Two-photon form factors of the peseudoscalar mesons in Phokhara Monte Carlo generator

Szymon Tracz

Institute of Physics, University of Silesia

September 4, 2017

Outline

- Monte Carlo generators base on reliable models are needed for data analysis and feasibility study.
- The knowledge of the transition form factors is important for calculation of hadronic light-by-light scattering part of anomalous magnetic moment of the muon.

$$
\begin{aligned}
a_\mu^{SM} &= 11659181.00 \pm 3.62 \\
a_\mu^{exp} &= 11659209.10 \pm 6.33
\end{aligned}
$$

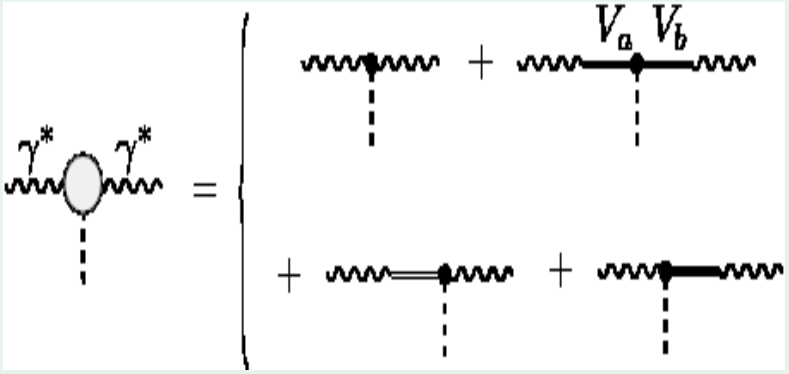
$$a_\mu^{exp} - a_\mu^{SM} = 28.1 \pm 7.3$$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$$

$$
\begin{aligned}
a_\mu^{had,HO} &= -9.83 \pm 0.04 \\
a_\mu^{had,LO} &= 692.23 \pm 2.54 \\
a_\mu^{had,LBL} &= 9.80 \pm 2.60
\end{aligned}
$$

Thomas Teubner talk on International Workshop on e^+e^- collisions from Phi to Psi 2017 in Mainz.
Muon g-2 Collaboration (G.W. Bennett et al.), Phys. ReV. D 73, 072003 (2006) [hep-ex/0602035].

$\gamma^*\gamma^*\mathcal{P}$ transition



$$\mathcal{L}_{\gamma V} = -e\sum_{i=1}^{3} f_{V_i}\partial_\mu B_\nu\left(\rho_i^{\tilde{\mu}\nu} + \frac{1}{3}\mathbf{F}_{\omega_i}\tilde{\omega}_i^{\mu\nu} - \frac{\sqrt{2}}{3}\mathbf{F}_{\phi_i}\tilde{\phi}_i^{\mu\nu}\right), \tilde{V}_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\begin{aligned}\mathcal{L}_{\gamma\gamma\mathcal{P}} &= \frac{-e^2 N_c}{24\pi^2 \mathbf{f}_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu \partial_\alpha B_\beta\Big[\pi^0 + \eta(\frac{5}{3}\mathbf{C_q} - \frac{\sqrt{2}}{3}\mathbf{C_s}) \\ &+ \eta'(\frac{5}{3}\mathbf{C'_q} + \frac{\sqrt{2}}{3}\mathbf{C'_s})\Big] \qquad (1)\end{aligned}$$

J. Prades, Z. Phys. C **63** (1994) 491 Erratum: [Z. Phys. C **11** (1999) 571] doi:10.1007/BF01580330 [hep-ph/9302246].
H. Czyz, S. Ivashyn, A. Korchin and O. Shekhovtsova, Phys. Rev. D **85** (2012) 094010 doi:10.1103/PhysRevD.85.094010 [arXiv:1202.1171 [hep-ph]].

$$\mathcal{L}_{V\gamma\pi^0} = -\sum_{i=1}^{n} \frac{4\sqrt{2}e\mathbf{h_{V_i}}}{3f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \left(\rho_i^\mu + 3\mathbf{H_{\omega_i}}\omega_i^\mu - \frac{3}{\sqrt{2}}\mathbf{A_i^{\pi^0}}\phi_i^\mu\right)\partial^\nu \pi^0$$

$$\begin{aligned}\mathcal{L}_{VV\pi^0} &= -\sum_{i=1}^{n} \frac{4\sigma_{\mathbf{V_i}}}{f_\pi}\epsilon_{\mu\nu\alpha\beta}\Big[\frac{1}{\mathbf{F_{\omega_i}}}\pi^0\partial^\mu\omega_i^\nu\partial^\alpha\rho_i^\beta + \frac{3(\mathbf{F_{\omega_i}H_{\omega_i}} - 1 - \mathbf{A^{\pi^0}_{\omega\phi_i}})}{2\mathbf{F_{\omega_i}}^2}\pi^0\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta \\ &+ \frac{3(\mathbf{A_i^{\pi^0}} - \mathbf{A^{\pi^0}_{\omega\phi_i}}/\mathbf{F_{\phi_i}})}{4\mathbf{F_{\phi_i}}}\pi^0\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta - \frac{3\mathbf{A^{\pi^0}_{\phi\omega_i}}}{\sqrt{2}\mathbf{F_{\omega_i}F_{\phi_i}}}\pi^0\partial^\mu\phi_i^\nu\partial^\alpha\omega_i^\beta\Big],\end{aligned}$$

$$\mathcal{L}_{V\gamma\eta} = -\sum_{i=1}^{n} \frac{4\sqrt{2}e\mathbf{h_{V_i}}}{3f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \Big[(3\rho_i^\mu + \omega_i^\mu)\mathbf{C_q} + 2\phi_i^\mu\mathbf{C_s} - (\frac{5}{\sqrt{2}}\mathbf{C_q} - \mathbf{C_s})\mathbf{A_i^\eta}\phi_i^\mu\Big]\partial^\nu\eta$$

$$\begin{aligned}\mathcal{L}_{VV\eta} &= -\sum_{i=1}^{n} \frac{4\sigma_{\mathbf{V_i}}}{f_\pi}\epsilon_{\mu\nu\alpha\beta}\eta\Big[(\partial^\mu\rho_i^\nu\partial^\alpha\rho_i^\beta + \frac{1}{\mathbf{F_{\omega_i}}}\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta)\frac{1}{2}\mathbf{C_q} - \frac{9\mathbf{A^\eta_{\omega\phi_i}}}{\mathbf{F_{\omega_i}}^2}\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta \\ &- \frac{1}{\mathbf{F_{\phi_i}}}\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta\frac{1}{\sqrt{2}}\mathbf{C_s} - \frac{9\mathbf{A^\eta_{\omega\phi_i}}}{2\mathbf{F_{\phi_i}}^2}\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta + \frac{\mathbf{A_i^\eta}}{6\mathbf{F_{\phi_i}}}(\frac{15}{2}\mathbf{C_q} - \frac{3}{\sqrt{2}}\mathbf{C_s})\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta - \frac{9\sqrt{2}\mathbf{A^\eta_{\omega\phi_i}}}{\mathbf{F_{\omega_i}F_{\phi_i}}}\partial^\mu\phi_i^\nu\partial^\alpha\omega_i^\beta\Big],\end{aligned}$$

η':$\mathbf{C_q} \to \mathbf{C_q'}$, $\mathbf{C_s} \to -\mathbf{C_s'}$, $\mathbf{A^{\eta'}_{\omega\phi_i}} = 0$, $\mathbf{A_i^\eta} \to \mathbf{A_i^{\eta'}}$

Two-photon form factor for π^0

$$
\begin{aligned}
F_{\gamma^*\gamma^*\pi^0}(t_1,t_2) \quad = \quad & -\frac{N_c}{12\pi^2 f_\pi} + \sum_{i=1}^{3} \frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi} t_1 \left(D_{\rho_i}(t_1) + \mathbf{F_{\omega_i}}\mathbf{H_{\omega_i}} D_{\omega_i}(t_1) + \mathbf{A_i^{\pi 0}}\mathbf{F_{\phi_i}} D_{\phi_i}(t_1) \right) \\
+ \quad & \sum_{i=1}^{3} \frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi} t_2 \left(D_{\rho_i}(t_2) + \mathbf{F_{\omega_i}}\mathbf{H_{\omega_i}} D_{\omega_i}(t_2) + \mathbf{A_i^{\pi 0}}\mathbf{F_{\phi_i}} D_{\phi_i}(t_2) \right) \\
- \quad & \sum_{i=1}^{3} \frac{4\sigma_{\mathbf{V_i}}\mathbf{f_{V_i}^2}}{3f_\pi} t_1 t_2 \Big(D_{\rho_i}(t_2) D_{\omega_i}(t_1) + D_{\rho_i}(t_1) D_{\omega_i}(t_2) + \left(\mathbf{A_i^{\pi 0}}\mathbf{F_{\phi_i}} - \mathbf{A_{\omega\phi_i}^{\pi 0}} \right) D_{\phi_i}(t_1) D_{\phi_i}(t_2) \\
+ \quad & \left(\mathbf{F_{\omega_i}}\mathbf{H_{\omega_i}} - 1 - \mathbf{A_{\omega\phi_i}^{\pi 0}} \right) D_{\omega_i}(t_1) D_{\omega_i}(t_2) + \mathbf{A_{\omega\phi_i}^{\pi 0}} \left(D_{\omega_i}(t_1) D_{\phi_i}(t_2) + D_{\omega_i}(t_2) D_{\phi_i}(t_1) \right) \Big)
\end{aligned}
$$

$\mathbf{H_{\omega_i}}$, $\mathbf{F_{\phi_i}} = \mathbf{1}$, $\mathbf{A_{\omega\phi_i}^{\pi 0}} = \mathbf{0}$ for $\mathbf{i} = \mathbf{2}, \mathbf{3}$, $\mathbf{A_3^{\pi^0}} = \mathbf{0}$. The $\mathbf{D_{V_i}}$ - vector meson propagators.

