

Theory review of tau physics

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Matter To The Deepest 2017

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Outline :

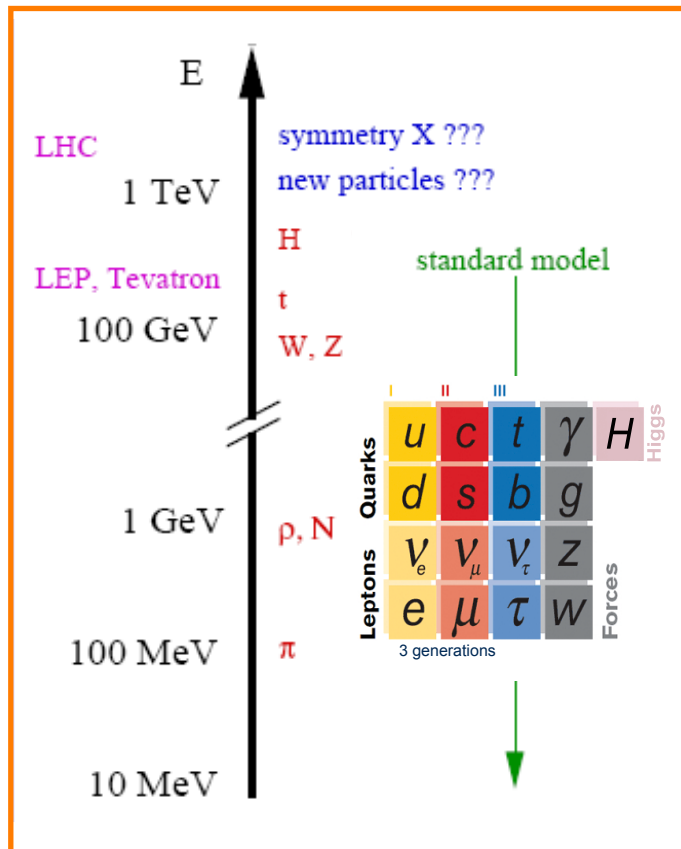
1. τ lepton as a laboratory to explore the Standard Model and possible extensions
2. Extraction of V_{us} from hadronic τ decays and test of the CKM unitarity
3. Lepton Flavour Violation
4. Conclusion and outlook

NB: several very interesting topics not covered:
lepton universality, CP violation, $g-2$, EDMs, neutrinos, etc...

1. Introduction and Motivation

1.1 Quest for New Physics

- New era in particle physics :
➡ (unexpected) *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*
- *Where do we look?* Everywhere! ➡ search for New Physics with *broad search strategy* given lack of clear indications on the SM-EFT boundaries (*both in energies and effective couplings*)



➡ Key unique role of *Tau physics*

1.2 Tau decays

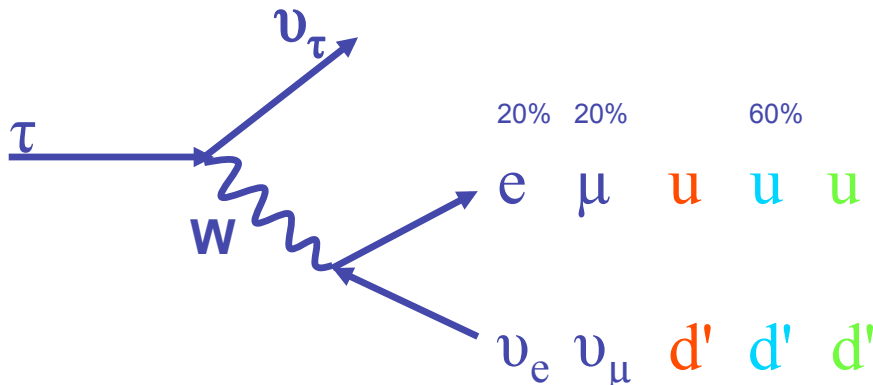
➔ Key unique role of *Tau physics*

$$m_\tau = 1.77682(16) \text{ GeV}$$

and

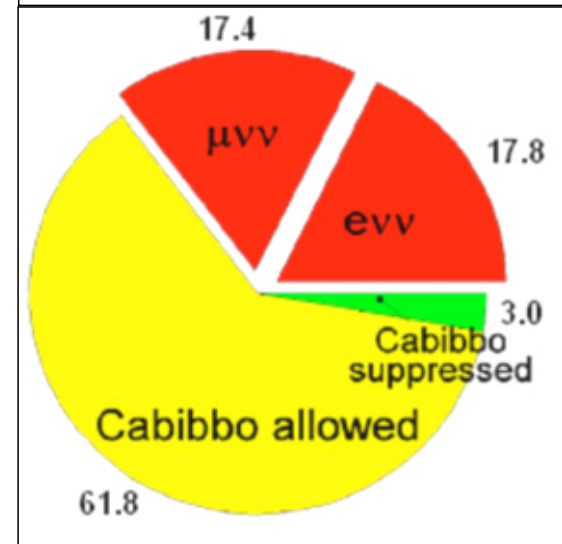
$$\tau_\tau = 2.096(10) \cdot 10^{-13} \text{ s}$$

- Naive prediction:



$$|d'\rangle = V_{ud}|d\rangle + V_{us}|s\rangle$$

Including QED & QCD corrections:

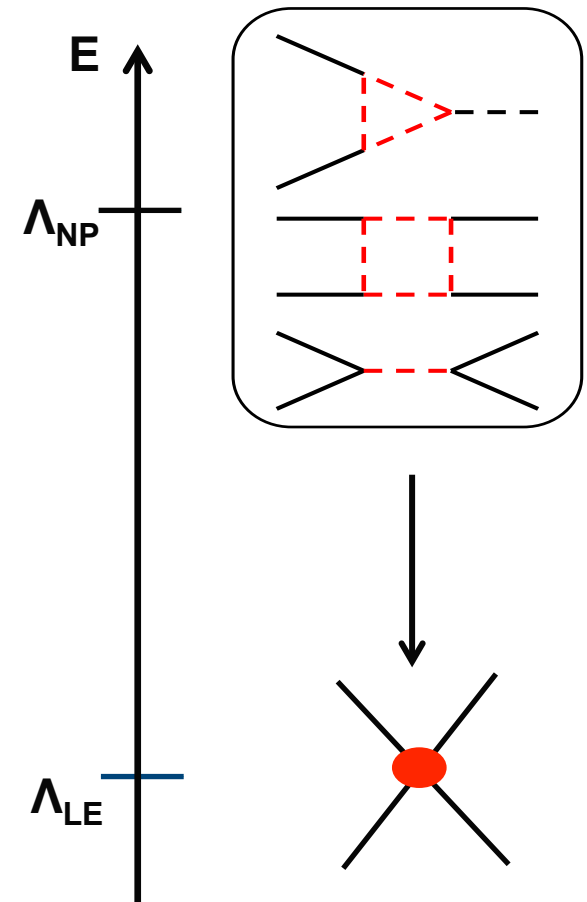


1.3 τ lepton as a unique probe of new physics

- Unique probe of *Lepton Universality* and *Charged Lepton Flavour Violation*
No SM background
Indirect probe of flavor-violating NP occurring at energies not directly accessible at accelerators:

– Kaon physics: $\frac{s\bar{d}s\bar{d}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^5 \text{ TeV}$
 $[\epsilon_K]$

– Tau physics: $\frac{\tau\bar{\mu}f\bar{f}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^2 \text{ TeV}$
 $[\tau \rightarrow \mu\gamma]$



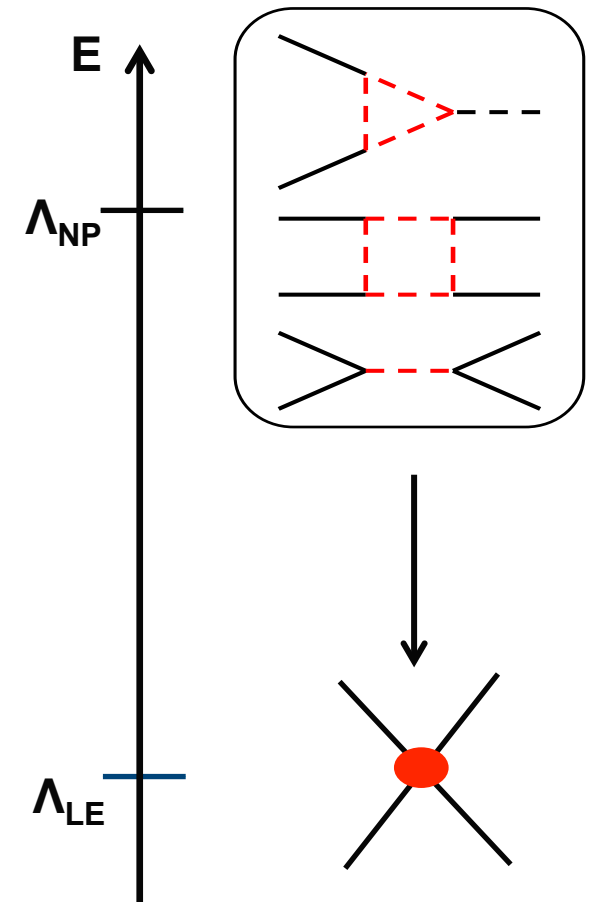
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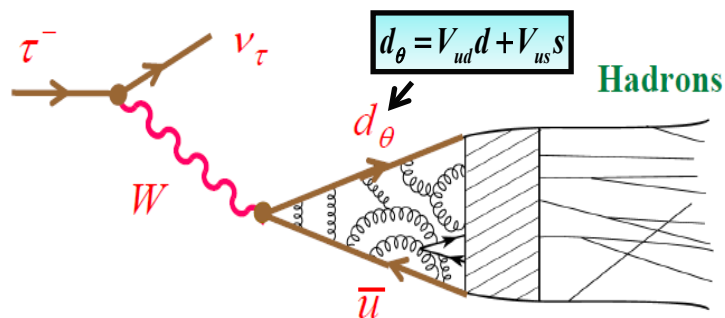
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- Tool to search for *New Physics* at *colliders*:
Ex: $h \rightarrow \tau\tau$, LFV in $h \rightarrow \tau\mu$



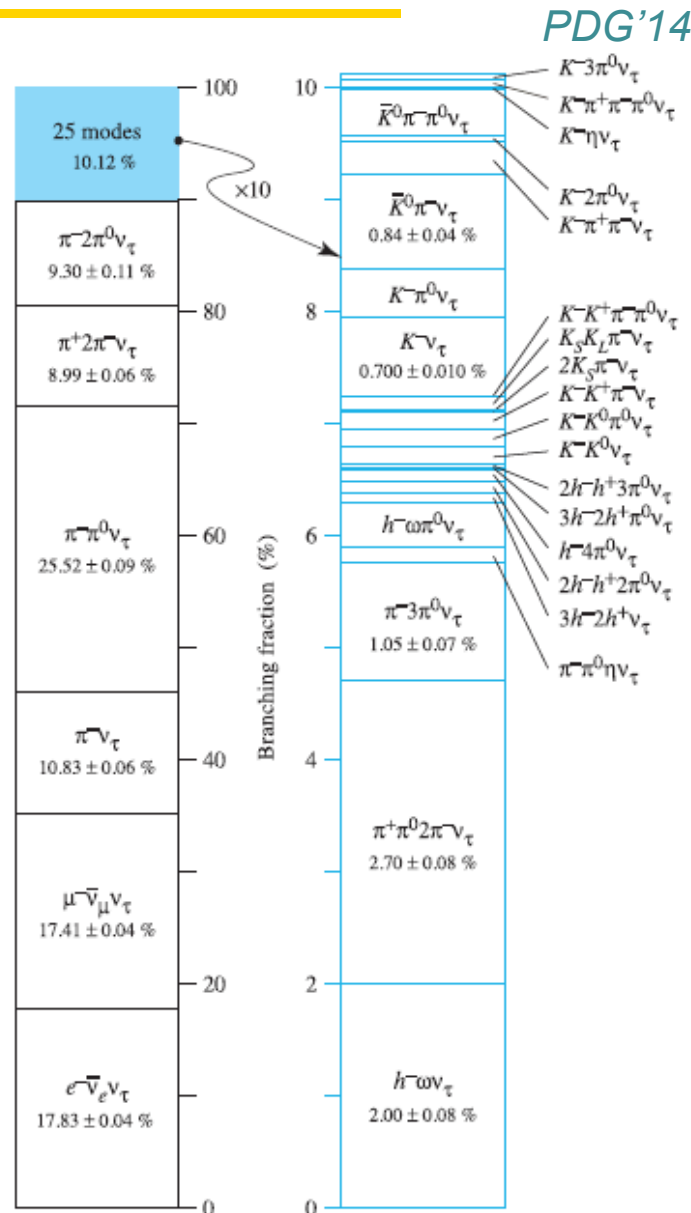
1.4 τ lepton as a unique probe for understanding QCD



- Studying its hadronic decays:
inclusive & *exclusive*
 - Unique probe of some of the *fundamental SM parameters*: α_S , $|V_{us}|$, m_s
 - Ideal set-up for the “R&D” of theory tools about *non perturbative* & *perturbative dynamics*:
OPE, Chiral Perturbation Theory, Resonances, large N_c , dispersion relations, lattice QCD, etc...



Improve our understanding of the SM and QCD at low energy



1.5 Experimental situation

- A lot of progress in tau physics since its discovery on all the items described before → important experimental efforts from *LEP, CLEO, B factories: Babar, Belle, BES, VEPP-2M, LHCb, neutrino experiments,...*
→ More to come from *LHCb, BES, VEPP-2M, Belle II, CMS, ATLAS*
- But τ physics has still potential
“*unexplored frontiers*”
→ deserve future exp. & th. efforts
- In the following, some selected examples

Experiment	Number of τ pairs
LEP	$\sim 3 \times 10^5$
CLEO	$\sim 1 \times 10^7$
BaBar	$\sim 5 \times 10^8$
Belle	$\sim 9 \times 10^8$
Belle II	$\sim 10^{12}$

2. V_{us} and CKM unitarity test

2.1 Probing the CKM mechanism

- The CKM Mechanism source of *Charge Parity Violation* in SM
- **Unitary 3x3 Matrix**, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates

CKM Matrix

Mass Eigenstates

$$\sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

2.2 Extraction of V_{us}

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}

- Fundamental parameter of the Standard Model

Check unitarity of the first row of the CKM matrix:

➡ *Cabibbo Universality*

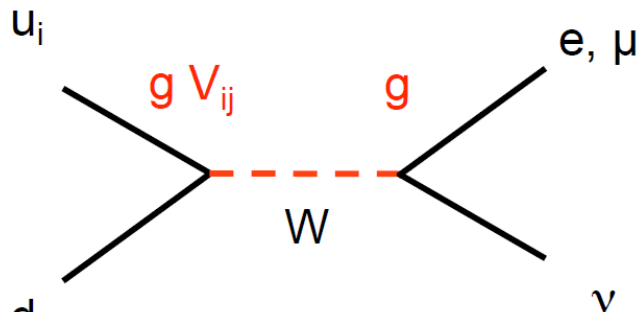
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$$

Negligible
(B decays)

- Input in UT analysis

- Look for *new physics*

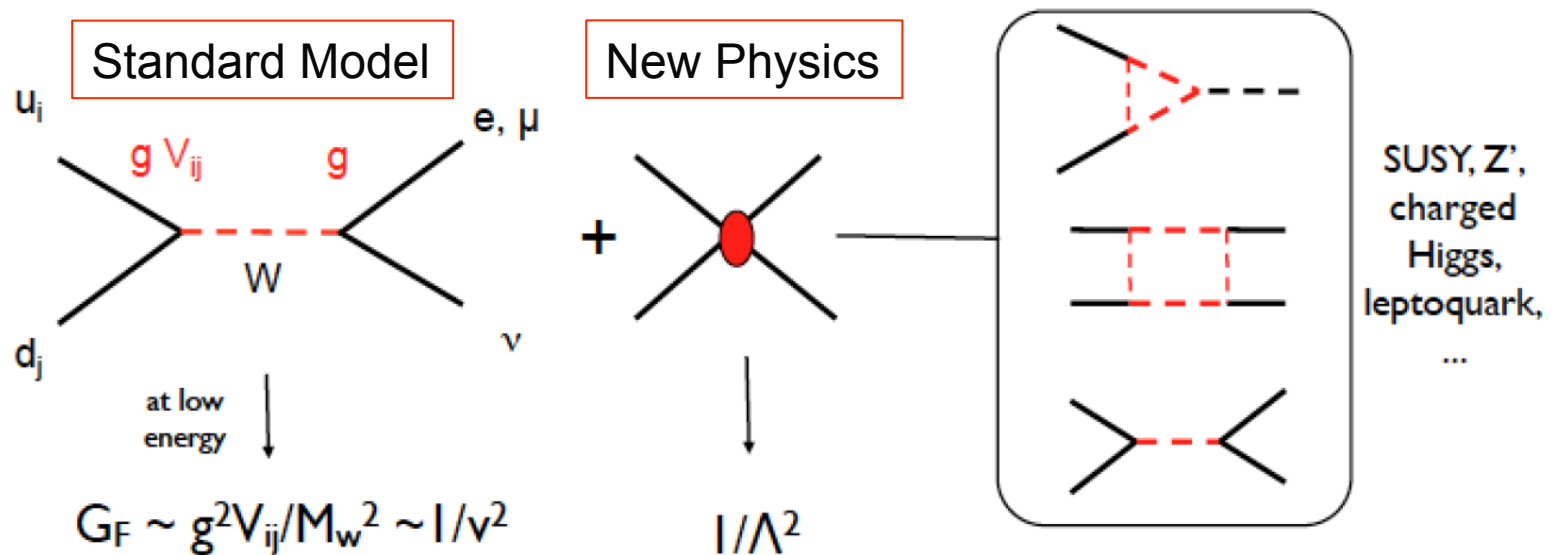
- In the Standard Model : W exchange ➡ only V-A structure



2.2 Extraction of V_{us}

- BSM: sensitive to tree-level and loop effects of a large class of models

➔ $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$



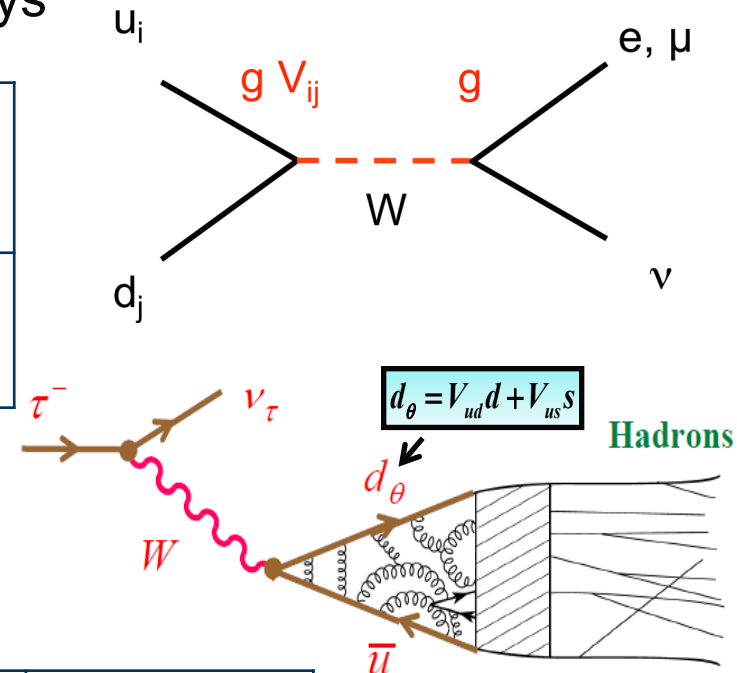
➔ BSM effects : $\Delta_{CKM} \sim (v/\Lambda)^2$

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed $K_{l 3}$, hadronic τ decays

2.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \rightarrow 0^+$ $\pi^\pm \rightarrow \pi^0 e \nu_e$	$n \rightarrow p e \nu_e$	$\pi \rightarrow l \nu_l$
V_{us}	$K \rightarrow \pi l \nu_l$	$\Lambda \rightarrow p e \nu_e$	$K \rightarrow l \nu_l$



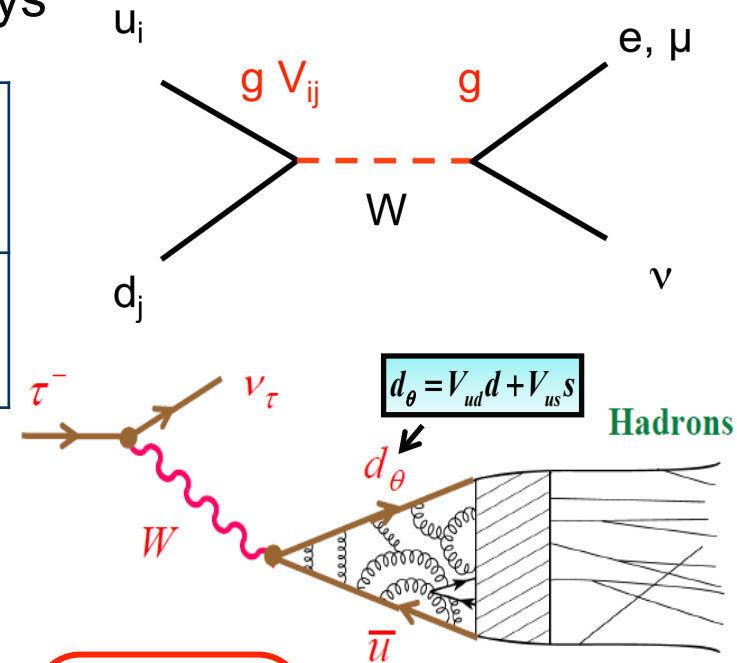
- From τ decays (crossed channel)

V_{ud}	$\tau \rightarrow \pi \pi \nu_\tau$		$\tau \rightarrow \pi \nu_\tau$	$\tau \rightarrow h_{NS} \nu_\tau$
V_{us}	$\tau \rightarrow K \pi \nu_\tau$		$\tau \rightarrow K \nu_\tau$	$\tau \rightarrow h_S \nu_\tau$ (inclusive)

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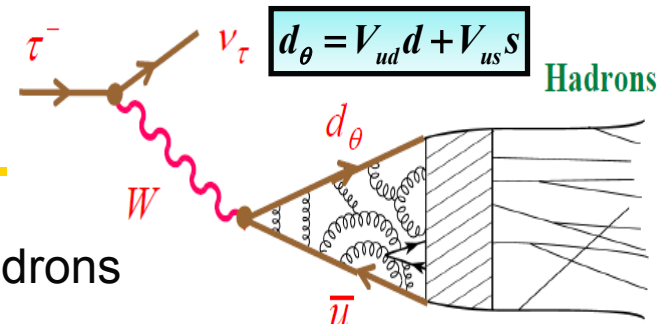
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V_{us}	$K \rightarrow \pi l \nu_l$	$\Lambda \rightarrow p e \nu_e$	$K \rightarrow l \nu_l$



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2.3 V_{us} from inclusive measurement

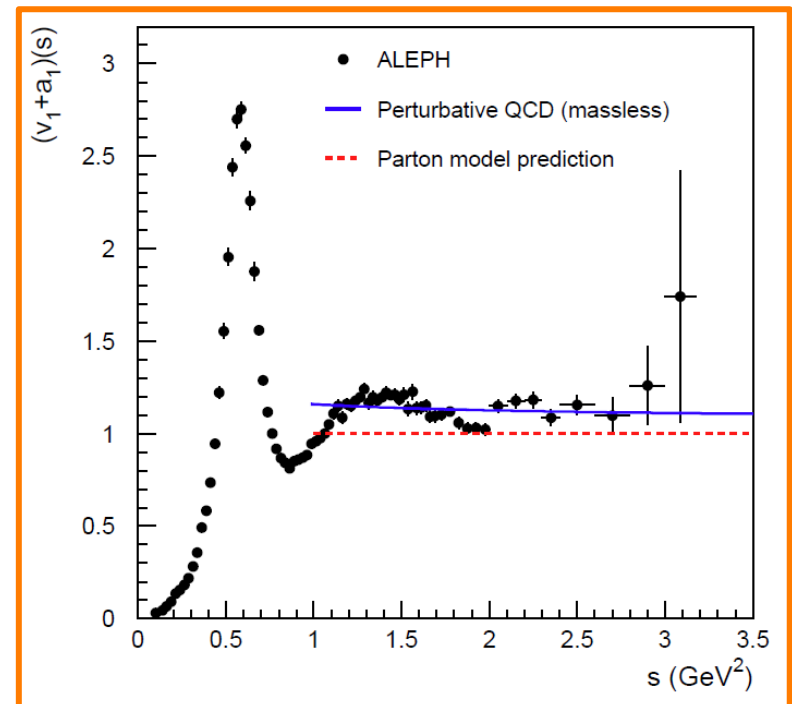


- Tau, the only lepton heavy enough to decay into hadrons
- $m_\tau \sim 1.77\text{GeV} > \Lambda_{QCD}$ $\Rightarrow$ use *perturbative tools: OPE...*
- Inclusive τ decays : $\tau \rightarrow (\bar{u}d, \bar{u}s)\nu_\tau$ $\Rightarrow$ fund. SM parameters $(\alpha_s(m_\tau), |V_{us}|, m_s)$

Davier et al'13

- We consider $\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons}_{S=0})$
- $\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons}_{S \neq 0})$
- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system $\Rightarrow$ huge *QCD activity*!

