Radiative corrections in bound states

Light-by-light scattering in the Lamb shift and the bound electron g factor

(Andrzej Czarnecki, Robert Szafron Phys.Rev. A94 (2016) no.6, 060501)

Enhanced QED correction to $B_{s,d} \to \mu^+\mu^-$

(Martin Beneke, Christoph Bobeth, Robert Szafron arXiv:1708.09152)

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Matter To The Deepest

Outline

- Precision searches for BSM physics
- Static observables (spectrum)
  - Light-by-light contribution to the Lamb shift
  - Bound electron g-factor and the LBL contribution
- Dynamical observables (decay rate)
  - $B_s \to \mu^+\mu^-$
- Conclusions

New Physics?

How can we discover BSM physics?

The answer is simple: we need an observable that can be computed in the SM, we need to measure it and find a discrepancy.

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Long-distance non-perturbative physics hides short-distance BSM contribution

Our best options are

- Precise low energy measurements dominated by QED effects
- Rare process suppressed/forbidden in the SM

First and fundamental task is to get a precise SM theoretical prediction!

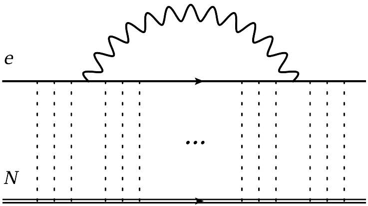
Example: muon $g-2$

Electron g-2 may be sensitive to the same New Physics $\delta g_e \sim \frac{m_e^2}{m_\mu^2}\delta g_\mu$, but a new source of α is needed

- Atomic spectroscopy ($R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$)
- Bound electron g
  - currently the best source of m_e
  - in the future also a source of α

We need QED corrections for the Lamb shift, and bound electron g-factor.
Current relative uncertainty for $1S-2S$ transition $\sim 10^{-15}$ and for bound g-factor $\sim 10^{-10}$; improvement expected soon.

Self-energy correction to hydrogen energy levels



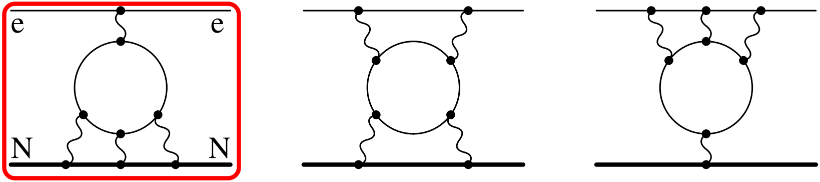
Can be organized as an expansion in powers of $\frac{\alpha}{\pi}$ (number of photons) and $Z\alpha$ (binding corrections)

$$\frac{\Delta E}{m_e} = \frac{\alpha}{\pi}\left(A_{41}(Z\alpha)^4 \ln(Z\alpha)^{-2} + A_{40}(Z\alpha)^4 + A_{50}(Z\alpha)^5 + \ldots\right) + \left(\frac{\alpha}{\pi}\right)^2\left(B_{40}(Z\alpha)^4 + B_{50}(Z\alpha)^5 + B_{63}(Z\alpha)^6 \ln^3(Z\alpha)^{-2} + \ldots\right) + \ldots$$

$A_{41} = \frac{4}{3}$ (Bethe Logarithm)

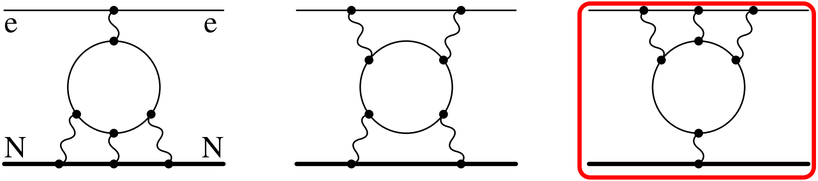
We focus on the light-by-light contributions.

Light-by-light contribution



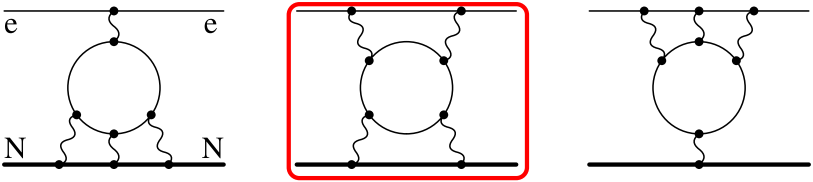
1. Wichmann-Kroll potential
   - $\mathcal{O}\left(\alpha(Z\alpha)^6\right)$: A_{60}
   - $\Delta E_{1S} = 2.5\mathrm{kHz}$ (Z=1)
   - [E. Wichmann and N. M. Kroll, 1954, 1956]