H. Czyz, S. Ivashyn, A. Korchin and O. Shekhovtsova, Phys. Rev. D **85** (2012) 094010 doi:10.1103/PhysRevD.85.094010 [arXiv:1202.1171 [hep-ph]].

Two-photon form factors for η (η')

$$
\begin{aligned}
F_{\gamma^*\gamma^*\eta}(t_1,t_2) \quad = \quad & -\frac{N_c}{12\pi^2 f_\pi}\Big(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s\Big) \\
& + \sum_{i=1}^{3}\frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi}t_1\Bigg(\Big(3C_q D_{\rho_i}(t_1) + \frac{1}{3}\mathbf{F}_{\omega_i}C_q D_{\omega_i}(t_1) - \frac{2\sqrt{2}}{3}C_s\mathbf{F}_{\phi_i}D_{\phi_i}(t_1)\Big) + \Big(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s\Big)\mathbf{A}_{\mathbf{i}}^{\eta}\mathbf{F}_{\phi_i}D_{\phi_i}(t_1)\Bigg) \\
& + \sum_{i=1}^{3}\frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi}t_2\Bigg(\Big(3C_q D_{\rho_i}(t_2) + \frac{1}{3}C_q\mathbf{F}_{\omega_i}D_{\omega_i}(t_2) - \frac{2\sqrt{2}}{3}C_s\mathbf{F}_{\phi_i}D_{\phi_i}(t_2)\Big) + \Big(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s\Big)\mathbf{A}_{\mathbf{i}}^{\eta}\mathbf{F}_{\phi_i}D_{\phi_i}(t_2)\Bigg) \\
& - \sum_{i=1}^{3}\frac{8\sigma_{\mathbf{V_i}}\mathbf{f}_{\mathbf{V_i}}^2}{f_\pi}t_1 t_2\Bigg[\Big(\frac{1}{2}C_q D_{\rho_i}(t_1)D_{\rho_i}(t_2) + \frac{1}{18}\mathbf{F}_{\omega_i}C_q D_{\omega_i}(t_1)D_{\omega_i}(t_2) - \frac{\sqrt{2}}{9}C_s\mathbf{F}_{\phi_i}D_{\phi_i}(t_1)D_{\phi_i}(t_2)\Big) \\
& \qquad + \quad \frac{\mathbf{A}_{\mathbf{i}}^{\eta}\mathbf{F}_{\phi_i}}{6}\Big(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s\Big)D_{\phi_i}(t_1)D_{\phi_i}(t_2) - \mathbf{A}_{\omega\phi_i}^{\eta}D_{\phi_i}(t_1)D_{\phi_i}(t_2) \\
& \qquad - \quad \mathbf{A}_{\omega\phi_i}^{\eta}D_{\omega_i}(t_1)D_{\omega_i}(t_2) + \mathbf{A}_{\omega\phi_i}^{\eta}\Big(D_{\omega_i}(t_1)D_{\phi_i}(t_2) + D_{\omega_i}(t_2)D_{\phi_i}(t_1)\Big)\Bigg]
\end{aligned}
$$

$$
F_{\gamma^*\gamma^*\eta}(t_1,t_2) = F_{\gamma^*\gamma^*\eta'}(t_1,t_2)\Big|_{C_q\to C_q',\ C_s\to -C_s'}
$$

$\mathbf{H}_{\omega_i}$, $\mathbf{F}_{\phi_i} = \mathbf{1}$, $\mathbf{A}_{\omega\phi_i}^{\eta} = \mathbf{0}$ for $\mathbf{i} = \mathbf{2}, \mathbf{3}$, $\mathbf{A}_{\mathbf{3}}^{\eta} = \mathbf{0}$, $\mathbf{A}_{\mathbf{3}}^{\eta'} = \mathbf{0}$. The $\mathbf{D}_{\mathbf{V_i}}$ - vector meson propagators.

H. Czyz, S. Ivashyn, A. Korchin and O. Shekhovtsova, Phys. Rev. D **85** (2012) 094010 doi:10.1103/PhysRevD.85.094010 [arXiv:1202.1171 [hep-ph]].

Asymptotic behavior

$$\lim_{\mathbf{t_1}\to\pm\infty} \mathbf{F}_{\gamma^*\gamma^*\mathcal{P}}(\mathbf{t_1},\mathbf{t_2})\Big|_{t_2=const} = 0$$

$$-\frac{N_c}{4\pi^2} + 4\sqrt{2}\sum_{i=1}^{3} h_{V_i} f_{V_i}(1 + F_{\omega_i} H_{\omega_i} + A_i^{\pi^0} F_{\phi_i}) = 0,$$

$$\sqrt{2} h_{V_i} f_{V_i} - \sigma_{V_i} f_{V_i}^2 = 0, \qquad i = 1, 2, 3$$

$$-\frac{N_c}{4\pi^2}(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s) + 4\sqrt{2}\sum_{i=1}^{3} h_{V_i} f_{V_i}\left[(3C_q + \frac{1}{3}F_{\omega_i}C_q - \frac{2\sqrt{2}}{3}C_s F_{\phi_i}) + (\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s)A_i^{\eta} F_{\phi_i}\right] = 0,$$

$$-\frac{N_c}{4\pi^2}(\frac{5}{3}C_q' + \frac{\sqrt{2}}{3}C_s') + 4\sqrt{2}\sum_{i=1}^{3} h_{V_i} f_{V_i}\left[(3C_q' + \frac{1}{3}F_{\omega_i}C_q' + \frac{2\sqrt{2}}{3}C_s' F_{\phi_i}) + (\frac{5}{3}C_q' + \frac{\sqrt{2}}{3}C_s')A_i^{\eta'} F_{\phi_i}\right] = 0.$$

We have chosen $\sigma_{\mathbf{V_1}}\mathbf{f}_{\mathbf{V_1}}^2$, $\sigma_{\mathbf{V_2}}\mathbf{f}_{\mathbf{V_2}}^2$, $\sigma_{\mathbf{V_3}}\mathbf{f}_{\mathbf{V_3}}^2$, $\mathbf{h}_{\mathbf{V_3}}\mathbf{f}_{\mathbf{V_3}}$, $\mathbf{A}_2^{\eta}$, $\mathbf{A}_2^{\eta'}$ to be determined by using asymptotic relations
G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22** (1980) 2157. doi:10.1103/PhysRevD.22.2157

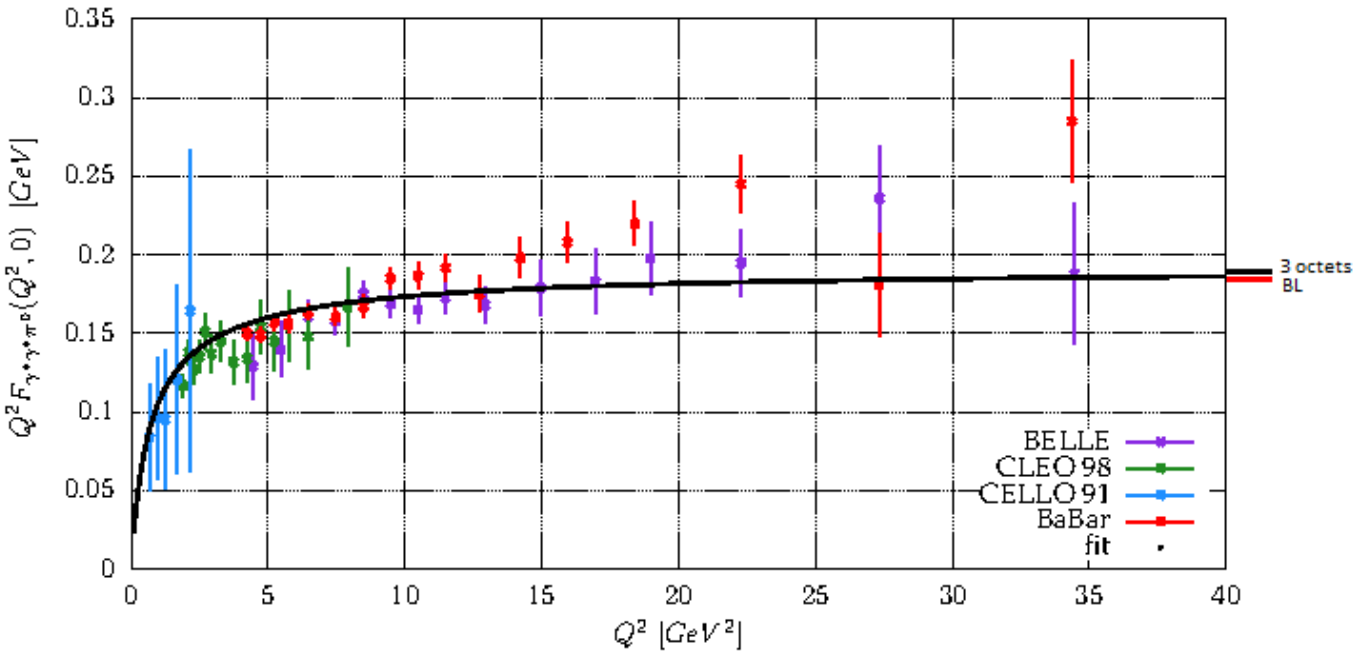
Fit the model parameters

Data in space-like region: BELLE, CLEO, CELLO, BaBar, ~~BaBar $F_{\gamma^*\gamma^*\pi^0}$~~
Data in time-like region: SND, CMD2, A2, BESIII, KLOE-2
Decay widths: $V \to e^+e^-$, $V \to \mathcal{P}\gamma$, $\mathcal{P} \to V\gamma$, $\mathcal{P} \to \gamma\gamma$
16 parameters: $\mathbf{h_{V_1}}$, $\mathbf{A_1^{\pi^0}}$, $\mathbf{A_2^{\pi^0}}$, $\mathbf{h_{V_2}}$ $\mathbf{f_{V_2}}$, $\mathbf{A_1^{\eta}}$, $\mathbf{A_1^{\eta'}}$, $\mathbf{f_{V_1}}$, $\mathbf{H_{\omega_1}}$, $\mathbf{F_{\omega_1}}$, $\mathbf{F_{\phi_1}}$, $\mathbf{A^{\pi^0}_{\phi\omega_1}}$, $\mathbf{A^{\eta}_{\phi\omega_1}}$, $\mathbf{C_q}$, $\mathbf{C_s}$, $\mathbf{C'_q}$, $\mathbf{C'_s}$