- Observable studied: $R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)}$

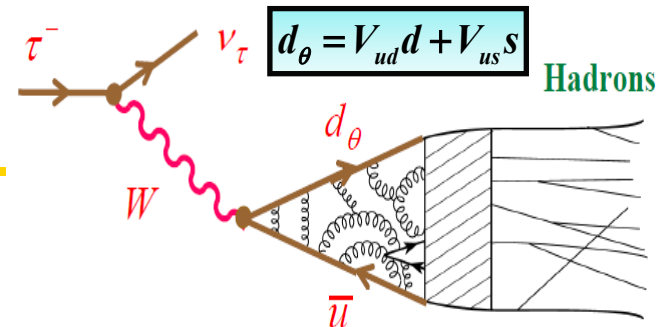


2.4 Theory

- $$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C$$
 parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$$

- $$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} \Rightarrow |V_{us}|$$



QCD switch

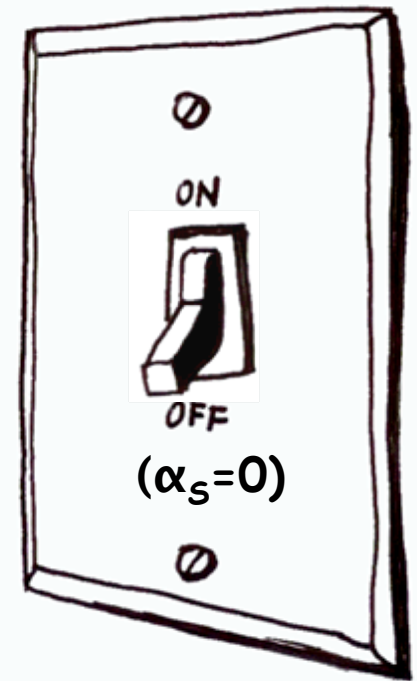


Figure from
M. González Alonso'13

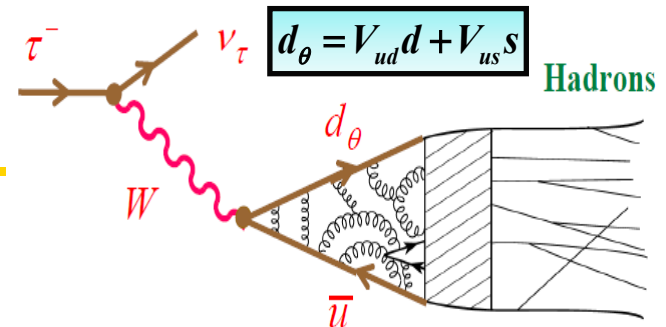
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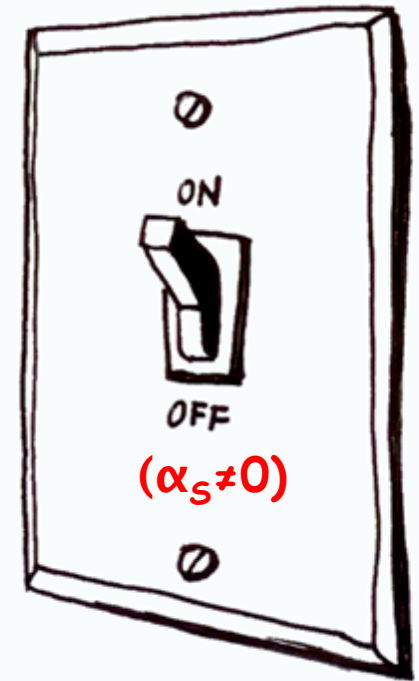
- $$R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$$

- Experimentally:

$$R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$$



QCD switch



2.4 Theory

- $$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C$$
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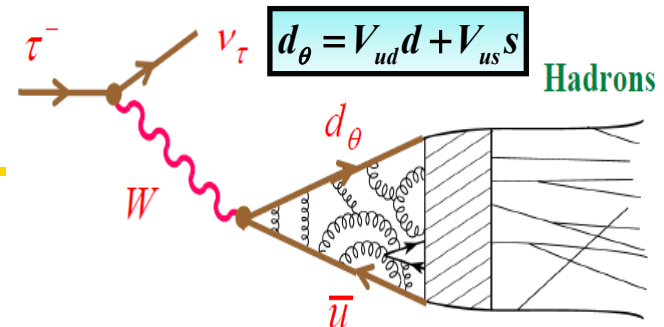
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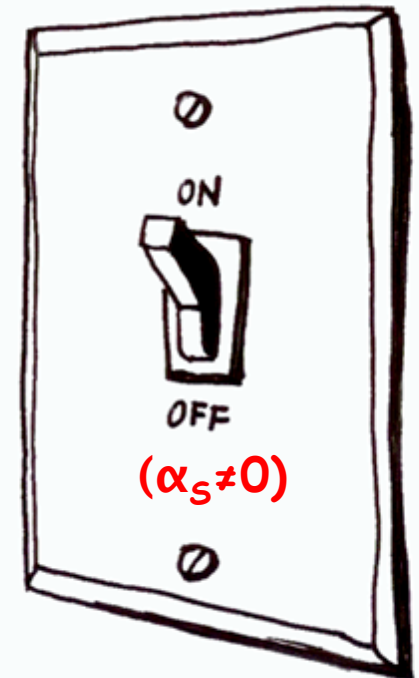
$$R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$$

- Due to *QCD corrections*:

$$R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + \mathcal{O}(\alpha_s)$$



QCD switch



2.4 Theory

- From the measurement of the spectral functions, extraction of α_S , $|V_{us}|$

- $$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C$$
naïve QCD prediction

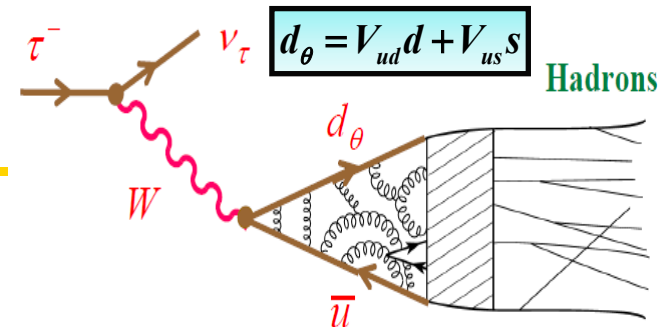
- Extraction of the strong coupling constant :

$$\begin{array}{c}
 \text{measured} \nearrow R_\tau^{NS} = |V_{ud}|^2 N_C + \text{calculated} \nwarrow O(\alpha_s) \xrightarrow{\text{yellow arrow}} \alpha_s
 \end{array}$$

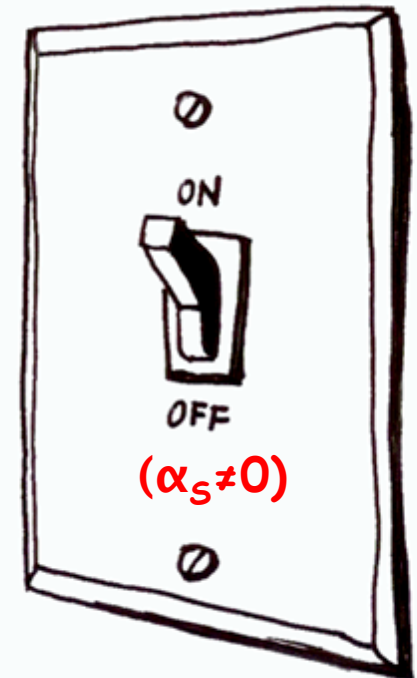
- Determination of V_{us} :

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)$$

- Main difficulty: compute the QCD corrections with the best accuracy



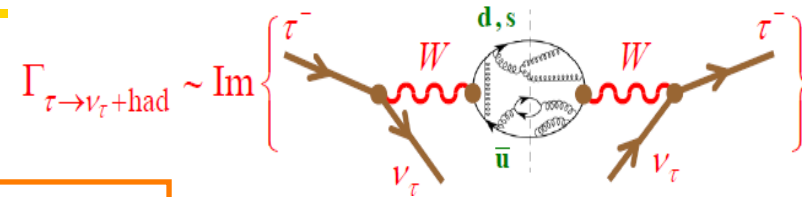
QCD switch



2.5 Calculation of the QCD corrections

- Calculation of R_τ :

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s+i\epsilon) + \text{Im} \Pi^{(0)}(s+i\epsilon) \right]$$



Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

→ Cauchy Theorem

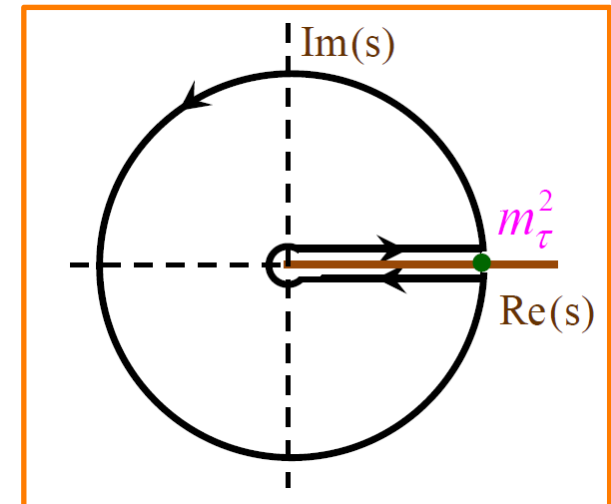
$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \Pi^{(1)}(s) + \Pi^{(0)}(s) \right]$$

- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\dots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s, \mu) \langle O_D(\mu) \rangle$$

Wilson coefficients

Operators



μ : separation scale between short and long distances

2.5 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ :

$$R_\tau(m_\tau^2) = N_C S_{EW} (1 + \delta_P + \delta_{NP})$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano & Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$ $a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS} (\propto m_u, m_d)$ but not for $R_\tau^S (\propto m_s)$

- $D \geq 4$: Non perturbative part, not known, *fitted from the data*

 Use of weighted distributions

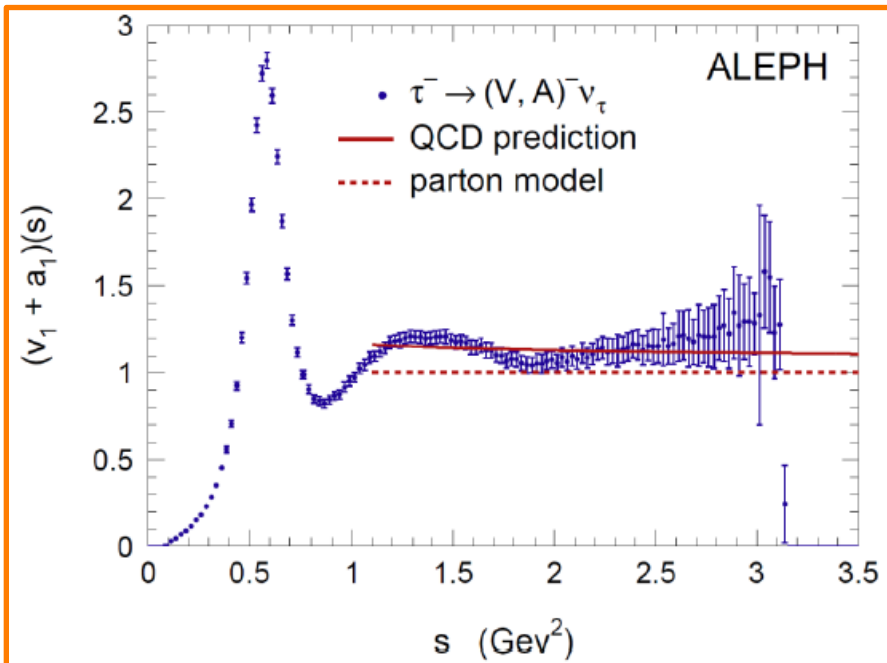
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Le Diberder&Pich'92

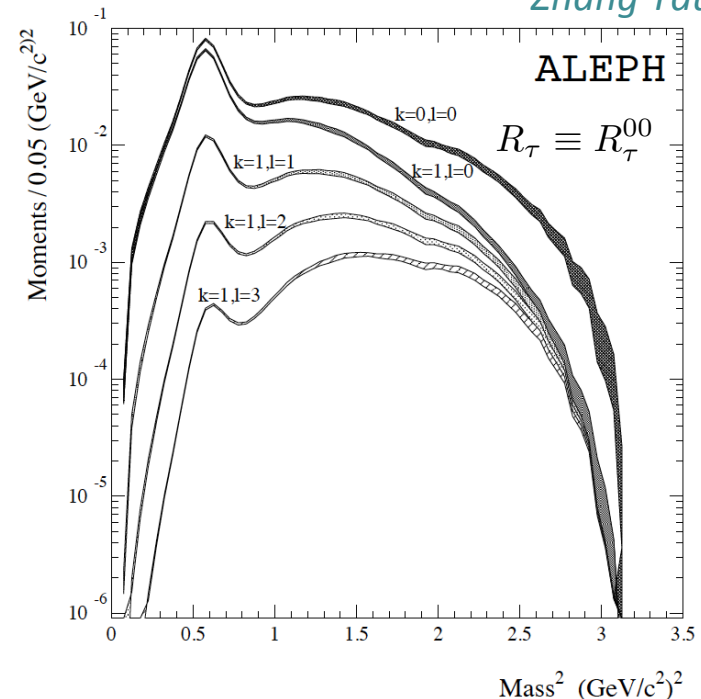
- $D \geq 4$: Non perturbative part, not known, *fitted from the data*
➡ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



2.5 Inclusive determination of V_{us}

- With QCD on:
$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + \mathcal{O}(\alpha_s)$$

- Use OPE:
$$R_\tau^{NS}(m_\tau^2) = N_C S_{EW} |V_{ud}|^2 (1 + \delta_P + \delta_{NP}^{ud})$$

$$R_\tau^S(m_\tau^2) = N_C S_{EW} |V_{us}|^2 (1 + \delta_P + \delta_{NP}^{us})$$

- $$\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32) \quad \text{Gamiz et al'07, Maltman'11}$$

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

HFAG'17

$$R_{\tau,S} = 0.1633(28)$$

$$R_{\tau,NS} = 3.4718(84)$$

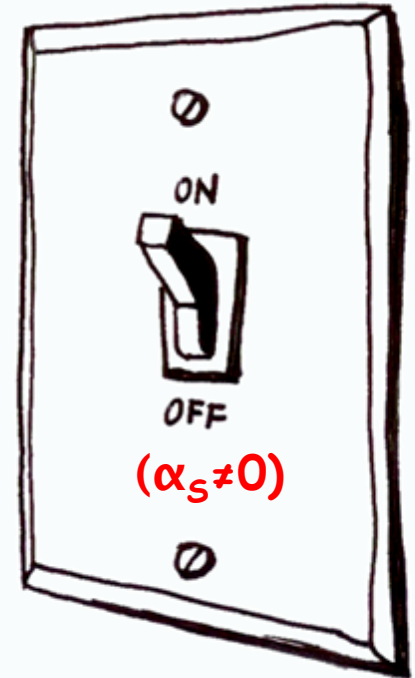
$$|V_{ud}| = 0.97417(21)$$

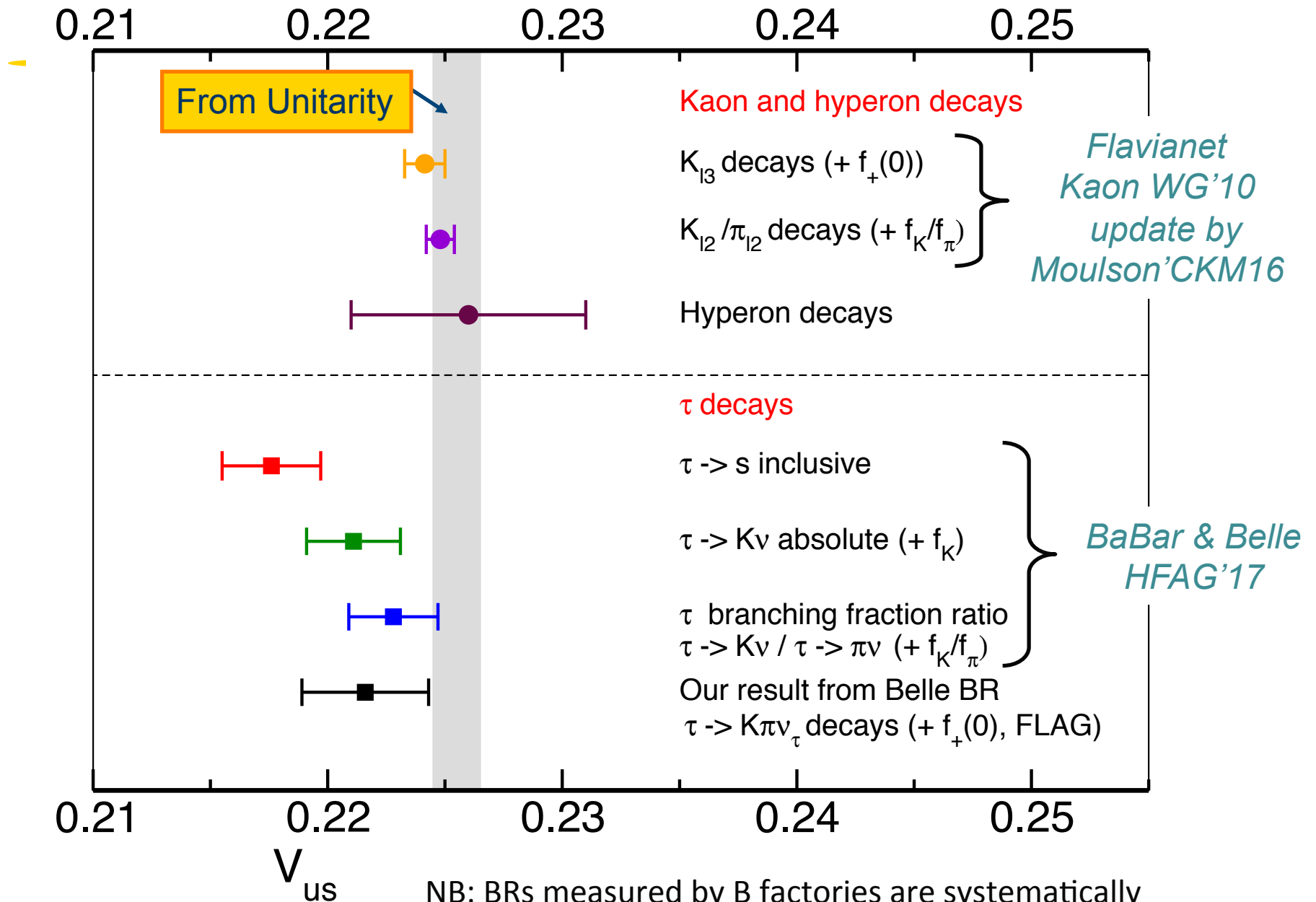


$$|V_{us}| = 0.2186 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}$$

3.1σ away from unitarity!

