New developments with the loop-tree duality

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Matter To The Deepest
Recent Developments In Physics Of Fundamental Interactions
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- S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J. C. Winter, "*From loops to trees by-passing Feynman's theorem*," JHEP **0809** (2008) 065 [arXiv:0804.3170 [hep-ph]].
- I. Bierenbaum, S. Catani, P. Draggiotis and G. Rodrigo, "*A Tree-Loop Duality Relation at Two Loops and Beyond*," JHEP **1010** (2010) 073 [arXiv:1007.0194 [hep-ph]].
- I. Bierenbaum, S. Buchta, P. Draggiotis, I. Malamos and G. Rodrigo, "*Tree-Loop Duality Relation beyond simple poles*," JHEP **1303** (2013) 025 [arXiv:1211.5048 [hep-ph]].
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- S. Buchta, G. Chachamis, P. Draggiotis and G. Rodrigo, "*Numerical implementation of the Loop-Tree Duality method*," EPJC **77** (2017) 274 [arXiv:**1510**.00187 [hep-ph]].
- R. J. Hernández-Pinto, G. F. R. Sborlini and G. Rodrigo, "***Towards gauge theories in four dimensions,***" JHEP **1602** (2016) 044 [arXiv:**1506.04617** [hep-ph]].
- G. F. R. Sborlini, F. Driencourt-Mangin, J. Hernández-Pinto and G. Rodrigo, "***Four dimensional unsubtraction from the loop-tree duality,***"JHEP **1608** (2016) 160 [arXiv:**1604.06699** [hep-ph]].
- G. F. R. Sborlini, F. Driencourt-Mangin and G. Rodrigo, "***Four dimensional unsubtraction with massive particles,***" JHEP **1610** (2016) 162 [arXiv:**1608.01584** [hep-ph]].
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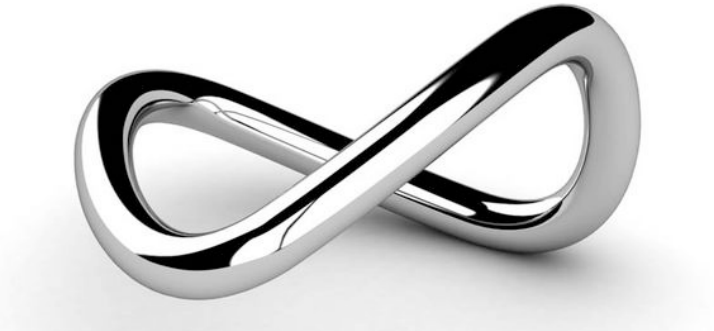

in **four** space-time dimensions

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Ultraviolet singularities (UV)

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soft singularities (IR)

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soft singularities (IR)

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and **threshold** singularities,
integrable but numerically unstable

in **four** space-time dimensions

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▪ Modify the dimensions of the space-time to **d = 4-2e**	

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▪ Virtual and real contributions are considered **separately:** phase-space with **different number of final-state particles**	▪ Virtual and real contributions are considered **simultaneously:** more efficient Monte Carlo implementation and fully differential

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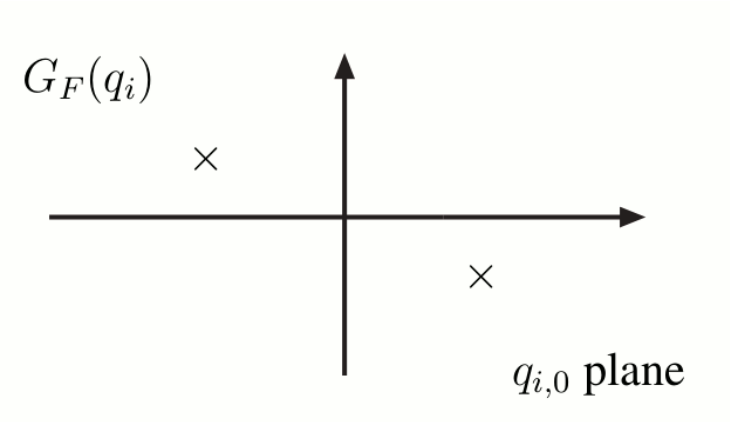
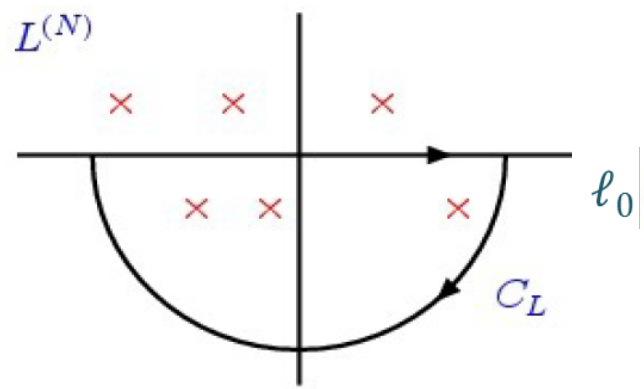
The loop-tree duality theorem