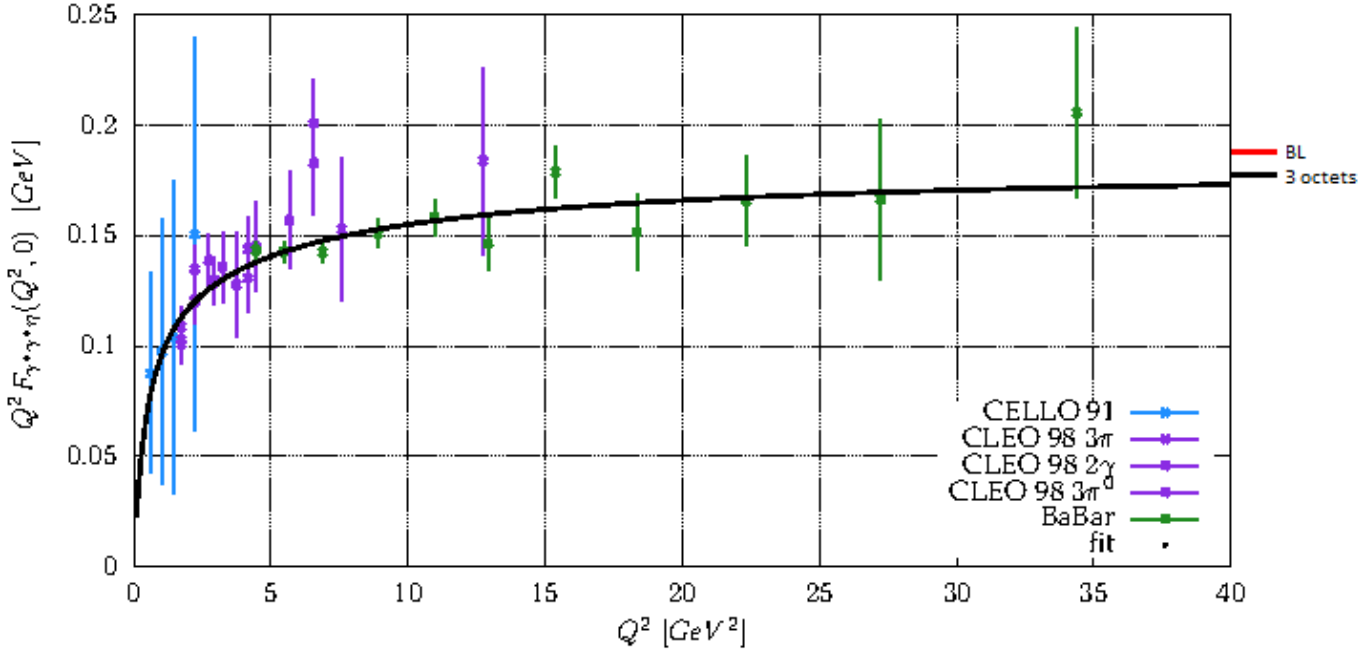
$$\chi^2 = \mathbf{429.60}$$
$$\mathbf{d.o.f} = \mathbf{527}$$

Width	exp [GeV]	th [GeV]
$\Gamma(\rho \to e^+e^-)$	$(7.04 \pm 0.06) \cdot 10^{-6}$	$7.05 \cdot 10^{-6}$
$\Gamma(\rho \to \pi^0\gamma)$	$(8.9 \pm 1.2) \cdot 10^{-5}$	$6.05 \cdot 10^{-5}$
$\Gamma(\omega \to e^+e^-)$	$(6.0 \pm 0.2) \cdot 10^{-7}$	$6.05 \cdot 10^{-7}$
$\Gamma(\omega \to \pi^0\gamma)$	$(7.0 \pm 0.2) \cdot 10^{-4}$	$7.66 \cdot 10^{-4}$
$\Gamma(\phi \to e^+e^-)$	$(1.26 \pm 0.02) \cdot 10^{-6}$	$1.275 \cdot 10^{-6}$
$\Gamma(\phi \to \pi^0\gamma)$	$(5.4 \pm 0.3) \cdot 10^{-6}$	$5.64 \cdot 10^{-6}$
$\Gamma(\rho \to \eta\gamma)$	$(4.5 \pm 0.3) \cdot 10^{-5}$	$3.91 \cdot 10^{-5}$
$\Gamma(\omega \to \eta\gamma)$	$(3.9 \pm 0.3) \cdot 10^{-6}$	$4.72 \cdot 10^{-6}$
$\Gamma(\phi \to \eta\gamma)$	$(5.6 \pm 0.1) \cdot 10^{-5}$	$5.71 \cdot 10^{-5}$
$\Gamma(\phi \to \eta'\gamma)$	$(2.67 \pm 0.09) \cdot 10^{-7}$	$2.66 \cdot 10^{-7}$
$\Gamma(\eta' \to \rho\gamma)$	$(5.8 \pm 0.3) \cdot 10^{-5}$	$5.19 \cdot 10^{-5}$
$\Gamma(\eta' \to \omega\gamma)$	$(5.4 \pm 0.5) \cdot 10^{-6}$	$5.16 \cdot 10^{-6}$
$\Gamma(\pi^0 \to \gamma\gamma)$	$(7.6 \pm 0.2) \cdot 10^{-9}$	$7.73 \cdot 10^{-9}$
$\Gamma(\eta \to \gamma\gamma)$	$(5.2 \pm 0.2) \cdot 10^{-7}$	$4.99 \cdot 10^{-7}$
$\Gamma(\eta' \to \gamma\gamma)$	$(4.4 \pm 0.3) \cdot 10^{-6}$	$4.61 \cdot 10^{-6}$

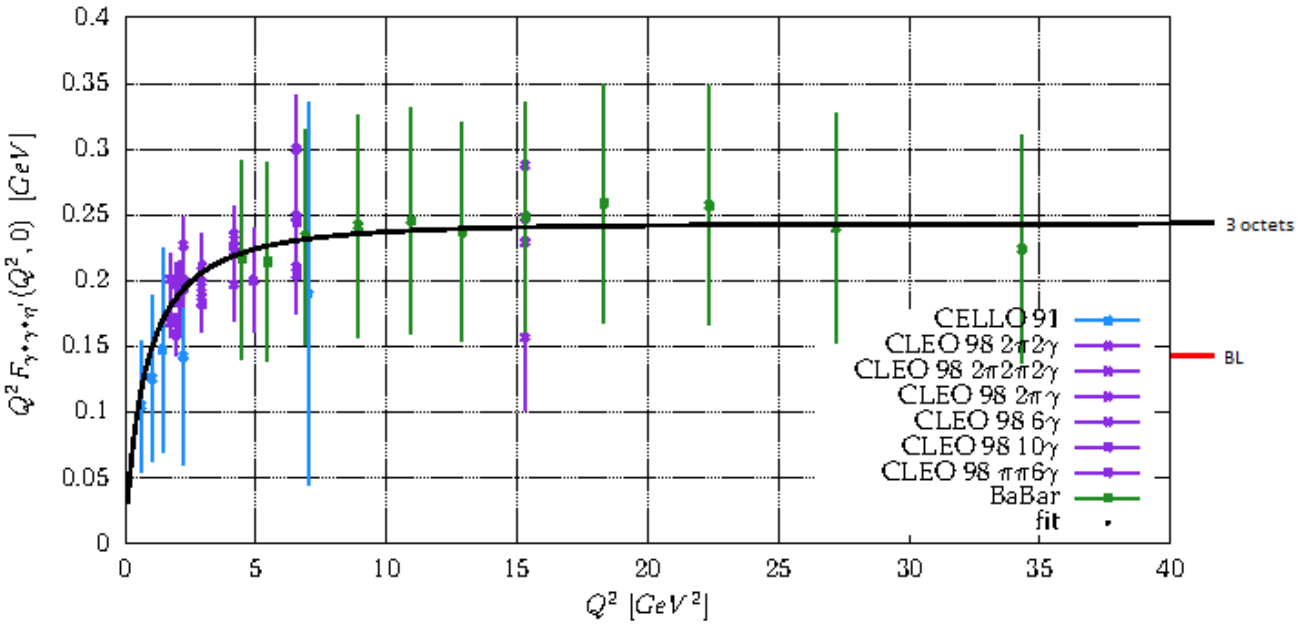
Transition form factor $\gamma^*\gamma^*\pi^0$ compared to the data