QCD switch





2.6 V_{us} using info on Kaon decays and $\tau \rightarrow K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$ $\rightarrow$ $(0.713 \pm 0.003)\%$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$ $\rightarrow$ $(0.471 \pm 0.018)\%$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$ $\rightarrow$ $(0.857 \pm 0.030)\%$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$ $\rightarrow$ $(2.967 \pm 0.060)\%$

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us}

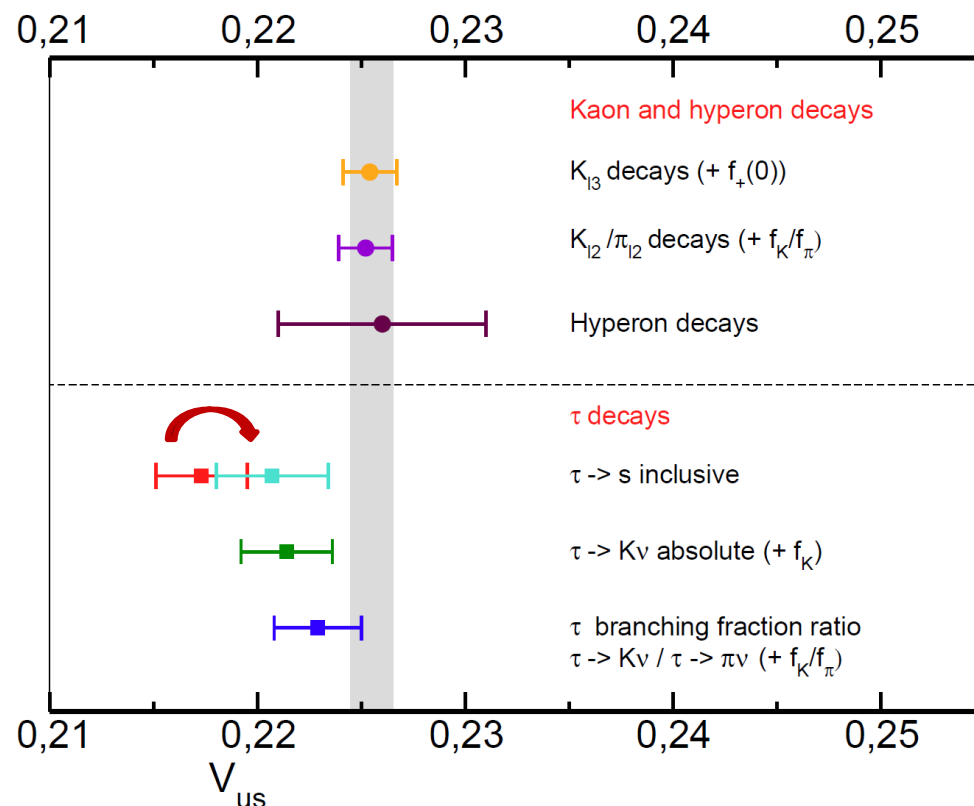
$\rightarrow$ Recent studies

R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:
 $\tau \rightarrow K\pi\nu_\tau$ Br + spectrum

$$|V_{us}| = 0.2229 \pm 0.0022_{\text{exp}} \pm 0.0004_{\text{theo}}$$

$\rightarrow$ need new data

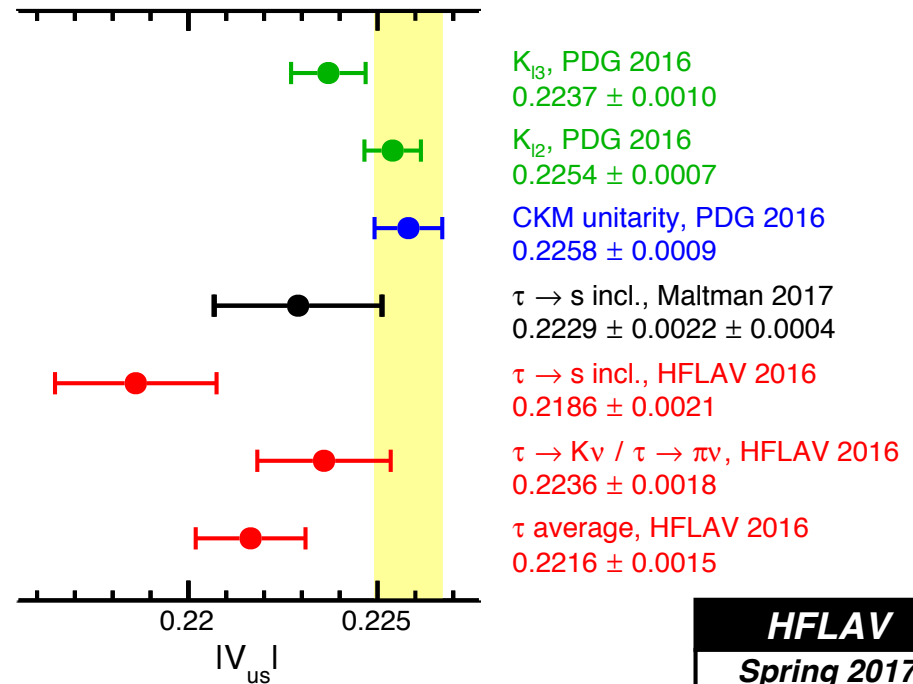


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$\rightarrow$ need new data

Very good prospect from Belle II, BES?

3. Charged Lepton-Flavour Violation

3.1 Tau LFV

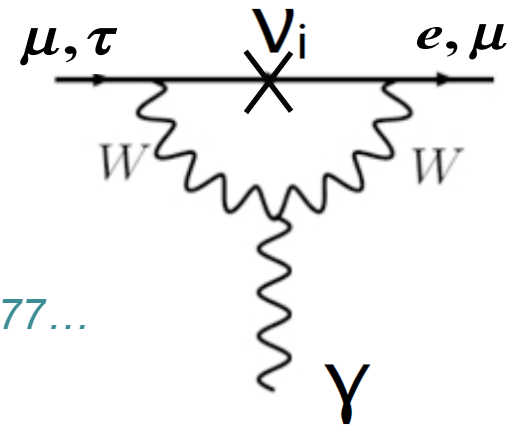
- Lepton Flavour Number is an « accidental » symmetry of the SM ($m_\nu=0$)
- In the **SM** with massive neutrinos effective CLFV vertices are tiny due to GIM suppression $\Rightarrow$ *unobservably small rates!*

E.g.: $\mu \rightarrow e\gamma$

$$Br(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U_{\mu i}^* U_{ei} \frac{\Delta m_{1i}^2}{M_W^2} \right|^2 < \mathbf{10^{-54}}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77...

$$[Br(\tau \rightarrow \mu\gamma) < \mathbf{10^{-40}}]$$

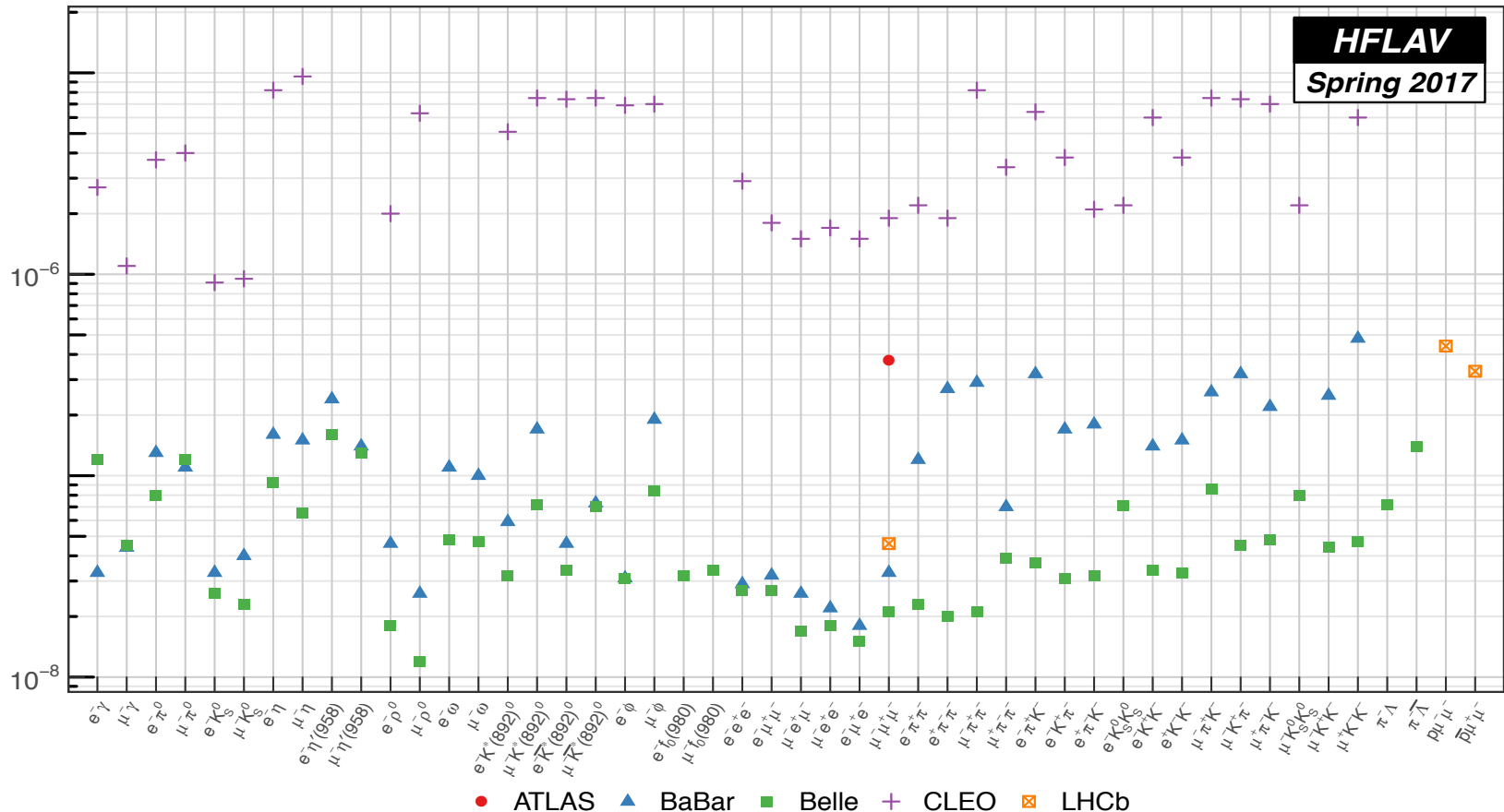


- Extremely *clean probe of beyond SM physics*
- In New Physics models: seazible effects
Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

3.2 Tau LFV

- Several processes: $\tau \rightarrow \ell \gamma$, $\tau \rightarrow \ell_{\alpha} \bar{\ell}_{\beta} \ell_{\beta}$, $\tau \rightarrow \ell Y$
 $\swarrow P, S, V, P\bar{P}, \dots$

90% CL upper limits on τ LFV decays



- 48 LFV modes studied at Belle and BaBar

- ## 90% CL upper limits on τ LFV decays



- ## LHCb, Belle II?

3.3 Effective Field Theory approach

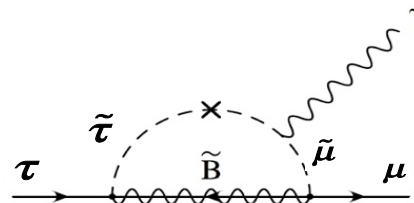
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

- Build all $D > 5$ LFV operators:

➤ Dipole:

$$\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$$

e.g.



See e.g.

Black, Han, He, Sher'02

Brignole & Rossi'04

Dassinger et al.'07

Matsuzaki & Sanda'08

Giffels et al.'08

Crivellin, Najjari, Rosiek'13

Petrov & Zhuridov'14

Cirigliano, Celis, E.P.'14

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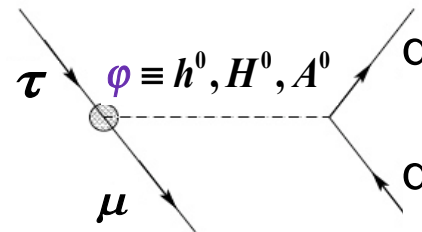
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:

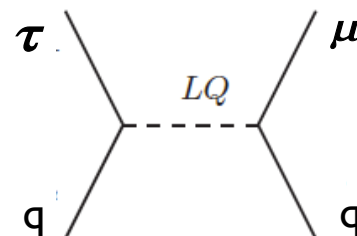
➤ Dipole: $\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$

- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$\mathcal{L}_{eff}^{S,V} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$ e.g.



$$\Gamma \equiv 1$$



$$\Gamma \equiv \gamma^\mu$$

3.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

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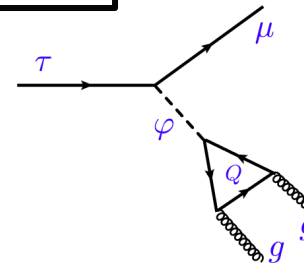
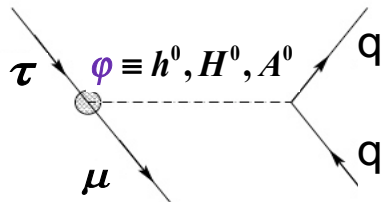
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➤ Dipole: $\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$

➤ Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^S \supset -\frac{C_{S,Y}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$

- Integrating out heavy quarks generates *gluonic operator*

$$\frac{1}{\Lambda^2} \bar{\mu} P_{L,R} \tau Q Q \bar{Q} \rightarrow \mathcal{L}_{eff}^G \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau G_{\mu\nu}^a G_a^{\mu\nu}$$



3.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

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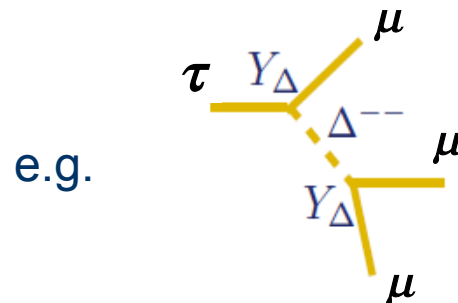
Cirigliano, Celis, E.P.'14

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➤ Dipole: $\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$

➤ Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^S \supset -\frac{C_{S,Y}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$

➤ 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,Y}^{4\ell}}{\Lambda^2} \bar{\mu} \Gamma P_{L,R} \tau \bar{\mu} \Gamma P_{L,R} \mu$



$$\Gamma \equiv 1, \gamma^\mu$$

3.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

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➤ Lepton-gluon (Scalar, Pseudo-scalar): $\mathcal{L}_{eff}^G \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau G_{\mu\nu}^a G_a^{\mu\nu}$

➤ 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,Y}^{4\ell}}{\Lambda^2} \bar{\mu} \Gamma P_{L,R} \tau \bar{\mu} \Gamma P_{L,R} \mu$

- Each UV model generates a *specific pattern* of them

$$\Gamma \equiv 1, \gamma^\mu$$

3.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \rightarrow 3\mu$	$\tau \rightarrow \mu\gamma$	$\tau \rightarrow \mu\pi^+\pi^-$	$\tau \rightarrow \mu K \bar{K}$	$\tau \rightarrow \mu\pi$	$\tau \rightarrow \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	—	—	—	—	—
O_D	✓	✓	✓	✓	—	—
O_V^q	—	—	✓ (I=1)	✓ (I=0,1)	—	—
O_S^q	—	—	✓ (I=0)	✓ (I=0,1)	—	—
O_{GG}	—	—	✓	✓	—	—
O_A^q	—	—	—	—	✓ (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	—	—	—	—	—	✓

- The notion of “*best probe*” (process with largest decay rate) is *model dependent*
- If observed, compare rate of processes ➡ key handle on *relative strength* between operators and hence on the *underlying mechanism*

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O_D	✓	✓	✓	✓	—	—
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O_{GG}	—	—	✓	✓	—	—
O_A^q	—	—	—	—	✓ (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	—	—	—	—	—	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!
- But need reliable determinations of the hadronic part:
form factors and *decay constants* (e.g. $f_\eta, f_{\eta'}$)

3.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \rightarrow 3\mu$	$\tau \rightarrow \mu\gamma$	$\tau \rightarrow \mu\pi^+\pi^-$	$\tau \rightarrow \mu K\bar{K}$	$\tau \rightarrow \mu\pi$	$\tau \rightarrow \mu\eta^{(\prime)}$
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O_D	✓	✓	✓	✓	—	—
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O_{GG}	—	—	✓	✓	—	—
O_A^q	—	—	—	—	✓ (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	—	—	—	—	—	✓

- Form factors for $\tau \rightarrow \mu(e)\pi\pi$ determined using *dispersive techniques*

Donoghue, Gasser, Leutwyler'90

- Hadronic part:

Moussallam'99

Daub et al'13

Celis, Cirigliano, E.P.'14

$$H_\mu = \langle \pi\pi | (V_\mu - A_\mu) e^{iL_{QCD}} | 0 \rangle = (\text{Lorentz struct.})_\mu^i F_i(s)$$

with

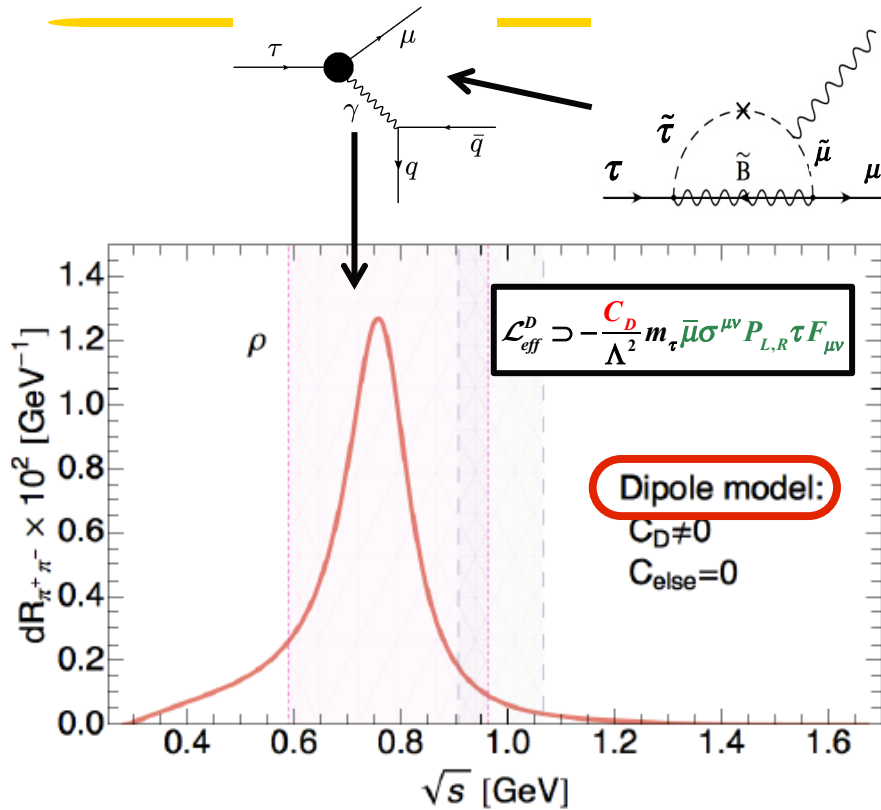
$$s = (p_{\pi^+} + p_{\pi^-})^2$$

- 2-channel unitarity condition is solved with
I=0 S-wave $\pi\pi$ and KK scattering data as input
 $n = \pi\pi, K\bar{K}$

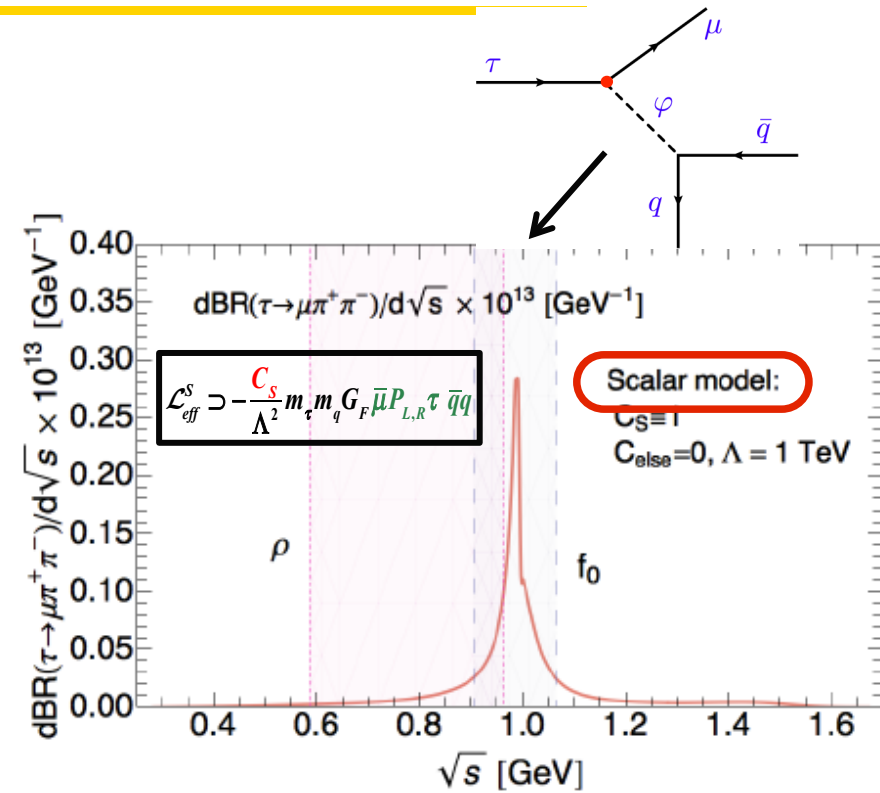
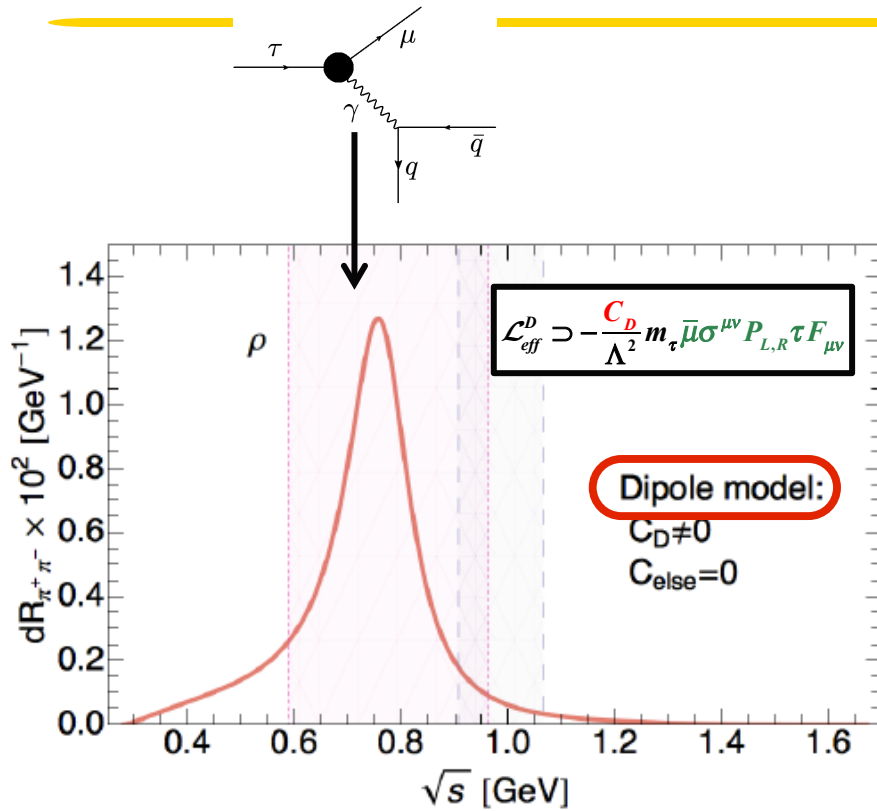
$$\text{Im}F_n(s) = \sum_{m=1}^2 T_{nm}^*(s) \sigma_m(s) F_m(s)$$