[Catani et al. 2008]



Cauchy residue theorem
in the loop energy complex plane



Feynman Propagator +i0:
positive frequencies are propagated forward in time, and negative backward

selects residues with definite **positive energy and negative imaginary part** (indeed in any coordinate system)

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

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One-loop integrals (or scattering amplitudes in any relativistic, local and unitary QFT) represented as a linear combination of *N* **single-cut phase-space** integrals

$$\int_\ell \prod G_F(q_i) = -\sum \int_\ell \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

- $\tilde{\delta}(q_i) = i\,2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i;q_j) = \dfrac{1}{q_j^2 - m_j^2 - i0\,\eta\,k_{ji}}$ **dual propagator,** $k_{ji} = q_j - q_i$

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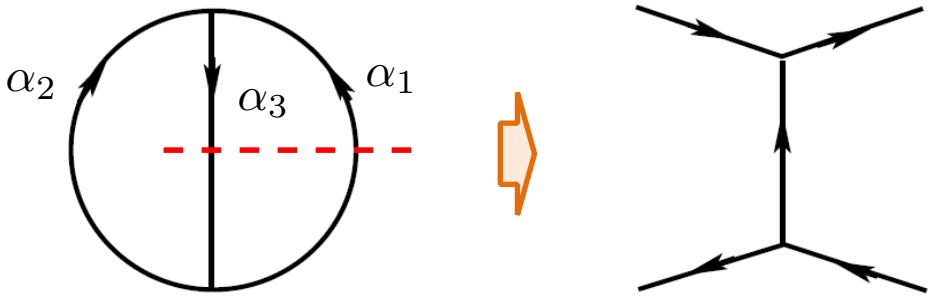
- Lorentz-covariant dual prescription with η a **future-like** vector; from now $\eta^\mu = (1,\mathbf{0})$ only the **sign** matters

LTD at two-loops and beyond

- **Iterative** application of LTD at higher orders

$$\int_{\ell_1}\int_{\ell_2} G_F(\alpha_1\cup\alpha_2\cup\alpha_3) = \int_{\ell_1}\int_{\ell_2} [G_D(\alpha_1)G_D(\alpha_2\cup\alpha_3) + G_D(-\alpha_1\cup\alpha_2)G_D(\alpha_3) - G_D(\alpha_1)G_F(\alpha_2)G_D(\alpha_3)]$$



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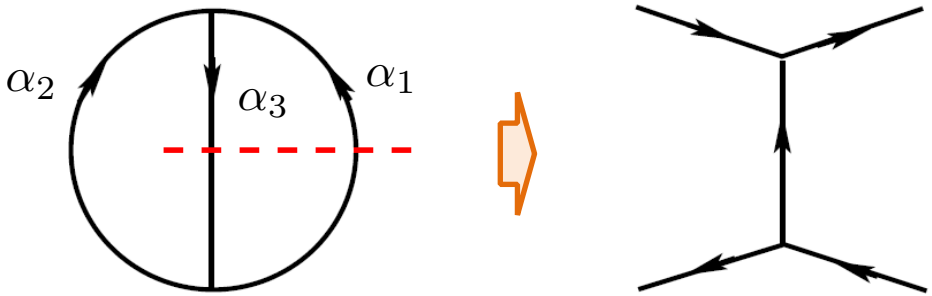

