Operator forms for 3N scattering

Kacper Topolnicki
Jacek Golak
Roman Skibiński
Henryk Witała
Yuriy Volkotrub

3-8 IX 2017

WFAiS UJ

- Few (2,3) body (nucleon) systems
- $|\,\text{nucleon}\rangle = |\,\text{momentum}\ \ \text{isospin}\ \ \text{spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, ...equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\text{ nucleon}\rangle = |\text{ momentum isospin spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, …equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\,\text{nucleon}\rangle = |\,\text{momentum isospin spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, ...equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\,\text{nucleon}\rangle = |\,\text{momentum isospin spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, ... equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\,\text{nucleon}\rangle = |\,\text{momentum}\ \ \text{isospin}\ \ \text{spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, . . . equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\text{ nucleon}\rangle = |\text{ momentum isospin spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, ...equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\text{ nucleon}\rangle = |\text{ momentum isospin spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, ... equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

- Few (2,3) body (nucleon) systems
- $|\text{ nucleon}\rangle = |\text{ momentum isospin spin}\rangle$
- Small energies, non-relativistic QM (we hope to include relativistic corrections)
- Directly solve: Schrödinger, Lippmann-Schwinger, Fadeev, …equations using a numerical approach
- Use effective (2,3) nuclear forces
- Use fenomenological potentials (e.g. AV18) or derived from ChEFT (Bonn, Bohum) - find parameters
- New calculation schemes

PARTIAL WAVES - EXAMPLE

- For the moment let's focus on the 2N system.
- Let's try to calculate the 2N transition operator.

PARTIAL WAVES - SYMMETRIZATION

$$
[\check{V}] = \begin{array}{cc}
\begin{array}{ccccccccccccccc}
|(00)0\rangle & |(11)0\rangle & |(01)1\rangle & |(10)1\rangle & |(11)1\rangle & |(21)1\rangle & |(11)2\rangle & |(20)2\rangle & |(21)2\rangle & |(31)2\rangle & |(21)3\rangle & |(30)3\rangle & |(31)3\rangle & |(41)3\rangle & \cdots
\end{array} & \\
\left[\begin{array}{ccccccccccccccc}
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right] &
\begin{array}{l}
|(00)0\rangle \\ |(11)0\rangle \\ |(01)1\rangle \\ |(10)1\rangle \\ |(11)1\rangle \\ |(21)1\rangle \\ |(11)2\rangle \\ |(20)2\rangle \\ |(21)2\rangle \\ |(31)2\rangle \\ |(21)3\rangle \\ |(30)3\rangle \\ |(31)3\rangle \\ |(41)3\rangle \\ \vdots
\end{array}
\end{array}
$$

PARTIAL WAVES - SYMMETRIZATION

- $\check{V}$ is the 2N potential.
- Each ■ lives in a subspace with given orbital angular momentum l, spin s and total angular momentum j and different momentum magnitude states:

$$\langle |\boldsymbol{p}'|(l's')j' \mid \ldots \mid |\boldsymbol{p}|(ls)j \rangle$$

- Impose pairity, time reversal and rotational symmetry ...

PARTIAL WAVES - SYMMETRIZATION

- $\check{V}$ is the 2N potential.
- Each $\blacksquare$ lives in a subspace with given orbital angular momentum l, spin s and total angular momentum j and different momentum magnitude states:

$$\langle |\boldsymbol{p}'|(l's')j' \mid \ldots \mid |\boldsymbol{p}|(ls)j\rangle$$

- Impose pairity, time reversal and rotational symmetry ...

PARTIAL WAVES - SYMMETRIZATION

$$
\begin{array}{ccc}
 & \begin{array}{ccccccccccccccc} |(00)0\rangle & |(11)0\rangle & |(01)1\rangle & |(10)1\rangle & |(11)1\rangle & |(21)1\rangle & |(11)2\rangle & |(20)2\rangle & |(21)2\rangle & |(31)2\rangle & |(21)3\rangle & |(30)3\rangle & |(31)3\rangle & |(41)3\rangle & \cdots \end{array} & \\
[\check{V}] = & \begin{bmatrix}
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix} & \begin{array}{c} |(00)0\rangle \\ |(11)0\rangle \\ |(01)1\rangle \\ |(10)1\rangle \\ |(11)1\rangle \\ |(21)1\rangle \\ |(11)2\rangle \\ |(20)2\rangle \\ |(21)2\rangle \\ |(31)2\rangle \\ |(21)3\rangle \\ |(30)3\rangle \\ |(31)3\rangle \\ |(41)3\rangle \\ \vdots \end{array}
\end{array}
$$

PARTIAL WAVES - SYMMETRIZATION

$$
[\check{V}] = \begin{array}{cc}
\begin{array}{ccccccccccccccc}
\mid(00)0\rangle & \mid(11)0\rangle & \mid(01)1\rangle & \mid(10)1\rangle & \mid(11)1\rangle & \mid(21)1\rangle & \mid(11)2\rangle & \mid(20)2\rangle & \mid(21)2\rangle & \mid(31)2\rangle & \mid(21)3\rangle & \mid(30)3\rangle & \mid(31)3\rangle & \mid(41)3\rangle & \cdots
\end{array} & \\
\begin{bmatrix}
\blacksquare & & & \blacksquare & & & & \blacksquare & & & & \blacksquare & & & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
\blacksquare & & & \blacksquare & & & & \blacksquare & & & & \blacksquare & & & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
\blacksquare & & & \blacksquare & & & & \blacksquare & & & & \blacksquare & & & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
\blacksquare & & & \blacksquare & & & & \blacksquare & & & & \blacksquare & & & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix} &
\begin{array}{l}
\mid(00)0\rangle \\ \mid(11)0\rangle \\ \mid(01)1\rangle \\ \mid(10)1\rangle \\ \mid(11)1\rangle \\ \mid(21)1\rangle \\ \mid(11)2\rangle \\ \mid(20)2\rangle \\ \mid(21)2\rangle \\ \mid(31)2\rangle \\ \mid(21)3\rangle \\ \mid(30)3\rangle \\ \mid(31)3\rangle \\ \mid(41)3\rangle \\ \vdots
\end{array}
\end{array}
$$

PARTIAL WAVES - SYMMETRIZATION

$$
[\check{V}] = \begin{array}{cc}
\begin{array}{ccccccccccccccc}
|(00)0\rangle & |(11)0\rangle & |(01)1\rangle & |(10)1\rangle & |(11)1\rangle & |(21)1\rangle & |(11)2\rangle & |(20)2\rangle & |(21)2\rangle & |(31)2\rangle & |(21)3\rangle & |(30)3\rangle & |(31)3\rangle & |(41)3\rangle & \cdots
\end{array} & \\
\left[\begin{array}{ccccccccccccccc}
\blacksquare & & & & & & & & & & & & & & \cdots \\
 & \blacksquare & & & & & & & & & & & & & \cdots \\
 & & \blacksquare & & \blacksquare & \blacksquare & & & & & & & & & \cdots \\
 & & & \blacksquare & & & & & & & & & & & \cdots \\
 & & \blacksquare & & \blacksquare & \blacksquare & & & & & & & & & \cdots \\
 & & \blacksquare & & \blacksquare & \blacksquare & & & & & & & & & \cdots \\
 & & & & & & \blacksquare & & \blacksquare & \blacksquare & & & & & \cdots \\
 & & & & & & & \blacksquare & & & & & & & \cdots \\
 & & & & & & \blacksquare & & \blacksquare & \blacksquare & & & & & \cdots \\
 & & & & & & \blacksquare & & \blacksquare & \blacksquare & & & & & \cdots \\
 & & & & & & & & & & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & & & & & & & & & & & \blacksquare & & & \cdots \\
 & & & & & & & & & & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
 & & & & & & & & & & \blacksquare & & \blacksquare & \blacksquare & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right] &
\begin{array}{l}
|(00)0\rangle \\ |(11)0\rangle \\ |(01)1\rangle \\ |(10)1\rangle \\ |(11)1\rangle \\ |(21)1\rangle \\ |(11)2\rangle \\ |(20)2\rangle \\ |(21)2\rangle \\ |(31)2\rangle \\ |(21)3\rangle \\ |(30)3\rangle \\ |(31)3\rangle \\ |(41)3\rangle \\ \vdots
\end{array}
\end{array}
$$