Light-by-light contribution



2. Dirac form factor
   - $\mathcal{O}\left(\alpha^3(Z\alpha)^4\right)$: C_{40}
   - [K. Melnikov and T. van Ritbergen, 2000]

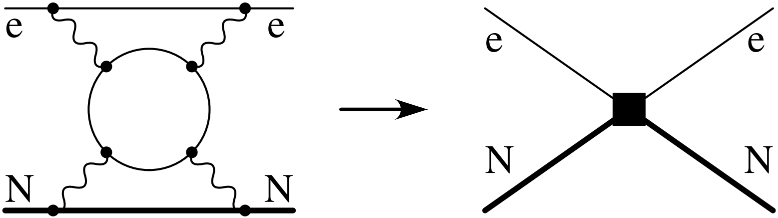
Light-by-light contribution



3. $\mathcal{O}\left(\alpha^2(Z\alpha)^5\right)$: B_{50}
   - $\Delta E_{1\mathrm{S}} = -5.3\mathrm{kHz}$ (Z=1)
   - [M.I.Eides,H.Grotch,and P.Peble, 1994; K. Pachucki 1993, 1994]

A given diagram may contribute also to higher orders in $Z\alpha$. In the third case, the higher order contribution is *logarithmically enhanced* $\rightarrow B_{61}$.

δ-contribution



- $\mathcal{O}\left(\alpha^2(Z\alpha)^5\right)$
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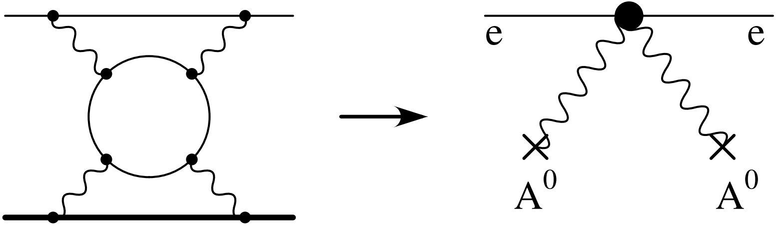
- [M.I.Eides,H.Grotch,and P.Peble, 1994; K. Pachucki 1993, 1994, M. Dowling, J. Mondejar, J. H. Piclum, and A. Czarnecki 2011]

Logarithmic contribution



$$\vec{E}^2 \sim \frac{(Z\alpha)^2}{r^4}$$

The matrix element of this operator is logarithmically divergent

$$\Delta E_{nS} = \chi_{\mathrm{LBL}} \left\langle \vec{E}^2 \right\rangle_{nS} = \frac{(Z\alpha)^6}{n^3} \ln(Z\alpha)^2 4\chi_{\mathrm{LBL}}$$

with the matching coefficient

$$\chi_{\mathrm{LBL}} = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{43}{144} - \frac{133}{3456}\pi^2\right)$$

[A. Czarnecki, R.S., 2016]

Significance of the LBL correction

Total corrections at $\mathcal{O}\left(\alpha^2(Z\alpha)^6 \ln(Z\alpha)\right)$ [K. Pachucki 2001, U. D. Jentschura, A. Czarnecki, and K. Pachucki, 2005] are much larger than the LBL contribution.

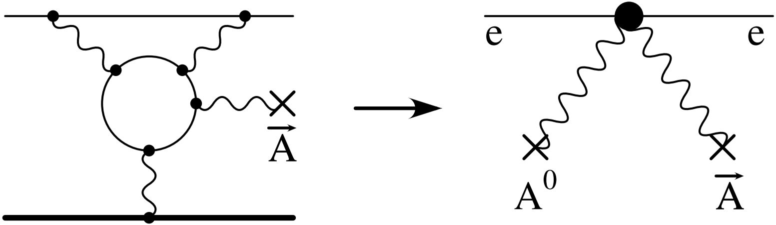
LBL correction decreases $1S-2S$ by $280\mathrm{Hz}$; experimental accuracy is $10\mathrm{Hz}$. Other transitions are measured with accuracy $\sim \mathrm{kHz}$.

Theory of hydrogen spectrum has to be further checked!