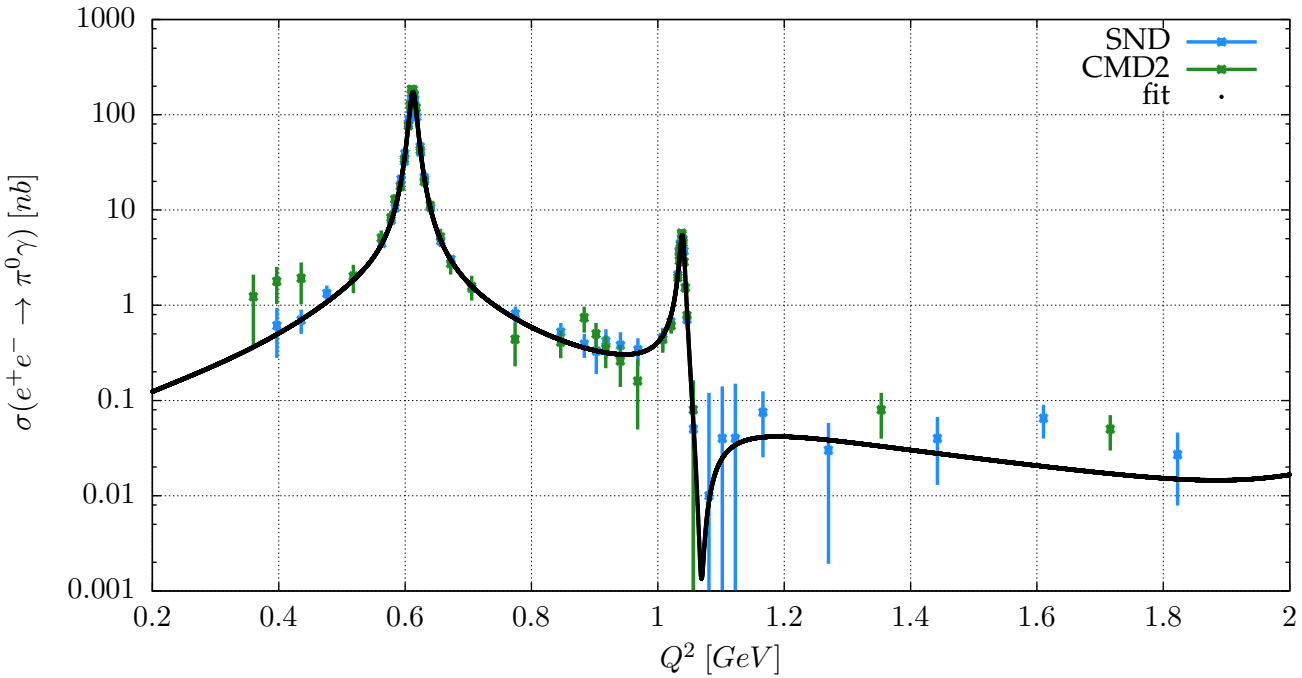
Transition form factor $\gamma^*\gamma^*\eta$ compared to the data



Transition form factor $\gamma^*\gamma\eta'$ compared to the data



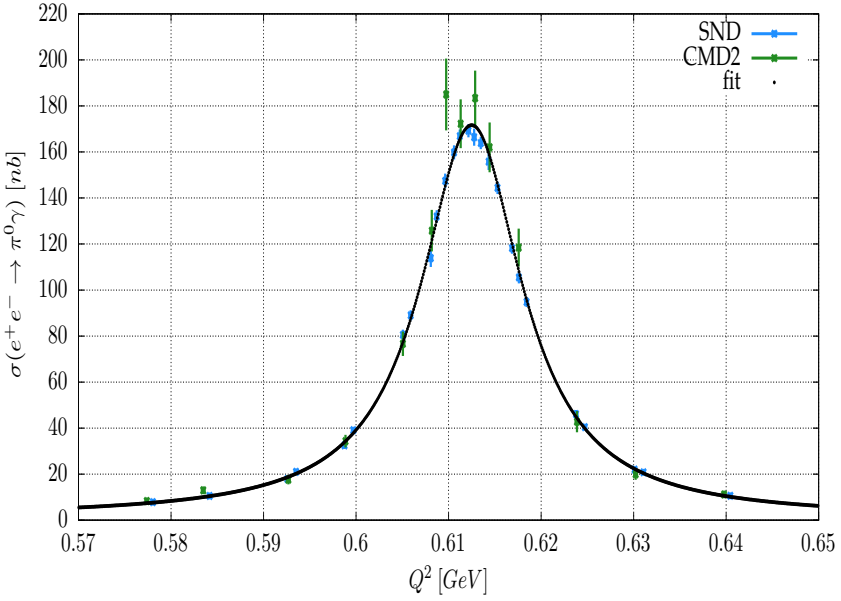
Experimental data for $\sigma(e^+e^- \to \pi^0\gamma)$ compared to the model predictions

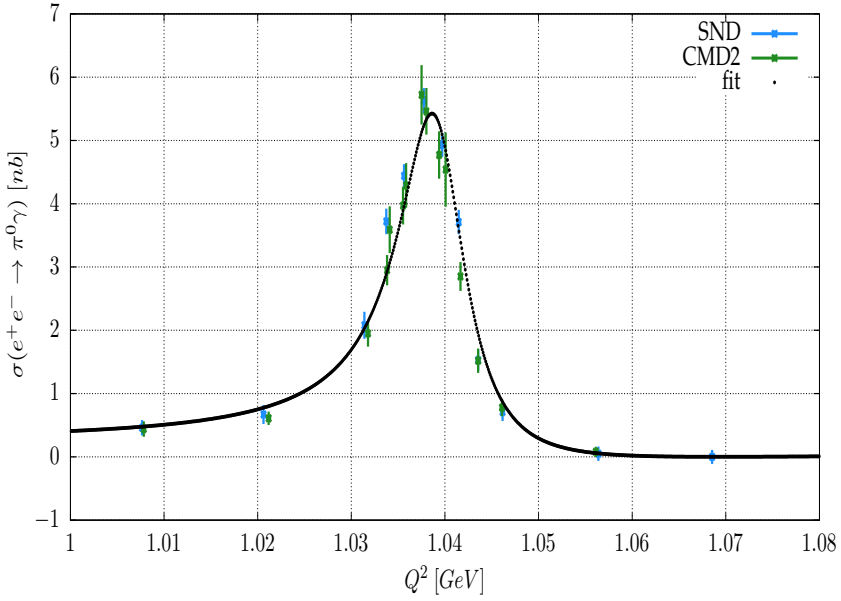


R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **605** (2005) 26 doi:10.1016/j.physletb.2004.11.020 [hep-ex/0409030].

M. N. Achasov *et al.* [SND Collaboration], Phys. Rev. D **93** (2016) no.9, 092001 doi:10.1103/PhysRevD.93.092001 [arXiv:1601.08061 [hep-ex]].

Experimental data for $\sigma(e^+e^- \to \pi^0\gamma)$ compared to the model predictions

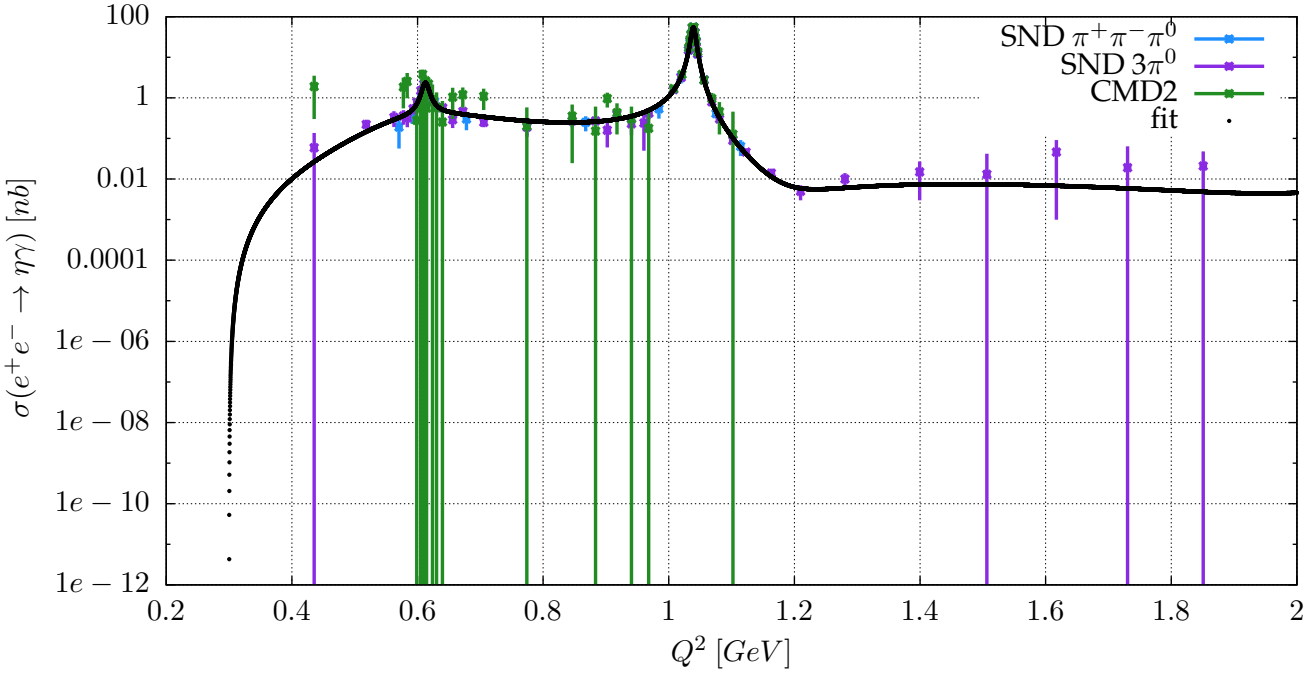




R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **605** (2005) 26 doi:10.1016/j.physletb.2004.11.020 [hep-ex/0409030].

M. N. Achasov *et al.* [SND Collaboration], Phys. Rev. D **93** (2016) no.9, 092001 doi:10.1103/PhysRevD.93.092001 [arXiv:1601.08061 [hep-ex]].