PARTIAL WAVES - SYMMETRIZATION

$$
\begin{array}{cc}
 & \begin{array}{ccccccccccccccc} |(00)0\rangle & |(11)0\rangle & |(01)1\rangle & |(10)1\rangle & |(11)1\rangle & |(21)1\rangle & |(11)2\rangle & |(20)2\rangle & |(21)2\rangle & |(31)2\rangle & |(21)3\rangle & |(30)3\rangle & |(31)3\rangle & |(41)3\rangle & \cdots \end{array} \\
[\check{V}] = & \left[\begin{array}{ccccccccccccccc}
\blacksquare & & & & & & & & & & & & & & \cdots \\
 & \blacksquare & & & & & & & & & & & & & \cdots \\
 & & \blacksquare & & & \blacksquare & & & & & & & & & \cdots \\
 & & & \blacksquare & & & & & & & & & & & \cdots \\
 & & & & \blacksquare & & & & & & & & & & \cdots \\
 & & \blacksquare & & & \blacksquare & & & & & & & & & \cdots \\
 & & & & & & \blacksquare & & & \blacksquare & & & & & \cdots \\
 & & & & & & & \blacksquare & & & & & & & \cdots \\
 & & & & & & & & \blacksquare & & & & & & \cdots \\
 & & & & & & \blacksquare & & & \blacksquare & & & & & \cdots \\
 & & & & & & & & & & \blacksquare & & & \blacksquare & \cdots \\
 & & & & & & & & & & & \blacksquare & & & \cdots \\
 & & & & & & & & & & & & \blacksquare & & \cdots \\
 & & & & & & & & & & \blacksquare & & & \blacksquare & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right]
\begin{array}{l} |(00)0\rangle \\ |(11)0\rangle \\ |(01)1\rangle \\ |(10)1\rangle \\ |(11)1\rangle \\ |(21)1\rangle \\ |(11)2\rangle \\ |(20)2\rangle \\ |(21)2\rangle \\ |(31)2\rangle \\ |(21)3\rangle \\ |(30)3\rangle \\ |(31)3\rangle \\ |(41)3\rangle \\ \vdots \end{array}
\end{array}
$$

PARTIAL WAVES - CALCULATION

- Perform PWD on each operator of, eg., LSE $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.
- Solve the resulting linear equations.

PARTIAL WAVES - CALCULATION

- Perform PWD on each operator of, eg., LSE $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.
- Solve the resulting linear equations.

PARTIAL WAVES - CALCULATION

- Perform PWD on each operator of, eg., LSE $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.
- Solve the resulting linear equations.

PARTIAL WAVES - PROS AND CONS

- +
  - Battle tested.
  - Small numerical workload.
- −
  - Implementation requires heavily oscilating functions.
  - It is not always obvious how many partial waves need to be taken into account.
  - This is more complicated for three or more particles and different coupling schemes.
  - Convergence problems for higher energies.

PARTIAL WAVES - PROS AND CONS

- +
  - Battle tested.
  - Small numerical workload.
- −
  - Implementation requires heavily oscilating functions.
  - It is not always obvious how many partial waves need to be taken into account.
  - This is more complicated for three or more particles and different coupling schemes.
  - Convergence problems for higher energies.

PARTIAL WAVES - PROS AND CONS

- +
  - Battle tested.
  - Small numerical workload.
- −
  - Implementation requires heavily oscilating functions.
  - It is not always obvious how many partial waves need to be taken into account.
  - This is more complicated for three or more particles and different coupling schemes.
  - Convergence problems for higher energies.

PARTIAL WAVES - PROS AND CONS

- $+$
  - Battle tested.
  - Small numerical workload.
- $-$
  - Implementation requires heavily oscilating functions.
  - It is not always obvious how many partial waves need to be taken into account.
  - This is more complicated for three or more particles and different coupling schemes.
  - Convergence problems for higher energies.

PARTIAL WAVES - PROS AND CONS

- $+$
  - Battle tested.
  - Small numerical workload.
- $-$
  - Implementation requires heavily oscilating functions.
  - It is not always obvious how many partial waves need to be taken into account.
  - This is more complicated for three or more particles and different coupling schemes.
  - Convergence problems for higher energies.

3D - EXAMPLE

- Lets take the 2N transition operator.
- Assume we are working in momentum space with $\boldsymbol{p}' = (p'_x, p'_y, p'_z)$ being the final and $\boldsymbol{p} = (p_x, p_y, p_z)$ being the initial momentum of the two nucleons.

3D - EXAMPLE

- Lets take the 2N transition operator.
- Assume we are working in momentum space with $\boldsymbol{p}' = (p'_x, p'_y, p'_z)$ being the final and $\boldsymbol{p} = (p_x, p_y, p_z)$ being the initial momentum of the two nucleons.

3D - EXAMPLE

- Lets take the 2N transition operator.
- Assume we are working in momentum space with $\boldsymbol{p}' = (p'_x, p'_y, p'_z)$ being the final and $\boldsymbol{p} = (p_x, p_y, p_z)$ being the initial momentum of the two nucleons.

3D - EXAMPLE

- Lets take the 2N transition operator.
- Assume we are working in momentum space with $\boldsymbol{p}' = (p'_x, p'_y, p'_z)$ being the final and $\boldsymbol{p} = (p_x, p_y, p_z)$ being the initial momentum of the two nucleons.

3D - SIZE OF THE PROBLEM

- We would like to calculate the full transition operator. This is equivalent to calculating, for every $\boldsymbol{p}'$ and every $\boldsymbol{p}$, the matrix element $\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle$.
- This matrix element is an operator in spin space and has the form:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle \right] =$$

$$\begin{bmatrix} t_{11}(p'_x, p'_y, p'_z, p_x, p_y, p_z) & t_{12}(\dots) & t_{13}(\dots) & t_{14}(\dots) \\ t_{21}(\dots) & t_{22}(\dots) & t_{23}(\dots) & t_{24}(\dots) \\ t_{31}(\dots) & t_{32}(\dots) & t_{33}(\dots) & t_{34}(\dots) \\ t_{41}(\dots) & t_{42}(\dots) & t_{43}(\dots) & t_{44}(\dots) \end{bmatrix}.$$

- It must satisfy the LSE equation $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.

3D - SIZE OF THE PROBLEM

- We would like to calculate the full transition operator. This is equivalent to calculating, for every $\boldsymbol{p}'$ and every $\boldsymbol{p}$, the matrix element $\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle$.
- This matrix element is an operator in spin space and has the form:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle \right] =$$

$$\begin{bmatrix} t_{11}(p'_x, p'_y, p'_z, p_x, p_y, p_z) & t_{12}(\dots) & t_{13}(\dots) & t_{14}(\dots) \\ t_{21}(\dots) & t_{22}(\dots) & t_{23}(\dots) & t_{24}(\dots) \\ t_{31}(\dots) & t_{32}(\dots) & t_{33}(\dots) & t_{34}(\dots) \\ t_{41}(\dots) & t_{42}(\dots) & t_{43}(\dots) & t_{44}(\dots) \end{bmatrix} .$$

- It must satisfy the LSE equation $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.

3D - SIZE OF THE PROBLEM

- We would like to calculate the full transition operator. This is equivalent to calculating, for every $\boldsymbol{p}'$ and every $\boldsymbol{p}$, the matrix element $\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle$.
- This matrix element is an operator in spin space and has the form:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle \right] =$$

$$\begin{bmatrix} t_{11}(p'_x, p'_y, p'_z, p_x, p_y, p_z) & t_{12}(\dots) & t_{13}(\dots) & t_{14}(\dots) \\ t_{21}(\dots) & t_{22}(\dots) & t_{23}(\dots) & t_{24}(\dots) \\ t_{31}(\dots) & t_{32}(\dots) & t_{33}(\dots) & t_{34}(\dots) \\ t_{41}(\dots) & t_{42}(\dots) & t_{43}(\dots) & t_{44}(\dots) \end{bmatrix}.$$

- It must satisfy the LSE equation $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.

3D - SIZE OF THE PROBLEM

- We would like to calculate the full transition operator. This is equivalent to calculating, for every $\boldsymbol{p}'$ and every $\boldsymbol{p}$, the matrix element $\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle$.
- This matrix element is an operator in spin space and has the form:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle\right] =$$

$$\begin{bmatrix} t_{11}(p'_x, p'_y, p'_z, p_x, p_y, p_z) & t_{12}(\ldots) & t_{13}(\ldots) & t_{14}(\ldots) \\ t_{21}(\ldots) & t_{22}(\ldots) & t_{23}(\ldots) & t_{24}(\ldots) \\ t_{31}(\ldots) & t_{32}(\ldots) & t_{33}(\ldots) & t_{34}(\ldots) \\ t_{41}(\ldots) & t_{42}(\ldots) & t_{43}(\ldots) & t_{44}(\ldots) \end{bmatrix}.$$

- It must satisfy the LSE equation $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$.