Measurements of $1S-2S$ transition in He^+ can provide a test of bound-state QED. [M. Herrmann et al. 2009]

LBL correction - bound electron g factor

Calculation of the LBL correction to the bound electron g is similar to Lamb



$$\mathcal{L}_{\mathrm{NRQED}} \supset \frac{\psi^\dagger (\vec{\sigma} \cdot \vec{B})(\vec{\nabla} \cdot \vec{E})\psi}{m_e^3}$$

The LBL correction (not included in previous evaluation of $(Z\alpha)^4 \left(\frac{\alpha}{\pi}\right)^2$ terms)

$$\delta g_e = (Z\alpha)^4 \left(\frac{\alpha}{\pi}\right)^2 \frac{16 - 19\pi^2}{108}$$

[A. Czarnecki, R.S., 2016]

Rare SM processes

Flavor violating processes are typically suppressed and offer a chance to probe BSM physics at scales well above the reach of the LHC.

On the lepton side, the most important processes are

- $\mu \to e\gamma$
- muon - electron conversion (note enhanced QED corrections to bound muon decay spectrum [A. Czarnecki, R.S., 2015])

Processes with quarks are usually contaminated by non-perturbative long-distance QCD effects, however there are exceptions.

Leptons vs quarks

NP effects can be parametrized with SM EFT

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{i,n} \frac{C_i}{\Lambda^n} \mathcal{O}_{i,n}$$

In the leptonic case $\mathcal{A}_{\mathrm{SM}}$ is negligible due to smallness of neutrino masses.

$$|\mathcal{A}_{\mathrm{SM}} + \mathcal{A}_{\mathrm{NP}}|^2 \sim |\mathcal{A}_{\mathrm{NP}}|^2 \sim \frac{1}{\Lambda^4}$$

In the quark case, the interference term gives the dominant NP effect

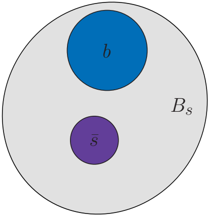
$$|\mathcal{A}_{\mathrm{SM}} + \mathcal{A}_{\mathrm{NP}}|^2 \sim |\mathcal{A}_{SM}|^2 + 2\mathrm{Re}\left[\mathcal{A}_{\mathrm{SM}}\mathcal{A}_{\mathrm{NP}}\right] \sim 1 + \frac{1}{\Lambda^2}$$

Better sensitivity but requires precise knowledge of the SM contribution.

$B_s \to \mu^+\mu^-$

In the SM the process is

- loop suppressed (FCNC)
- helicity suppressed (scalar meson decaying into energetic muons)
- purely leptonic final state allows for a prescience SM prediction, QCD contained in f_B



This decay has been observed by LHCb and CMS
$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathrm{LHCb}} = (3.0^{+0.7}_{-0.6}) \cdot 10^{-9}$ [LHCb, 2017]

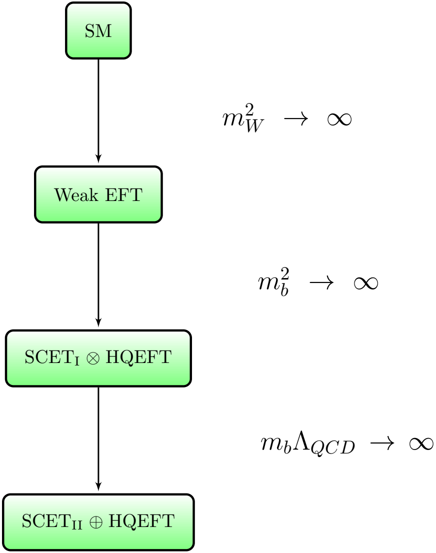
SM helicity suppression makes it very sensitive to BSM scalar interactions.

Scales in the problem

Leptonic decay of B_s is a multi-scale problem

- Electroweak scale m_W
- Hard scale m_b
- Hard-collinear scale $\sqrt{m_b \Lambda_{\mathrm{QCD}}}$
- Soft scale Λ_{QCD}
- Collinear scale m_μ

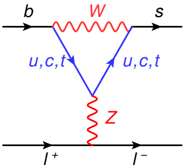
We take $\Lambda_{\mathrm{QCD}} \sim m_\mu$ so the soft scale of HQEFT is also a soft scale of $\mathrm{SCET_I}$



$B_s \to \mu^+\mu^-$ in the SM

Weak interaction EFT - integrate out the EW scale, expansion in $\frac{m_b}{m_W}$, RG evolution to $\mu_b \sim m_b$ [C. Bobeth, P. Gambino, M. Gorbahn, and U. Haisch, 2004; T. Huber, E. Lunghi, M. Misiak, and D. Wyler,2006]

