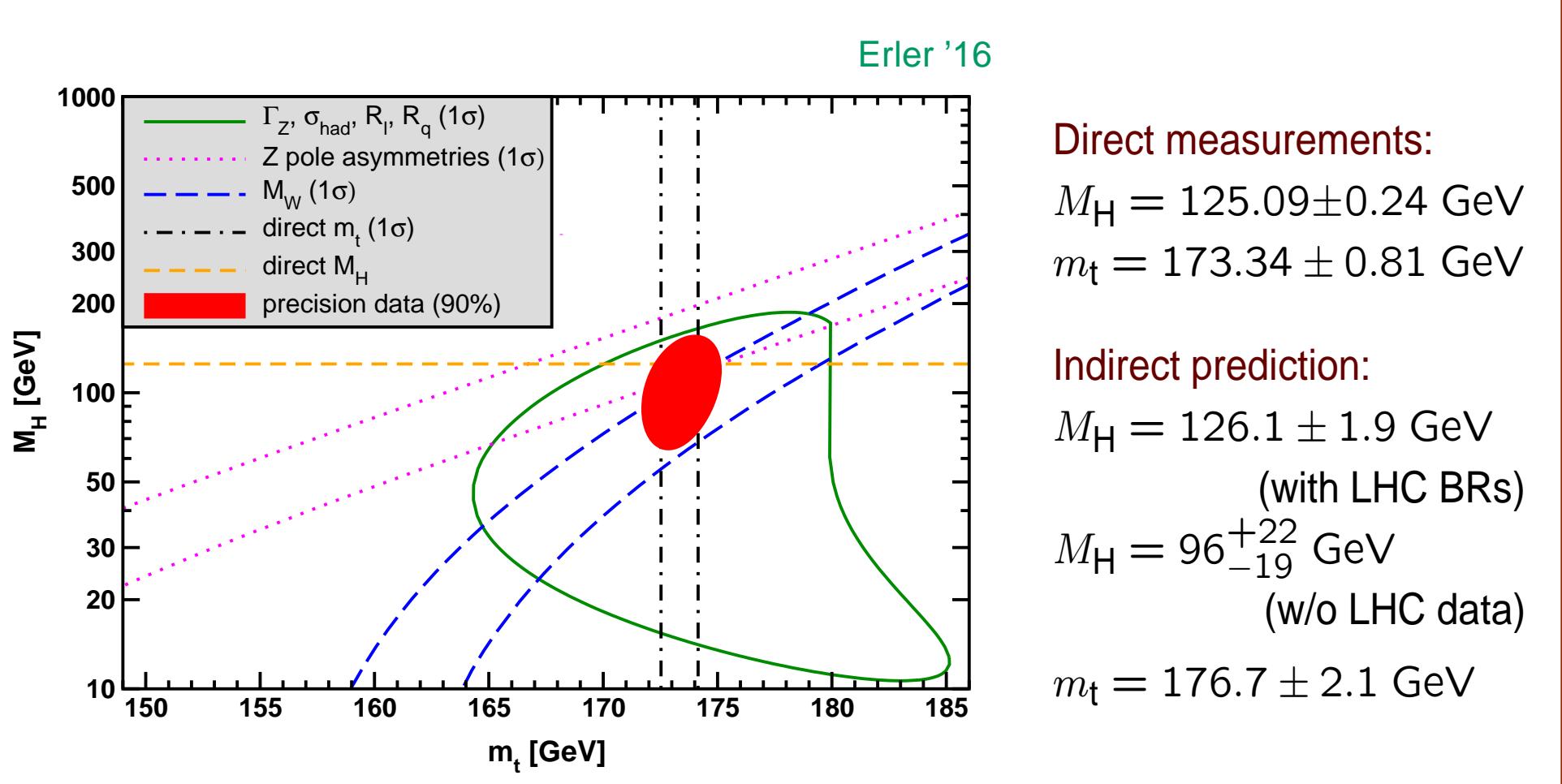
1. Current status of electroweak precision observables