3D - SIZE OF THE PROBLEM

- We need to calculate 16 functions of 6 real parameters that satisfy the LSE.
- **If each argument is discretized over** 32 **points then** $16 \times 32^6 \approx 17.2 \times 10^9$ **complex numbers to represent** $\check{t}$**.**
- We know that the solution has to satisfy appropriate symmetries.
- Can we use this to simplify the problem?

3D - SIZE OF THE PROBLEM

- We need to calculate 16 functions of 6 real parameters that satisfy the LSE.
- **If each argument is discretized over** 32 **points then** $16 \times 32^6 \approx 17.2 \times 10^9$ **complex numbers to represent** $\check{t}$**.**
- We know that the solution has to satisfy appropriate symmetries.
- Can we use this to simplify the problem?

3D - SIZE OF THE PROBLEM

- We need to calculate 16 functions of 6 real parameters that satisfy the LSE.
- **If each argument is discretized over** 32 **points then** $16 \times 32^6 \approx 17.2 \times 10^9$ **complex numbers to represent** $\check{t}$**.**
- We know that the solution has to satisfy appropriate symmetries.
- Can we use this to simplify the problem?

3D - SIZE OF THE PROBLEM

- We need to calculate 16 functions of 6 real parameters that satisfy the LSE.
- **If each argument is discretized over** 32 **points then** $16 \times 32^6 \approx 17.2 \times 10^9$ **complex numbers to represent** $\check{t}$**.**
- We know that the solution has to satisfy appropriate symmetries.
- Can we use this to simplify the problem?

3D - SIZE OF THE PROBLEM

- We need to calculate 16 functions of 6 real parameters that satisfy the LSE.
- **If each argument is discretized over** 32 **points then** $16 \times 32^6 \approx 17.2 \times 10^9$ **complex numbers to represent** $\check{t}$**.**
- We know that the solution has to satisfy appropriate symmetries.
- Can we use this to simplify the problem?

3D - SYMMETRIZATION

- The general operator form of the two nucleon potential and transition operator is well known [Phys. Rev. 96 1654 (1954)].
- The matrix element in momentum space can be written as a linear combination of 6 scalar functions t_i and spin operators $[w_i]$:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle\right] = \sum_{i=1}^{6} t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}}) \left[\check{w}_i(\boldsymbol{p}, \boldsymbol{p})\right].$$

3D - SYMMETRIZATION

- The general operator form of the two nucleon potential and transition operator is well known [Phys. Rev. 96 1654 (1954)].
- The matrix element in momentum space can be written as a linear combination of 6 scalar functions t_i and spin operators $[w_i]$:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p}\rangle\right] = \sum_{i=1}^{6} t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}}) \left[\check{w}_i(\boldsymbol{p}, \boldsymbol{p})\right].$$

3D - SYMMETRIZATION

- The general operator form of the two nucleon potential and transition operator is well known [Phys. Rev. 96 1654 (1954)].
- The matrix element in momentum space can be written as a linear combination of 6 scalar functions t_i and spin operators $[w_i]$:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle\right] = \sum_{i=1}^{6} t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}}) \left[\check{w}_i(\boldsymbol{p}, \boldsymbol{p})\right].$$

3D - SYMMETRIZATION

- Instead of calculating 16 functions of 6 real variables we now only need to calculate 6 functions of 3 variables.
- **From** $16 \times 32^6 \approx 17.2 \times 10^9$ **to** $6 \times 32^3 \approx 16.4 \times 10^4$ **complex numbers for** $\check{t}$**.**
- Couple orders of magnitude less numerical work!

3D - SYMMETRIZATION

- Instead of calculating 16 functions of 6 real variables we now only need to calculate 6 functions of 3 variables.
- **From** $16 \times 32^6 \approx 17.2 \times 10^9$ **to** $6 \times 32^3 \approx 16.4 \times 10^4$ **complex numbers for** $\check{t}$**.**
- Couple orders of magnitude less numerical work!

3D - SYMMETRIZATION

- Instead of calculating 16 functions of 6 real variables we now only need to calculate 6 functions of 3 variables.
- **From** $16 \times 32^6 \approx 17.2 \times 10^9$ **to** $6 \times 32^3 \approx 16.4 \times 10^4$ **complex numbers for** $\check{t}$**.**
- Couple orders of magnitude less numerical work!

3D - PROS AND CONS

- $+$
  - More precision at higher energies.
  - Calculations can be easily modified to use different potentials.
  - Operator fomrms (operators and states) significantly reduce numerical workload.
- $-$
  - We are running out of operator forms!
  - **Can we construct new symmetric operator forms? Can this be generalized to systems of three or more particles?**

3D - PROS AND CONS

- $+$
  - More precision at higher energies.
  - Calculations can be easily modified to use different potentials.
  - Operator fomrms (operators and states) significantly reduce numerical workload.
- $-$
  - We are running out of operator forms!
  - **Can we construct new symmetric operator forms? Can this be generalized to systems of three or more particles?**

3D - PROS AND CONS

- +
  - More precision at higher energies.
  - Calculations can be easily modified to use different potentials.
  - Operator fomrms (operators and states) significantly reduce numerical workload.
- −
  - We are running out of operator forms!
  - **Can we construct new symmetric operator forms? Can this be generalized to systems of three or more particles?**

3D - PROS AND CONS

- $+$
  - More precision at higher energies.
  - Calculations can be easily modified to use different potentials.
  - Operator fomrms (operators and states) significantly reduce numerical workload.
- $-$
  - We are running out of operator forms!
  - **Can we construct new symmetric operator forms? Can this be generalized to systems of three or more particles?**

3D - PROS AND CONS

- $+$
  - More precision at higher energies.
  - Calculations can be easily modified to use different potentials.
  - Operator fomrms (operators and states) significantly reduce numerical workload.
- $-$
  - We are running out of operator forms!
  - Can we construct new symmetric operator forms? Can this be generalized to systems of three or more particles?

3D - PROS AND CONS

- $+$
  - More precision at higher energies.
  - Calculations can be easily modified to use different potentials.
  - Operator fomrms (operators and states) significantly reduce numerical workload.
- $-$
  - We are running out of operator forms!
  - **Can we construct new symmetric operator forms? Can this be generalized to systems of three or more particles?**

INVARIANCE UNDER SPATIAL ROTATIONS

- If $\check{R}$ is a spatial rotation, we require that operator $\check{X}$:

$$\check{R}^{-1}\check{X}\check{R} = \check{X}.$$

- Ideally we would like to fit the operator into an operator form:

$$\langle \ldots \mid \check{X} \mid \ldots \rangle = \sum x_i \left[\check{O}_i \right].$$

Here x is a scalar function of momenta and O is an operator in spin space and $\ldots$ are momenta.

- How can a rotation invariant operator be constructed?

INVARIANCE UNDER SPATIAL ROTATIONS

- If $\check{R}$ is a spatial rotation, we require that operator $\check{X}$:

$$\check{R}^{-1}\check{X}\check{R} = \check{X}.$$

- Ideally we would like to fit the operator into an operator form:

$$\langle \ldots | \check{X} | \ldots \rangle = \sum x_i \left[\check{O}_i \right].$$

Here x is a scalar function of momenta and O is an operator in spin space and $\ldots$ are momenta.

- How can a rotation invariant operator be constructed?

INVARIANCE UNDER SPATIAL ROTATIONS

- If $\check{R}$ is a spatial rotation, we require that operator $\check{X}$:

$$\check{R}^{-1}\check{X}\check{R} = \check{X}.$$

- Ideally we would like to fit the operator into an operator form:

$$\langle \ldots \mid \check{X} \mid \ldots \rangle = \sum x_i \left[\check{O}_i \right].$$

  Here x is a scalar function of momenta and O is an operator in spin space and ... are momenta.

- How can a rotation invariant operator be constructed?