- NLO EW [C. Bobeth, M. Gorbahn, E. Stamou, 2014]
- NNLO QCD [T. Hermann, M. Misiak, M. Steinhauser, 2013]



$$\mathcal{L}_{\Delta B=1} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i Q_i + \text{h.c.}$$

$$\begin{aligned} Q_9 &= \frac{\alpha_{\rm em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell) \\ Q_{10} &= \frac{\alpha_{\rm em}}{4\pi}\left(\bar{q}\gamma^\mu P_L b\right)\left(\bar{\ell}\gamma_\mu\gamma_5 \ell\right) \\ Q_7 &= \frac{e}{16\pi^2} m_b \left(\bar{q}\sigma^{\mu\nu} P_R b\right) F_{\mu\nu} \end{aligned}$$

QED corrections in the QCD bond-states

The final state has no strong interaction – QCD is contained in the decay constant

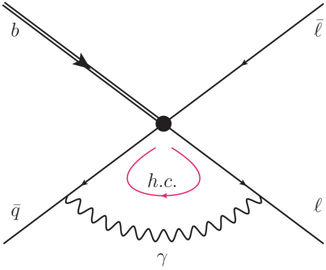
$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle = if_{B_q}p^\mu$$

This is no longer true when QED effects are included – non-local time ordered products have to be evaluated

$$\langle 0|\int d^4x\, T\{j_{\mathrm{QED}}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

This can be done for QED bound-states but QCD is non-perturbative at low scales

SCET approach



Modes

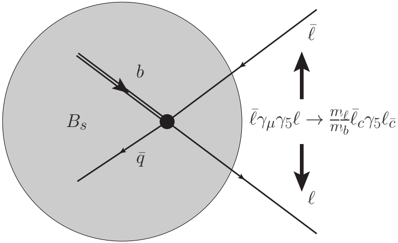
- Hard collinear, $p^2 \sim \Lambda_{\mathrm{QCD}} m_b$
- Collinear, $p^2 \sim \Lambda^2_{\mathrm{QCD}} \sim m^2_\mu$
- Soft $p^2 \sim \Lambda^2_{\mathrm{QCD}}$

EFT approach allows to perform systematic expansion in $\frac{\Lambda_{\mathrm{QCD}}}{m_b}$ Two step matching is required: Effective weak interaction $\rightarrow$ $SCET_I$ $\rightarrow$ $SCET_{II}$

In each case, the quark has a hard-collinear virtuality – *soft gluons are power suppressed*

Helicity suppression

Can the helicity suppression be relaxed?



For $m_\ell \to 0$ the amplitude has to vanish

Annihilation and helicity flip take place at the same point $r \lesssim \frac{1}{m_b}$

Helicity suppression

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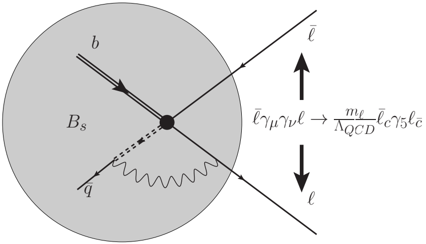
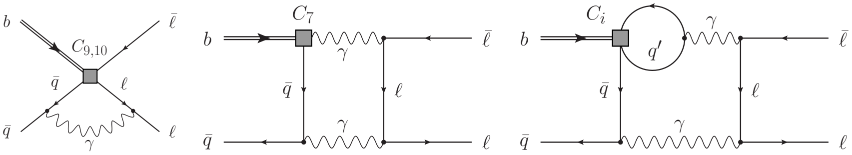


Annihilation and helicity flip can be separated by $r \sim \frac{1}{\sqrt{m_b \Lambda_{\mathrm{QCD}}}}$

It is still short distance effect, since the size of the meson is $r \sim \frac{1}{\Lambda_{\mathrm{QCD}}}$

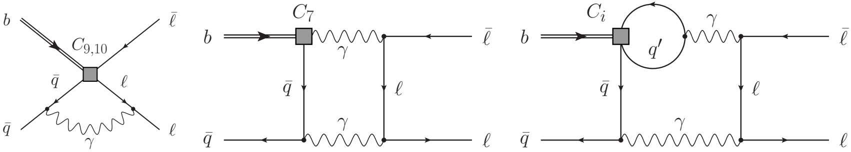
For $m_\ell \to 0$ the amplitude still vanishes

The correction



$$
\begin{aligned}
i\mathcal{A} \;=\;& m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell m_B\, f_{B_q}\mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
& \times \Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2) \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right] \\
& - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2 \frac{m_b\omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \Bigg\}
\end{aligned}
$$

The correction



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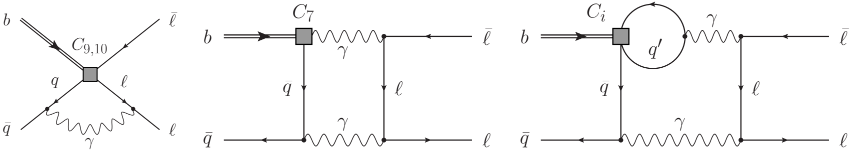
\end{aligned}
$$