2. Bosonic $\mathcal{O}(\alpha^2)$ corrections

3. Results

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables

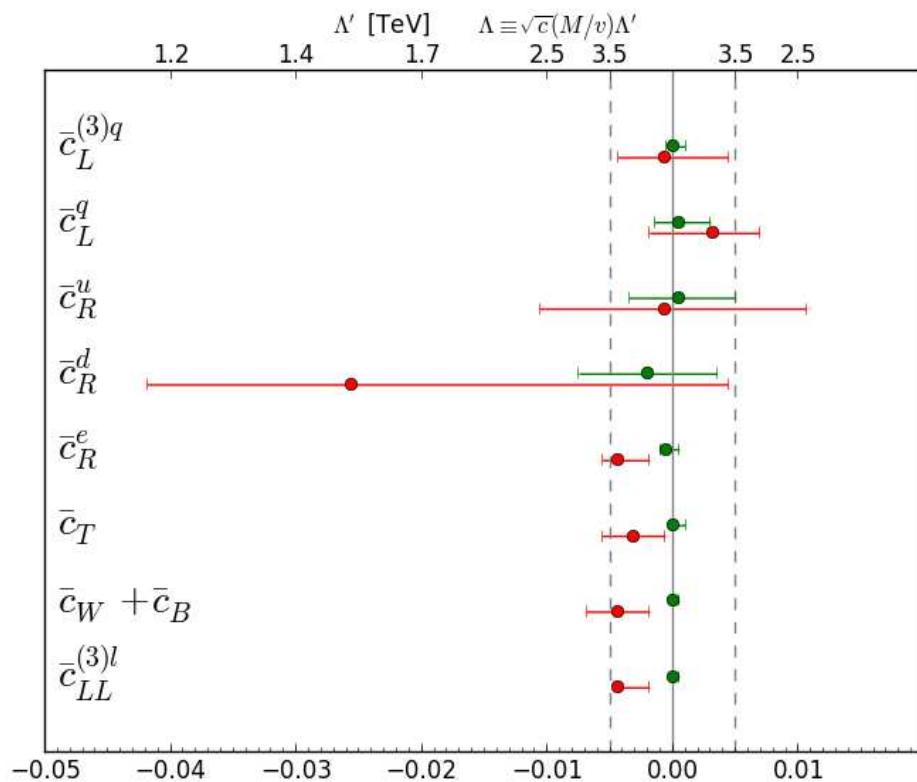


Constraints on some dim-6 operators

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Assuming flavor universality:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \quad (\Lambda \gg M_Z)$$



$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$$

$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$$

$$\mathcal{O}_{LL}^{(3)e} = (\bar{L}_L^e \sigma^a \gamma_\mu L_L^e)(\bar{L}_L^e \sigma^a \gamma^\mu L_L^e)$$

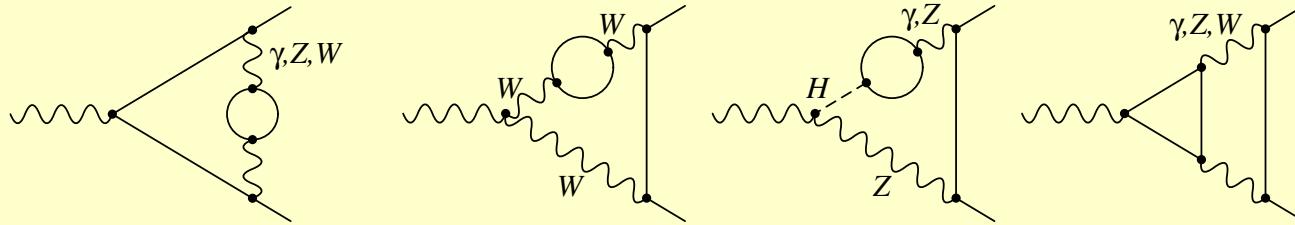
$$\mathcal{O}_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_R \gamma^\mu f_R)$$

$$\mathcal{O}_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_L \gamma^\mu F_L)$$

$$\mathcal{O}_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_L \sigma_a \gamma^\mu F_L)$$

Pomaral, Riva '13
Ellis, Sanz, You '14

Known corrections to Δr , $\sin^2 \theta_{\text{eff}}^f$, $g_V f$, $g_A f$:



- Complete NNLO corrections (Δr , $\sin^2 \theta_{\text{eff}}^\ell$) Freitas, Hollik, Walter, Weiglein '00
Awramik, Czakon '02; Onishchenko, Veretin '02
Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
 - “Fermionic” NNLO corrections ($g_V f$, $g_A f$) Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98
Freitas '13,14
 - Partial 3/4-loop corrections to ρ/T -parameter
 $\mathcal{O}(\alpha_t \alpha_s^2)$, $\mathcal{O}(\alpha_t^2 \alpha_s)$, $\mathcal{O}(\alpha_t \alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95
Faisst, Kühn, Seidensticker, Veretin '03
Boughezal, Tausk, v. d. Bij '05
Schröder, Steinhauser '05; Chetyrkin et al. '06
Boughezal, Czakon '06
- $(\alpha_t \equiv \frac{y_t^2}{4\pi})$

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	$\alpha^3, \alpha^2 \alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2 \alpha_s, \alpha \alpha_s^2$
σ_{had}^0	41540 ± 37 pb	6 pb	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2 \alpha_s$
$R_b \equiv \Gamma_Z^b / \Gamma_Z^{\text{had}}$	0.21629 ± 0.00066	0.00015	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2 \alpha_s$
$\sin^2 \theta_{\text{eff}}^\ell$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^3, \alpha^2 \alpha_s$

- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
 - Count prefactors (α, N_c, N_f, \dots)
 - Extrapolation of perturbative series
 - Renormalization scale dependence
 - Renormalization scheme dependence
- Also parametric error from external inputs ($m_t, m_b, \alpha_s, \Delta \alpha_{\text{had}}, \dots$)

Example: Error estimation for Γ_Z

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- Geometric perturbative series

$$(\alpha_t \equiv y_t^2/4\pi)$$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2 \alpha_s) - \mathcal{O}(\alpha_t^2 \alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha \alpha_s^3) - \mathcal{O}(\alpha_t \alpha_s^3) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{\text{bos}}^2) \sim \mathcal{O}(\alpha_{\text{bos}})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{\text{bos}}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$

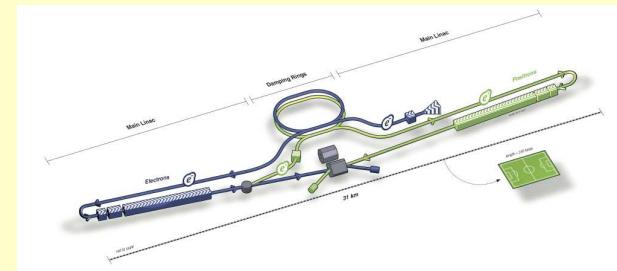
$$\mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \Gamma_Z \frac{\alpha n_{\text{lg}}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta \Gamma_Z \approx 0.5 \text{ MeV}$

Future high-energy e^+e^- colliders

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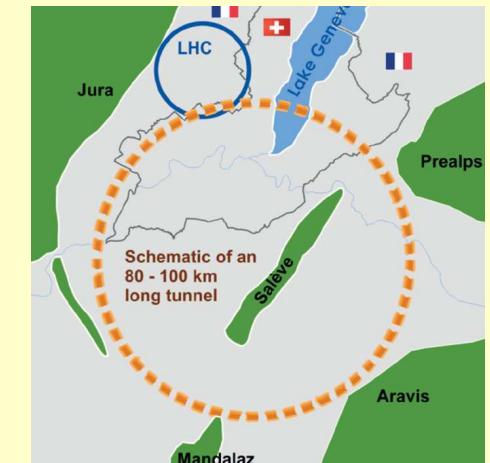
- International Linear Collider (ILC)
Int. lumi at $\sqrt{s} \sim M_Z$: 50–100 fb $^{-1}$



- Circular Electron-Positron Collider (CEPC)
Int. lumi at $\sqrt{s} \sim M_Z$: $2 \times 150 \text{ fb}^{-1}$



- Future Circular Collider (FCC-ee)
Int. lumi at $\sqrt{s} \sim M_Z$: $> 2 \times 30 \text{ ab}^{-1}$