INVARIANCE UNDER SPATIAL ROTATIONS

- If $\check{R}$ is a spatial rotation, we require that operator $\check{X}$:

$$\check{R}^{-1}\check{X}\check{R} = \check{X}.$$

- Ideally we would like to fit the operator into an operator form:

$$\langle \ldots \mid \check{X} \mid \ldots \rangle = \sum x_i \left[\check{O}_i\right].$$

  Here x is a scalar function of momenta and O is an operator in spin space and $\ldots$ are momenta.

- How can a rotation invariant operator be constructed?

INVARIANCE UNDER SPATIAL ROTATIONS

- Let's generalize a little bit and incorporate the dependance on the total momentum $\boldsymbol{K}$.
- Boulding blocks (actually any number of momenta and spin vectors can be used):

$$\mathbb{T} = \{\check{\boldsymbol{p}}', \check{\boldsymbol{p}}, \check{\boldsymbol{K}}, \check{\boldsymbol{\sigma}}(1), \check{\boldsymbol{\sigma}}(2)\}.$$

- In principle we have to consider all scalar combinations of the elements from $\mathbb{T}$. For example if $\boldsymbol{v}_i \in \mathbb{T}$ we could use:

$$(\check{\boldsymbol{v}}_1 \times (\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_3)) \cdot (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times (\check{\boldsymbol{v}}_6 \times \check{\boldsymbol{v}}_7)))$$

or

$$(\check{\boldsymbol{v}}_1 \times \check{\boldsymbol{v}}_2) \cdot (\check{\boldsymbol{v}}_3 \times (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times \check{\boldsymbol{v}}_6))) .$$

INVARIANCE UNDER SPATIAL ROTATIONS

- Let's generalize a little bit and incorporate the dependance on the total momentum $\boldsymbol{K}$.
- Boulding blocks (actually any number of momenta and spin vectors can be used):

$$\mathbb{T} = \{\check{\boldsymbol{p}}', \check{\boldsymbol{p}}, \check{\boldsymbol{K}}, \check{\boldsymbol{\sigma}}(1), \check{\boldsymbol{\sigma}}(2)\}.$$

- In principle we have to consider all scalar combinations of the elements from $\mathbb{T}$. For example if $\boldsymbol{v}_i \in \mathbb{T}$ we could use:

$$(\check{\boldsymbol{v}}_1 \times (\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_3)) \cdot (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times (\check{\boldsymbol{v}}_6 \times \check{\boldsymbol{v}}_7)))$$

or

$$(\check{\boldsymbol{v}}_1 \times \check{\boldsymbol{v}}_2) \cdot (\check{\boldsymbol{v}}_3 \times (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times \check{\boldsymbol{v}}_6)))\,.$$

INVARIANCE UNDER SPATIAL ROTATIONS

- Let's generalize a little bit and incorporate the dependance on the total momentum $\boldsymbol{K}$.
- Boulding blocks (actually any number of momenta and spin vectors can be used):

$$\mathbb{T} = \{\check{\boldsymbol{p}}', \check{\boldsymbol{p}}, \check{\boldsymbol{K}}, \check{\boldsymbol{\sigma}}(1), \check{\boldsymbol{\sigma}}(2)\}.$$

- In principle we have to consider all scalar combinations of the elements from $\mathbb{T}$. For example if $\boldsymbol{v}_i \in \mathbb{T}$ we could use:

$$(\check{\boldsymbol{v}}_1 \times (\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_3)) \cdot (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times (\check{\boldsymbol{v}}_6 \times \check{\boldsymbol{v}}_7)))$$

or

$$(\check{\boldsymbol{v}}_1 \times \check{\boldsymbol{v}}_2) \cdot (\check{\boldsymbol{v}}_3 \times (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times \check{\boldsymbol{v}}_6))) .$$

INVARIANCE UNDER SPATIAL ROTATIONS

- Let's generalize a little bit and incorporate the dependance on the total momentum $\boldsymbol{K}$.
- Boulding blocks (actually any number of momenta and spin vectors can be used):

$$\mathbb{T} = \{\check{\boldsymbol{p}}', \check{\boldsymbol{p}}, \check{\boldsymbol{K}}, \check{\boldsymbol{\sigma}}(1), \check{\boldsymbol{\sigma}}(2)\}.$$

- In principle we have to consider all scalar combinations of the elements from $\mathbb{T}$. For example if $\boldsymbol{v}_i \in \mathbb{T}$ we could use:

$$(\check{\boldsymbol{v}}_1 \times (\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_3)) \cdot (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times (\check{\boldsymbol{v}}_6 \times \check{\boldsymbol{v}}_7)))$$

or

$$(\check{\boldsymbol{v}}_1 \times \check{\boldsymbol{v}}_2) \cdot (\check{\boldsymbol{v}}_3 \times (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times \check{\boldsymbol{v}}_6))) \, .$$

INVARIANCE UNDER SPATIAL ROTATIONS

- Simple vector identities lead to:

$$
\begin{aligned}
(\check{\boldsymbol{v}}_1 \times (\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_3)) \cdot (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times (\check{\boldsymbol{v}}_6 \times \check{\boldsymbol{v}}_7)))
&= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7) \\
&-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_3 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7) \\
&+(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_3 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7) \\
&-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7)
\end{aligned}
$$

and

$$
\begin{aligned}
(\check{\boldsymbol{v}}_1 \times \check{\boldsymbol{v}}_2) \cdot (\check{\boldsymbol{v}}_3 \times (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times \check{\boldsymbol{v}}_6)))
&= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5) \\
&-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5) \\
&-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6) \\
&+(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6).
\end{aligned}
$$

INVARIANCE UNDER SPATIAL ROTATIONS

- Simple vector identities lead to:

$$
\begin{aligned}
(\check{\boldsymbol{v}}_1 \times (\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_3)) \cdot (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times (\check{\boldsymbol{v}}_6 \times \check{\boldsymbol{v}}_7))) \\
= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7) \\
-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_3 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7) \\
+(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_3 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7) \\
-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7)
\end{aligned}
$$

and

$$
\begin{aligned}
(\check{\boldsymbol{v}}_1 \times \check{\boldsymbol{v}}_2) \cdot (\check{\boldsymbol{v}}_3 \times (\check{\boldsymbol{v}}_4 \times (\check{\boldsymbol{v}}_5 \times \check{\boldsymbol{v}}_6))) \\
= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5) \\
-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5) \\
-(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6) \\
+(\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6).
\end{aligned}
$$

INVARIANCE UNDER SPATIAL ROTATIONS

- A general observation can be made: Any scalar expression constructed from operators in $\mathbb{T}$ can be constructed from a combination of operators in the set $\mathbb{V}$:

$$\mathbb{V} = \{\check{1}, \check{\boldsymbol{v}}_i \cdot \check{\boldsymbol{v}}_j\ ,\ (\check{\boldsymbol{v}}_i \times \check{\boldsymbol{v}}_j) \cdot \check{\boldsymbol{v}}_k\}.$$

- For example, from the previous slide, we have the following CHAINS of operators of length 3:

$$\begin{aligned}
[\check{C}_1] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_2] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_3 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_3] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_3 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_4] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_5] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5),\\
[\check{C}_6] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5),\\
[\check{C}_7] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6),\\
[\check{C}_8] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6).
\end{aligned}$$

INVARIANCE UNDER SPATIAL ROTATIONS

- A general observation can be made: Any scalar expression constructed from operators in $\mathbb{T}$ can be constructed from a combination of operators in the set $\mathbb{V}$:

$$\mathbb{V} = \{\check{1}, \check{\boldsymbol{v}}_i \cdot \check{\boldsymbol{v}}_j \ , \ (\check{\boldsymbol{v}}_i \times \check{\boldsymbol{v}}_j) \cdot \check{\boldsymbol{v}}_k\}.$$