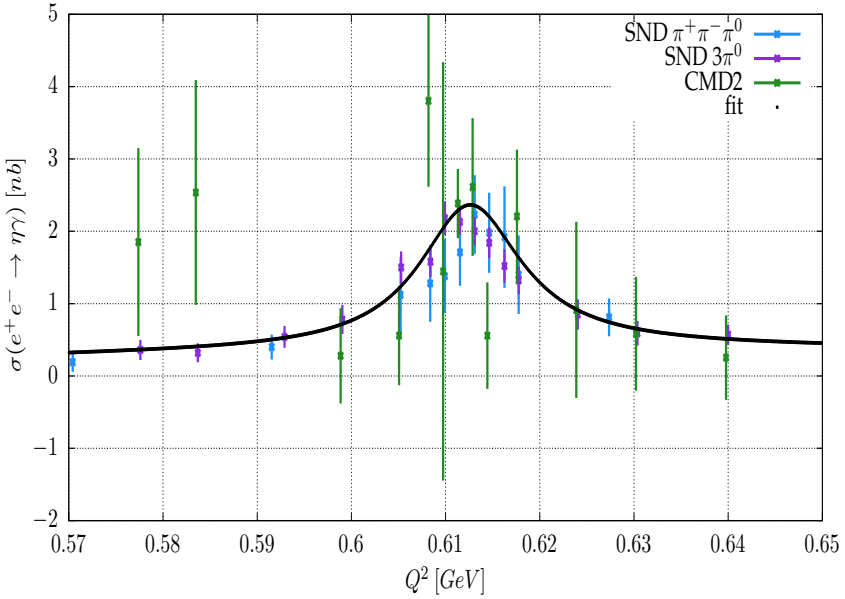
Experimental data for $\sigma(e^+e^- \to \eta\gamma)$ compared to the model predictions

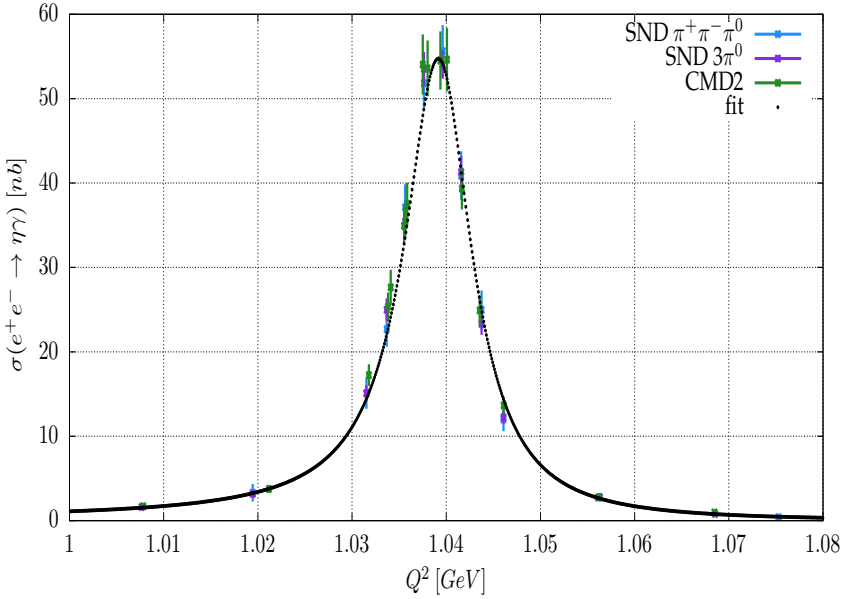


R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **605** (2005) 26 doi:10.1016/j.physletb.2004.11.020 [hep-ex/0409030].

M. N. Achasov *et al.* [SND Collaboration], Phys. Rev. D **93** (2016) no.9, 092001 doi:10.1103/PhysRevD.93.092001 [arXiv:1601.08061 [hep-ex]].

Experimental data for $\sigma(e^+e^- \to \eta\gamma)$ compared to the model predictions

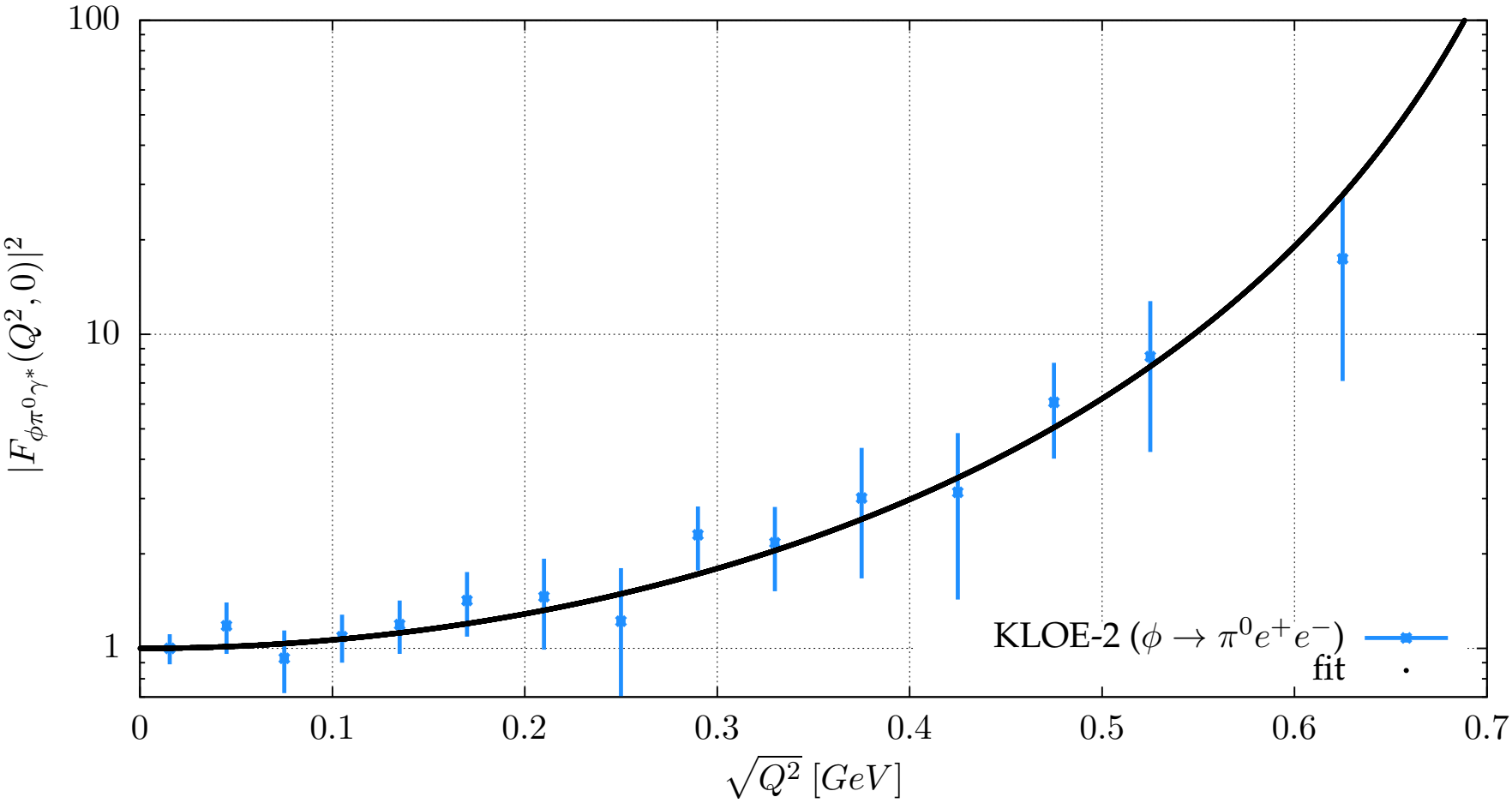




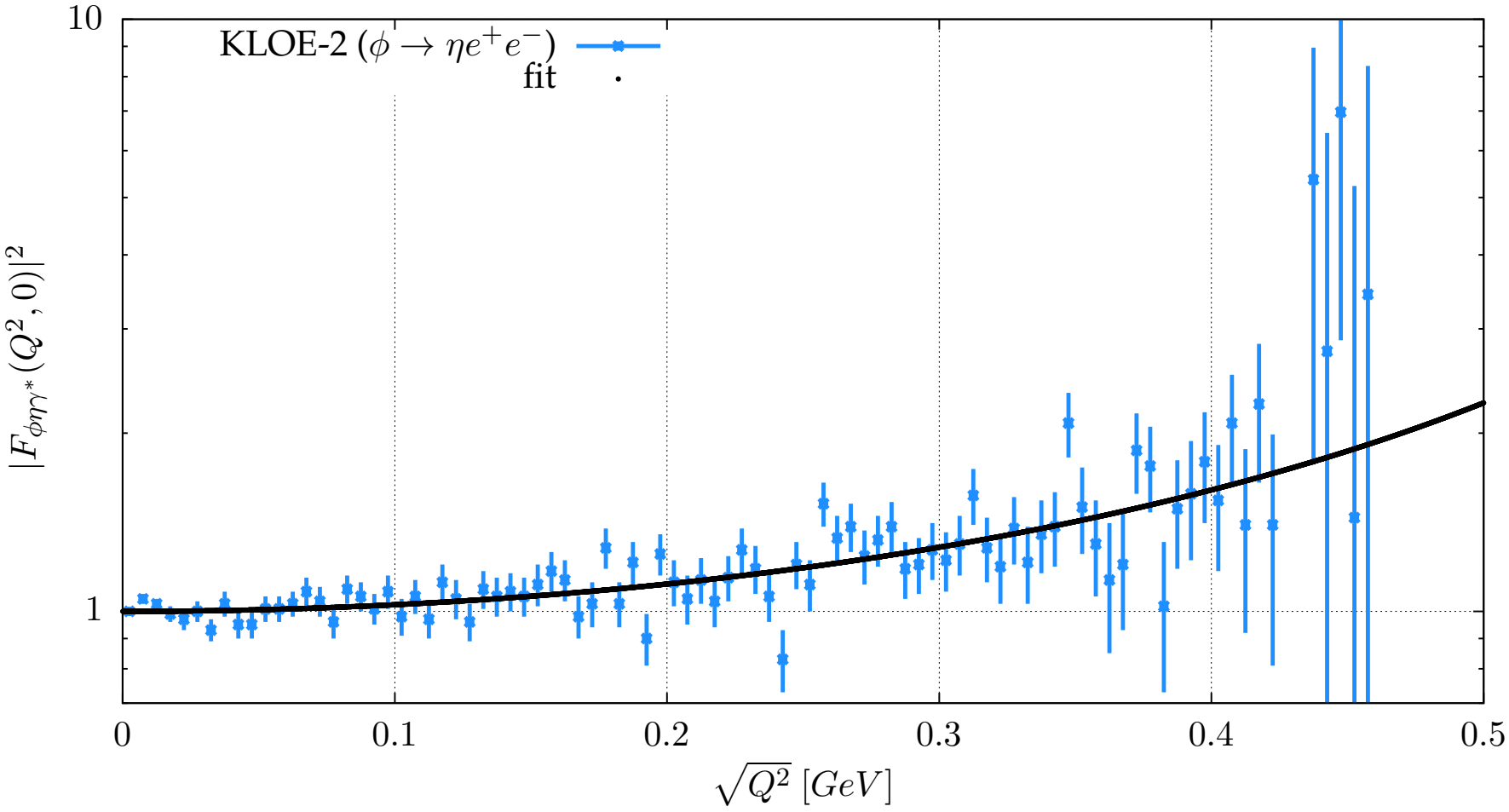
R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **605** (2005) 26 doi:10.1016/j.physletb.2004.11.020 [hep-ex/0409030].