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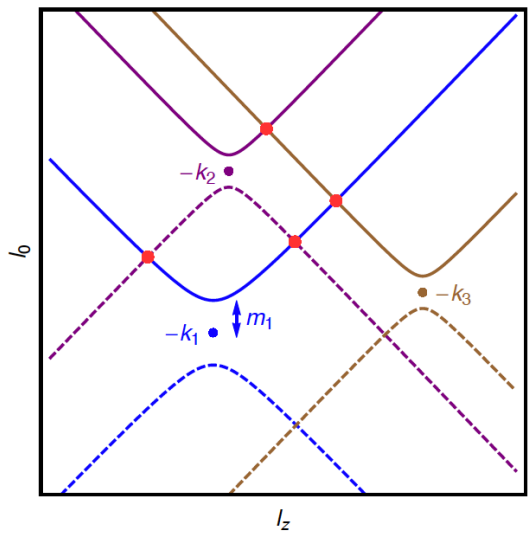
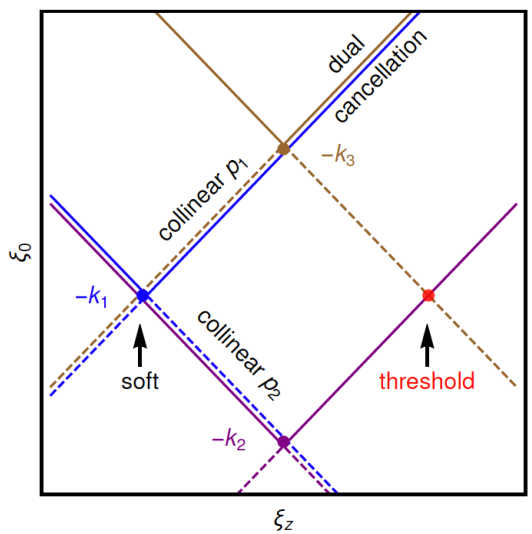
- However, the on-shell loop momenta still **unconstrained**

Singularities of the loop integrand

Buchta et al, arXiv:1405.7850





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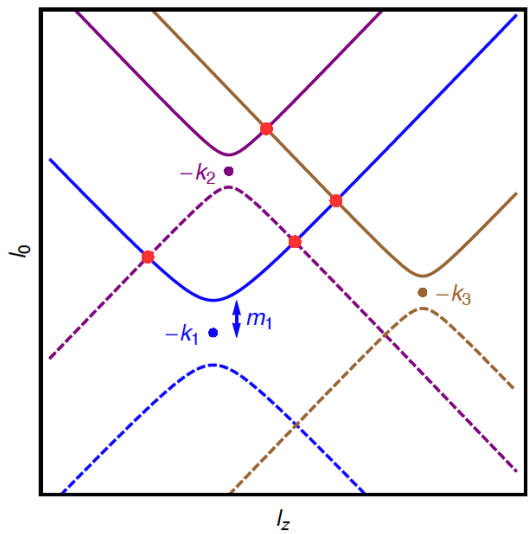

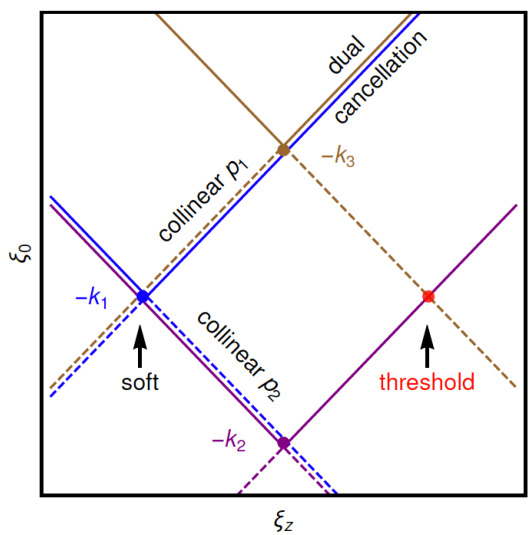

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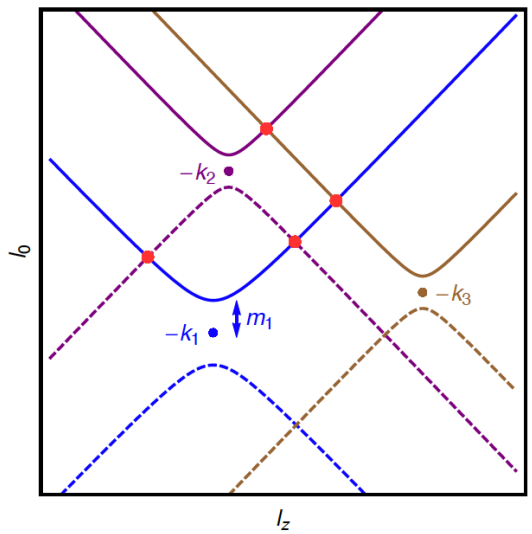