- For example, from the previous slide, we have the following CHAINS of operators of length 3:

$$\begin{aligned}
[\check{C}_1] &= (\check{v}_1 \cdot \check{v}_3)(\check{v}_2 \cdot \check{v}_5)(\check{v}_4 \times \check{v}_6 \cdot \check{v}_7),\\
[\check{C}_2] &= (\check{v}_1 \cdot \check{v}_2)(\check{v}_3 \cdot \check{v}_5)(\check{v}_4 \times \check{v}_6 \cdot \check{v}_7),\\
[\check{C}_3] &= (\check{v}_1 \cdot \check{v}_2)(\check{v}_4 \cdot \check{v}_5)(\check{v}_3 \times \check{v}_6 \cdot \check{v}_7),\\
[\check{C}_4] &= (\check{v}_1 \cdot \check{v}_3)(\check{v}_4 \cdot \check{v}_5)(\check{v}_2 \times \check{v}_6 \cdot \check{v}_7),\\
[\check{C}_5] &= (\check{v}_1 \cdot \check{v}_6)(\check{v}_2 \cdot \check{v}_3)(\check{v}_4 \cdot \check{v}_5),\\
[\check{C}_6] &= (\check{v}_1 \cdot \check{v}_3)(\check{v}_2 \cdot \check{v}_6)(\check{v}_4 \cdot \check{v}_5),\\
[\check{C}_7] &= (\check{v}_1 \cdot \check{v}_5)(\check{v}_2 \cdot \check{v}_3)(\check{v}_4 \cdot \check{v}_6),\\
[\check{C}_8] &= (\check{v}_1 \cdot \check{v}_3)(\check{v}_2 \cdot \check{v}_5)(\check{v}_4 \cdot \check{v}_6).
\end{aligned}$$

INVARIANCE UNDER SPATIAL ROTATIONS

- A general observation can be made: Any scalar expression constructed from operators in $\mathbb{T}$ can be constructed from a combination of operators in the set $\mathbb{V}$:

$$\mathbb{V} = \{\check{\mathbb{1}}, \check{\boldsymbol{v}}_i \cdot \check{\boldsymbol{v}}_j \ , \ (\check{\boldsymbol{v}}_i \times \check{\boldsymbol{v}}_j) \cdot \check{\boldsymbol{v}}_k\}.$$

- For example, from the previous slide, we have the following CHAINS of operators of length 3:

$$\begin{aligned}
[\check{C}_1] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_2] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_3 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_3] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_2)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_3 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_4] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \times \check{\boldsymbol{v}}_6 \cdot \check{\boldsymbol{v}}_7),\\
[\check{C}_5] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5),\\
[\check{C}_6] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_6)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_5),\\
[\check{C}_7] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6),\\
[\check{C}_8] &= (\check{\boldsymbol{v}}_1 \cdot \check{\boldsymbol{v}}_3)(\check{\boldsymbol{v}}_2 \cdot \check{\boldsymbol{v}}_5)(\check{\boldsymbol{v}}_4 \cdot \check{\boldsymbol{v}}_6).
\end{aligned}$$

INVARIANCE UNDER SPATIAL ROTATIONS

- How get from this to the operator form?
- In principle we have to include CHAINS of operators constructed from $\mathbb{V}$ with any number of links?

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p}\boldsymbol{K} \rangle = \ldots + \sum_{i=1}^{8} x_i \left[\check{C}_i \right] + \ldots$$

- Infinitely many terms?

INVARIANCE UNDER SPATIAL ROTATIONS

- How get from this to the operator form?
- In principle we have to include CHAINS of operators constructed from $\mathbb{V}$ with any number of links?

$$\langle \boldsymbol{p}'\boldsymbol{K} \mid \check{X} \mid \boldsymbol{p}\boldsymbol{K}\rangle = \ldots + \sum_{i=1}^{8} x_i \left[\check{C}_i\right] + \ldots$$

- Infinitely many terms?

INVARIANCE UNDER SPATIAL ROTATIONS

- How get from this to the operator form?
- In principle we have to include CHAINS of operators constructed from $\mathbb{V}$ with any number of links?

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p} \boldsymbol{K} \rangle = \ldots + \sum_{i=1}^{8} x_i \left[\check{C}_i \right] + \ldots$$

- Infinitely many terms?

INVARIANCE UNDER SPATIAL ROTATIONS

- How get from this to the operator form?
- In principle we have to include CHAINS of operators constructed from $\mathbb{V}$ with any number of links?

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p}\boldsymbol{K} \rangle = \ldots + \sum_{i=1}^{8} x_i \left[\check{C}_i \right] + \ldots$$

- Infinitely many terms?

INVARIANCE UNDER SPATIAL ROTATIONS

- No! [*Eur. Phys. J.* A 52:188 (2016)]
- The operator form for $\check{X}$:

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p} \boldsymbol{K} \rangle = \sum_{i=1}^{16} x_i \left[\check{O}_i(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{K}) \right] .$$

- $\check{X}$ could be the potential, transition operator with relativistic corrections ...
- The set $\mathbb{T}$ can be extended ...

INVARIANCE UNDER SPATIAL ROTATIONS

- No! [*Eur. Phys. J.* A 52:188 (2016)]
- The operator form for $\check{X}$:

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p} \boldsymbol{K} \rangle = \sum_{i=1}^{16} x_i \left[\check{O}_i(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{K}) \right] .$$

- $\check{X}$ could be the potential, transition operator with relativistic corrections ...
- The set $\mathbb{T}$ can be extended ...

INVARIANCE UNDER SPATIAL ROTATIONS

- No! [*Eur. Phys. J.* A 52:188 (2016)]
- The operator form for $\check{X}$:

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p} \boldsymbol{K} \rangle = \sum_{i=1}^{16} x_i \left[\check{O}_i(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{K}) \right].$$

- $\check{X}$ could be the potential, transition operator with relativistic corrections ...
- The set $\mathbb{T}$ can be extended ...

INVARIANCE UNDER SPATIAL ROTATIONS

- No! [*Eur. Phys. J.* A 52:188 (2016)]
- The operator form for $\check{X}$:

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p} \boldsymbol{K} \rangle = \sum_{i=1}^{16} x_i \left[\check{O}_i(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{K}) \right] .$$

- $\check{X}$ could be the potential, transition operator with relativistic corrections ...
- The set $\mathbb{T}$ can be extended ...

INVARIANCE UNDER SPATIAL ROTATIONS

- No! [*Eur. Phys. J.* A 52:188 (2016)]
- The operator form for $\check{X}$:

$$\langle \boldsymbol{p}' \boldsymbol{K} \mid \check{X} \mid \boldsymbol{p} \boldsymbol{K} \rangle = \sum_{i=1}^{16} x_i \left[\check{O}_i(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{K}) \right] .$$

- $\check{X}$ could be the potential, transition operator with relativistic corrections ...
- The set $\mathbb{T}$ can be extended ...

ADDING ADDITIONAL SYMMETRIES

- We can use a simple symmetrization procedure.
- Let $\mathbb{D}$ be a grup of transformations constructed from **parity, time reversal, Hermitian conjugate and two particle exchange**.
- A symmetric operator is obtained using:

$$\check{X} \to \sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}.$$

- Applying any $\check{T} \in \mathbb{D}$ to $\sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}$ returns the same operator.

ADDING ADDITIONAL SYMMETRIES

- We can use a simple symmetrization procedure.
- Let $\mathbb{D}$ be a grup of transformations constructed from **parity, time reversal, Hermitian conjugate and two particle exchange**.
- A symmetric operator is obtained using:

$$\check{X} \to \sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}.$$

- Applying any $\check{T} \in \mathbb{D}$ to $\sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}$ returns the same operator.

ADDING ADDITIONAL SYMMETRIES

- We can use a simple symmetrization procedure.
- Let $\mathbb{D}$ be a grup of transformations constructed from **parity, time reversal, Hermitian conjugate and two particle exchange**.
- A symmetric operator is obtained using:

$$\check{X} \to \sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}.$$

- Applying any $\check{T} \in \mathbb{D}$ to $\sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}$ returns the same operator.

ADDING ADDITIONAL SYMMETRIES

- We can use a simple symmetrization procedure.
- Let $\mathbb{D}$ be a grup of transformations constructed from **parity, time reversal, Hermitian conjugate and two particle exchange**.
- A symmetric operator is obtained using:

$$\check{X} \to \sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}.$$

- Applying any $\check{T} \in \mathbb{D}$ to $\sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}$ returns the same operator.

ADDING ADDITIONAL SYMMETRIES

- We can use a simple symmetrization procedure.
- Let $\mathbb{D}$ be a grup of transformations constructed from **parity, time reversal, Hermitian conjugate and two particle exchange**.
- A symmetric operator is obtained using:

$$\check{X} \to \sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}.$$

- Applying any $\check{T} \in \mathbb{D}$ to $\sum_{\check{T} \in \mathbb{D}} \check{T}\check{X}$ returns the same operator.

ADDING ADDITIONAL SYMMETRIES

- If this is done carefully [*Eur. Phys. J.* A 52:188 (2016)], a new general form for operators that have rotation invariance and are symmetric with respect to $\mathbb{D}$ can be constructed.
- Additionall symmetry conditions on the scalar functions appear.
- The problem becomes more complicated if there are three particles involved [*Phys. Rev. C* 87,054007 (2013)] . . .