3.5 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays

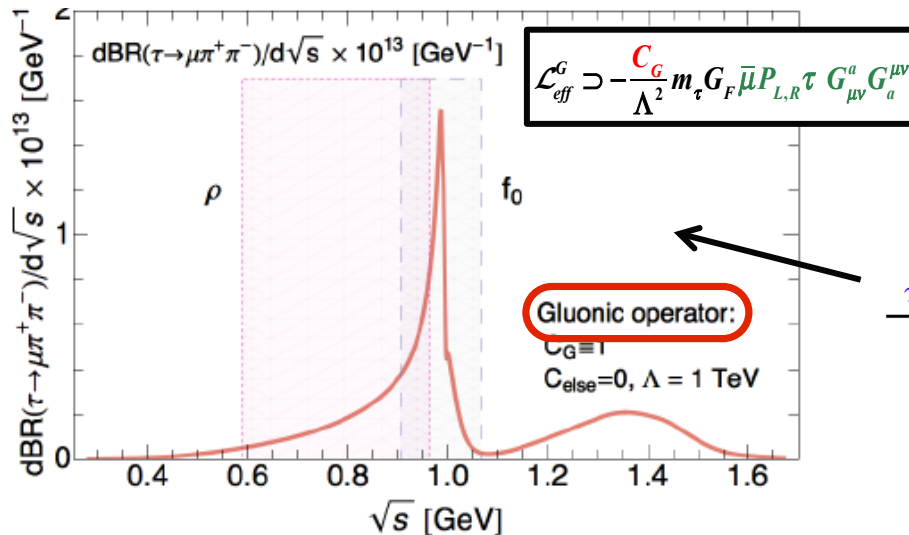
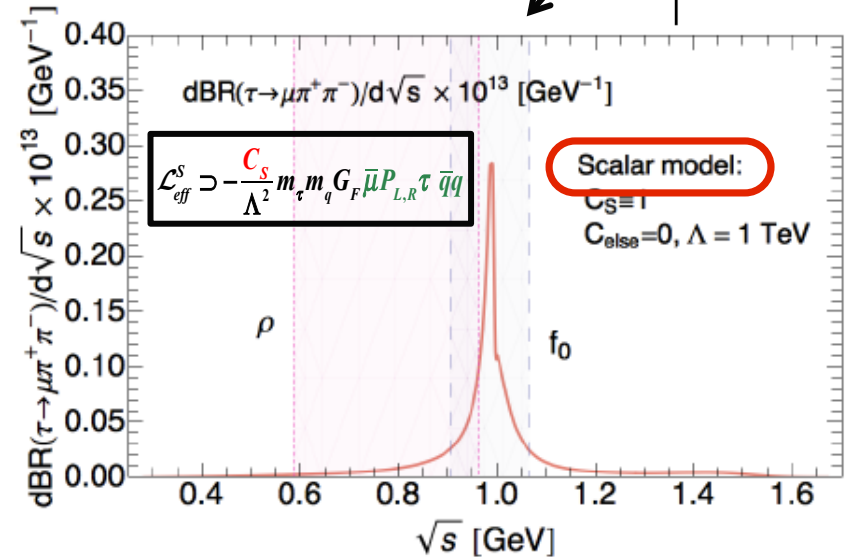
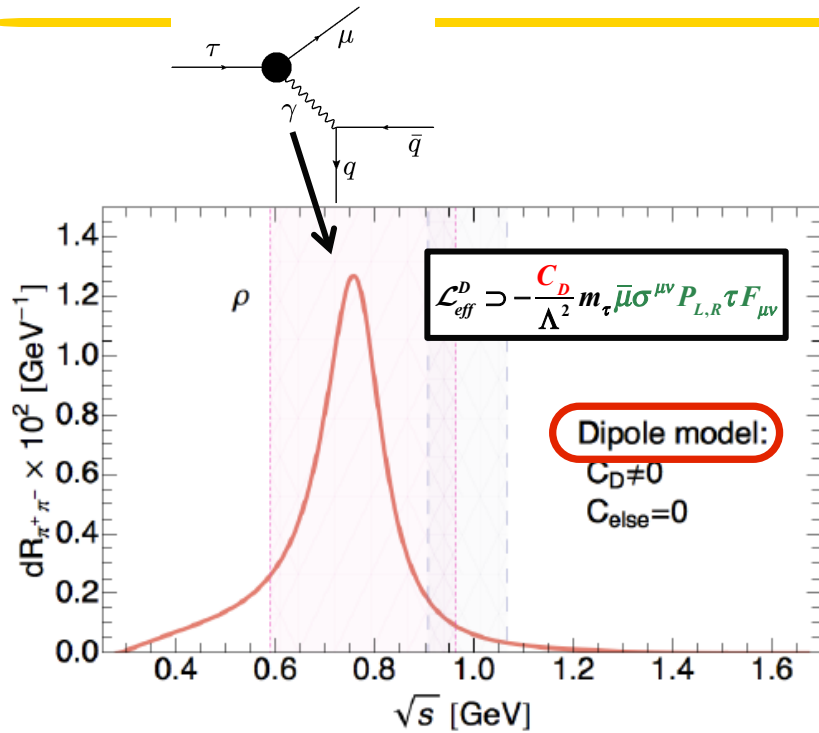
Celis, Cirigliano, E.P.'14



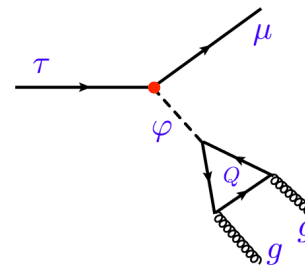
3.5 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays



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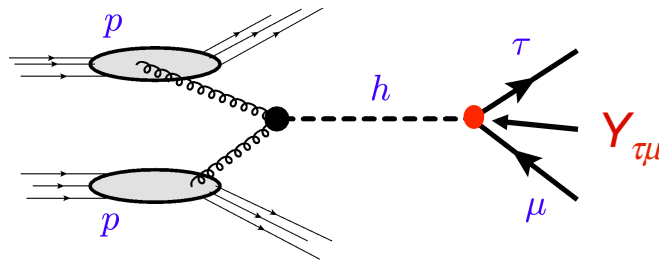
Different distributions according to the **operator!**



3.6 Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2} (\bar{f}_L^i f_R^j H) H^\dagger H \quad \Rightarrow \quad -Y_{ij} (\bar{f}_L^i f_R^j) h$$

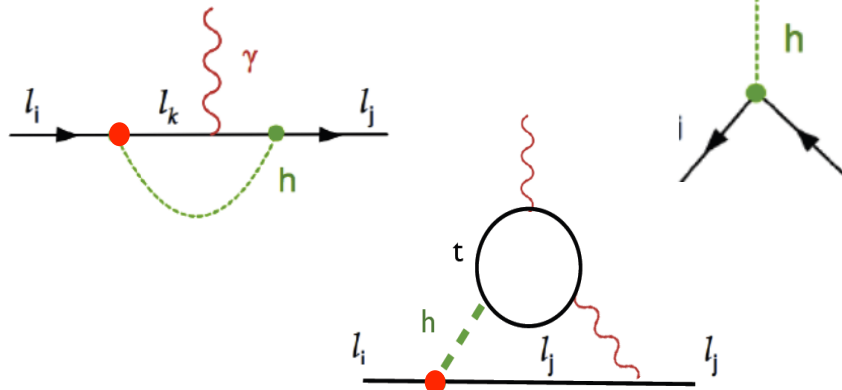
- High energy : LHC



In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v} \delta_{ij}$

Hadronic part treated with perturbative QCD

- Low energy : D, S operators

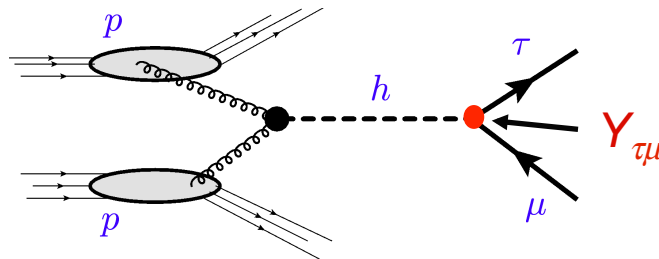


Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

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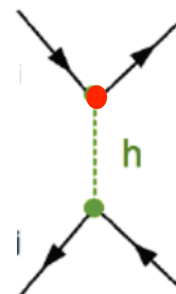
- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2} (\bar{f}_L^i f_R^j H) H^\dagger H \quad \Rightarrow \quad -Y_{ij} (\bar{f}_L^i f_R^j) h$$

- High energy : LHC

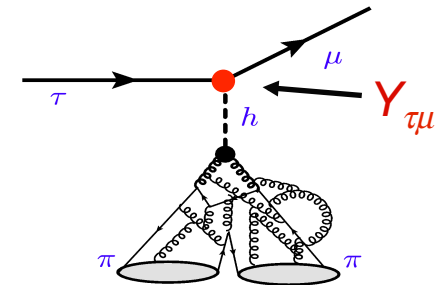


Hadronic part treated with perturbative QCD

Reverse the process

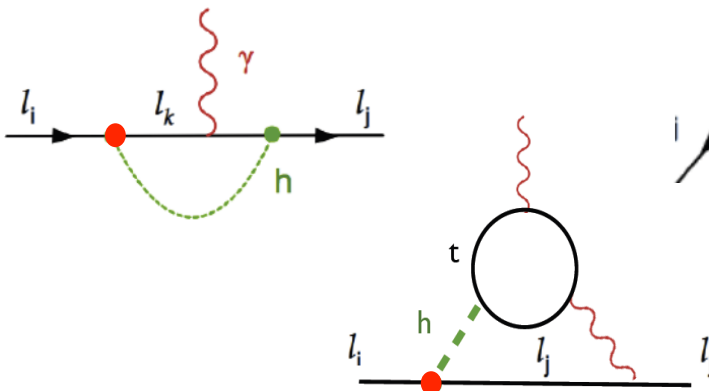


+



Hadronic part treated with non-perturbative QCD

- Low energy : D, S, G operators



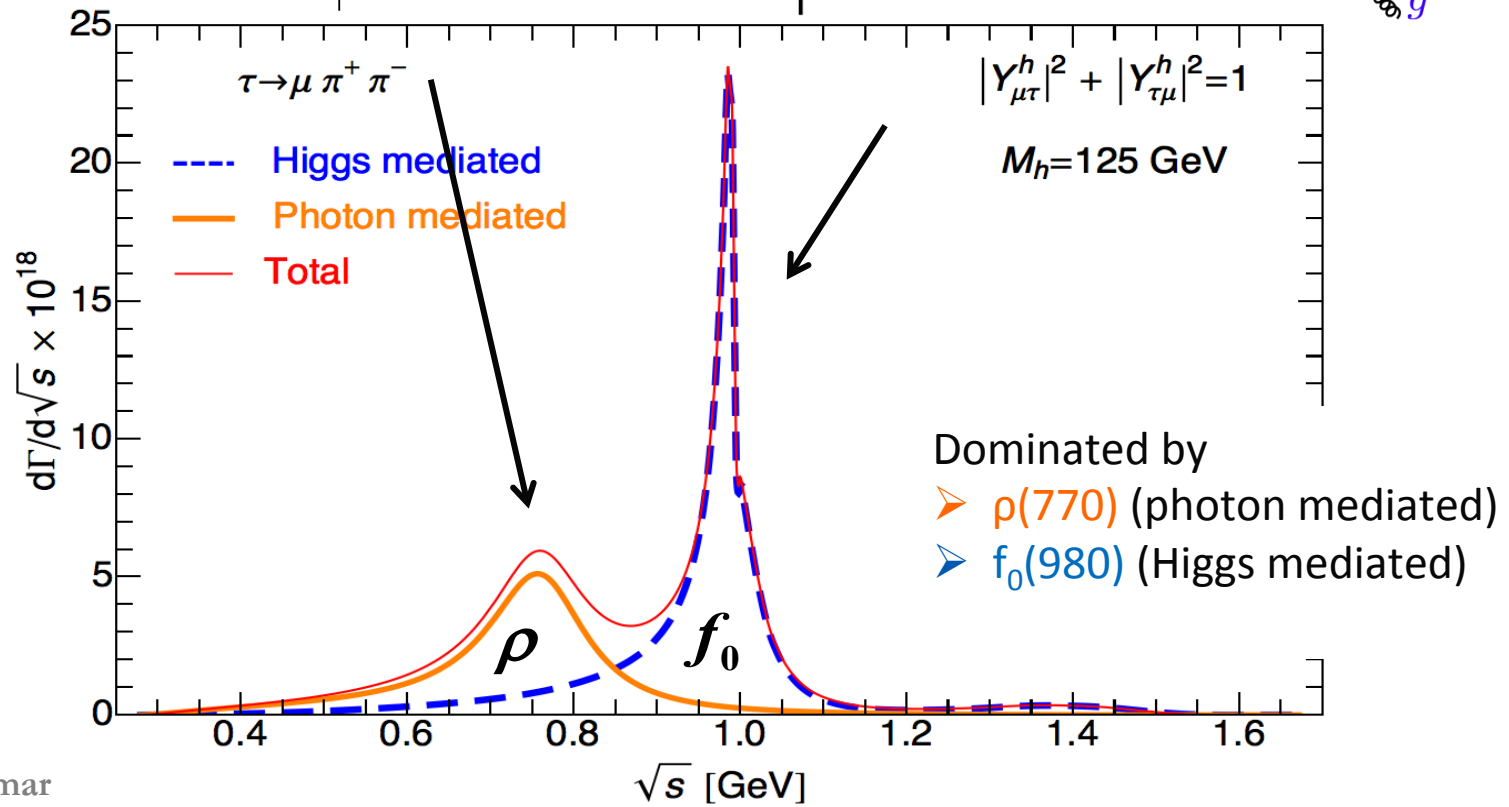
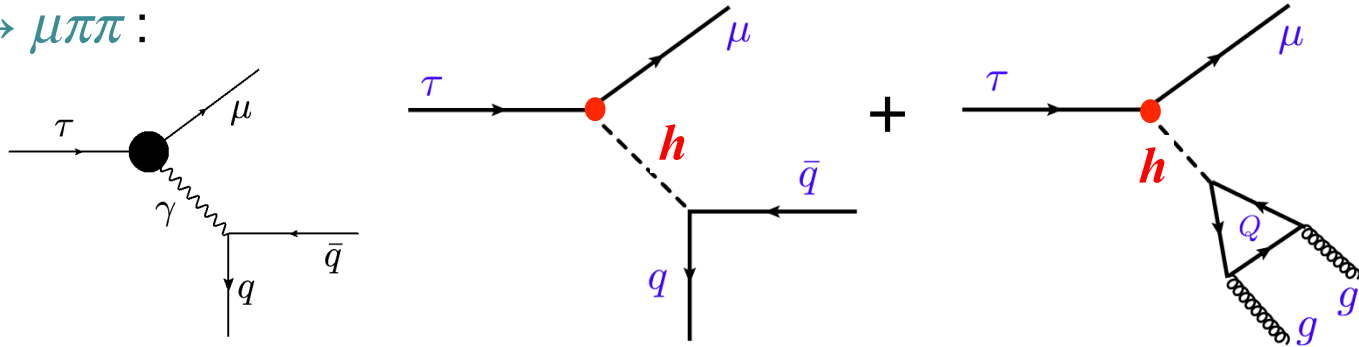
Goudelis, Lebedev, Park'11
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Constraints in the $\tau\mu$ sector

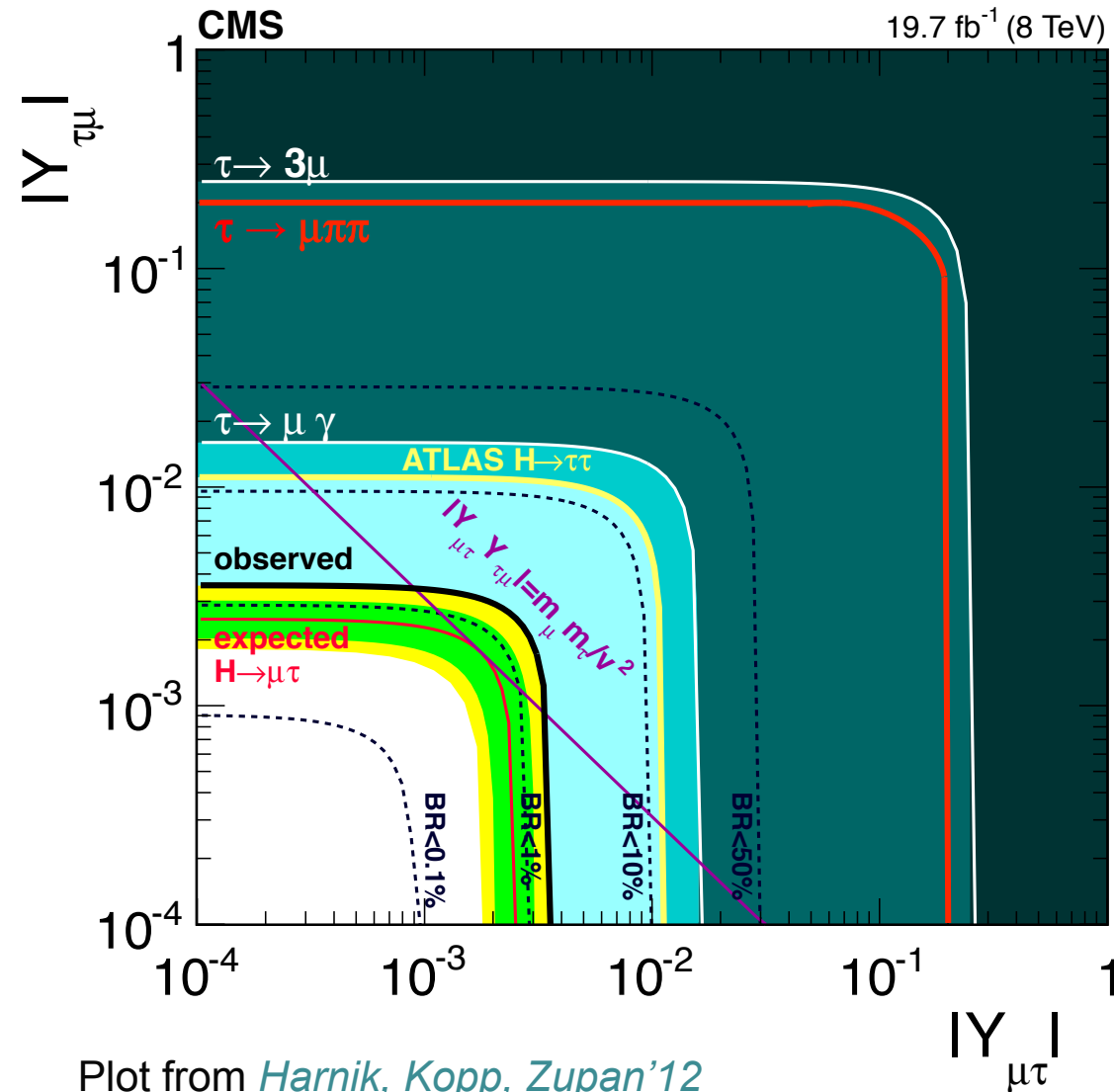
Cirigliano, Celis, E.P.'14

- At low energy

➤ $\tau \rightarrow \mu \pi \pi$:



Constraints in the $\tau\mu$ sector



- Constraints from LE:
 - $\tau \rightarrow \mu\gamma$: best constraints but loop level
 - sensitive to UV completion of the theory
 - $\tau \rightarrow \mu\pi\pi$: tree level diagrams
 - robust handle on LFV
- Constraints from HE:
 - LHC** wins for $\tau\mu$!
- Opposite situation for μe !
- For LFV Higgs and nothing else: LHC bound

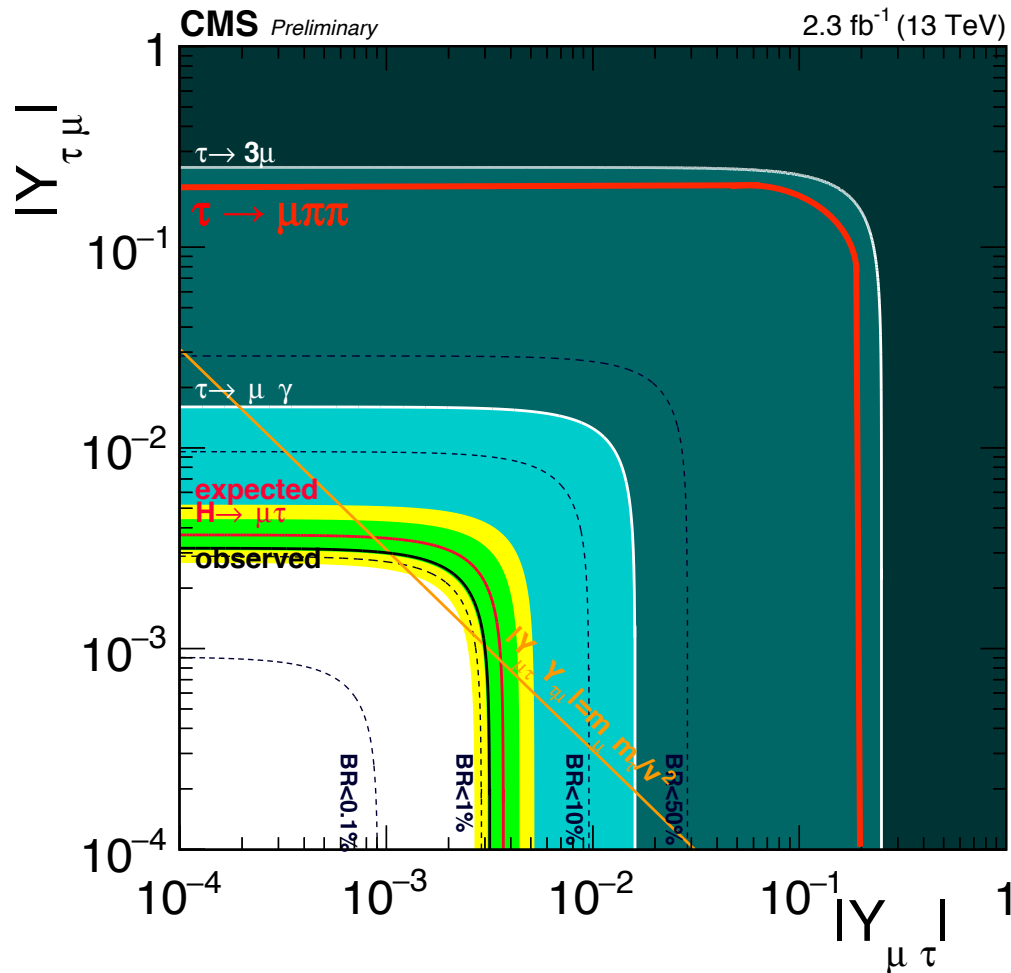
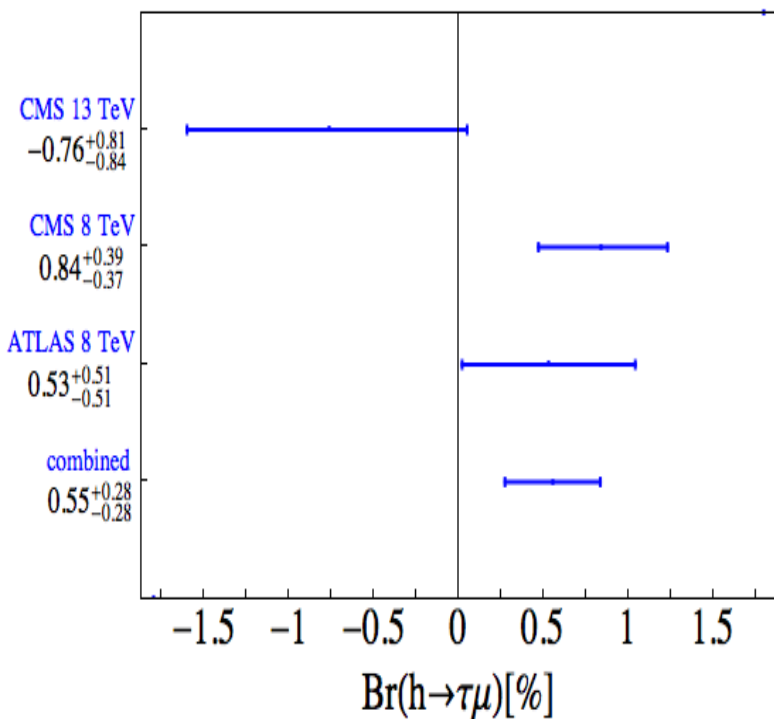