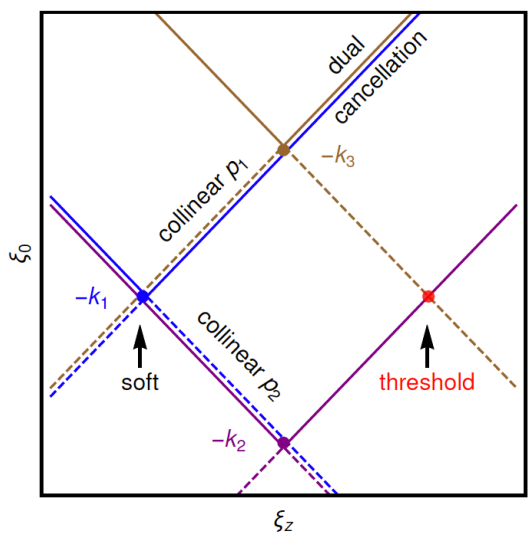

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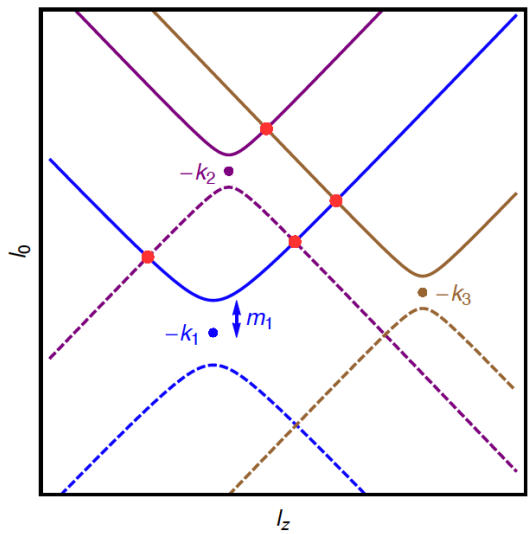

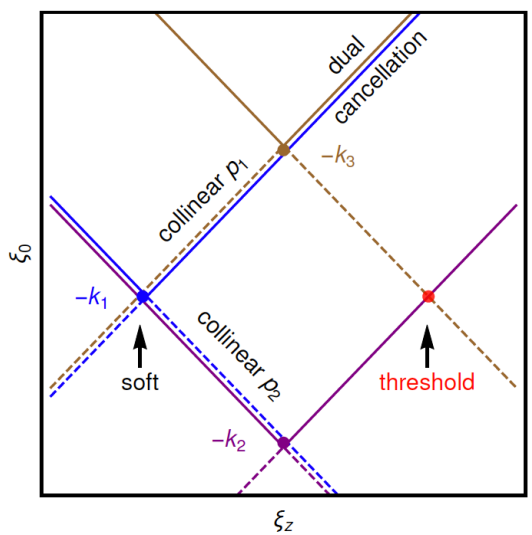

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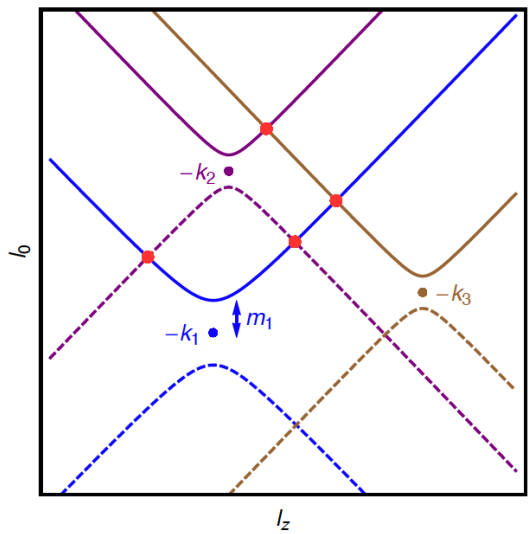