ADDING ADDITIONAL SYMMETRIES

- If this is done carefully [*Eur. Phys. J.* A 52:188 (2016)], a new general form for operators that have rotation invariance and are symmetric with respect to $\mathbb{D}$ can be constructed.
- Additionall symmetry conditions on the scalar functions appear.
- The problem becomes more complicated if there are three particles involved [*Phys. Rev. C* 87,054007 (2013)] . . .

ADDING ADDITIONAL SYMMETRIES

- If this is done carefully [*Eur. Phys. J.* A 52:188 (2016)], a new general form for operators that have rotation invariance and are symmetric with respect to $\mathbb{D}$ can be constructed.
- Additionall symmetry conditions on the scalar functions appear.
- The problem becomes more complicated if there are three particles involved [*Phys. Rev. C* 87,054007 (2013)] . . .

ADDING ADDITIONAL SYMMETRIES

- If this is done carefully [*Eur. Phys. J.* A 52:188 (2016)], a new general form for operators that have rotation invariance and are symmetric with respect to $\mathbb{D}$ can be constructed.
- Additionall symmetry conditions on the scalar functions appear.
- The problem becomes more complicated if there are three particles involved [*Phys. Rev. C* 87,054007 (2013)] ...

NUCLEON DEUTERON SCATTERING

- N-d elastic scattering and breakup description via the 3N Faddeev equation

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{T}.$$

$$\check{P} = \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}.$$

- Use only first order terms:

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{G}_0\check{P}\check{t}\check{P} + \ldots \approx \check{t}\check{P}.$$

- Calculate observables in the breakup channel ($\langle \phi_o \mid$ - three free nucleons, $\mid \phi \rangle$ - deuteron and free nucleon):

$$\langle \phi_0 \mid \check{u}_0 \mid \phi \rangle = \langle \phi_0 \mid (1 + \check{P})\check{t}\check{P} \mid \phi \rangle.$$

- Calculate observables in the elastic channel:

$$\langle \phi' \mid \check{u} \mid \phi \rangle = \langle \phi' \mid \check{P}\check{G}_0^{-1} + \check{P}\check{t}\check{P} \mid \phi \rangle.$$

NUCLEON DEUTERON SCATTERING

- N-d elastic scattering and breakup description via the 3N Faddeev equation

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{T}.$$

$$\check{P} = \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}.$$

- Use only first order terms:

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{G}_0\check{P}\check{t}\check{P} + \ldots \approx \check{t}\check{P}.$$

- Calculate observables in the breakup channel ($\langle \phi_o \mid$ - three free nucleons, $\mid \phi \rangle$ - deuteron and free nucleon):

$$\langle \phi_0 \mid \check{u}_0 \mid \phi \rangle = \langle \phi_0 \mid (1 + \check{P})\check{t}\check{P} \mid \phi \rangle.$$

- Calculate observables in the elastic channel:

$$\langle \phi' \mid \check{u} \mid \phi \rangle = \langle \phi' \mid \check{P}\check{G}_0^{-1} + \check{P}\check{t}\check{P} \mid \phi \rangle.$$

NUCLEON DEUTERON SCATTERING

- N-d elastic scattering and breakup description via the 3N Faddeev equation

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{T}.$$
$$\check{P} = \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}.$$

- Use only first order terms:

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{G}_0\check{P}\check{t}\check{P} + \ldots \approx \check{t}\check{P}.$$

- Calculate observables in the breakup channel ($\langle \phi_o \mid$ - three free nucleons, $\mid \phi \rangle$ - deuteron and free nucleon):

$$\langle \phi_0 \mid \check{u}_0 \mid \phi \rangle = \langle \phi_0 \mid (1 + \check{P})\check{t}\check{P} \mid \phi \rangle.$$

- Calculate observables in the elastic channel:

$$\langle \phi' \mid \check{u} \mid \phi \rangle = \langle \phi' \mid \check{P}\check{G}_0^{-1} + \check{P}\check{t}\check{P} \mid \phi \rangle.$$

NUCLEON DEUTERON SCATTERING

- N-d elastic scattering and breakup description via the 3N Faddeev equation

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{T}.$$
$$\check{P} = \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}.$$

- Use only first order terms:

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{t}\check{G}_0\check{P}\check{t}\check{P} + \ldots \approx \check{t}\check{P}.$$

- Calculate observables in the breakup channel ($\langle\phi_o \mid$ - three free nucleons, $\mid \phi\rangle$ - deuteron and free nucleon):

$$\langle\phi_0 \mid \check{u}_0 \mid \phi\rangle = \langle\phi_0 \mid (1 + \check{P})\check{t}\check{P} \mid \phi\rangle.$$

- Calculate observables in the elastic channel:

$$\langle\phi' \mid \check{u} \mid \phi\rangle = \langle\phi' \mid \check{P}\check{G}_0^{-1} + \check{P}\check{t}\check{P} \mid \phi\rangle.$$

NUCLEON DEUTERON SCATTERING

- Compare 3D approach and "battle tested" PWD approach.

ELASTIC SCATTERING:

BREAKUP:

- Initially the neutron: $\boldsymbol{q}_i$. In the final state, the neutron: $\boldsymbol{q}_f$.
- Scattering parametrized by $\theta_{c.m.}$.

- **In the final state the Jacobi momenta: $\boldsymbol{p}^f$ and $\boldsymbol{q}^f$.**
- **Scattering parametrized by the kinematic curve parameter S: $\boldsymbol{p}^f(S)$, $\boldsymbol{q}^f(S)$.**

NUCLEON DEUTERON SCATTERING

- Compare 3D approach and “battle tested” PWD approach.

ELASTIC SCATTERING:

BREAKUP:

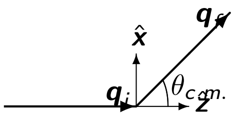
- Initially the neutron: $\boldsymbol{q}_i$. In the final state, the neutron: $\boldsymbol{q}_f$.
- Scattering parametrized by $\theta_{c.m.}$.

- In the final state the Jacobi momenta: $\boldsymbol{p}^f$ and $\boldsymbol{q}^f$.
- Scattering parametrized by the kinematic curve parameter S: $\boldsymbol{p}^f(S)$, $\boldsymbol{q}^f(S)$.

NUCLEON DEUTERON SCATTERING

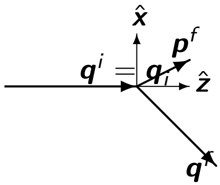
- Compare 3D approach and "battle tested" PWD approach.

ELASTIC SCATTERING:



- Initially the neutron: $\boldsymbol{q}_i$. In the final state, the neutron: $\boldsymbol{q}_f$.
- Scattering parametrized by $\theta_{c.m.}$.

BREAKUP:



- **In the final state the Jacobi momenta: $\boldsymbol{p}^f$ and $\boldsymbol{q}^f$.**
- **Scattering parametrized by the kinematic curve parameter S: $\boldsymbol{p}^f(S)$, $\boldsymbol{q}^f(S)$.**

NUCLEON DEUTERON SCATTERING

- Results for breakup [Eur. Phys. J. A 51:132 (2015)].
- Deuteron and nucleon vector analyzing powers (A_y^d, A_y^N) and the deuteron tensor analyzing powers (A_{xx}, A_{yy}, A_{zz}) LAB energy 190 MeV.
- Solid line - 3D results.
- The dashed-dotted, dotted and dashed lines - PWD results with max. total anguler momentum $21/2, 23/2, 25/2$ and max. $2-3$ angular momentum 8.