$$BR(\tau \rightarrow \mu\gamma) < 2.2 \times 10^{-9}$$

$$BR(\tau \rightarrow \mu\pi\pi) < 1.5 \times 10^{-11}$$

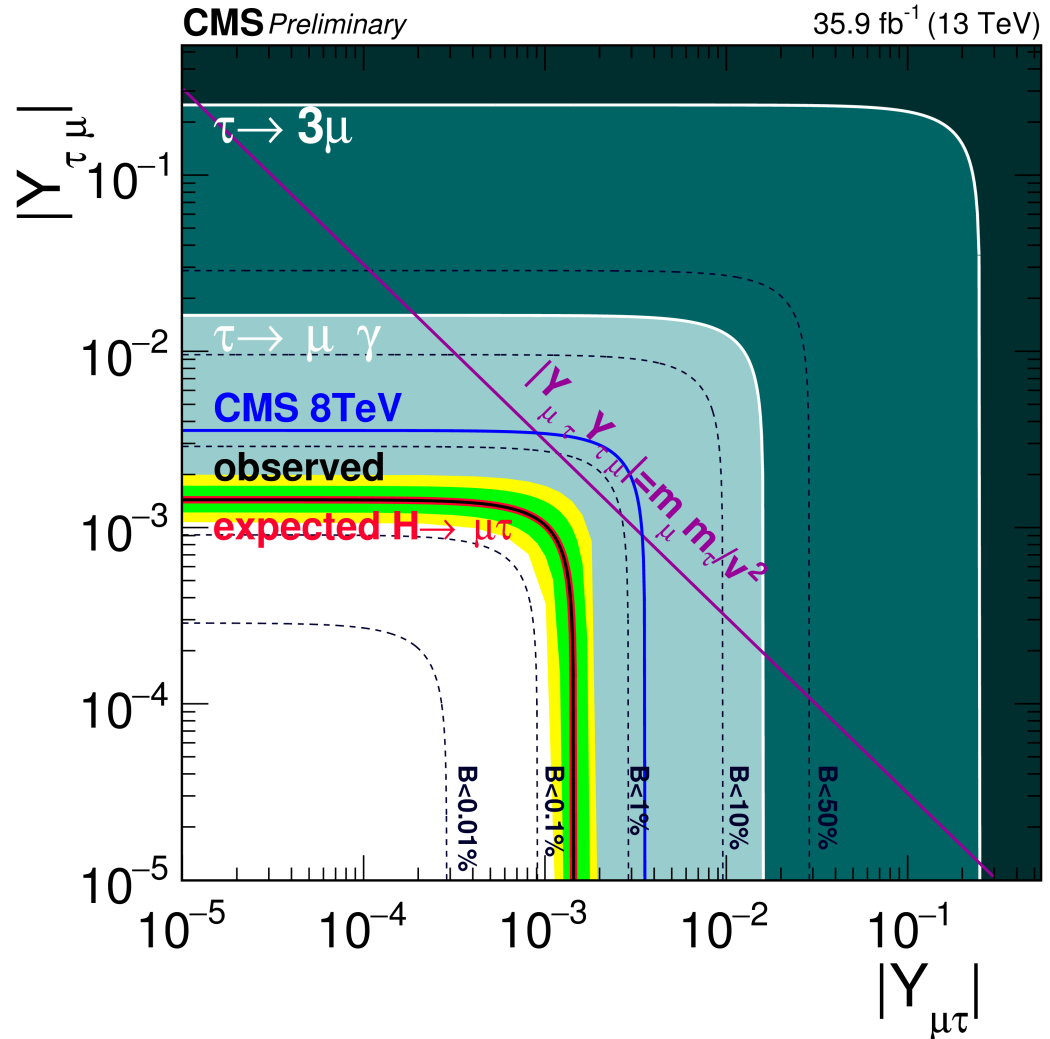
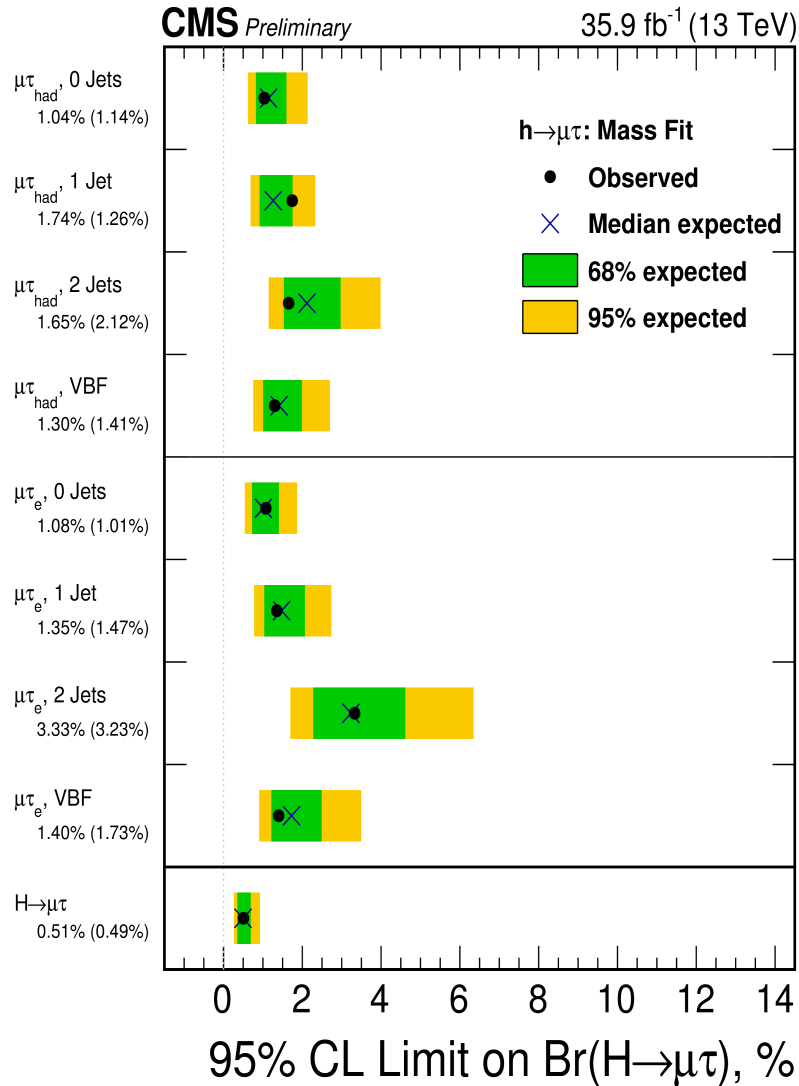
Hint of New Physics in $h \rightarrow \tau\mu$?

CMS'16



Hint of New Physics in $h \rightarrow \tau\mu$?

CMS'17



4. Conclusion and outlook

4.1 Conclusion

- Tau physics is a very rich field: test QCD and EW, neutrino physics, etc..
- Several interesting anomalies: LFU, V_{us} , Higgs LFV, CPV in $\tau \rightarrow K\pi\nu_\tau$, $g-2$
- Important experimental activities: Belle, BaBar, LHCb, BESIII, VEPP
- Intense theoretical activities : QCD, new physics
- A lot of *very interesting physics* remains to be done in the tau sector!

4.2 Outlook

- 45 billion $\tau^+\tau^-$ pairs in full dataset from $\sigma(\tau^+\tau^-)_{E=\Upsilon(4S)} = 0.9 \text{ nb}$ @Belle II
- B2TiP initiative: define the first set of measurements to be performed at Belle II, <https://confluence.desy.de/display/BI/B2TiP+WebHome>
- **Golden/Silver modes** for the Tau, Low Multiplicity and EW working group

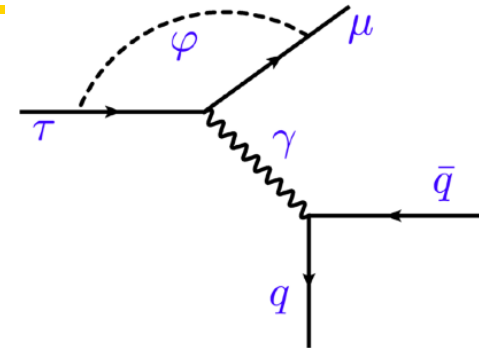
Process	Observable	Theory	Sys. limit (Discovery) [ab ⁻¹]	vs LHCb/BESIII	vs Belle	Anomaly	NP
● $\tau \rightarrow \mu\gamma$	$Br.$	***	-	***	***	*	***
● $\tau \rightarrow lll$	$Br.$	***	-	***	***	*	***
● $\tau \rightarrow K\pi\nu$	A_{CP}	***	-	***	***	**	**
● $e^+e^- \rightarrow \gamma A' (\rightarrow \text{invisible})$	σ	***	-	***	***	*	***
● $e^+e^- \rightarrow \gamma A' (\rightarrow \ell^+\ell^-)$	σ	***	-	***	***	*	***
● π form factor	$g-2$	**	-	***	**	**	***
● ISR $e^+e^- \rightarrow \pi\pi$ g-2	$g-2$	**	-	***	***	**	***

5. Back-up

3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Photon mediated contribution requires the pion vector form factor:

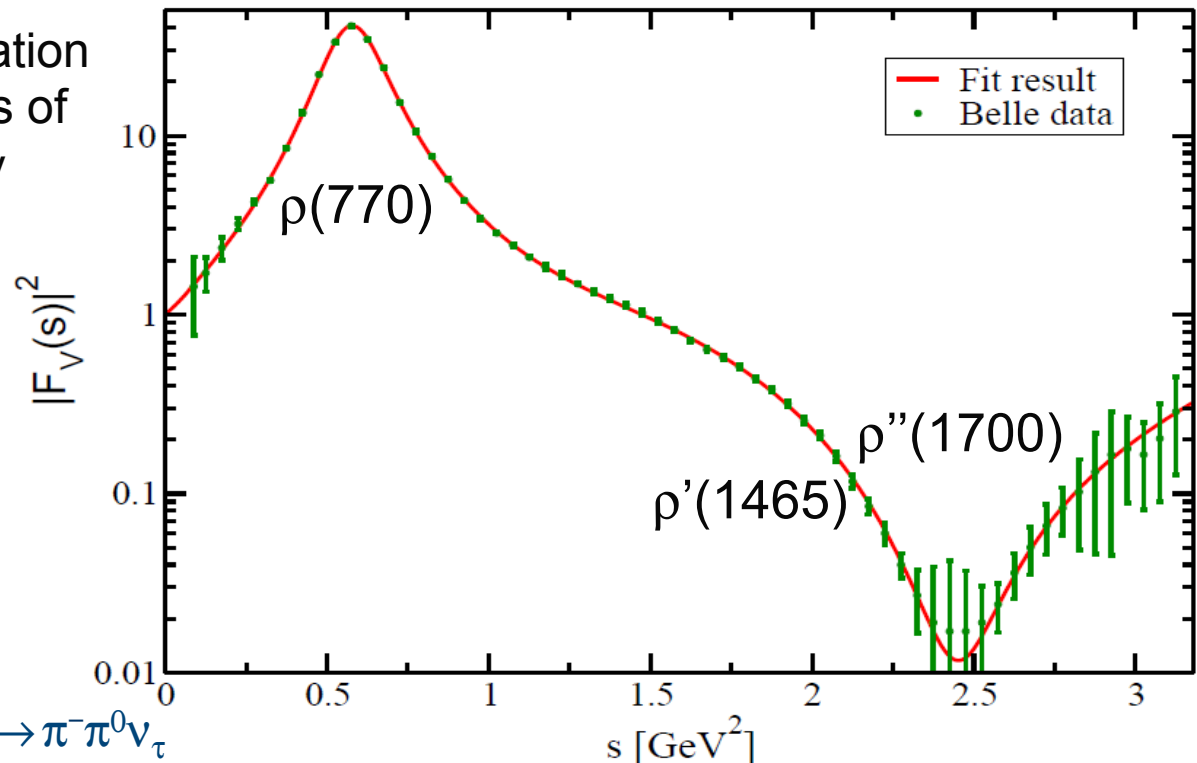
$$\langle \pi^+(p_{\pi^+}) \pi^-(p_{\pi^-}) | \frac{1}{2} (\bar{u} \gamma^\alpha u - \bar{d} \gamma^\alpha d) | 0 \rangle \equiv F_V(s) (p_{\pi^+} - p_{\pi^-})^\alpha$$



- Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

Gasser, Meißner'91
Guerrero, Pich'97
Oller, Oset, Palomar'01
Pich, Portolés '08
Gómez Dumm&Roig'13
 ...

- Determined from a fit to the Belle data on $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$



Celis, Cirigliano, E.P.'14

Determination of $F_V(s)$

- Vector form factor
 - Precisely known from experimental measurements
 $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$ (isospin rotation)
 - Theoretically: Dispersive parametrization for $F_V(s)$

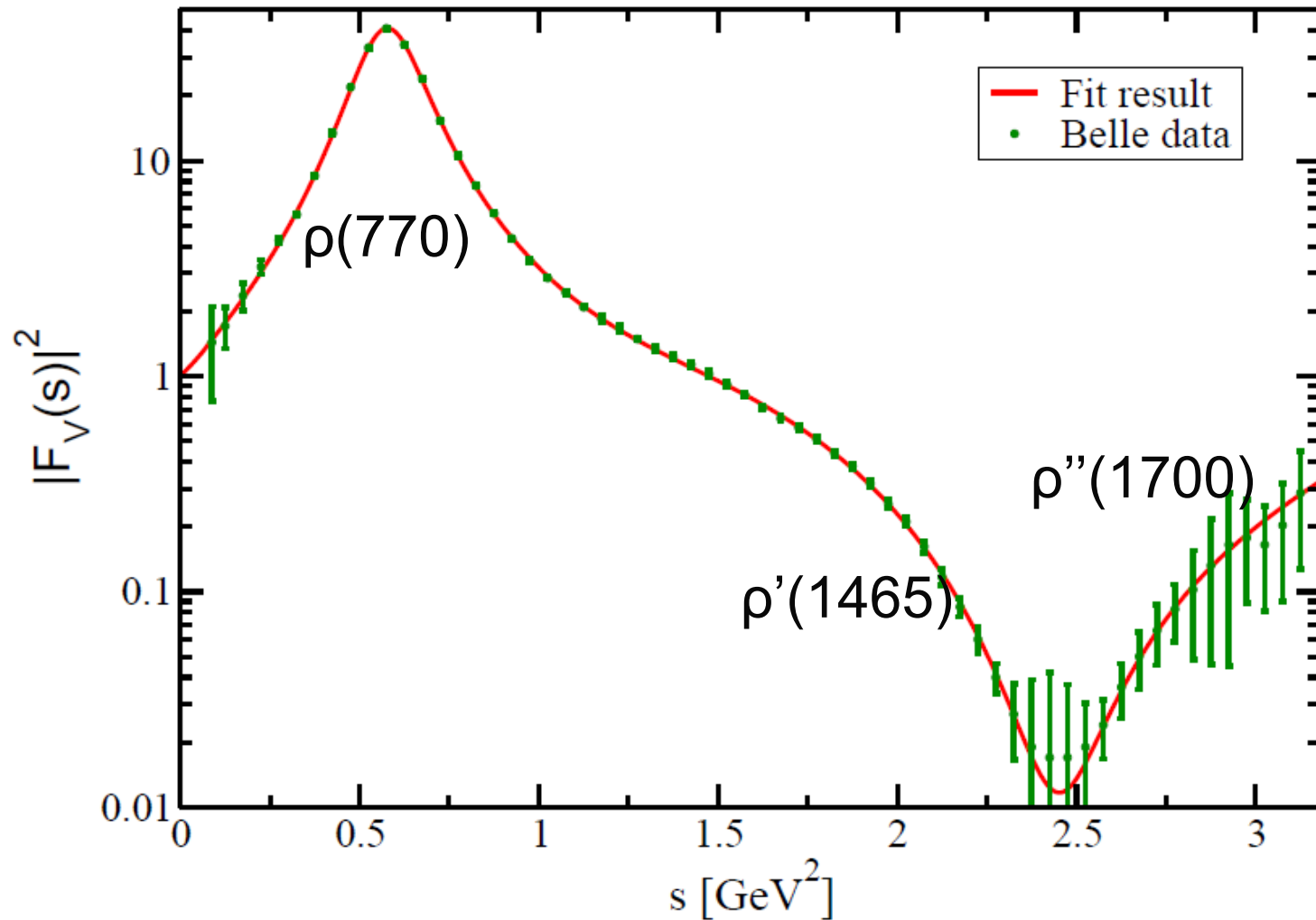
*Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13*

$$F_V(s) = \exp \left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2} (\lambda_V'' - \lambda_V'^2) \left(\frac{s}{m_\pi^2} \right)^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s' - s - i\epsilon)} \right]$$

Extracted from a model including
3 resonances $\rho(770)$, $\rho'(1465)$
and $\rho''(1700)$ fitted to the data

- Subtraction polynomial + phase determined from a *fit* to the
Belle data $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$

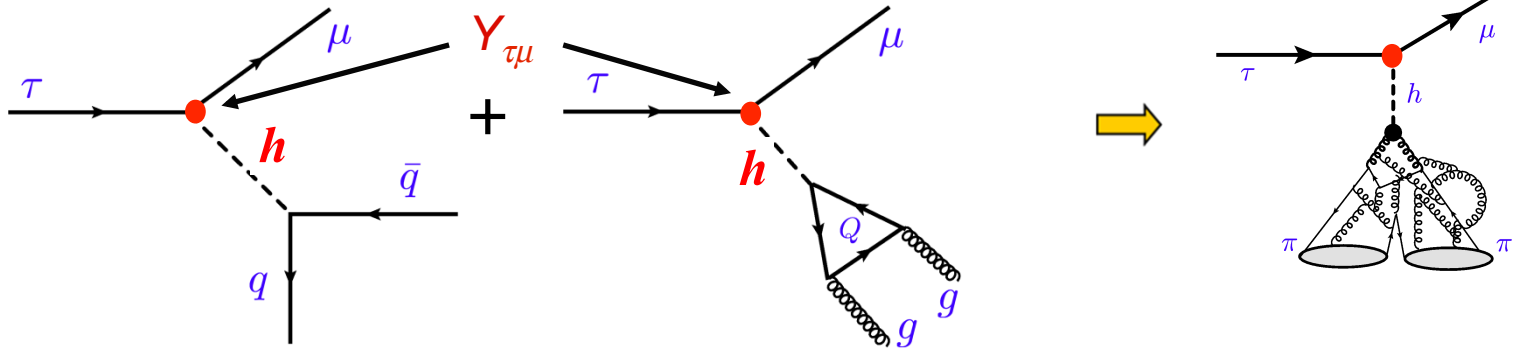
Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Tree level Higgs exchange



$$\langle \pi^+ \pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$$

$$\langle \pi^+ \pi^- | \theta_\mu^\mu | 0 \rangle \equiv \theta_\pi(s)$$

$$\langle \pi^+ \pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$s = (p_{\pi^+} + p_{\pi^-})^2$$

Voloshin'85

$$\theta_\mu^\mu = -9 \frac{\alpha_s}{8\pi} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q}q$$

$$\frac{d\Gamma(\tau \rightarrow \mu \pi^+ \pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{(|Y_{\tau\mu}^h|^2 + |Y_{\mu\tau}^h|^2)}{M_h^4 v^2} |\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)|^2$$

$$f(y_q^h)$$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs $\Rightarrow$ *Coupled channel analysis*

up to $\sqrt{s} \sim 1.4$ GeV

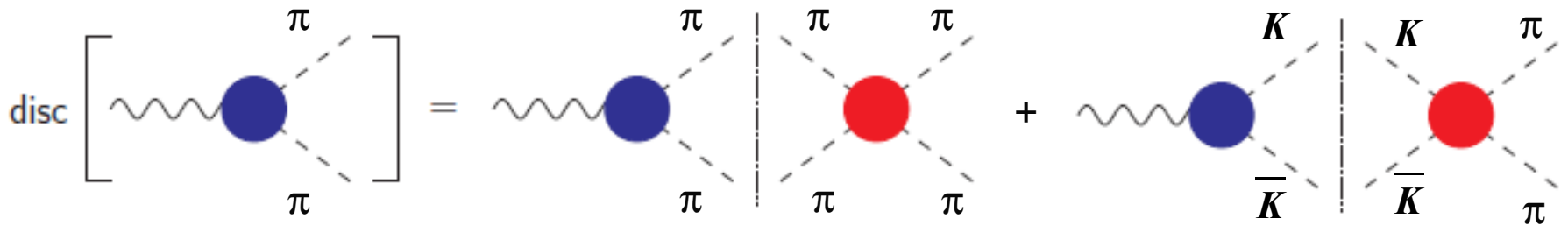
Inputs: $I=0$, S-wave $\pi\pi$ and KK data

Donoghue, Gasser, Leutwyler'90

Moussallam'99

Daub et al'13

- Unitarity:



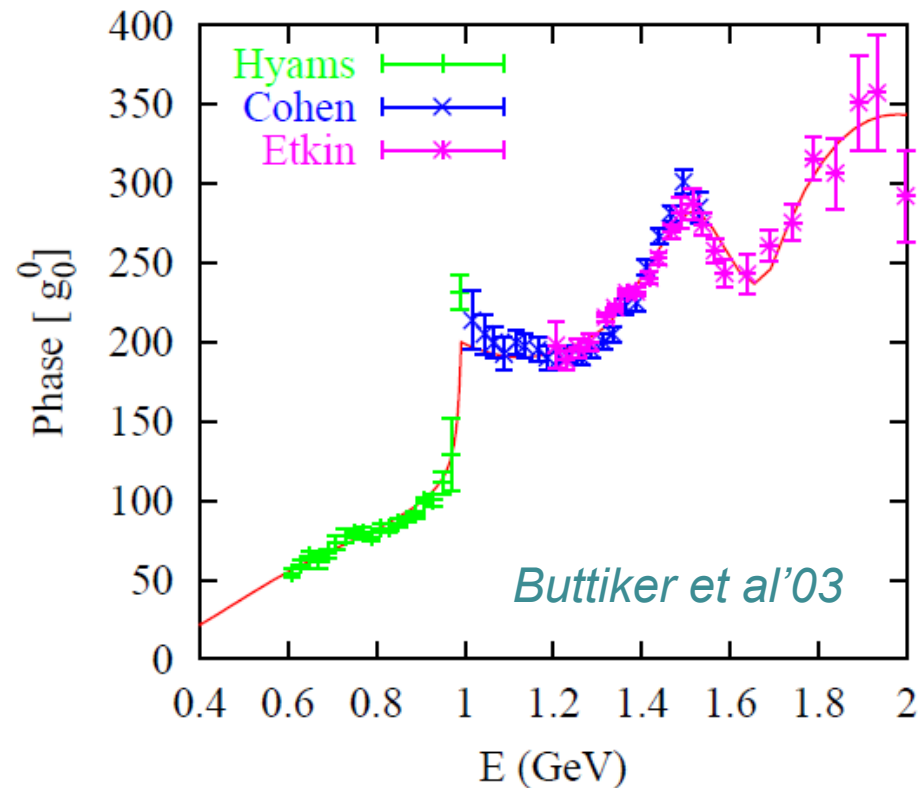
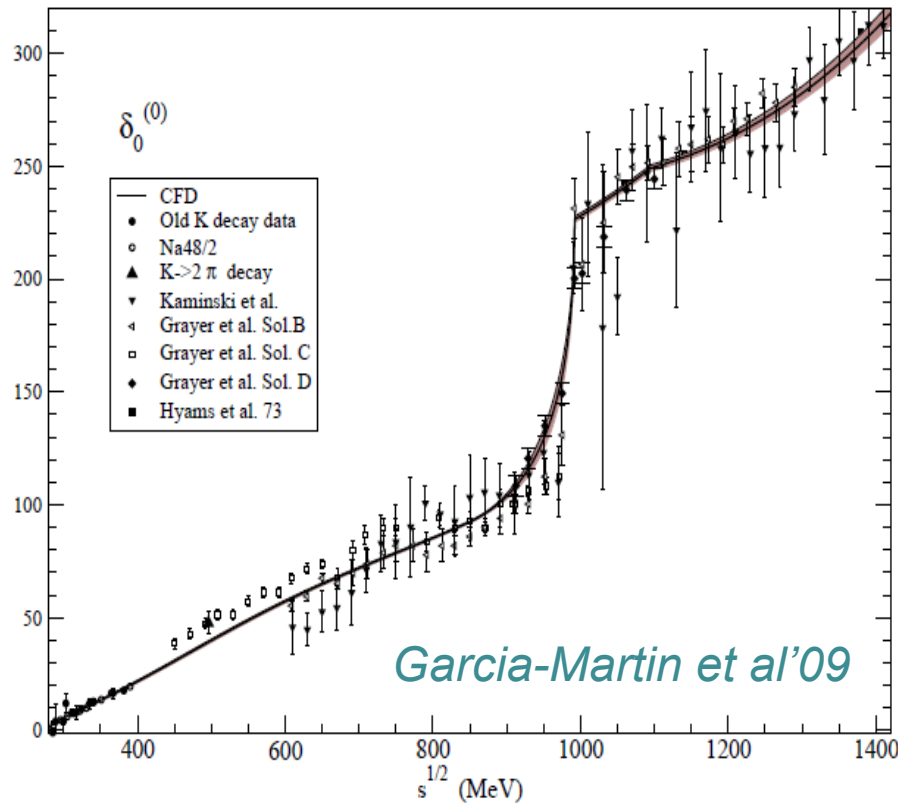
$$\text{Im}F_n(s) = \sum_{m=1}^2 T_{nm}^*(s) \sigma_m(s) F_m(s)$$

$$n = \pi\pi, K\bar{K}$$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \rightarrow \pi\pi, KK$