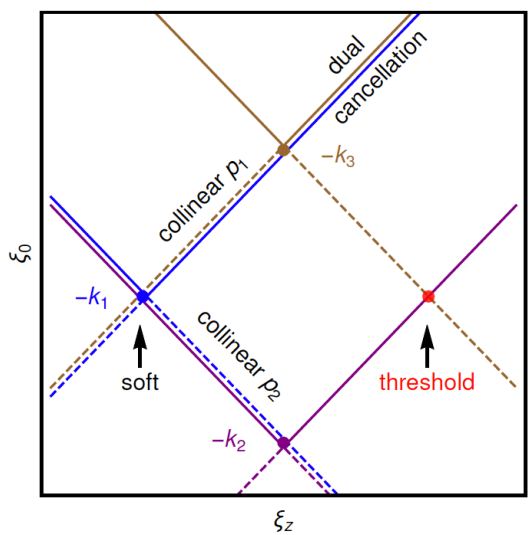

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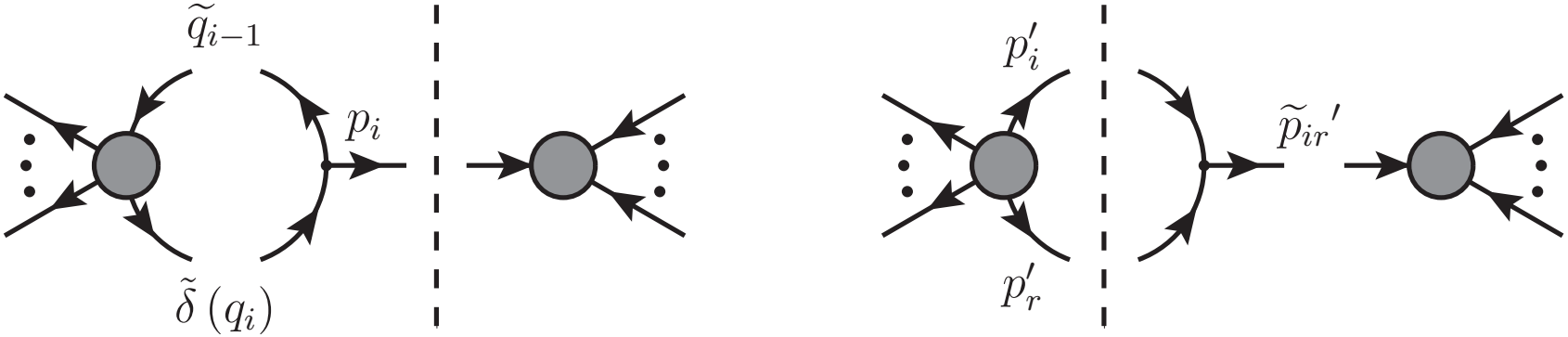
IR and threshold singularities are restricted to a **compact region** of the loop three-momentum

Momentum mapping



- Motivated by the **factorization properties of QCD**: assuming q_i^μ on-shell, and close to collinear with p_i^μ ,we define the momentum mapping

$$p_r'^\mu = q_i^\mu \ , \qquad q_{i,0} < p_{i,0}$$

$$p_i'^\mu = p_i^\mu - q_i^\mu + \alpha_i\, p_j^\mu \ , \qquad \alpha_i = \frac{(q_i - p_i)^2}{2p_j \cdot (q_i - p_i)} \ ,$$

$$p_j'^\mu = (1-\alpha_i)\, p_j^\mu \ , \qquad p_k'^\mu = p_k^\mu \ , \qquad k \neq i, j$$

- All the primed momenta (real process) on-shell and momentum conservation
- p_i^μ is the **emitter**, p_j^μ the **spectator** needed to absorb momentum recoil

Massive particles

Sborlini, Driencourt-Mangin,GR, arXiv:**1608.01584**

- Rewrite **emitter** and **spectator** in terms of two massless momenta

$$p_i^\mu = \beta_+ \, \hat{p}_i^\mu + \beta_- \, \hat{p}_j^\mu$$
$$p_j^\mu = (1-\beta_+) \, \hat{p}_i^\mu + (1-\beta_-) \, \hat{p}_j^\mu \qquad \hat{p}_i^\mu + \hat{p}_j^\mu = p_i^\mu + p_j^\mu$$

- Mapping and phase-space partition formally equal to the massless case: determine **mapping parameters from on-shell conditions**

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- **Quasi-collinear configurations** are conveniently mapped such that the massless limit is smooth

UV renormalisation: local subtraction

- Expand propagators and numerators around a UV propagator [Weinzierl et al.]

$$G_F(q_i) = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0} + \ldots \qquad q_{\rm UV} = \ell + k_{\rm UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{\rm MS}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}$$

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- Dual representation needs to deal with **multiple poles** [Bierenbaum et al.]