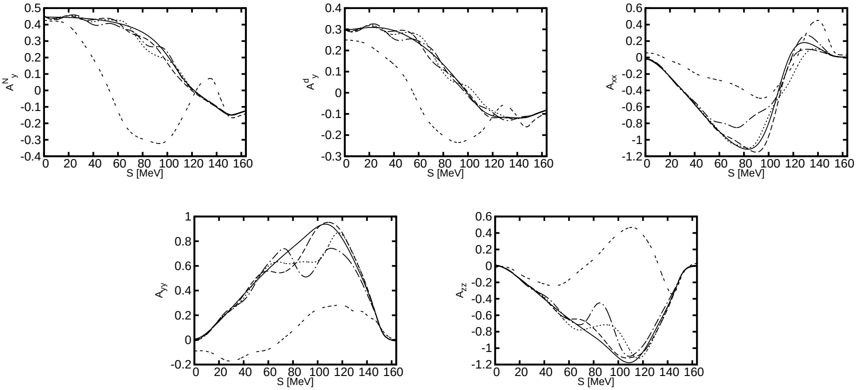
NUCLEON DEUTERON SCATTERING

- Results for breakup [Eur. Phys. J. A 51:132 (2015)].
- Deuteron and nucleon vector analyzing powers (A_y^d, A_y^N) and the deuteron tensor analyzing powers (A_{xx}, A_{yy}, A_{zz}) LAB energy 190 MeV.
- Solid line - 3D results.
- The dashed-dotted, dotted and dashed lines - PWD results with max. total anguler momentum $21/2, 23/2, 25/2$ and max. $2-3$ angular momentum 8.

NUCLEON DEUTERON SCATTERING

- Results for breakup [Eur. Phys. J. A 51:132 (2015)].
- Deuteron and nucleon vector analyzing powers (A_y^d, A_y^N) and the deuteron tensor analyzing powers (A_{xx}, A_{yy}, A_{zz}) LAB energy 190 MeV.
- Solid line - 3D results.
- The dashed-dotted, dotted and dashed lines - PWD results with max. total anguler momentum $21/2, 23/2, 25/2$ and max. $2-3$ angular momentum 8.

NUCLEON DEUTERON SCATTERING



FULL 3D CALCULATION

- Most general approach for $\check{T}$, no rotational symmetry.
- In general $(8 \times 8 = 64) \times 64$ functions of 4 vector arguments - the Jacobi momnta in the initial and final states.
- **Every argument as a** 32 **element list leads to** $64 \times 64 \times 32^{12} \approx 47.2 \times 10^{21}$ **complex numbers to represent** $\check{T}$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach for $\check{T}$, no rotational symmetry.
- In general $(8 \times 8 = 64) \times 64$ functions of 4 vector arguments - the Jacobi momnta in the initial and final states.
- **Every argument as a** 32 **element list leads to** $64 \times 64 \times 32^{12} \approx 47.2 \times 10^{21}$ **complex numbers to represent** $\check{T}$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach for $\check{T}$, no rotational symmetry.
- In general $(8 \times 8 = 64) \times 64$ functions of 4 vector arguments - the Jacobi momnta in the initial and final states.
- **Every argument as a** 32 **element list leads to** $64 \times 64 \times 32^{12} \approx 47.2 \times 10^{21}$ **complex numbers to represent** $\check{T}$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach for $\check{T}$, no rotational symmetry.
- In general $(8 \times 8 = 64) \times 64$ functions of 4 vector arguments - the Jacobi momnta in the initial and final states.
- **Every argument as a** 32 **element list leads to** $64 \times 64 \times 32^{12} \approx 47.2 \times 10^{21}$ **complex numbers to represent** $\check{T}$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach for $\check{T}$, no rotational symmetry.
- In general $(8 \times 8 = 64) \times 64$ functions of 4 vector arguments - the Jacobi momnta in the initial and final states.
- **Every argument as a** 32 **element list leads to** $64 \times 64 \times 32^{12} \approx 47.2 \times 10^{21}$ **complex numbers to represent** $\check{T}$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach to $\check{T} \mid \phi\rangle$.
- 8×8 functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 32^9 \approx 22.3 \times 10^{14}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach to $\check{T} \mid \phi\rangle$.
- 8×8 functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 32^9 \approx 22.3 \times 10^{14}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach to $\check{T} \mid \phi\rangle$.
- 8×8 functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 32^9 \approx 22.3 \times 10^{14}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach to $\check{T} \mid \phi\rangle$.
- 8×8 functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 32^9 \approx 22.3 \times 10^{14}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Not practical.

FULL 3D CALCULATION

- Most general approach to $\check{T} \mid \phi\rangle$.
- 8×8 functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 32^9 \approx 22.3 \times 10^{14}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Not practical.

FULL 3D CALCULATION

- Rotation invariant operator form for $\check{T} \mid \phi\rangle$.
- 64×8 **scalar** functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 8 \times 32^6 \approx 55.0 \times 10^{10}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Realistic.

FULL 3D CALCULATION

- Rotation invariant operator form for $\check{T} \mid \phi\rangle$.
- 64×8 **scalar** functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 8 \times 32^6 \approx 55.0 \times 10^{10}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$.
- Realistic.

FULL 3D CALCULATION

- Rotation invariant operator form for $\check{T} \mid \phi\rangle$.
- 64×8 **scalar** functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 8 \times 32^6 \approx 55.0 \times 10^{10}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$.
- Realistic.

FULL 3D CALCULATION

- Rotation invariant operator form for $\check{T} \mid \phi\rangle$.
- 64×8 **scalar** functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 8 \times 32^6 \approx 55.0 \times 10^{10}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Realistic.

FULL 3D CALCULATION

- Rotation invariant operator form for $\check{T} \mid \phi\rangle$.
- 64×8 **scalar** functions of 3 vector arguments - the Jacobi momnta in the final state and the free nucleon momentum.
- **Every argument as a** 32 **element list leads to** $64 \times 8 \times 32^6 \approx 55.0 \times 10^{10}$ **complex numbers to represent** $\check{T} \mid \phi\rangle$**.**
- Realistic.

ANATOMY OF THE SCATTERING STATE

- Observation, the scattering state in the Faddeev equations has the general form:

$$\langle \boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi \rangle = \left[\check{X}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \right] \mid s \rangle$$

- Rotation symmetry $\rightarrow \check{X}$ has rotational symmetry.
- We can use the algorithm from [*Eur. Phys. J.* A 52:188 (2016)].

ANATOMY OF THE SCATTERING STATE

- Observation, the scattering state in the Faddeev equations has the general form:

$$\langle \boldsymbol{pq} \mid \check{T} \mid \phi \rangle = \left[\check{X}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \right] \mid s \rangle$$

- Rotation symmetry $\rightarrow$ $\check{X}$ has rotational symmetry.
- We can use the algorithm from [Eur. Phys. J. A 52:188 (2016)].

ANATOMY OF THE SCATTERING STATE

- Observation, the scattering state in the Faddeev equations has the general form:

$$\langle \boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi \rangle = \left[\check{X}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \right] \mid s \rangle$$

- Rotation symmetry $\rightarrow \check{X}$ has rotational symmetry.
- We can use the algorithm from [*Eur. Phys. J.* A 52:188 (2016)].

ANATOMY OF THE SCATTERING STATE

- Observation, the scattering state in the Faddeev equations has the general form:

$$\langle \boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi\rangle = \left[\check{X}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0)\right] \mid s\rangle$$

- Rotation symmetry $\rightarrow$ $\check{X}$ has rotational symmetry.
- We can use the algorithm from [*Eur. Phys. J.* A 52:188 (2016)].

GENERAL FORM

- Finally:

$$\langle \boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi\rangle = \sum_{\gamma^{3N}} \sum_{r=1}^{64} \tau_r^{\gamma^{3N}}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) \mid \gamma^{3N}\rangle \otimes \left(\check{O}_r(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) \mid \alpha\rangle\right)$$

- $\check{T} \mid \phi\rangle = \check{t}\check{P} \mid \phi\rangle + \check{t}\check{G}_0\check{P}\check{T} \mid \phi\rangle \rightarrow \tau = \tilde{\tau} + \check{Q}\tau$ [*Phys. Rev. C 96 014611 (2017)*]
- We can use the same methods as in other "three dimensional" problems - Arnoldi algorithm.

GENERAL FORM

- Finally:

$$\langle \boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi\rangle = \sum_{\gamma^{3N}} \sum_{r=1}^{64} \tau_r^{\gamma^{3N}}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \mid \gamma^{3N}\rangle \otimes \left(\check{O}_r(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \mid \alpha\rangle\right)$$

- $\check{T} \mid \phi\rangle = \check{t}\check{P} \mid \phi\rangle + \check{t}\check{G}_0\check{P}\check{T} \mid \phi\rangle \rightarrow \tau = \tilde{\tau} + \check{Q}\tau$ [*Phys. Rev. C 96 014611 (2017)*]
- We can use the same methods as in other "three dimensional" problems - Arnoldi algorithm.