- A large number of theoretical analyses *Descotes-Genon et al'01*, *Kaminsky et al'01*, *Buttiker et al'03*, *Garcia-Martin et al'09*, *Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* $\Rightarrow$ **reconstruct T matrix**

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\text{Im} X_n^{(N+1)}(s) = \sum_{m=1}^2 \text{Re} \{ T_{nm}^* \sigma_m(s) X_m^{(N)} \}$$

→

$$\text{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s' - s} \text{Im} X_n^{(N+1)}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \rightarrow 1/s$ (*Brodsky & Lepage*) + ChPT:

Feynman-Hellmann theorem: $\Rightarrow$

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$\begin{aligned} M_{\pi^+}^2 &= (m_u + m_d) B_0 + O(m^2) \\ M_{K^+}^2 &= (m_u + m_s) B_0 + O(m^2) \\ M_{K^0}^2 &= (m_d + m_s) B_0 + O(m^2) \end{aligned} \Rightarrow$$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \dots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \dots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \dots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \dots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$\begin{aligned} M_{\pi^+}^2 &= (m_u + m_d) B_0 + O(m^2) \\ M_{K^+}^2 &= (m_u + m_s) B_0 + O(m^2) \\ M_{K^0}^2 &= (m_d + m_s) B_0 + O(m^2) \end{aligned} \quad \Rightarrow \quad \begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \dots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \dots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \dots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \dots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
 Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1) M_\pi^2$$

$$\Delta_K(0) = 1_{-0.05}^{+0.15} (M_K^2 - 1/2 M_\pi^2)$$

Dreiner, Hanart, Kubis, Meissner'13

Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- General solution

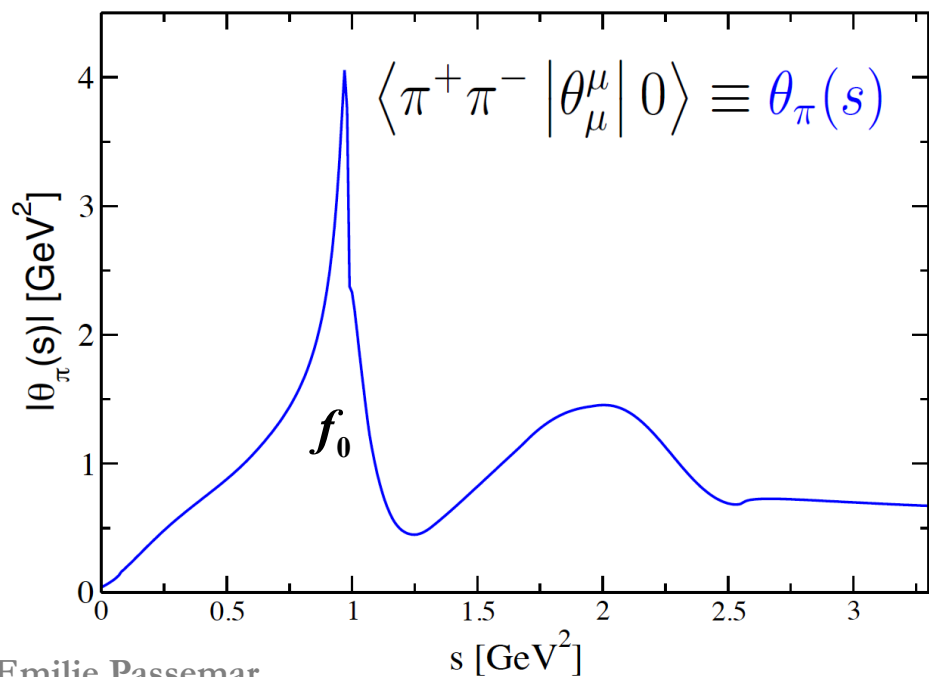
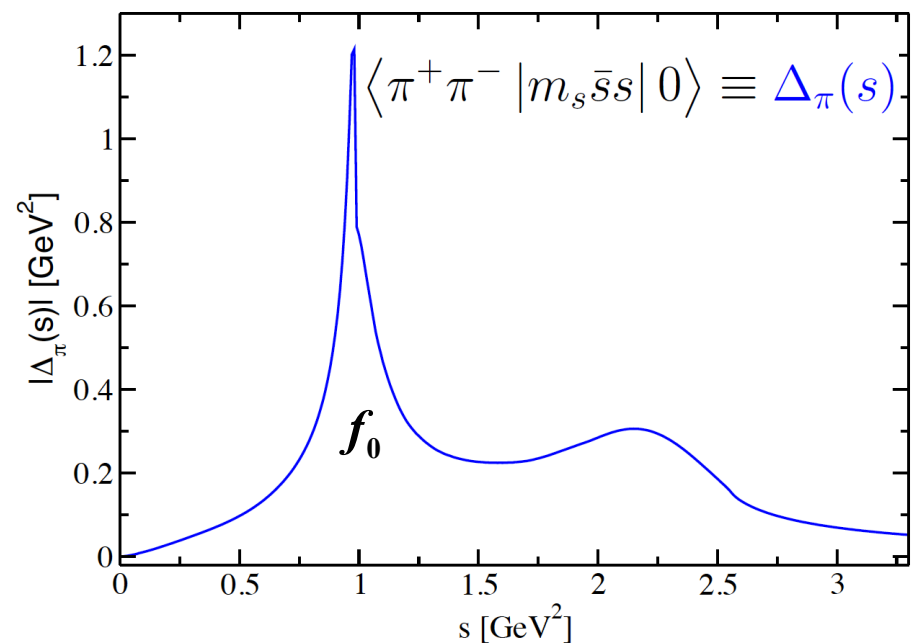
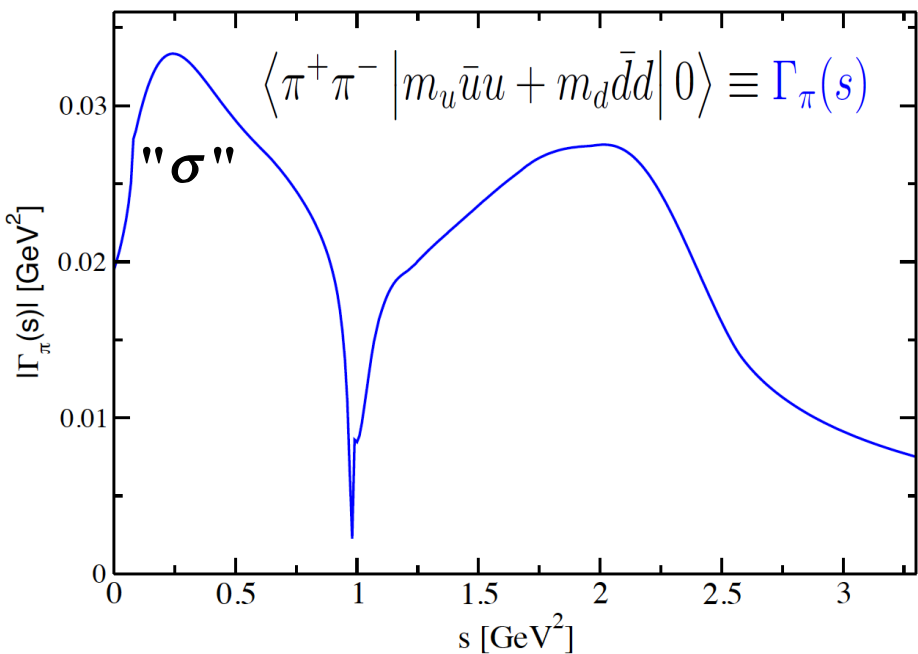
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}}F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

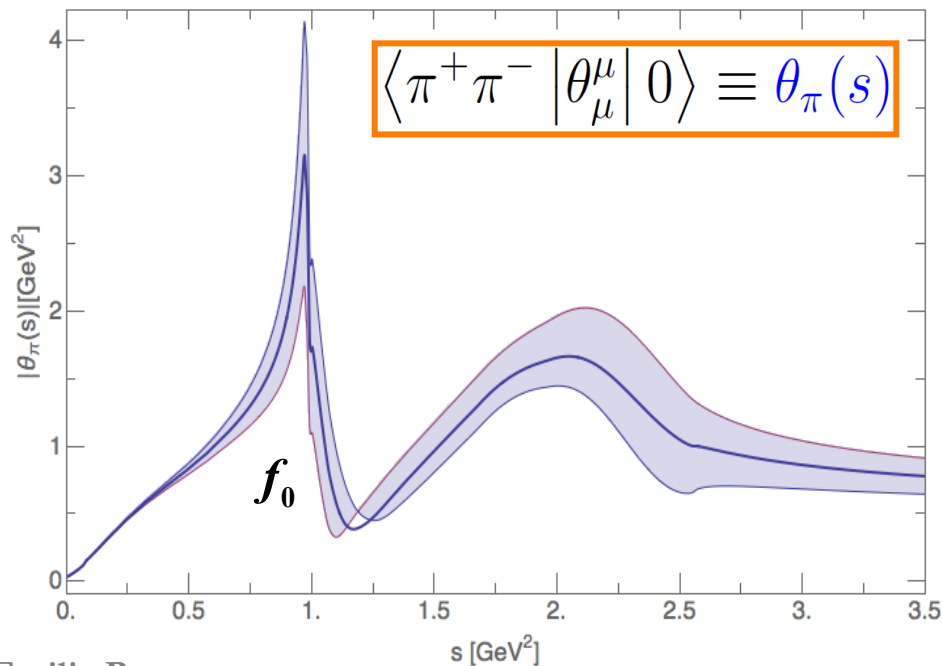
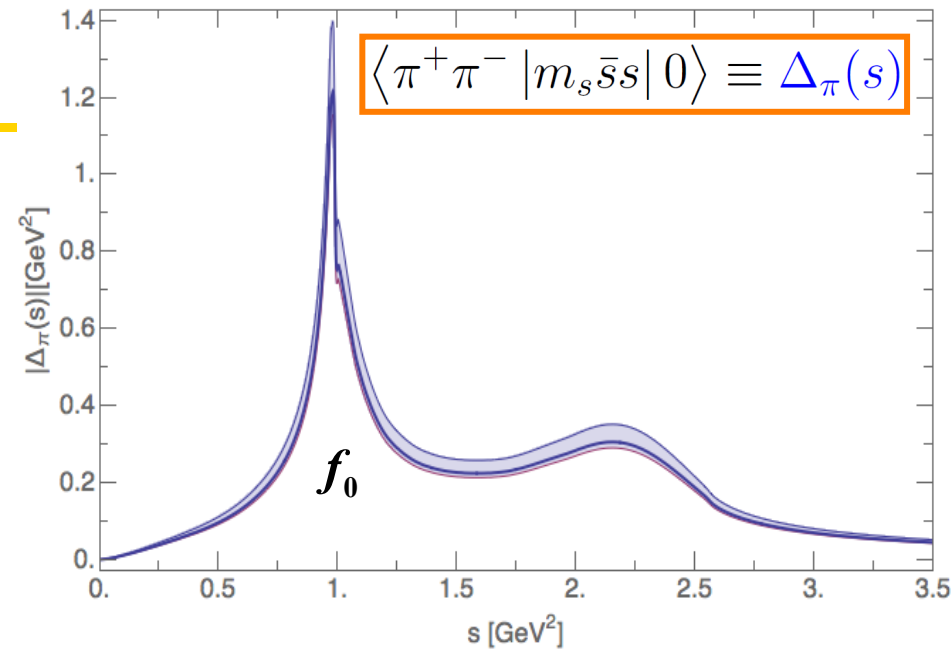
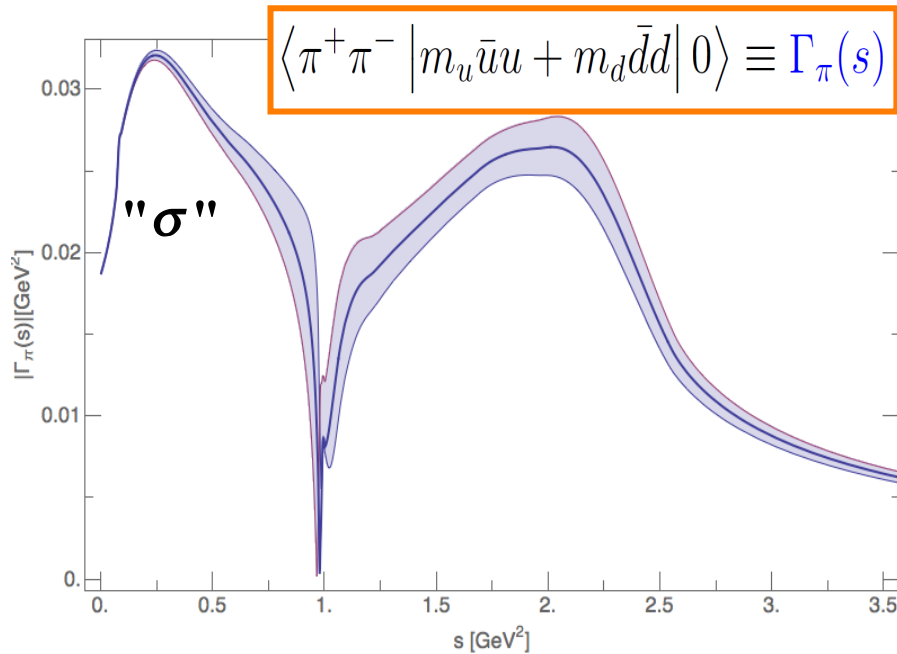
- For θ_p enforcing the asymptotic constraint is not consistent with ChPT
The unsubtracted DR is not saturated by the 2 states

 Relax the constraints and match to ChPT

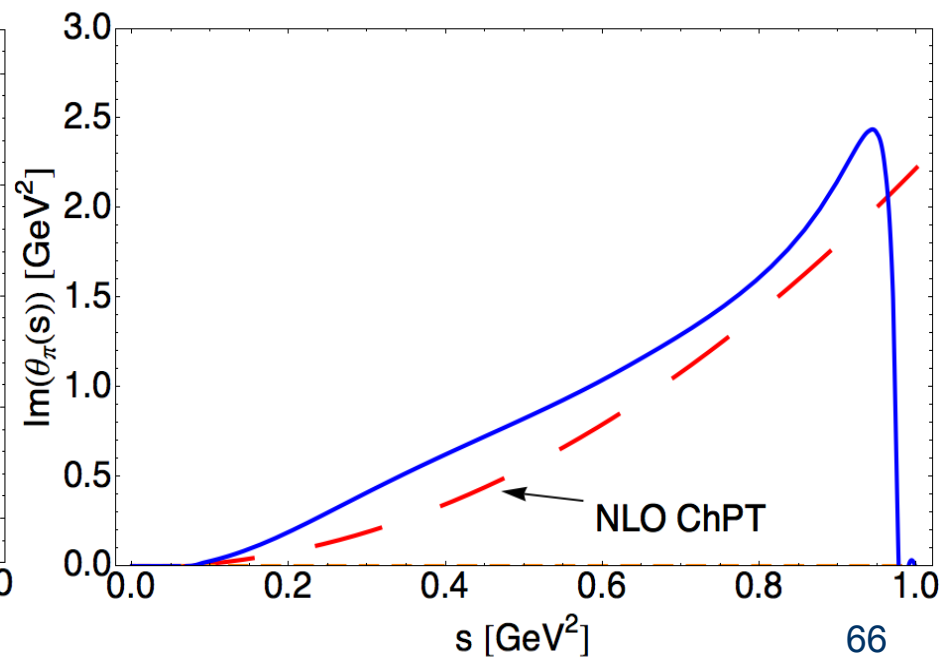
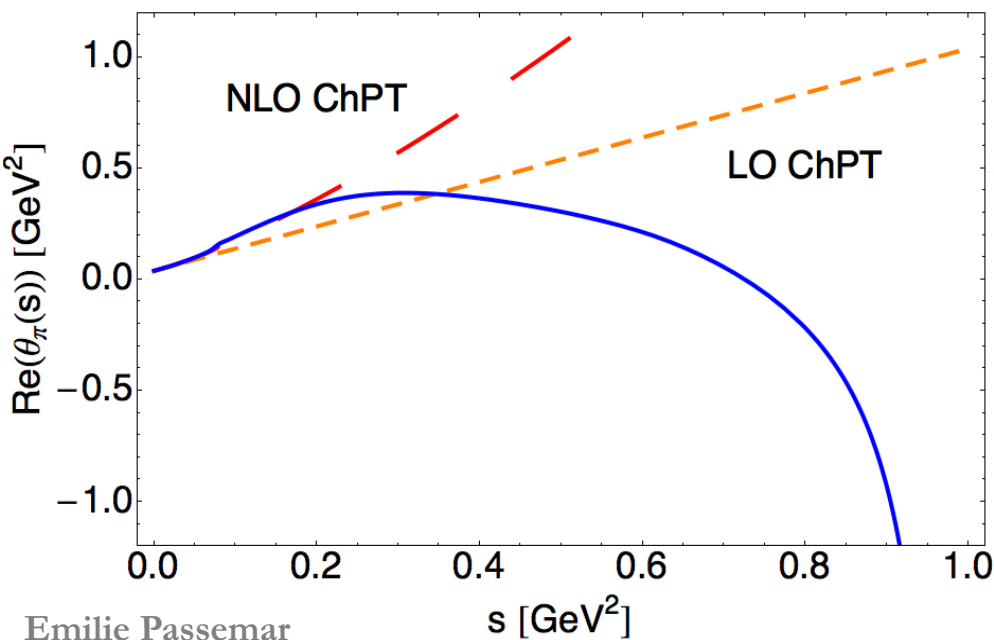
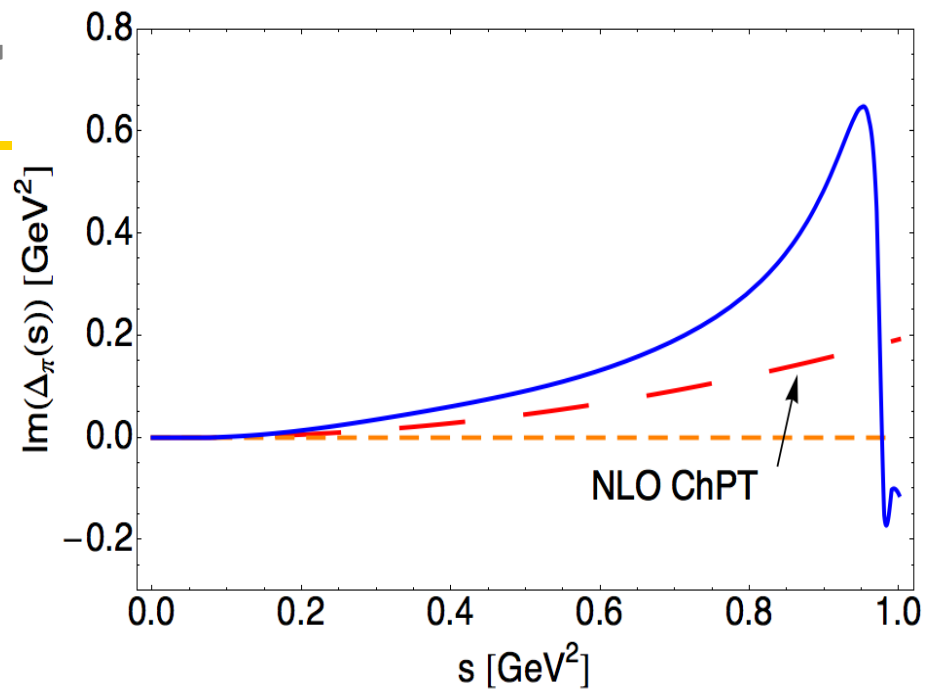
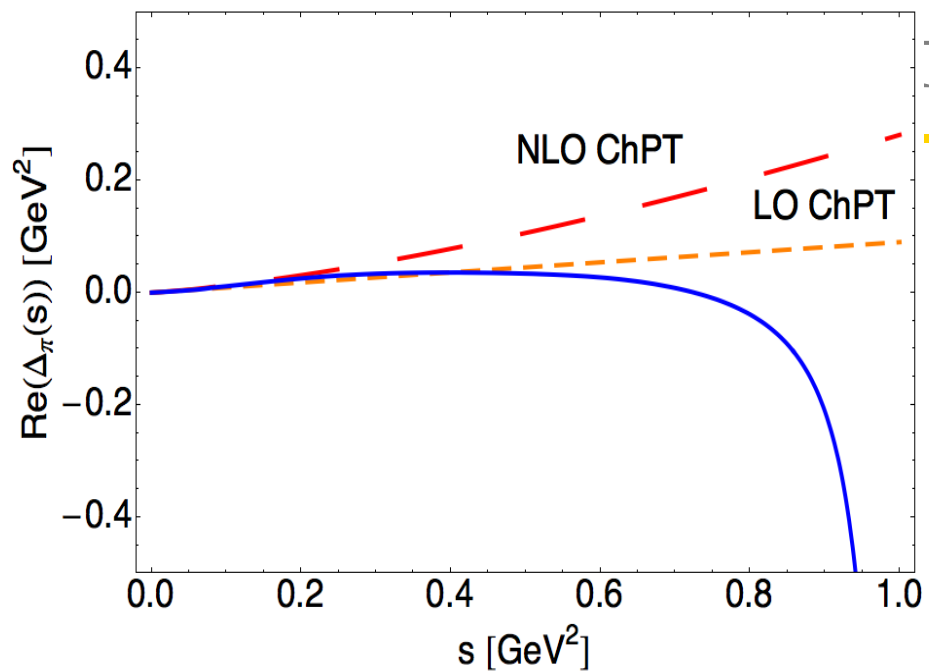
$$P_\theta(s) = 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1 \right) s$$

$$Q_\theta(s) = \frac{4}{\sqrt{3}}M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3}M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2 \right) s$$





- Uncertainties:
 - Varying s_{cut} (1.4 GeV² - 1.8 GeV²)
 - Varying the matching conditions
 - T matrix inputs



5. CPV in tau decays

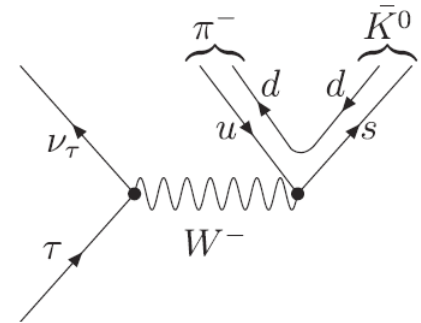
5.1 $\tau \rightarrow K\pi\nu_\tau$ CP violating asymmetry

- $$A_\varrho = \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0 \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S^0 \nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0 \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S^0 \nu_\tau)}$$

$$= |p|^2 - |q|^2 \approx (0.36 \pm 0.01)\% \quad \text{in the SM}$$

Bigi & Sanda'05

Grossman & Nir'11



$$|K_S^0\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

$$|K_L^0\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$$

$$\langle K_L | K_S \rangle = |p|^2 - |q|^2 \approx 2\text{Re}(\epsilon_K)$$

- Experimental measurement :

BaBar'11

$$A_{\varrho \text{ exp}} = (-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}})\%$$



2.8σ

from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

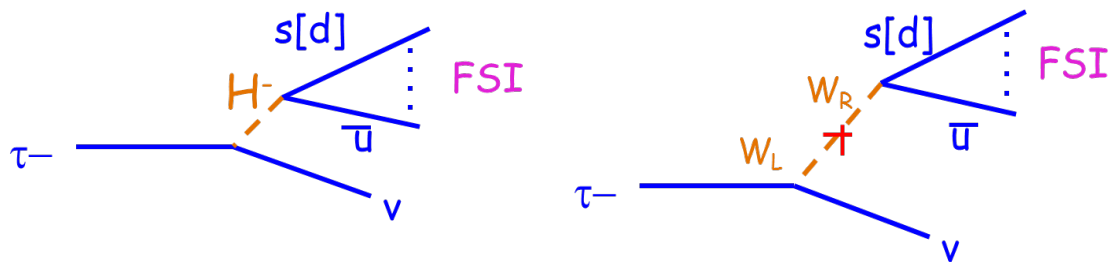
Grossman & Nir'11

$$A_D = \frac{\Gamma(D^+ \rightarrow \pi^+ K_S^0) - \Gamma(D^- \rightarrow \pi^- K_S^0)}{\Gamma(D^+ \rightarrow \pi^+ K_S^0) + \Gamma(D^- \rightarrow \pi^- K_S^0)} = (-0.54 \pm 0.14)\%$$

*Belle, Babar,
CLOE, FOCUS*

5.1 $\tau \rightarrow K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L - W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14*)?