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde\delta(q_{\rm UV})}{2\left(q_{{\rm UV},0}^{(+)}\right)^2}$$

$$q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

Hernández-Pinto, Sborlini, GR, arXiv:1506.04617

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- Integration on the UV on-shell hyperboloid: loop three-momentum unconstrained, but **loop contributions suppressed** for loop energies larger than $\mu_{\rm UV}$

Self-energy corrections

- **Wave function** corrections usually **ignored for massless partons**, but they feature non-trivial IR/UV behaviour, required to disentangle both regions, indeed necessary to map the squares of the real amplitudes in the IR

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- **Unintegrated wave-function and mass renormalisation: e.g. quarks**

$$\Delta Z_2(p_1) = -g_S^2 C_F \int_\ell G_F(q_1)\, G_F(q_3) \left((d-2)\frac{q_1\cdot p_2}{p_1\cdot p_2} + 4M^2 \left(1 - \frac{q_1\cdot p_2}{p_1\cdot p_2}\right) G_F(q_3) \right)$$

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- smooth massless limit

Self-energy corrections

- **Wave function** corrections usually **ignored for massless partons**, but they feature non-trivial IR/UV behaviour, required to disentangle both regions, indeed necessary to map the squares of the real amplitudes in the IR

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- and **subtract the UV**

$$\Delta Z_2^{\rm UV}(p_1) = -(d-2) g_S^2 C_F \int_\ell (G_F(q_{\rm UV}))^2 \left(1 + \frac{q_{\rm UV}\cdot p_2}{p_1\cdot p_2}\right) \times \left(1 - G_F(q_{\rm UV})(2 q_{\rm UV}\cdot p_1 + \mu_{\rm UV}^2)\right)$$

LTD unsubtraction: multi-leg

Sborlini, Driencourt-Mangin, Hernández-Pinto, GR, arXiv:**1604.06699**

- The **dual representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_m d\sigma_{\rm V}^{(1,R)} = \sum_{i=1}^{N} \int_m \int_\ell 2\,{\rm Re}\,\langle \mathcal{M}_N^{(0)} | \mathcal{M}_N^{(1,R)}(\tilde{\delta}(q_i))\rangle\, \mathcal{O}_N(\{p_j\})$$

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$$\sum \mathcal{R}_i(q_i,p_i) = \sum \prod_{jk\neq ir} \theta(y'_{jk} - y'_{ir}) = 1$$

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- The real contribution **mapped** to the Born kinematics + loop three-momentum

$$\int_{m+1} d\sigma_{\rm R}^{(1)} = \sum_{i=1}^{N} \int_{m+1} |\mathcal{M}_{N+1}^{(0)}(q_i,p_i)|^2\, \mathcal{R}_i(q_i,p_i)\, \mathcal{O}_{N+1}(\{p'_j\})$$