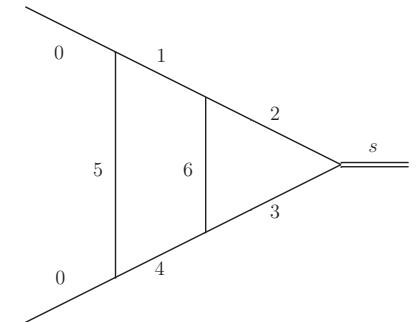
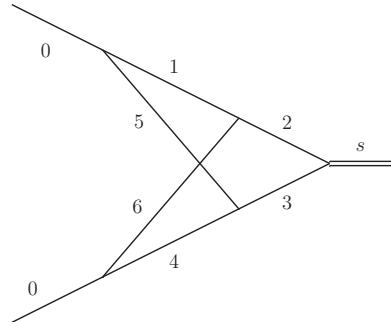
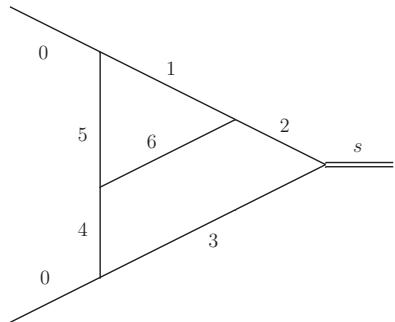
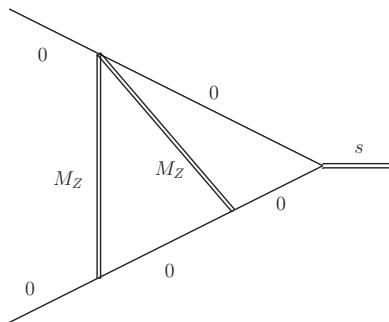
	Measurement error				Intrinsic theory	
	Current	ILC	CEPC	FCC-ee	Current	Future [†]
M_W [MeV]	15	3–4	3	1	4	1
Γ_Z [MeV]	2.3	0.8	0.5	0.1	0.5	0.2
R_b [10^{-5}]	66	14	17	6	15	7
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	16	1	2.3	0.6	4.5	1.5

→ Existing theoretical calculations adequate for LEP/SLC/LHC,
but not ILC/CEPC/FCC-ee!

[†] **Theory scenario:** $\mathcal{O}(\alpha_{\text{bos}}^2)$, $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$
 $(N_f^n = \text{at least } n \text{ closed fermion loops})$

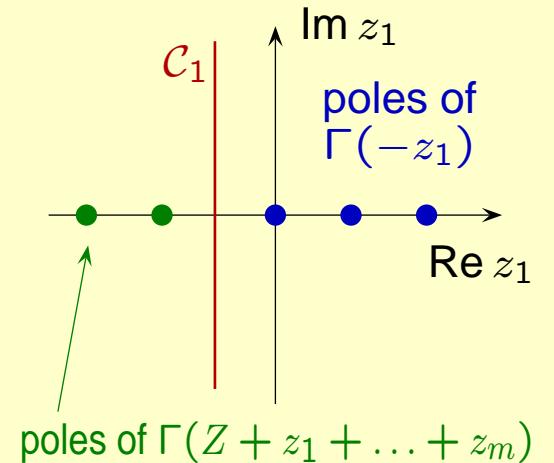
- Two-loop diagrams without closed fermion loops
- On-shell renormalization
- Self-energies (incl. from renormlization) and vertices with sub-loop bubbles using dispersion relation technique

S. Bauberger et al. '95
Awramik, Czakon, Freitas '06
- Non-trivial vertex diagrams:
 - Sector decomposition (FESTA 3 / SecDec 3) Smirnov '14; Borowka et al. '15
 - Mellin-Barnes representations (MB / AMBRE 3 / MBnumerics) Czakon '06
Dubovyk, Gluza, Riemann '15; Usovitsch '17
 - No tensor reduction (besides trivial cancellations)
 → $\mathcal{O}(1000)$ different two-loop vertex integrals



Transform Feynman integral with Mellin-Barnes representation

$$\begin{aligned} \frac{1}{(A_0 + \dots + A_m)^Z} &= \frac{1}{(2\pi i)^m} \int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m \\ &\times A_1^{z_1} \cdots A_m^{z_m} A_0^{-Z - z_1 - \dots - z_m} \\ &\times \frac{\Gamma(-z_1) \cdots \Gamma(-z_m) \Gamma(Z + z_1 + \dots + z_m)}{\Gamma(Z)}, \end{aligned}$$