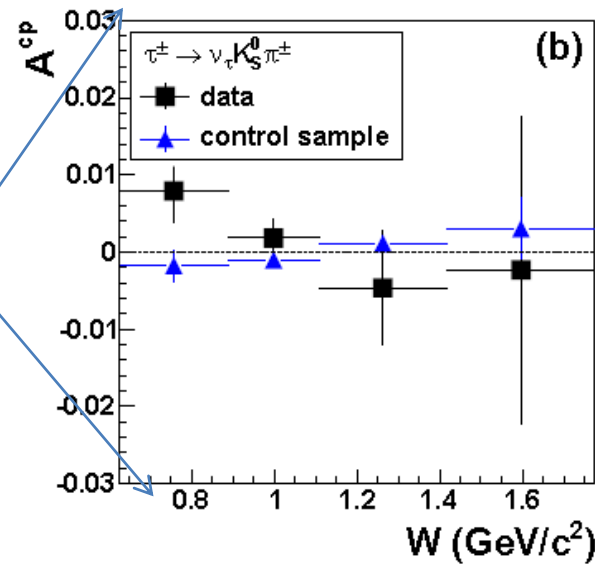
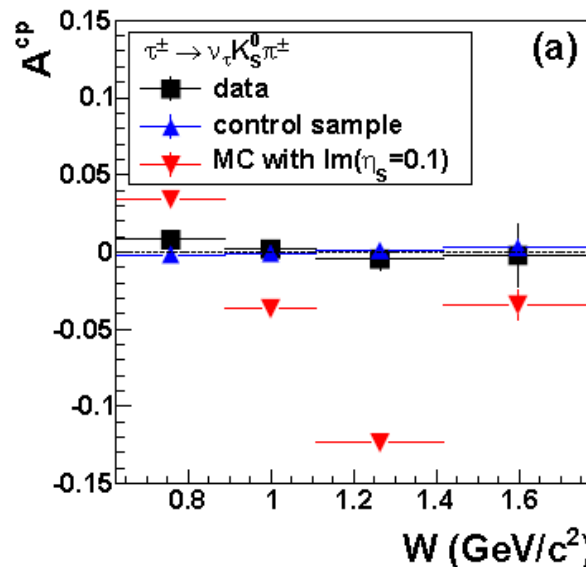
Bigi'Tau12

Very difficult to explain!

- Problem with this measurement? $\Rightarrow$ It would be great to have other experimental measurements from *Belle, BES III or Tau-Charm factory*

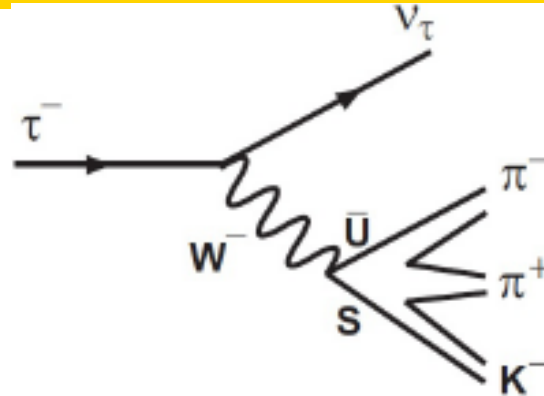
Belle'11

- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by *CLEO* and *Belle*
 $\Rightarrow$ Belle does not see any asymmetry at the **0.2 - 0.3% level**



5.2 Three body CP asymmetries

- Ex: $\tau \rightarrow K\pi\pi\nu_\tau$



- A variety of CPV observables can be studied :
 $\tau \rightarrow K\pi\pi\nu_\tau$, $\tau \rightarrow \pi\pi\pi\nu_\tau$ rate, angular asymmetries, triple products,....

*e.g., Choi, Hagiwara and Tanabashi'98
Kiers, Little, Datta, London et al., '08
Mileo, Kiers and, Szykman'14*

Same principle as in charm, *see Bevan'15*

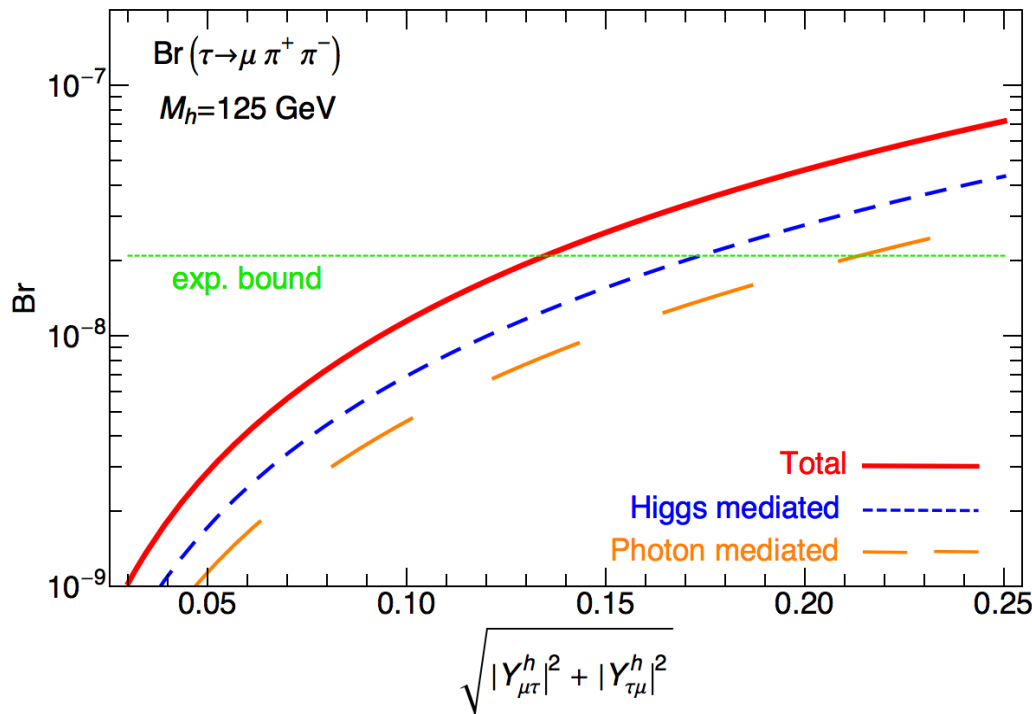
Difficulty : Treatment of the hadronic part

Hadronic final state interactions have to be taken into account!

➡ Disentangle weak and strong phases

- More form factors, more asymmetries to build but same principles as for 2 bodies

3.5 Results



Bound:

$$\sqrt{|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2} \leq 0.13$$

Process	(BR $\times 10^8$) 90% CL	$\sqrt{ Y_{\mu\tau}^h ^2 + Y_{\tau\mu}^h ^2}$	Operator(s)
$\tau \rightarrow \mu \gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \rightarrow \mu \mu \mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \rightarrow \mu \pi^+ \pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \rightarrow \mu \rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \rightarrow \mu \pi^0 \pi^0$	< 1.4×10^3 [87]	< 6.3	Scalar, Gluon

Less stringent
but more robust
handle on LFV
Higgs couplings

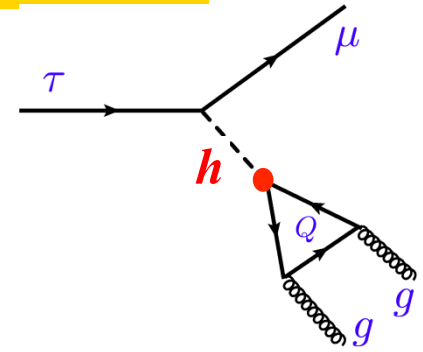
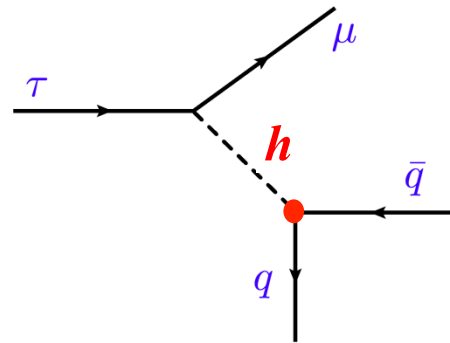
?

3.5 What if $\tau \rightarrow \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan
@ KEK-FF2014FALL

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!



- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \rightarrow \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \rightarrow \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$

$$Br(\tau \rightarrow e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \rightarrow e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

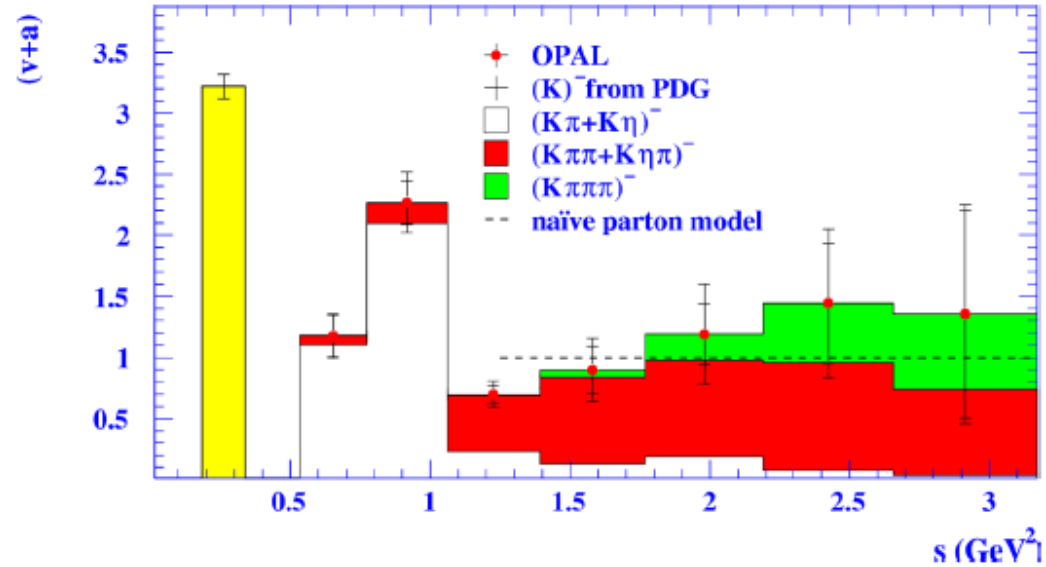
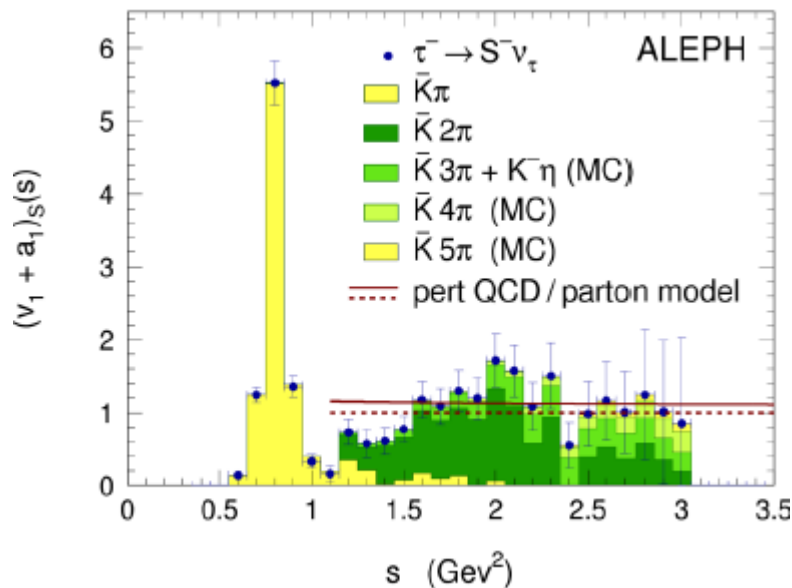
$$Br(\tau \rightarrow \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \rightarrow \mu\pi^0\pi^0) < 1.5 \times 10^{-8}$$
$$Br(\tau \rightarrow e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \rightarrow e\pi^0\pi^0) < 2.1 \times 10^{-7}$$

below present experimental limits!

- If discovered $\Rightarrow$ among other things **upper limit** on $Y_{u,d,s}$!
 $\Rightarrow$ Interplay between high-energy and low-energy constraints!

3.6 Prospects : τ strange Brs

- Experimental measurements of the strange spectral functions not very precise



➡ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller $\tau \rightarrow$ K branching ratios ➡ smaller $R_{\tau,S}$ ➡ smaller V_{us}

$$R_{\tau}^S|_{\text{old}} = 0.1686(47)$$



$$R_{\tau}^S|_{\text{new}} = 0.1615(28)$$

$$|V_{us}|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}$$



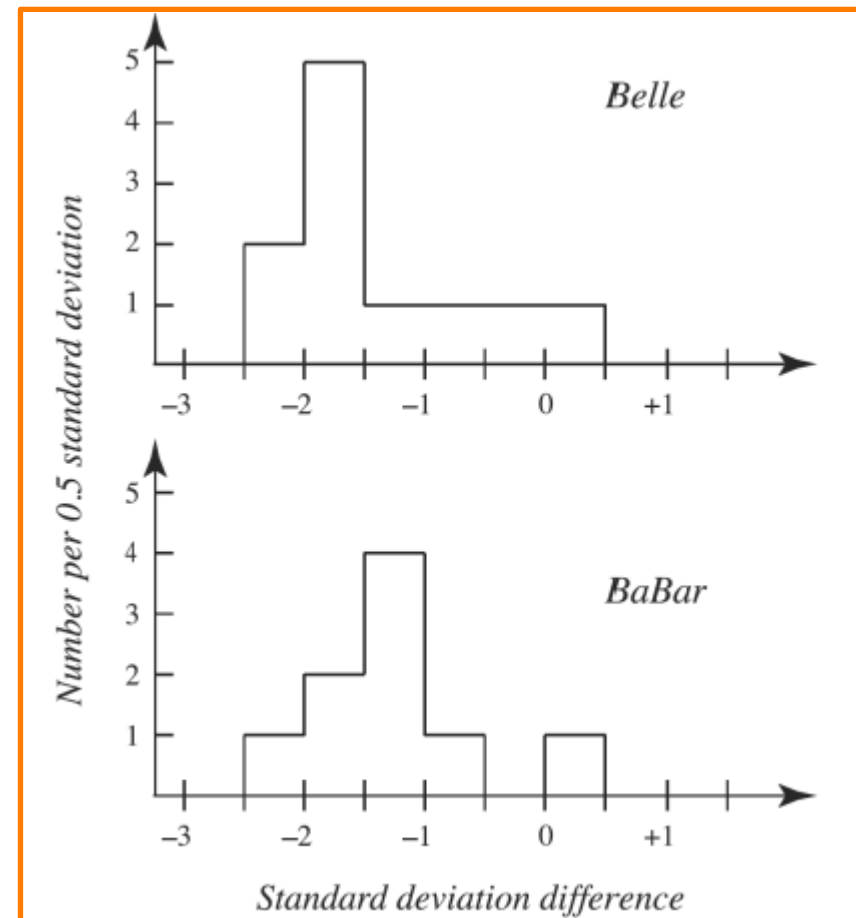
$$|V_{us}|_{\text{new}} = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}$$

3.6 Prospects : τ strange Brs

- PDG 2014*: « Eighteen of the 20 B -factory branching fraction measurements are smaller than the non- B -factory values. The average normalized difference between the two sets of measurements is -1.30 » (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)

- Measured modes by the 2 B factories:

Mode	BaBar – Belle Normalized Difference ($\# \sigma$)
$\pi^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	+1.4
$K^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	-2.9
$K^- K^+ \pi^- \nu_\tau$	-2.9
$K^- K^+ K^- \nu_\tau$	-5.4
$\eta K^- \nu_\tau$	-1.0
$\phi K^- \nu_\tau$	-1.3



2.2 Experimental situation

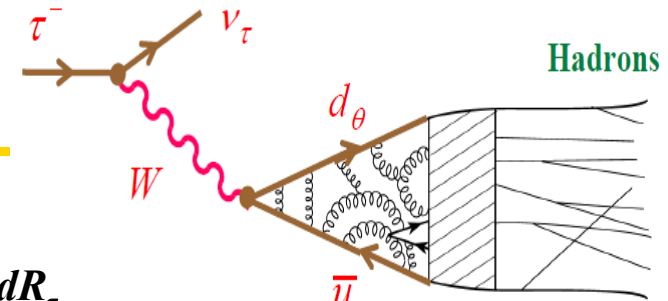
- Observable studied

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)}$$

$$\text{and } \frac{dR_\tau}{ds}$$

- Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$



Zhang'Tau14

$$v_1/a_1 [\tau^- \rightarrow V^- / A^- \nu_\tau] \propto \frac{\text{BR}[\tau^- \rightarrow V^- / A^- \nu_\tau]}{\text{BR}[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau]} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \frac{m_\tau^2}{(1-s/m_\tau^2)^2 (1+2s/m_\tau^2)}$$

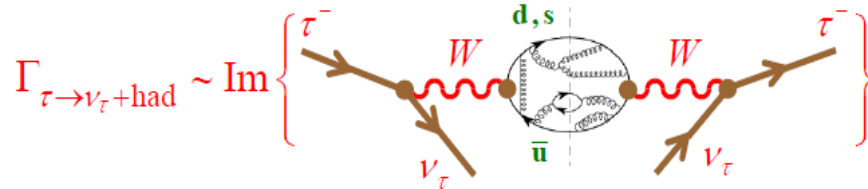
Vector/Axial-vector
spectral functions

branching fractions mass spectrum kinematic factor

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ :



$$\text{Im}\Pi_{\bar{u}d,V/A}^{(1)}(s) = \frac{1}{2\pi}v_1/a_1(s)$$

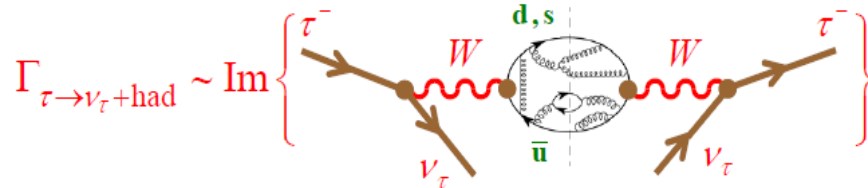


$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\Pi^{(1)}(s+i\epsilon) + \text{Im}\Pi^{(0)}(s+i\epsilon) \right]$$

2.4 Calculation of the QCD corrections

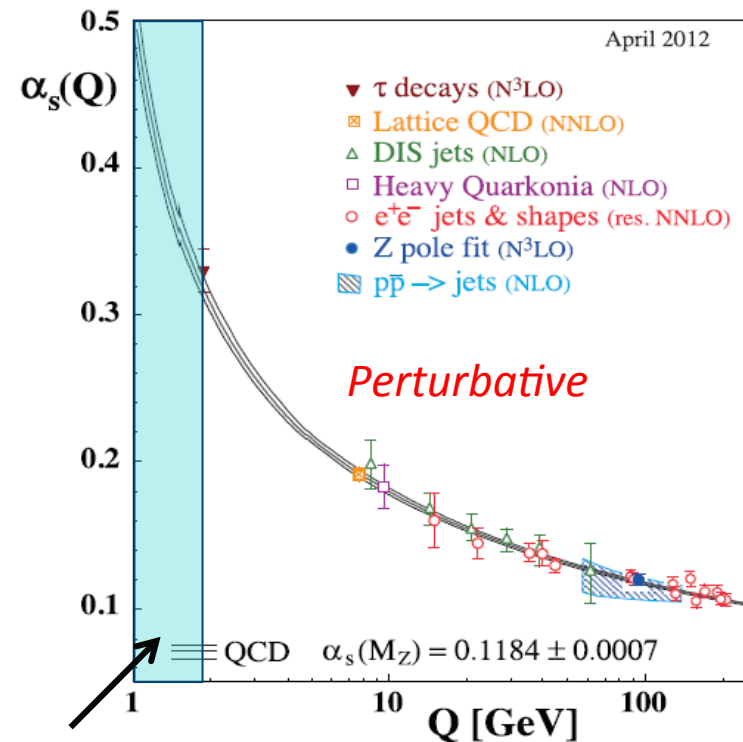
Braaten, Narison, Pich'92

- Calculation of R_τ :



$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s + i\epsilon) + \text{Im} \Pi^{(0)}(s + i\epsilon) \right]$$

- We are in the *non-perturbative* region: we do not know how to compute!
- Trick: use the analytical properties of Π !