- with

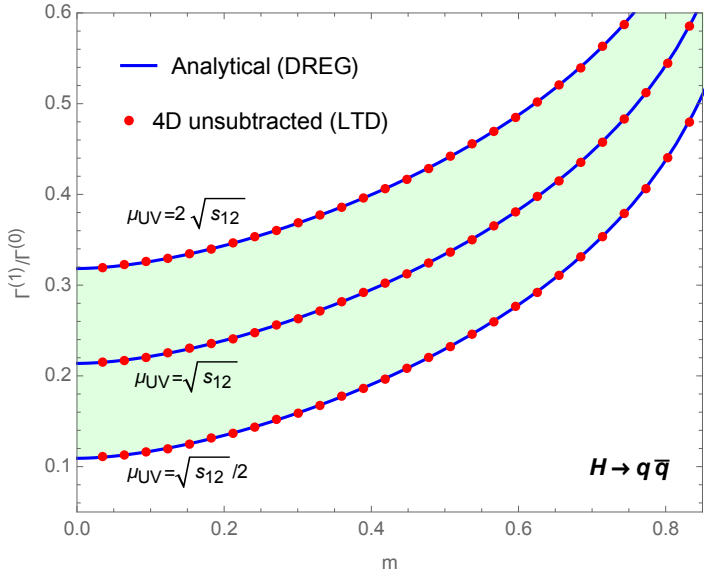
$$p'^{\mu}_r = q^{\mu}_i \ ,$$

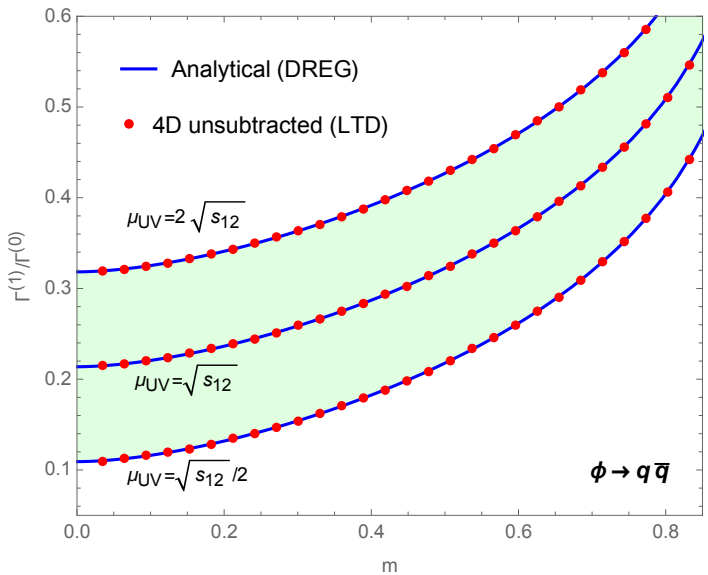
$$p'^{\mu}_i = p^{\mu}_i - q^{\mu}_i + \alpha_i\, p^{\mu}_j \ , \qquad \alpha_i = \frac{(q_i - p_i)^2}{2p_j\cdot(q_i - p_i)} \ ,$$

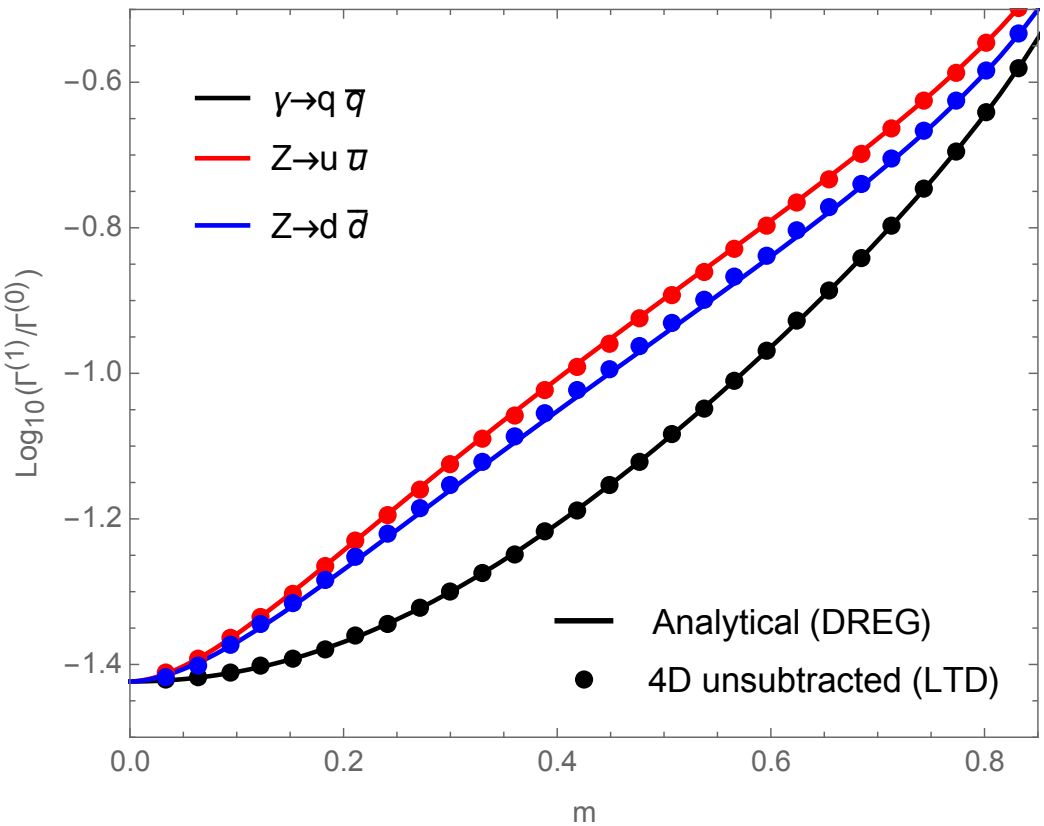
$$p'^{\mu}_j = (1-\alpha_i)\, p^{\mu}_j \ , \qquad p'^{\mu}_k = p^{\mu}_k \ , \qquad k \neq i,j$$