- Tree level amplitude

The correction



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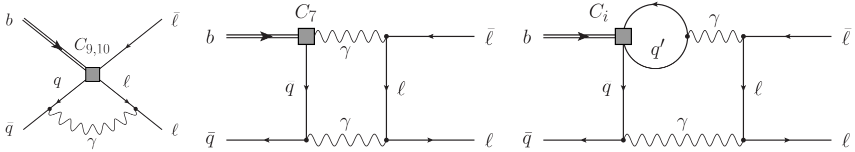
\end{aligned}
$$

- Helicity suppression × power enhancement factor

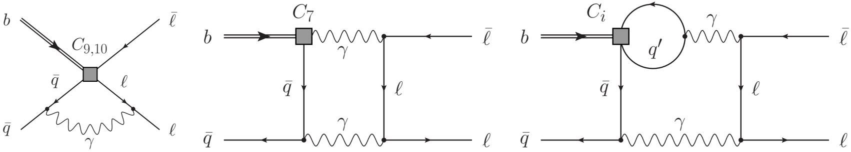
The correction



$$\begin{aligned} i\mathcal{A} \;=\;& m_\ell f_{B_q}\mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5\ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell m_B\, f_{B_q}\mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\ &\times\Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2) \int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega) \left[\ln\frac{m_b\omega}{m_\ell^2} + \ln\frac{u}{1-u}\right] \\ &- Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega) \left[\ln^2\frac{m_b\omega}{m_\ell^2} - 2\ln\frac{m_b\omega}{m_\ell^2} + \frac{2\pi^2}{3}\right]\Bigg\} \end{aligned}$$

- Convolution from the hard-scale matching

The correction



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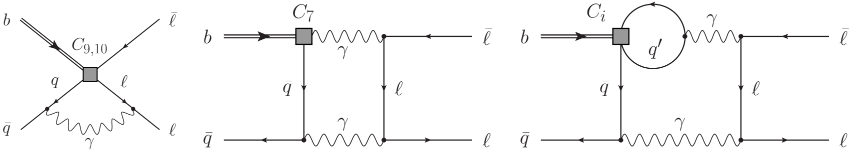
\end{aligned}
$$

- Convolution with the light-cone distribution function

The correction



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\begin{aligned}
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& \left. - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2 \frac{m_b\omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \right\}
\end{aligned}
$$

- Double logarithmic enhancement due to endpoint singularity

Non-perturbative contribution

Non-perturbative physics is encoded in the moments of B-meson light-cone distribution function [M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, 1999]

$$\frac{1}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega, \mu),$$

$$\frac{\sigma_n(\mu)}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\mu_0}{\omega}\, \phi_{B+}(\omega, \mu)$$

$\lambda_B(1\text{ GeV}) = (275 \pm 75)$ MeV
$\sigma_1(1\text{ GeV}) = 1.5 \pm 1$
$\sigma_2(1\text{ GeV}) = 3 \pm 2$
Power - enhancement factor

$$m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \ln^k \omega \sim \frac{m_B}{\lambda_B} \times \sigma_k$$

Numerical predictions

Total correction:

$$-(0.3 - 1.1)\%$$

thanks to cancellation between C_7 and C_9 part. Central value: $-0.6\% = 1.1\% - 1.7\%$ (C_7, C_9 parts).

Uncertainty comes form λ_B, σ_1, σ_2.

New prediction for the branching ratio [M. Beneke, C. Bobeth, R. S., 2017]

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} \quad = \quad (3.57 \pm 0.17) \cdot 10^{-9}$$

Uncertainty:

- parametric: ± 0.167 (now dominates but it is expected to be reduced in the future)
- non-parametric non-QED: ± 0.043
- QED $^{+0.022}_{-0.030}$ ($\sim 0.84\%$)

Previous estimate of QED uncertainty was 0.3%, obtained by scale variation method. This uncertainty is still present.

Conclusions

- Radiative corrections in bound states can have surprisingly complex pattern
- Spectroscopic measurements serve as the most precise source of fundamental constants and they can also facilitate discovery of new physics
- Theory of hydrogen energy levels has to be further scrutinized
- QED correction to QCD bound states can exhibit power enhancement that cannot be anticipated without detailed computation
- Radiative corrections can mimic New Physics signal
- Systematic progress is possible thanks to EFT approach (NRQED, HQEFT, SCET)