M. N. Achasov *et al.*, Phys. Rev. D **74** (2006) 014016 doi:10.1103/PhysRevD.74.014016 [hep-ex/0605109].

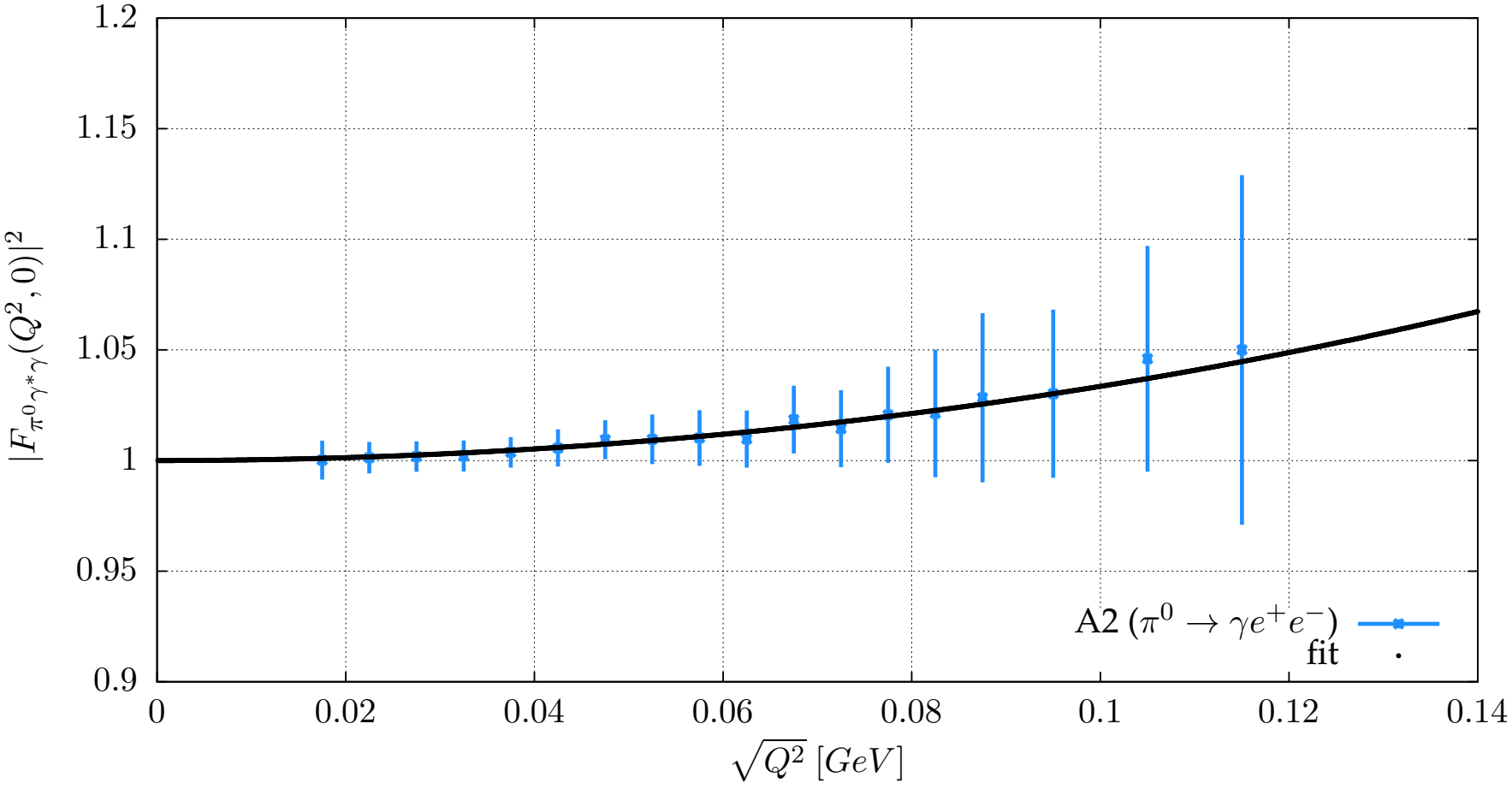
Form factor $\phi\pi^0\gamma^*$ compared to the data



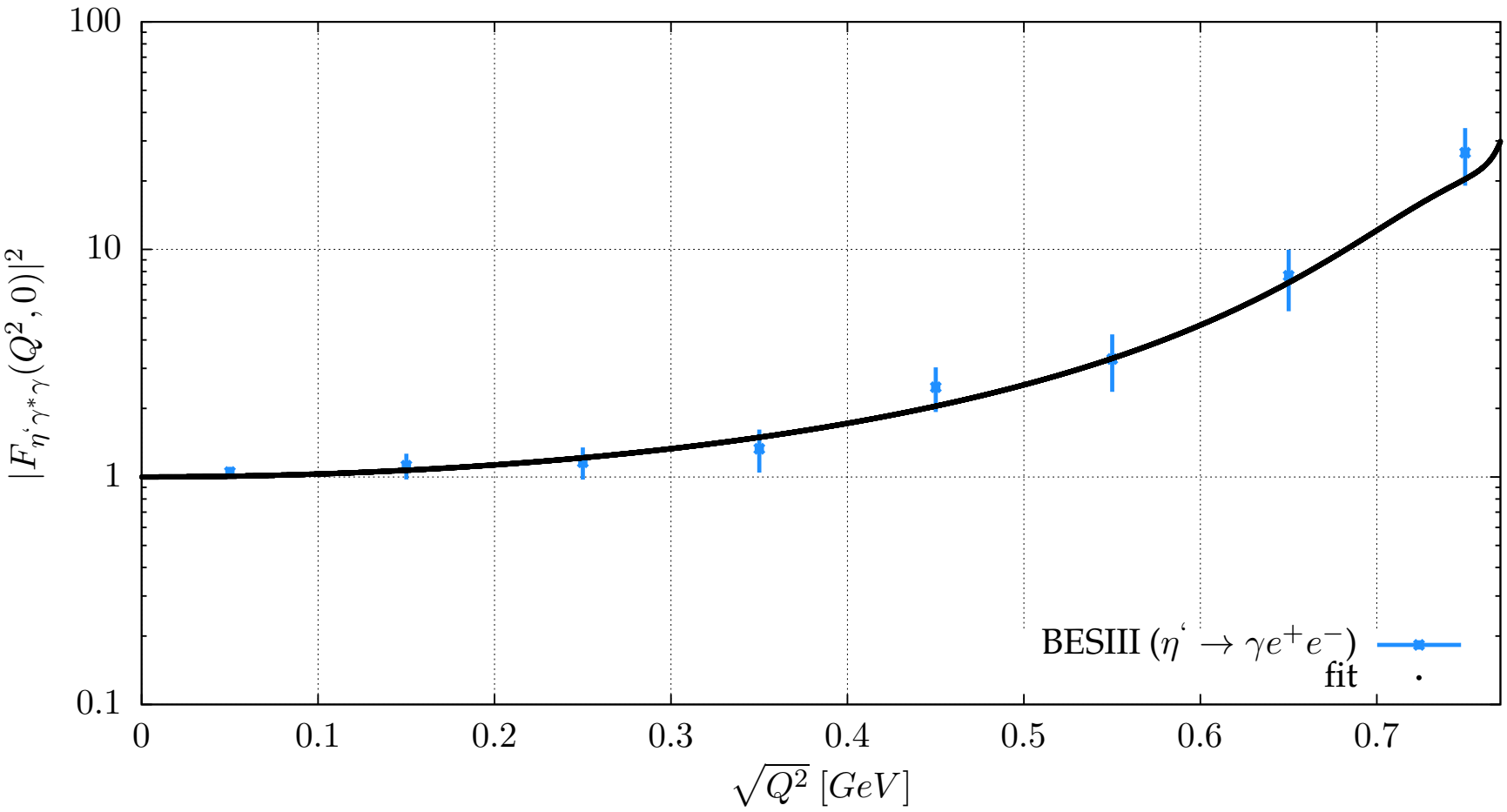
Form factor $\phi\eta\gamma^*$ compared to the data