Benchmark application: $A^* \to q\bar{q}(g)$

Sborlini, Driencourt-Mangin,GR, arXiv:**1608.01584**







- Excellent agreement with analytic DREG
- Efficient numerical implementation
- Smooth massless limit

Higgs boson interactions to gg and $\gamma\gamma$

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- **Golden channels** for production and decay of the Higgs boson

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- **Golden channels** for production and decay of the Higgs boson
- One-loop corrections are UV and IR finite due to the **absence of a direct interaction** at tree-level in the SM
- However, DREG or another **regularisation/ renormalisation scheme still required** for their correct evaluation

Hgg [Wilczek, 1977; Georgi, Glashow, Machacek, Nanopoulos, 1978; Rizzo, 1980]

$H\gamma\gamma$ [Ellis, Gaillard, Nanopoulos, 1976; Ioffe, Khoze, 1978; Shifman, Vainshtein, Voloshin, Zakharov, 1979]

Summary Report of the Regularization Scheme Workstop/ Thinkstart, 13-16 Sep 2016, Zurich

Universality of dual amplitude

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- **Universality** and **compactness** of the dual representation

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - i0)^2} \right) \frac{s_{12}\, M_f^2}{(2\ell\cdot p_1)(2\ell\cdot p_2)}\, c_1^{(f)} \right.$$

$$\left. + \frac{2 s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - i0)^2}\, c_{23}^{(f)} \right] , \qquad q_{i,0}^{(+)} = \sqrt{\boldsymbol{q}_i^2 + M_f^2} \ , \qquad f = \phi, t, W$$

$$c_{23}^{(f)} = \frac{d-4}{d-2} \left(2, -4, 2(d-1) + \frac{s_{12}}{M_W^2} \right)$$

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- **Local renormalization:**

$$\mathcal{A}_{1,\mathrm{UV}}^{(1,f)} = -g_f \int_{\boldsymbol{\ell}} \frac{s_{12}}{4(q_{\mathrm{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\mathrm{UV},0}^{(+)})^2} \frac{3\mu_{\mathrm{UV}}^2}{d-4} \right) c_{23}^{(f)} = 0$$

$$q_{\mathrm{UV},0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + \mu_{\mathrm{UV}}^2}$$

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- **Naïve power counting:** unintegrated *W* amplitude much more singular in the UV than quark and scalar amplitudes
- **Local renormalization:** smooth four dimensional limit

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$$\boxed{\mathcal{A}_{1,\mathrm{R}}^{(1,f)}\Big|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\mathrm{UV}}^{(1,f)} \right)_{d=4}}$$

Dual amplitude in four space-time dimensions

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- The dual amplitude in **four space-time dimensions**

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$$\left. \times \frac{M_f^2}{(2\ell\cdot p_1)(2\ell\cdot p_2)} c_1^{(f)} + \frac{3\mu_{\mathrm{UV}}^2}{4(q_{\mathrm{UV},0}^{(+)})^5} \hat{c}_{23}^{(f)} \right]$$

integration measure and coefficients at *d*=4

$$c_1^{(f)} = \left(2, -4 + \frac{s_{12}}{M_t^2}, 6 - \frac{3s_{12}}{M_W^2}\right)$$
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- **Dyson** prescription would fail for *W* 

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Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

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- Circumvent **expansion by regions** [Smirnov, Beneke]

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a mapping of momenta between real and virtual kinematics.

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Outlook: fully differential multi-leg at NNLO (and beyond)