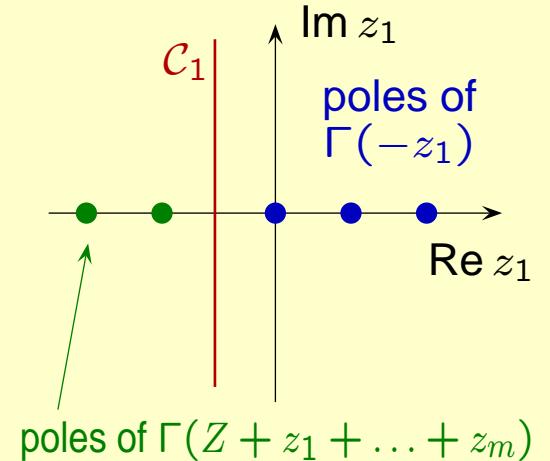


Transform Feynman integral with Mellin-Barnes representation

$$\frac{1}{(A_0 + \dots + A_m)^Z} = \frac{1}{(2\pi i)^m} \int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m$$

$$\times A_1^{z_1} \cdots A_m^{z_m} A_0^{-Z - z_1 - \dots - z_m}$$

$$\times \frac{\Gamma(-z_1) \cdots \Gamma(-z_m) \Gamma(Z + z_1 + \dots + z_m)}{\Gamma(Z)},$$

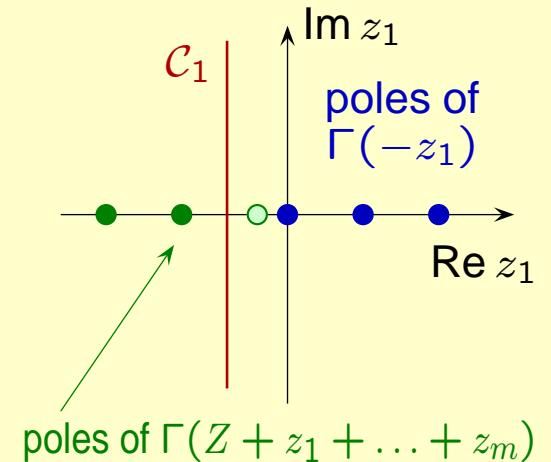


- Consistent choice of all \mathcal{C}_i often requires $\varepsilon \neq 0$
 $(Z = n + \epsilon)$

- For $\varepsilon \rightarrow 0$: residues from pole crossings
 $\rightarrow 1/\varepsilon^k$ terms

Czakon '06
Anastasiou, Daleo '06

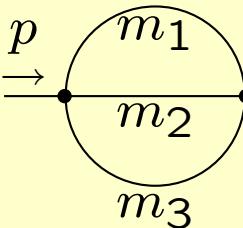
- Do remaining \mathcal{C}_i integrations numerically



$\varepsilon \rightarrow 0$

Mellin-Barnes representations

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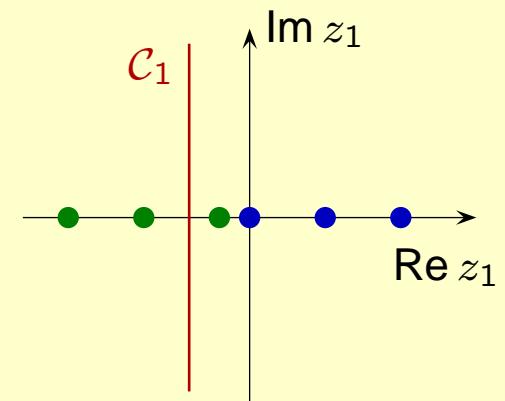


$$\begin{aligned}
 &= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 (m_1^2)^{-\varepsilon - z_1 - z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon + z_1 - z_3} (-p^2)^{z_3} \\
 &\quad \times \Gamma(-z_2) \Gamma(-z_3) \Gamma(1 + z_1 + z_2) \Gamma(z_3 - z_1) \\
 &\quad \times \frac{\Gamma(1 - \varepsilon - z_2) \Gamma(\varepsilon + z_1 + z_2) \Gamma(\varepsilon - 1 - z_1 + z_3)}{\Gamma(2 - \varepsilon + z_3)}
 \end{aligned}$$

$$z_3 = c_3 + iy_3, \quad y_i \in (-\infty, \infty)$$

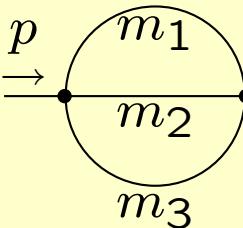
$$(-p^2)^{z_3} = \underbrace{(p^2)^{c_3+iy_3} e^{-i\pi c_3}}_{\text{oscillating}} \underbrace{e^{\pi y_3}}_{\text{div. for } y_3 \rightarrow \infty}$$

div. for $y_3 \rightarrow \infty$,
eventually over-
come by Γ funct.