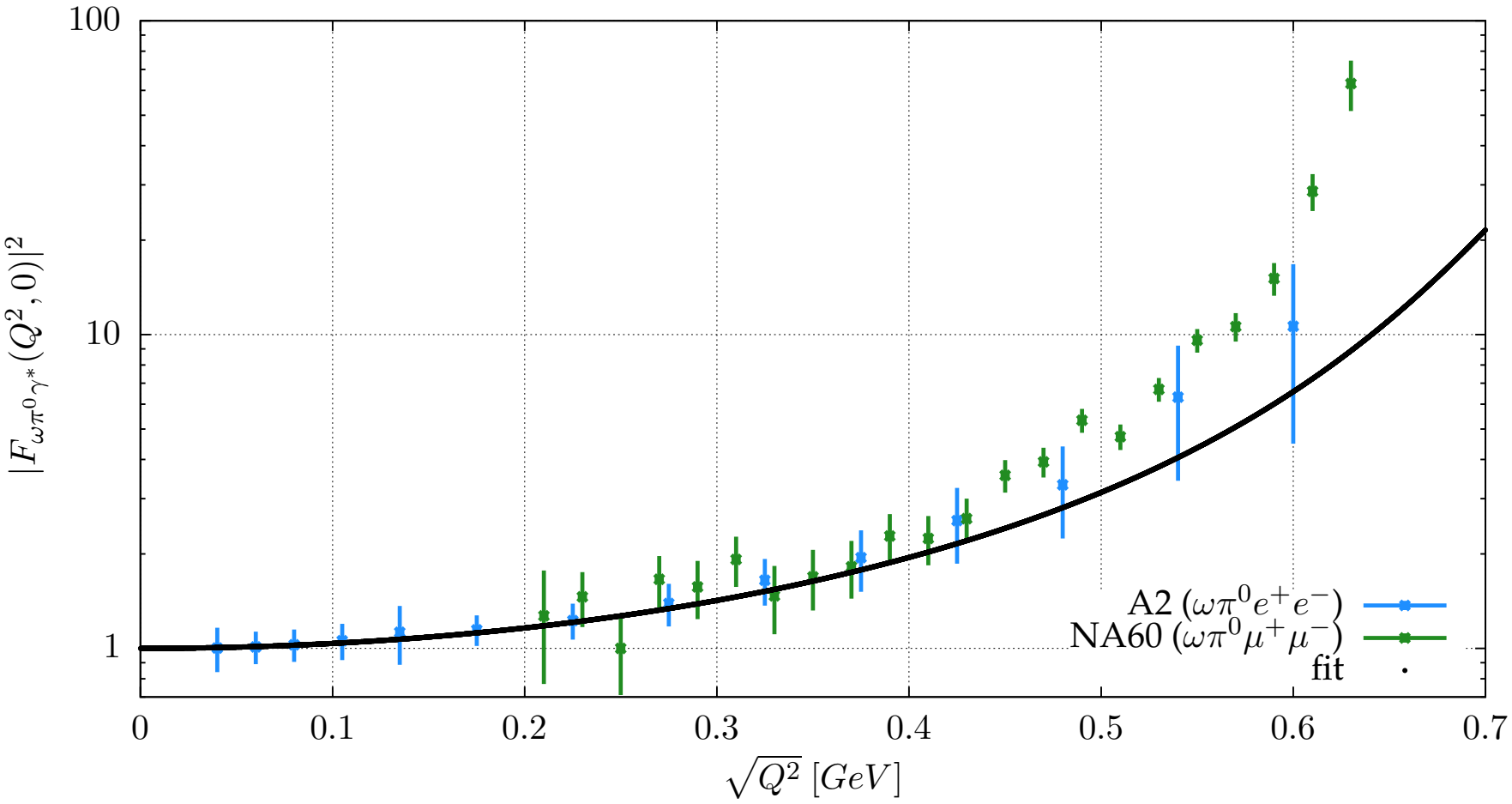
Form factor $\pi^0\gamma\gamma^*$ compared to the data



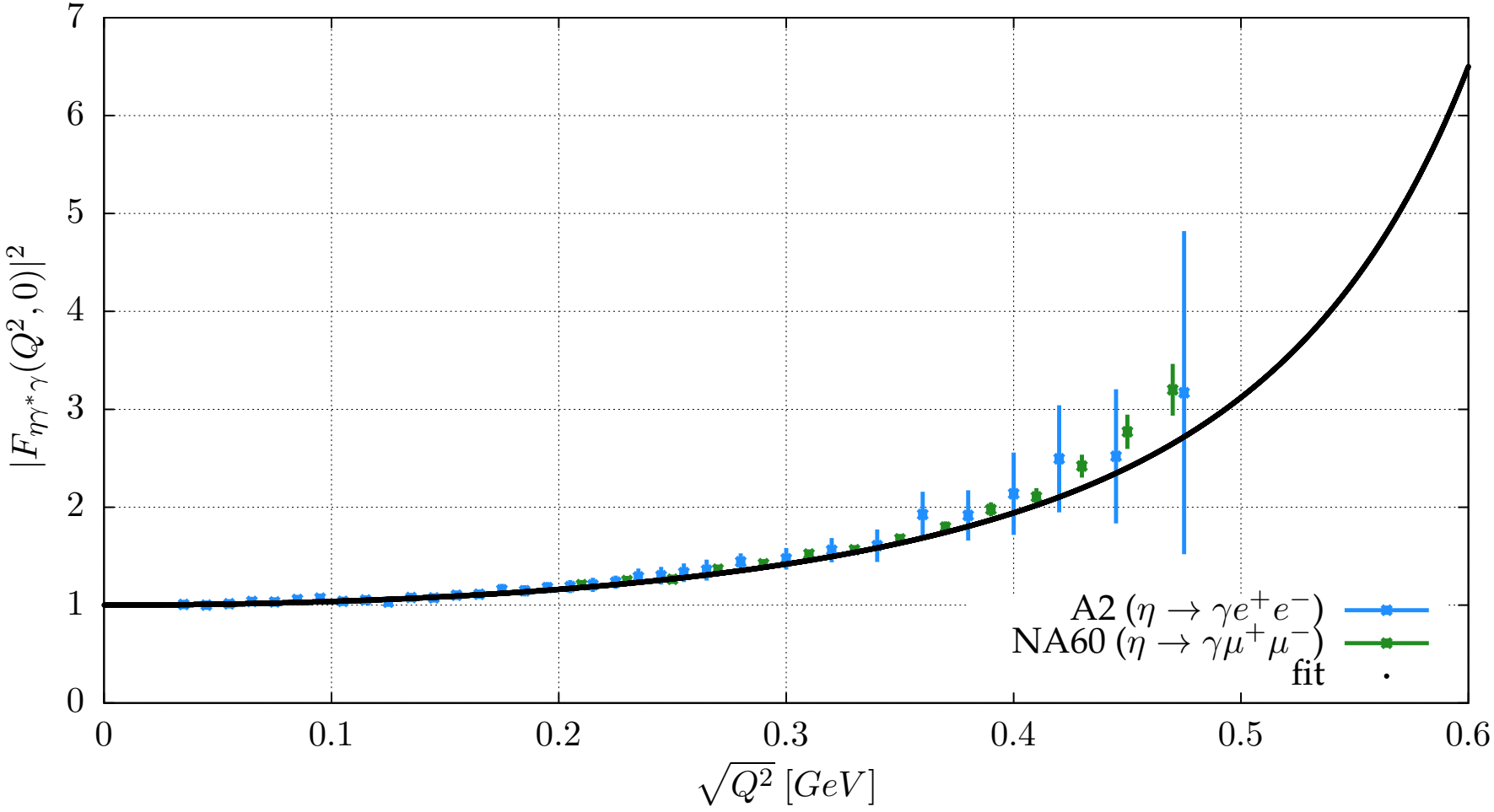
Form factor $\eta'\gamma\gamma^*$ compared to the data



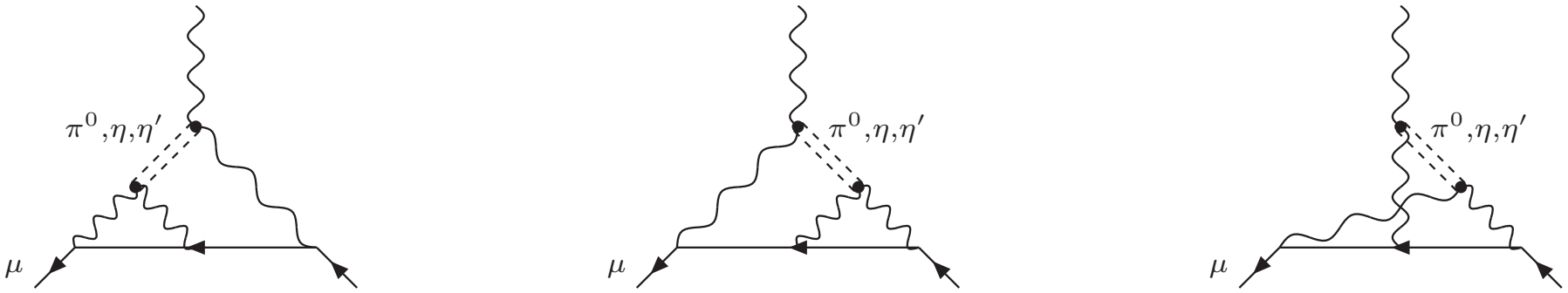
Form factor $\omega\pi^0\gamma^*$ compared to the data



Form factor $\eta\gamma\gamma^*$ compared to the data



HLBL contribution to g-2 of the muon



$$a_\mu^{LbL;P} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 dQ_2 \int_{-1}^{+1} dt\sqrt{1-t^2}\,Q_1^3 Q_2^3 [F_1 P_6 I_1(Q_1,Q_2,t) + F_2 P_7 I_2(Q_1,Q_2,t)]$$

F. Jegerlehner and A. Nyffeler, Phys. Rept. **477** (2009) 1 doi:10.1016/j.physrep.2009.04.003 [arXiv:0902.3360 [hep-ph]].