GENERAL FORM

- Finally:

$$\langle \boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi \rangle = \sum_{\gamma^{3N}} \sum_{r=1}^{64} \tau_r^{\gamma^{3N}}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \mid \gamma^{3N} \rangle \otimes \left(\check{O}_r(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \mid \alpha \rangle \right)$$

- $\check{T} \mid \phi \rangle = \check{t}\check{P} \mid \phi \rangle + \check{t}\check{G}_0 \check{P}\check{T} \mid \phi \rangle \to \tau = \tilde{\tau} + \check{Q}\tau$ [*Phys. Rev. C 96 014611 (2017)*]
- We can use the same methods as in other "three dimensional" problems - Arnoldi algorithm.

GENERAL FORM

- Finally:

$$\langle \boldsymbol{pq} \mid \check{T} \mid \phi \rangle = \sum_{\gamma^{3N}} \sum_{r=1}^{64} \tau_r^{\gamma^{3N}}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \mid \gamma^{3N} \rangle \otimes \left(\check{O}_r(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{q}_0) \mid \alpha \rangle \right)$$

- $\check{T} \mid \phi \rangle = \check{t}\check{P} \mid \phi \rangle + \check{t}\check{G}_0\check{P}\check{T} \mid \phi \rangle \rightarrow \tau = \tilde{\tau} + \check{Q}\tau$ [*Phys. Rev. C 96 014611 (2017)*]
- We can use the same methods as in other "three dimensional" problems - Arnoldi algorithm.

- Traditional and 3D calculations can benefit from a common template for the 3N force.
- General operator form available for local 3N potentials [*Eur. Phys. J. A 51 26 (2015)*].

- Traditional and 3D calculations can benefit from a common template for the 3N force.
- General operator form available for local 3N potentials [*Eur. Phys. J. A 51 26 (2015)*].

- Traditional and 3D calculations can benefit from a common template for the 3N force.
- General operator form available for local 3N potentials [*Eur. Phys. J. A 51 26 (2015)*].

- Spatial rotations.
- Isospin rotations.

- Spatial rotations.
- Isospin rotations.

- Spatial rotations.
- Isospin rotations.

- Time reflection.
- Parity inversion.
- Hermitian conjugation.
- Particle exchange ($\mathbb{S}_3$!).

- Time reflection.
- Parity inversion.
- Hermitian conjugation.
- Particle exchange ($\mathbb{S}_3$!).

- Time reflection.
- Parity inversion.
- Hermitian conjugation.
- Particle exchange ($\mathbb{S}_3$!).

- Time reflection.
- Parity inversion.
- Hermitian conjugation.
- Particle exchange ($\mathbb{S}_3$!).

- Time reflection.
- Parity inversion.
- Hermitian conjugation.
- Particle exchange ($\mathbb{S}_3$!).

- Discrete symmetry group is more complicated.
  - 2N potential - the particle exchange group is $\mathbb{Z}_2$.
  - General 3N potential - the particle exchange group is $\mathbb{S}_3$.
- Momentum dependence is more complicated.
  - Local 3N potential - two momentum transfer vectors.
  - General 3N potential - two Jacobi momenta in the initial state, two Jacobi momenta in the final state.

- Discrete symmetry group is more complicated.
  - 2N potential - the particle exchange group is $\mathbb{Z}_2$.
  - General 3N potential - the particle exchange group is $\mathbb{S}_3$.
- Momentum dependence is more complicated.
  - Local 3N potential - two momentum transfer vectors.
  - General 3N potential - two Jacobi momenta in the initial state, two Jacobi momenta in the final state.

- Discrete symmetry group is more complicated.
    - 2N potential - the particle exchange group is $\mathbb{Z}_2$.
    - General 3N potential - the particle exchange group is $\mathbb{S}_3$.
- Momentum dependence is more complicated.
    - Local 3N potential - two momentum transfer vectors.
    - General 3N potential - two Jacobi momenta in the initial state, two Jacobi momenta in the final state.

- Discrete symmetry group is more complicated.
  - 2N potential - the particle exchange group is $\mathbb{Z}_2$.
  - General 3N potential - the particle exchange group is $\mathbb{S}_3$.
- Momentum dependence is more complicated.
  - Local 3N potential - two momentum transfer vectors.
  - General 3N potential - two Jacobi momenta in the initial state, two Jacobi momenta in the final state.

- Discrete symmetry group is more complicated.
  - 2N potential - the particle exchange group is $\mathbb{Z}_2$.
  - General 3N potential - the particle exchange group is $\mathbb{S}_3$.
- Momentum dependence is more complicated.
  - Local 3N potential - two momentum transfer vectors.
  - General 3N potential - two Jacobi momenta in the initial state, two Jacobi momenta in the final state.

- Discrete symmetry group is more complicated.
  - Use approach from [*Eur. Phys. J. A 51 26 (2015)*].
  - Construct operators with definite transformation properties.
  - Require that the scalar functions have appropriate transformation properties and compensate.
- Momentum dependence is more complicated.
  - 320 operators in the isospin - spin space of the 3N system.
  - Adding total momentum dependence does not change the operator structure.
  - The set of operators in the final form is not unique.

- Discrete symmetry group is more complicated.
  - Use approach from [*Eur. Phys. J. A 51 26 (2015)*].
  - Construct operators with definite transformation properties.
  - Require that the scalar functions have appropriate transformation properties and compensate.
- Momentum dependence is more complicated.
  - 320 operators in the isospin - spin space of the 3N system.
  - Adding total momentum dependence does not change the operator structure.
  - The set of operators in the final form is not unique.

- Discrete symmetry group is more complicated.
  - Use approach from [*Eur. Phys. J. A 51 26 (2015)*].
  - Construct operators with definite transformation properties.
  - Require that the scalar functions have appropriate transformation properties and compensate.
- Momentum dependence is more complicated.
  - 320 operators in the isospin - spin space of the 3N system.
  - Adding total momentum dependence does not change the operator structure.
  - The set of operators in the final form is not unique.

- Discrete symmetry group is more complicated.
  - Use approach from [*Eur. Phys. J. A 51 26 (2015)*].
  - Construct operators with definite transformation properties.
  - Require that the scalar functions have appropriate transformation properties and compensate.
- Momentum dependence is more complicated.
  - 320 operators in the isospin - spin space of the 3N system.
  - Adding total momentum dependence does not change the operator structure.
  - The set of operators in the final form is not unique.

- Discrete symmetry group is more complicated.
  - Use approach from [*Eur. Phys. J. A 51 26 (2015)*].
  - Construct operators with definite transformation properties.
  - Require that the scalar functions have appropriate transformation properties and compensate.
- Momentum dependence is more complicated.
  - 320 operators in the isospin - spin space of the 3N system.
  - Adding total momentum dependence does not change the operator structure.
  - The set of operators in the final form is not unique.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p})\right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p})\right]$
- $\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q},\boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]$
- $\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q} \rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}) \right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K} \rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}) \right]$
- $\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q} \rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}) \right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K} \rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}) \right]$
- $\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q},\boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]$
- $\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K}\rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q},\boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]$
- $\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q} \rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}) \right]$
- $\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V}_{3N} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K} \rangle = \sum_{k=1}^{320} h_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{K}) \left[\check{S}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}) \right]$
- $\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}$ - Jacobi momenta in the final and initial state, $\boldsymbol{K}$ - total momentum of the system, h - scalar functions, $\check{S}$ - operators in the isospin - spin space of the 3N system.
- All the operators are given and have definite transformation properties under the discrete symmetries.
- All the scalar functions are required to have appropriate transformation properties in order to compensate - this can potentially lead to a reduction of memory requirements in the numerical implementation.
- Paper submitted to EPJ A with my choice of the final 320 operators.

- First order results for neutron deuteron scattering suggest that the 3D approach can be used to achieve convergence at higher energies.
- Constructing the general, rotation invariant form of $\check{T} \mid \phi\rangle$ can lead to efficient calculations.
- Including additional symmetries in the operator form of the scattering state is possible and might lead to a further reduction of numerical complexity.
- Including the three nucleon interaciton is also possible ... the general form of the three nucleon force needs to be developed.
- Possibility to add relativistic corrections to the calculations.

THANK YOU

The project was financed from the resources of the National Science Center (Poland) under grants No. 2016/21/D/ST2/01120, 2016/22/M/ST2/00173, 2013/11/N/ST2/03733 ,and 2013/10/M/ST2/00420. Some of the numerical work was performed at JSC Jülich.