Mellin-Barnes representations

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$$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 (m_1^2)^{-\varepsilon - z_1 - z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon + z_1 - z_3} (-p^2)^{z_3}$$

$$\times \Gamma(-z_2) \Gamma(-z_3) \Gamma(1 + z_1 + z_2) \Gamma(z_3 - z_1)$$

$$\times \frac{\Gamma(1 - \varepsilon - z_2) \Gamma(\varepsilon + z_1 + z_2) \Gamma(\varepsilon - 1 - z_1 + z_3)}{\Gamma(2 - \varepsilon + z_3)}$$

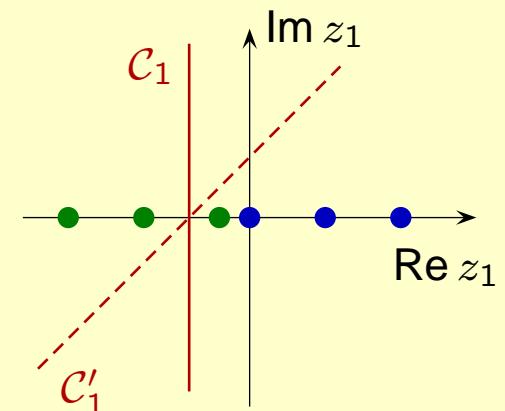
$$z_3 = c_3 + iy_3, \quad y_i \in (-\infty, \infty)$$

$$(-p^2)^{z_3} = \underbrace{(p^2)^{c_3+iy_3} e^{-i\pi c_3}}_{\text{oscillating}} \underbrace{e^{\pi y_3}}_{\text{div. for } y_3 \rightarrow \infty}$$

$$y_i \rightarrow y_i - i\theta$$

$$(-p^2)^{z_3} = (p^2)^{c_3+iy_3} e^{-i\pi(c_3+\theta y_i)} e^{(\pi+\theta \log p^2)y_3}$$

Huang, Freitas '10



Counter rotations not always successful:

$$\frac{1}{(2\pi i)^2} \int dz_1 dz_2 2(m^2)^{-2} \left(-\frac{p^2}{m^2}\right)^{-z_1-z_2} \times \frac{\Gamma(-z_2)\Gamma^3(1+z_2)\Gamma(-z_1-z_2)\Gamma(1+z_1+z_2)\Gamma(-1-z_1-2z_2)}{\Gamma(1-z_1)}$$

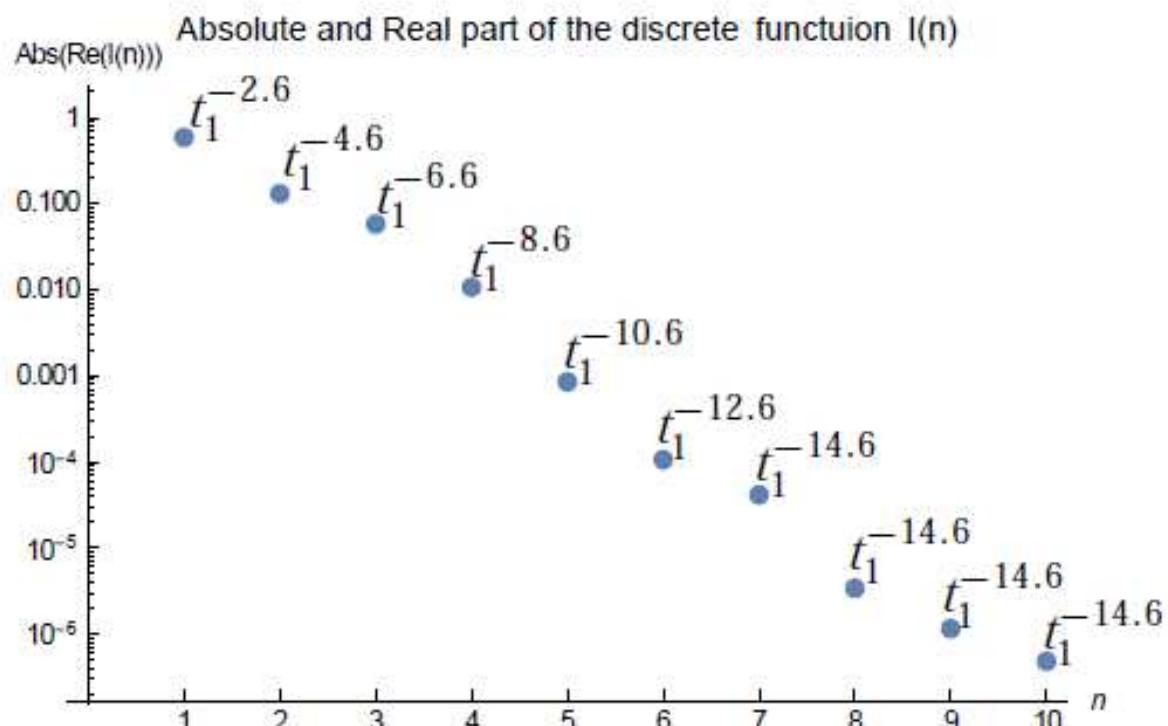
For $p^2 = m^2$ contour rotation has no effect