HLBL contribution to g-2 of the muon

Model	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	a_μ^{PS}
this work	59.07 ± 0.18	13.52 ± 0.10	12.95 ± 0.09	85.54 ± 0.37
[21]	57.4 ± 6.0	13.4 ± 1.6	11.9 ± 1.4	82.7 ± 6.4
[25]	58 ± 10	13 ± 1	12 ± 1	83 ± 12
[26]	76.5 ± 6.5	18 ± 1.4	18 ± 1.5	114 ± 10
[23, 27]	72 ± 12	14.5 ± 4.8	12.5 ± 4.2	99 ± 16
[7]	-	-	-	85 ± 13
[13]	$62.7 - 66.8$	-	-	-
[20]	65.0 ± 8.3	-	-	-

[21] M. Hayakawa and T. Kinoshita, Phys. Rev. D **57** (1998) 465 Erratum: [Phys. Rev. D **66** (2002) 019902] doi:10.1103/PhysRevD.57.465, 10.1103/PhysRevD.66.019902 [hep-ph/9708227].

[25] M. Knecht and A. Nyffeler, Phys. Rev. D **65** (2002) 073034 doi:10.1103/PhysRevD.65.073034 [hep-ph/0111058].

[26] K. Melnikov and A. Vainshtein, Phys. Rev. D **70** (2004) 113006 doi:10.1103/PhysRevD.70.113006 [hep-ph/0312226].

[23] A. Nyffeler, Phys. Rev. D **79** (2009) 073012 doi:10.1103/PhysRevD.79.073012.

[27] F. Jegerlehner and A. Nyffeler, Phys. Rept. **477** (2009) 1 doi:10.1016/j.physrep.2009.04.003.

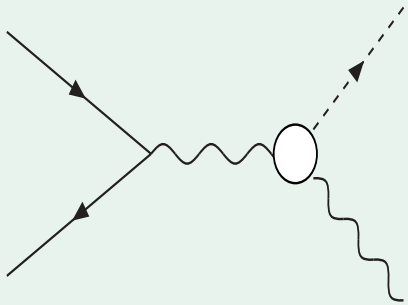
[7] J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **626** (2002) 410 doi:10.1016/S0550-3213(02)00074-3 [hep-ph/0112255].

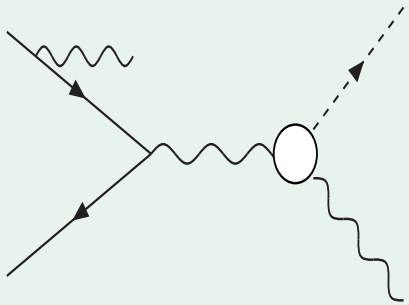
[13] A. E. Dorokhov and W. Broniowski, Phys. Rev. D **78** (2008) 073011 doi:10.1103/PhysRevD.78.073011.

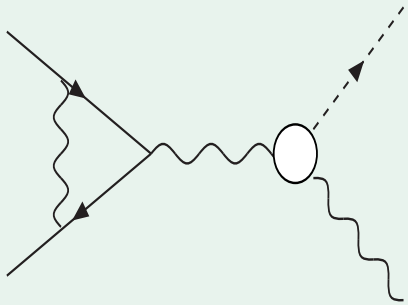
[20] A. GĂŠrardin, H. B. Meyer and A. Nyffeler, Phys. Rev. D **94** (2016) no.7, 074507 doi:10.1103/PhysRevD.94.074507.

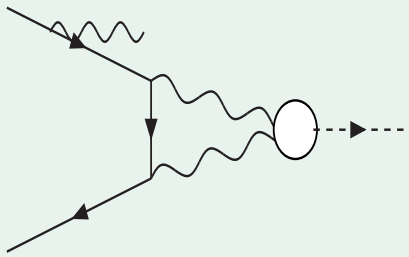
Implementation in PHOKHARA

- Scan mode in NLO
- Radiative return in LO

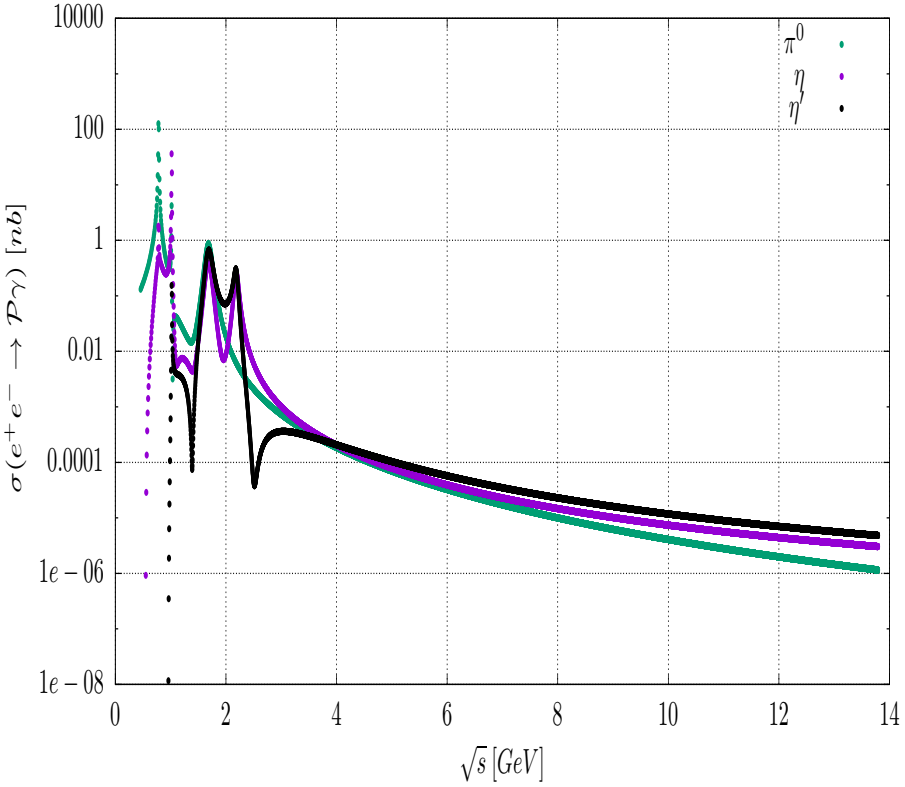


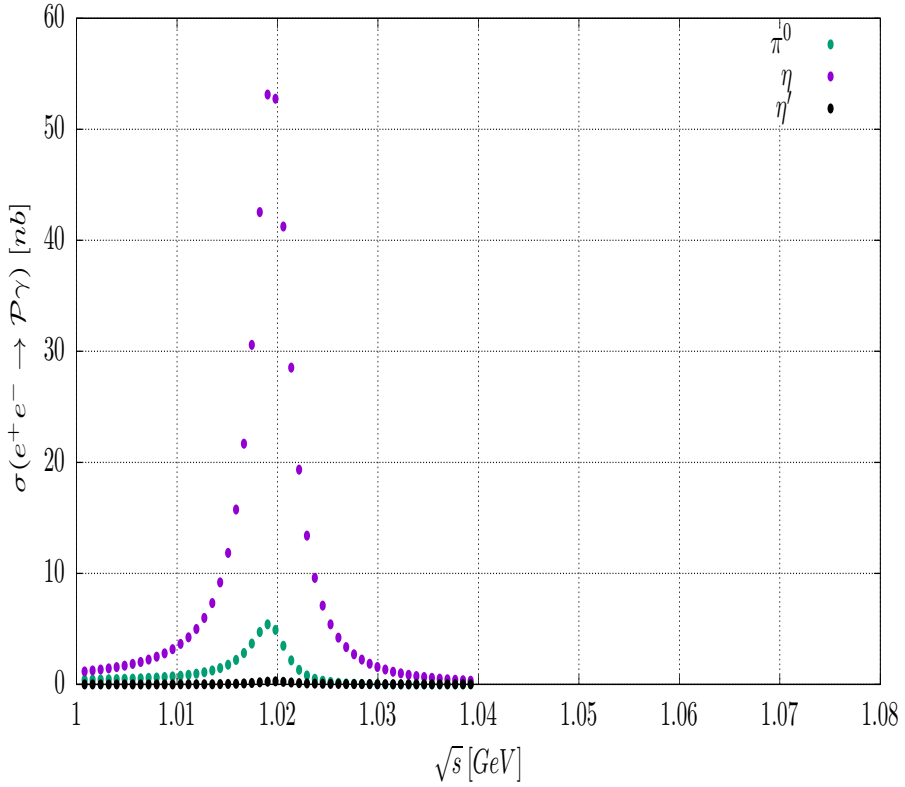






The model predictions for $e^+e^- \to \mathcal{P}\gamma$ cross section.





Final remarks

- The transition form factors of the pseudoscalar mesons have been constructed with parameters fitted to the experimental data in the time-like and space-like region.
- The model has been implemented in MC generator Phokhara, which allows for making predictions of $e^+e^- \to \mathcal{P}\gamma$ cross section.
- HLBL contribution to g-2 of the muon has been calculated

η-η' mixing parameters

$$C_q = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_8}\cos\theta_0 - \frac{1}{f_0}\sqrt{2}\sin\theta_8\right)$$

$$C_s = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_8}\sqrt{2}\cos\theta_0 + \frac{1}{f_0}\sin\theta_8\right)$$

$$C_q' = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_0}\sqrt{2}\cos\theta_8 + \frac{1}{f_8}\sin\theta_0\right)$$

$$C_s' = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_0}\cos\theta_8 - \frac{1}{f_8}\sqrt{2}\sin\theta_0\right)$$

T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58** (1998) 114006 doi:10.1103/PhysRevD.58.114006 [hep-ph/9802409].
T. Feldmann, Int. J. Mod. Phys. A **15** (2000) 159 doi:10.1142/S0217751X00000082 [hep-ph/9907491].