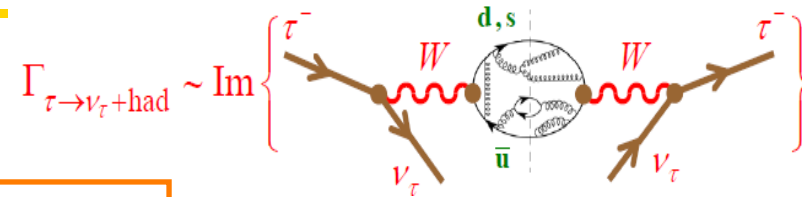


Non-Perturbative

2.4 Calculation of the QCD corrections

- Calculation of R_τ :

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s+i\epsilon) + \text{Im} \Pi^{(0)}(s+i\epsilon) \right]$$



Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

→ Cauchy Theorem

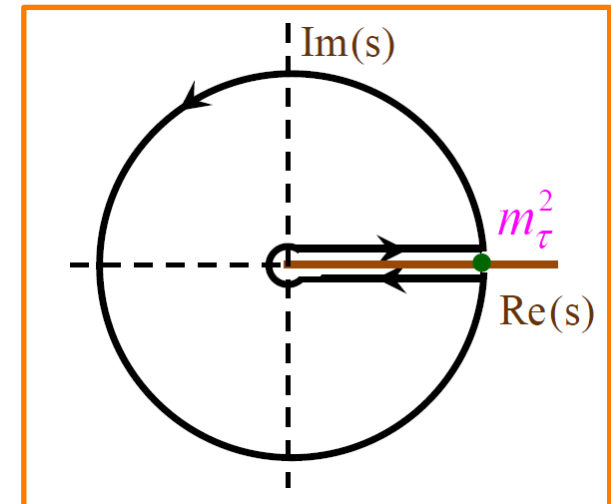
$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \Pi^{(1)}(s) + \Pi^{(0)}(s) \right]$$

- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\dots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s, \mu) \langle O_D(\mu) \rangle$$

Wilson coefficients

Operators



μ : separation scale between short and long distances

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ :

$$R_\tau(m_\tau^2) = N_C S_{EW} (1 + \delta_P + \delta_{NP})$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano & Sirlin'88, Braaten & Li'90, Erler'04*

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

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- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano & Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

Baikov, Chetyrkin, Kühn'08

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ :

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Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS} (\propto m_u, m_d)$ but not for $R_\tau^S (\propto m_s)$

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ :

$$R_\tau(m_\tau^2) = N_C S_{EW} (1 + \delta_P + \delta_{NP})$$

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- Perturbative part (D=0): $\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$ $a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS} (\propto m_u, m_d)$ but not for $R_\tau^S (\propto m_s)$

- $D \geq 4$: Non perturbative part, not known, *fitted from the data*



Use of weighted distributions

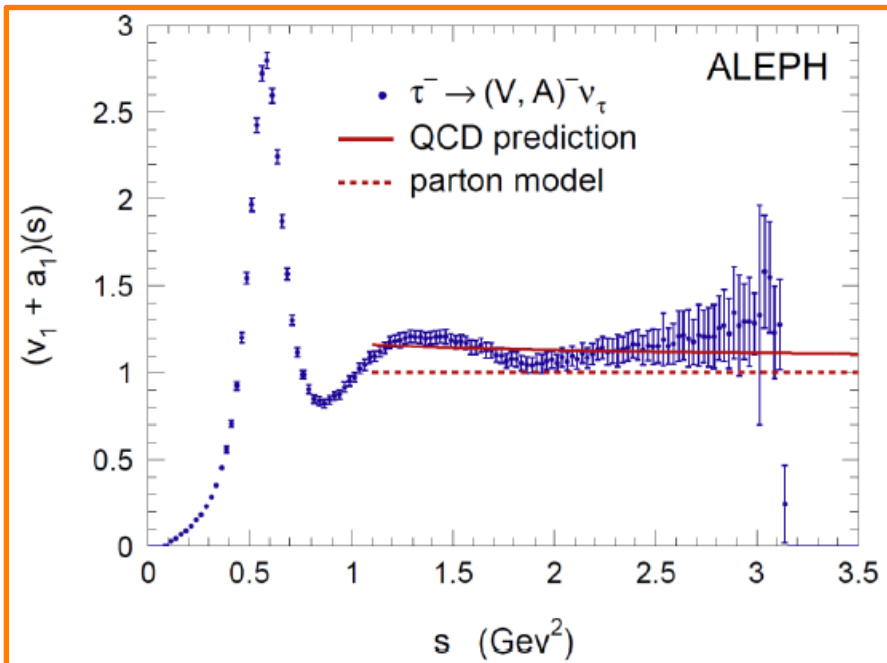
2.4 Calculation of the QCD corrections

Le Diberder&Pich'92

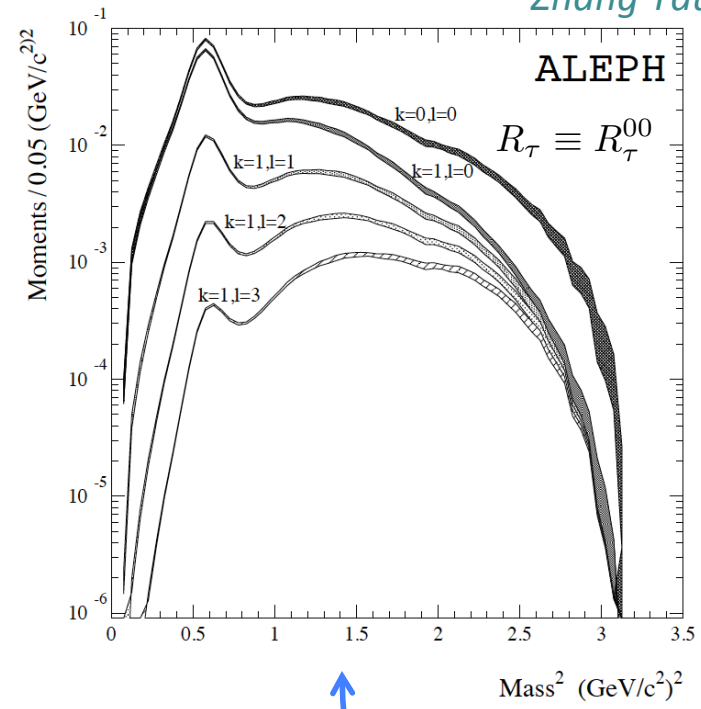
- $D \geq 4$: Non perturbative part, not known, *fitted from the data*
➡ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ :

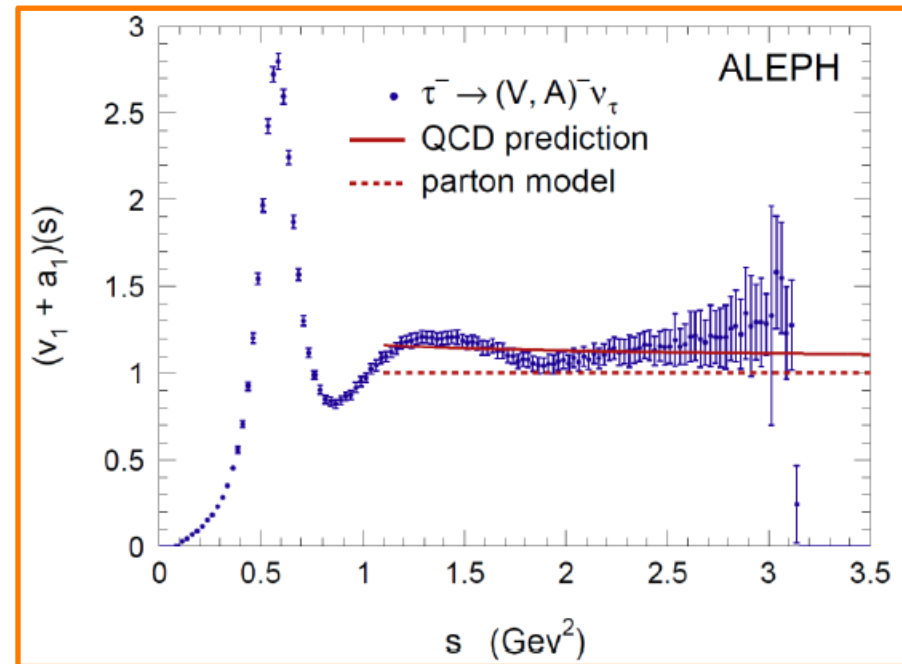
$$R_\tau(m_\tau^2) = N_C S_{EW} (1 + \delta_P + \delta_{NP})$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$
- Perturbative part (D=0): $\delta_P \approx 20\%$
- D=2: quark mass corrections, *neglected*
- D $\geq$ 4: Non perturbative part, not known, *fitted from the data*
Use of weighted distributions



$$\delta_{NP} = -0.0064 \pm 0.0013$$

Davier et al'14



- Small unknown NP part ($\delta_{NP} : 3\% \delta_P$) very precise extraction of α_s !

2.5 Results and determination of α_s

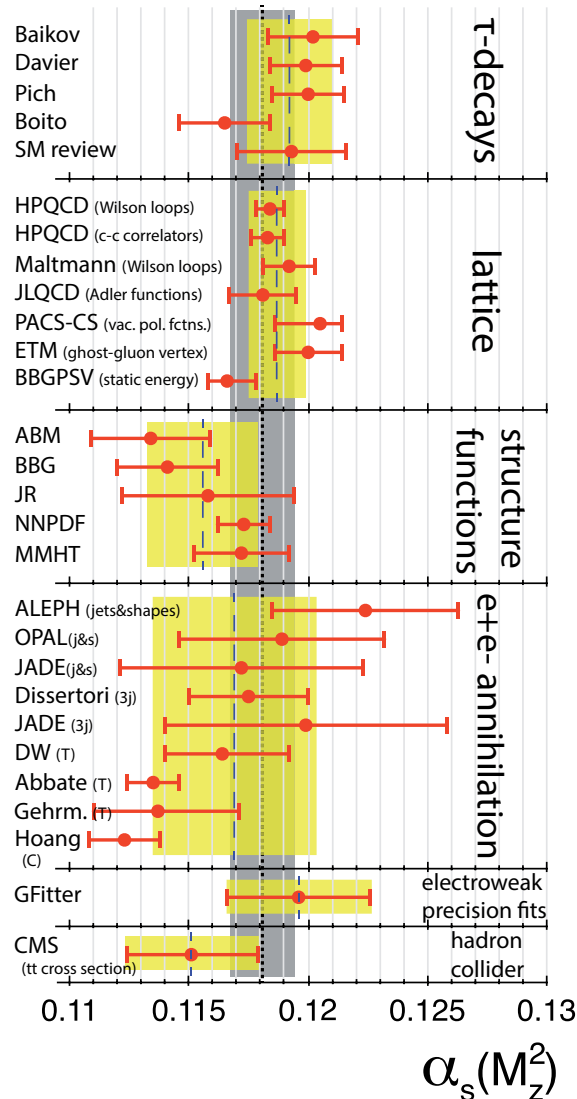
Pich'Tau14

Reference	Method	δ_{NP}	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT		0.1998 (43)	0.332 (16)	0.1202 (19)
Davier et al'14	CIPT, FOPT	− 0.0064 (13)	−	0.332 (12)	0.1199 (15)
Beneke-Jamin	BSR + FOPT	− 0.007 (3)	0.2042 (50)	0.316 (06)	0.1180 (08)
Maltman-Yavin	PWM + CIPT	+ 0.012 (18)	−	0.321 (13)	0.1187 (16)
Menke	CIPT, FOPT		0.2042 (50)	0.342 (11)	0.1213 (12)
Narison	CIPT, FOPT		−	0.324 (08)	0.1192 (10)
Caprini-Fischer	BSR + CIPT		0.2037 (54)	0.322 (16)	−
Abbas et al	IFOPT		0.2037 (54)	0.338 (10)	
Cvetič et al	β_{exp} + CIPT		0.2040 (40)	0.341 (08)	0.1211 (10)
Boito et al	CIPT, DV	− 0.002 (12)	−	0.347 (25)	0.1216 (27)
	FOPT, DV	− 0.004 (12)		0.325 (18)	0.1191 (22)
Pich'14	CIPT	− 0.0064 (13)	0.2014 (31)	0.342 (13)	0.1213 (14)
	FOPT			0.320 (14)	0.1187 (17)
Pich'14	CIPT, FOPT	− 0.0064 (13)	0.2014 (31)	0.332 (13)	0.1202 (15)

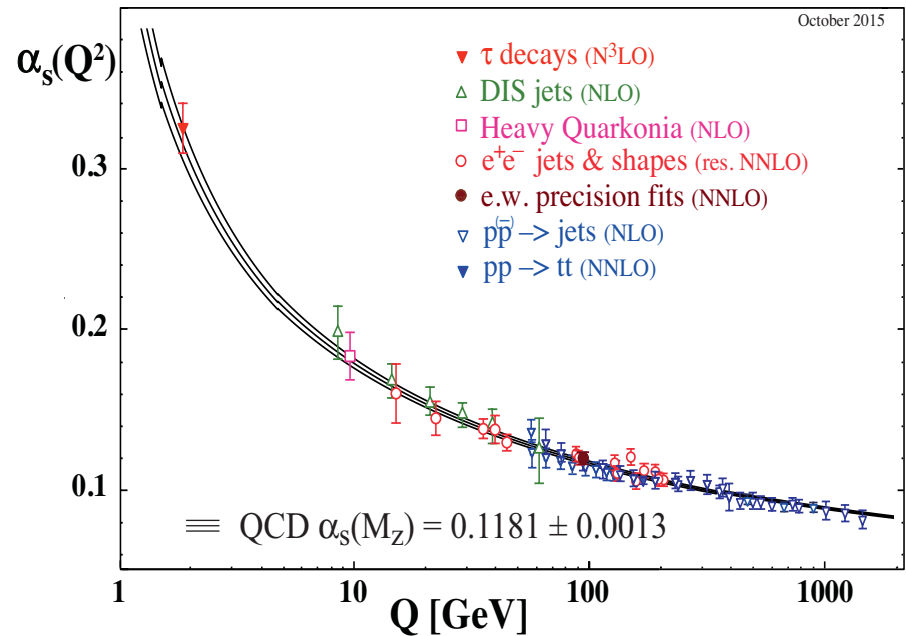
CIPT: Contour-improved perturbation theory
 FOPT: Fixed-order perturbation theory
 BSR: Borel summation of renormalon series
 IFOPT: Improved FOPT

β_{exp} : Expansion in derivatives of α_s (β function)
 PWM: Pinched-weight moments
 CIPTm: Modified CIPT (conformal mapping)
 DV: Duality violation (OPAL only)

3.4 Extraction of α_s



Bethke, Dissertori, Salam, PDG'15



- **Extraction of α_s** from hadronic τ very interesting : Moderate precision at the τ mass $\rightarrow$ very good precision at the Z mass
- Beautiful test of the QCD running

3.4 Extraction of α_s

- Several delicate points:
 - How to compute the perturbative part: CIPT vs. FOPT?
 - How to estimate the non perturbative contribution? Where do we truncate the expansion, what is the role of higher order condensates?
 - Which weights should we use?
 - What about duality violations?



A MITP topical workshop in Mainz: March 7-12, 2016

Determination of the fundamental parameters of QCD

- New data on spectral functions needed to help to answer some of these questions

3.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:

- Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_M)}$ with F_M dominant LFV mode for model M

- Spectra for > 2 bodies in the final state:

$$\frac{dBR(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}} \quad \text{and} \quad dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \rightarrow \mu\gamma)} \frac{d\Gamma(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}}$$

3.6 Model discriminating of BRs

- Studies in specific models

Buras et al.'10

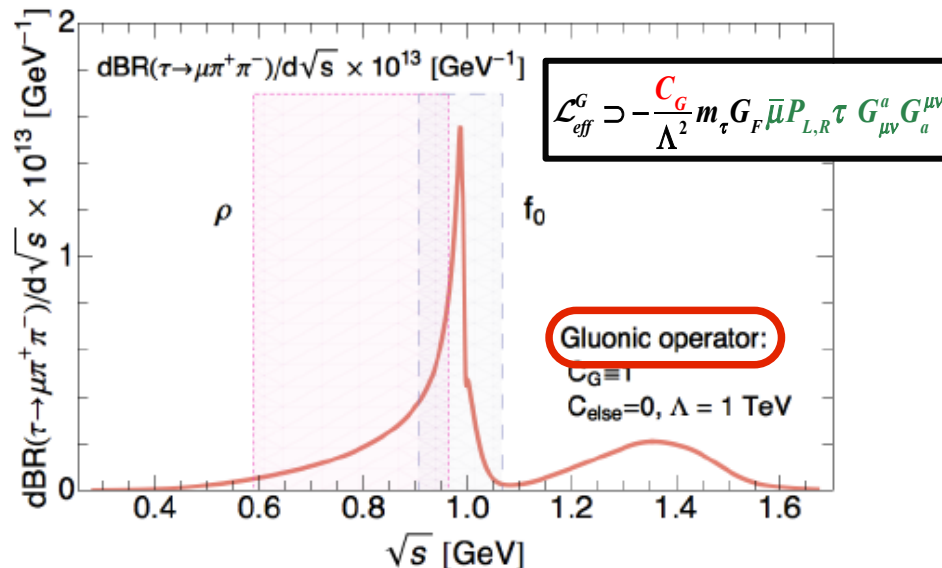
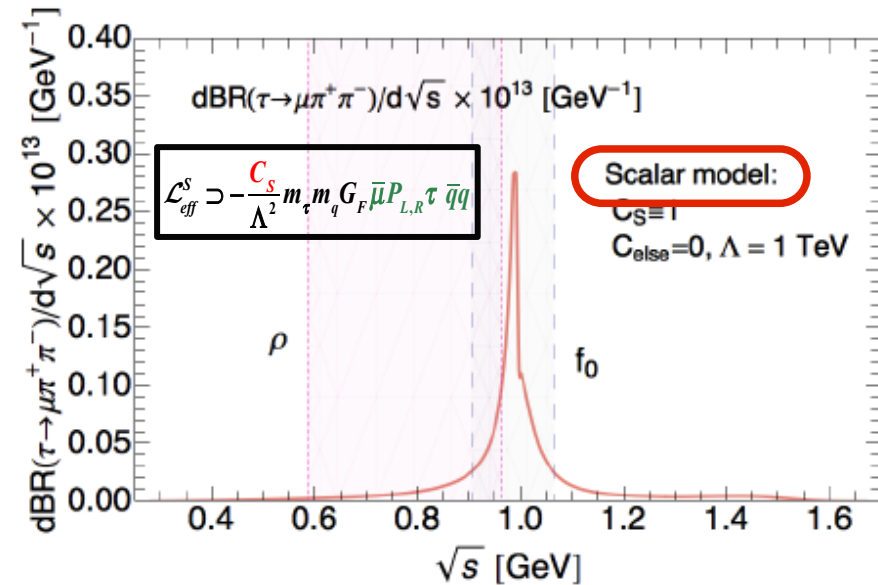
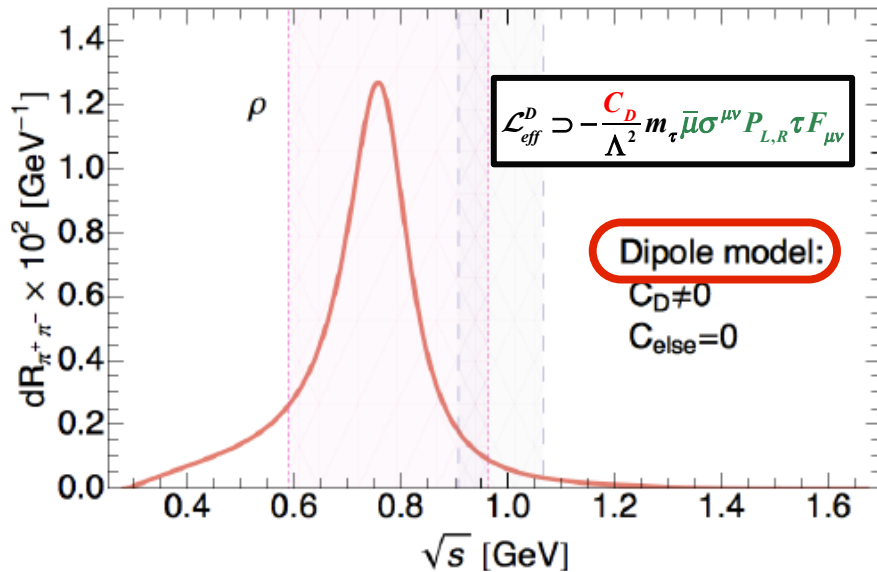
ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\text{Br}(\mu^- \rightarrow e^- e^+ e^-)}{\text{Br}(\mu \rightarrow e \gamma)}$	0.02...1	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	0.06...2.2
$\frac{\text{Br}(\tau^- \rightarrow e^- e^+ e^-)}{\text{Br}(\tau \rightarrow e \gamma)}$	0.04...0.4	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	0.07...2.2
$\frac{\text{Br}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{\text{Br}(\tau \rightarrow \mu \gamma)}$	0.04...0.4	$\sim 2 \cdot 10^{-3}$	0.06...0.1	0.06...2.2
$\frac{\text{Br}(\tau^- \rightarrow e^- \mu^+ \mu^-)}{\text{Br}(\tau \rightarrow e \gamma)}$	0.04...0.3	$\sim 2 \cdot 10^{-3}$	0.02...0.04	0.03...1.3
$\frac{\text{Br}(\tau^- \rightarrow \mu^- e^+ e^-)}{\text{Br}(\tau \rightarrow \mu \gamma)}$	0.04...0.3	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	0.04...1.4
$\frac{\text{Br}(\tau^- \rightarrow e^- e^+ e^-)}{\text{Br}(\tau^- \rightarrow e^- \mu^+ \mu^-)}$	0.8...2	~ 5	0.3...0.5	1.5...2.3
$\frac{\text{Br}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{\text{Br}(\tau^- \rightarrow \mu^- e^+ e^-)}$	0.7...1.6	~ 0.2	5...10	1.4...1.7
$\frac{\text{R}(\mu \text{Ti} \rightarrow e \text{Ti})}{\text{Br}(\mu \rightarrow e \gamma)}$	$10^{-3} \dots 10^2$	$\sim 5 \cdot 10^{-3}$	0.08...0.15	$10^{-12} \dots 26$



Disentangle the *underlying dynamics* of NP

3.7 Model discriminating of Spectra: $\tau \rightarrow \mu \pi \pi$

Celis, Cirigliano, E.P.'14



Very different distributions according to the *final hadronic state!*

NB: See also Dalitz plot analyses for $\tau \rightarrow \mu \mu \mu$

Dassinger et al.'07

1.1 The triumph of the Standard Model

- New era in particle physics :
  (unexpected) *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*
- Key results at LHC after run I + beginning of run II
 - *The Higgs boson* (last missing piece of the SM) has been found:
  it looks very standard
 - The Higgs boson is “*light*” ($m_h \sim 125$ GeV $\rightarrow$ not the heaviest SM particle)
 - *No “mass-gap”* above the SM spectrum (i.e. no unambiguous sign of NP up to ~ 1 TeV)
- *Was this unexpected?*
Not really!  *Consistent* with (pre-LHC) indications coming from indirect NP searches (*EWPO* + *flavour physics*)

1.2 Quest for New Physics

- *Shall we continue to test the Standard Model and search for New Physics?*

Yes!  Despite its phenomenological successes, the SM has some *deep* *unsolved* problems:

- hierarchy problem
- flavour pattern
- dark-matter, etc....
- *Strong interaction* not so well understood:
confinement, etc

1.2 Quest for New Physics

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Yes! ➡ Despite its phenomenological successes, the SM has some *deep*

unsolved problems:

- hierarchy problem
 - flavour pattern
 - dark-matter, etc....
 - **Strong interaction** not so well understood: confinement, etc
- Consider the SM as as an *effective theory*,
i.e. the limit –*in the accessible range of energies and effective couplings*–
of a more fundamental theory, with
 - new degrees of freedom
 - new symmetries