Shift countour: $z_1 = c_1 + iy_1, z_2 = c_2 + n + iy_2$

- Worst asymptotic behaviour of integrand for $y_1 \rightarrow -\infty, y_2 = 0$:
 $\sim y_1^{-2-2(c_2+n)}$ (for $n = 0$ and $c_2 = -0.7$: $\sim y_1^{-0.6}$)
- Pick up (finite number of) pole residues from contour shift

- Shifts improve asymptotic behaviour and size of numerical integral
- Automatic algorithms for finding suitable shifts in development
(MBnumerics)

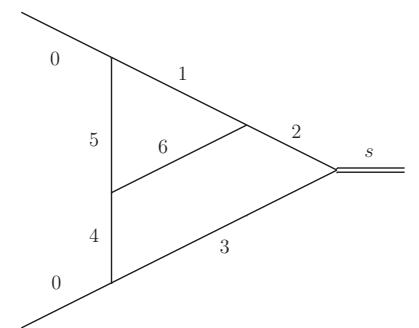
Usovitsch '17



$$m_1 = m_t, \quad m_5 = m_6 = M_W, \quad m_2 = m_3 = m_4 = 0$$

SecDec: (24 hours)

$$\begin{aligned} I_{\text{SD}} = & 1.541 + 0.2487 i + \frac{1}{\epsilon} (0.123615 - 1.06103 i) \\ & + \frac{1}{\epsilon^2} (-0.3377373796 - 5 \times 10^{-10} i) \end{aligned}$$



MBnumerics: (43 min.)

$$\begin{aligned} I_{\text{MB}} = & 1.541402128186602 + 0.248804198197504 i \\ & + \frac{1}{\epsilon} (0.12361459942846659 - 1.0610332704387688 i) \\ & + \frac{1}{\epsilon^2} (-0.33773737955057970 + 3.6 \times 10^{-17} i) \end{aligned}$$

$m_1 = M_Z$, rest zero

SecDec: error $\gg 1$

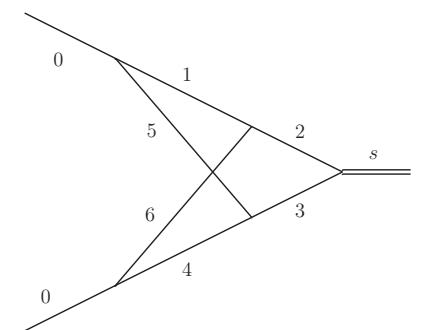
MBnumerics: (finite part)

$$-0.7785996083 - 4.12351260 i$$

Analytical:

Fleischer, Kotikov, Veretin '98

$$-0.7785996090 - 4.12351259 i$$



Sector decomposition:

- Fully automated for (almost) any multi-loop diagram
→ public tools available
- Numerical stability and precision difficult in some cases

Mellin-Barnes:

- Contour shift method applied successfully for 2-loop vertices
→ Good numerical precision
- Extension to more loops/legs possible, but more work needed
- Partial automatization possible, but full automatization difficult
(nested interdependent shifts for multi-dimensional integrals)
- Package MBnumerics under development

Usovitsch '17

- After removal of singularities through sector decomposition:

$$I_{\text{reg}}^{(1)} = \int_0^1 dx_1 \dots dx_{n-1} (A - i\epsilon)^{-k}$$

- Physical thresholds: A changes sign in ingration region

→ Problematic for numerical integrators

→ Deform integration into complex plane:

Nagy, Soper '06

$$x_i = z_i - i\lambda z_i(1 - z_i) \frac{\partial A}{\partial z_i}, \quad 0 \leq z_i \leq 1.$$

$$A(\vec{x}) = A(\vec{z}) - i\lambda \sum_i z_i(1 - z_i) \left(\frac{\partial A}{\partial z_i} \right)^2 + \mathcal{O}(\lambda^2).$$

Typical choice: $\lambda \sim 0.5 - 1$

- Potential issues:

→ $\partial A / \partial z_i$ may vanish in certain sub-spaces

→ Thresholds may be at edge of integration region

- Comparison of SecDec and MBtools results (successful for most integrals)
- Comparison for Euclidian kinematics (results for all integrals with SecDec)
- Comparison to analytical results for some single-scale and two-scale integrals
 - Fleischer, Kotikov, Veretin '99
 - Aglietti, Bonciani '03,04
 - Aglietti, Bonciani, Grassi, Remiddi '08
- Comparison to sub-loop dispersion relation method for integrals without soft divergencies and numerator terms
 - Bauberger, Berends, Böhm, Buza '95
 - Awramik, Czakon, Freitas '06

Inputs:

$$M_Z = 91.1876 \text{ GeV} \quad M_W = 80.385 \text{ GeV} \quad M_H = 125.1 \text{ GeV}$$
$$m_t = 173.2 \text{ GeV}$$

	Exp. error	$\mathcal{O}(\alpha_{\text{ferm}}^2)$	$\mathcal{O}(\alpha_{\text{bos}}^2)$
$\sin^2 \theta_{\text{eff}}^b$	0.016	0.86×10^{-4}	-0.22×10^{-4}
$\Gamma_Z \text{ [MeV]}$	2.3	8.2	in progress
$\sigma_{\text{had}}^0 \text{ [pb]}$	37	8.0	in progress
R_b	6.6×10^{-4}	-1.7×10^{-4}	in progress
R_ℓ	0.025	-0.027	in progress

Problem for dissemination of numerical result(s):

Large # of numerical integrals, slow evaluation

Solution: Fit formula with sufficient # of (motivated) terms to data grid spanning at least 2σ of each input parameter

$$\sin^2 \theta_{\text{eff}}^{\text{b}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\text{b}}),$$

$$\Delta\kappa_{\text{b}}^{(\alpha^2, \text{bos})} = k_0 + k_1 c_H + k_2 c_t + k_3 c_t^2 + k_4 c_H c_t + k_5 c_W,$$

$$c_H = \log \left(\frac{M_H}{M_Z} \times \frac{91.1876 \text{ GeV}}{125.1 \text{ GeV}} \right), \quad c_t = \left(\frac{m_t}{M_Z} \times \frac{91.1876 \text{ GeV}}{173.2 \text{ GeV}} \right)^2 - 1,$$

$$c_W = \left(\frac{M_W}{M_Z} \times \frac{91.1876 \text{ GeV}}{80.385 \text{ GeV}} \right)^2 - 1.$$

$$k_0 = -0.98605 \times 10^{-4}, \quad k_1 = 0.3342 \times 10^{-4}, \quad k_2 = 1.3882 \times 10^{-4},$$

$$k_3 = -1.7497 \times 10^{-4}, \quad k_4 = -0.4934 \times 10^{-4}, \quad k_5 = -9.930 \times 10^{-4}.$$

- **Electroweak precision tests at future e^+e^- colliders** (ILC, FCC-ee, CEPC): complete SM 2-loop and dominant 3-loop corrections mandatory
- **Sector decomposition:** very general, but numerical convergence sometimes slow and not guaranteed
- **Mellin-Barnes integrals with contour shifts:** powerful for accurate numerical evaluation for Minkowskian external momenta
- **Computation of complete 2-loop results underway** with missing bosonic contributions completed soon