THE DARK SIDE OF A BOSE GAS

In honor of M. Zralek

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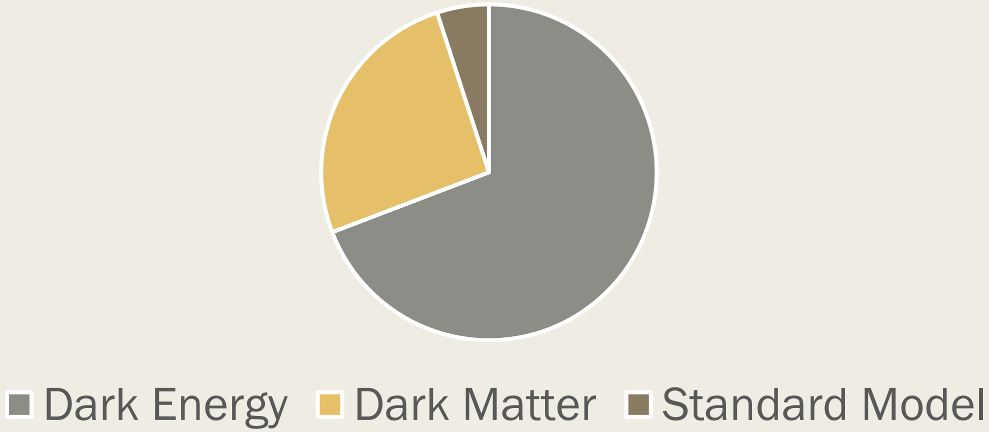
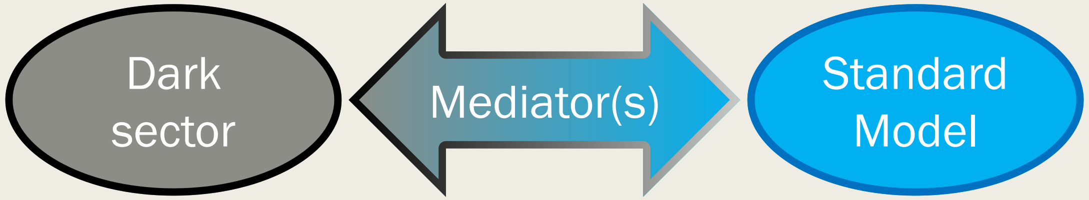
Introduction

- Latest cosmic census gives: ~23% or the universe contents made of Dark Matter

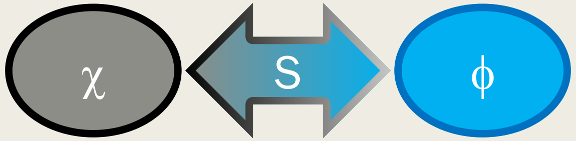


- Information obtained using gravitational effects → little information about $\mathcal{L}_{DM}$

- MANY models … simplest: one (or a small number) particle(s) mediate SM-dark interactions



Simplest of these: Higgs portal

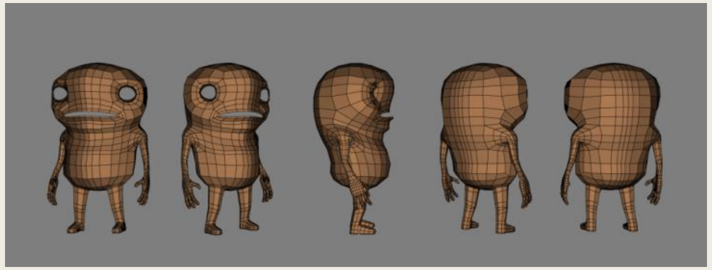


$$\Longrightarrow \quad |\chi|^2 |\phi|^2$$

where the mediator is a neutral scalar … that has been interrogated out.

- Most cases: χ = real scalar, for simplicity
- **Here:** χ = complex scalar ➔ global $U(1)_{dark}$ and a conserved charge.

The model



- Lagrangian:

$$\mathcal{L} = |\partial\chi|^2 - m_{\text{be}}|\chi|^2 - \lambda_{\text{be}}|\chi|^4 - \epsilon|\chi|^2|\phi|^2 + \mathcal{L}_{\text{SM}}$$

Large enough for DM equilibrium
Perturbativity: ≤ O(4 π)

Determines DM-SM equilibrium

- m_{be}: will consider a very wide range: from 10^{-20} eV to 1 TeV

  intended as toy model so I'll ignore naturality

- Two statistical systems: DM_χ and SM

  Equilibrium

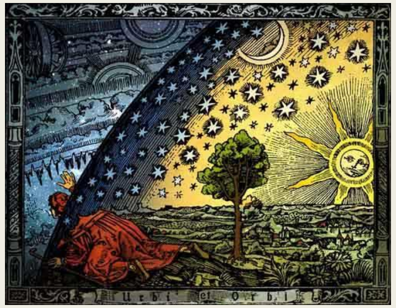
  – *DM_χ – SM: depending on $\boldsymbol{\varepsilon}$ and H*

  – *DM_χ throughout (T_χ always well defined)*

- DM_χ has a conserved charge ➔ $\boldsymbol{\mu} \neq 0$: asymmetric DM

- The universe: homogeneous, isotropic and flat.

Cosmology



- Energy, entropy and charge for the Bose gas

  Occupation numbers

particles $$n^{+}_{\text{be}} = \left(e^{(E-\mu)/T} - 1\right)^{-1} = \left(e^{x(\sqrt{u^2+1} - \varpi)} - 1\right)^{-1} ; \quad x = \frac{m_{\text{be}}}{T} , \ \varpi = \frac{\mu}{m_{\text{be}}}$$

antiparticles $$n^{-}_{\text{be}} = \left(e^{(E+\mu)/T} - 1\right)^{-1} = \left(e^{x(\sqrt{u^2+1} + \varpi)} - 1\right)^{-1}$$

- Chemical potential

$$|\mu| \le m_{\text{be}} ; \qquad |\mu| = m_{\text{be}} \implies$$ Bose-Einstein condensate (**BEC**)

- Thermodynamic quantities

(densities)

Conserved charge: $$q_{\rm be} = q_{\rm be}^{(c)} + {m_{\rm be}}^3 \nu_{\rm be}$$

condensate

excited states

Entropy: $$s_{\rm be} = {m_{\rm be}}^3 \sigma_{\rm be}$$

Energy: $$\rho_{\rm be} = m_{\rm be} q_{\rm be}^{(c)} + {m_{\rm be}}^4 r_{\rm be}$$

$$\nu_{\rm be} = \frac{1}{2\pi^2}\int_0^\infty du\, u^2 (n_{\rm be}^+ - n_{\rm be}^-)$$

$$\sigma_{\rm be} = \frac{1}{2\pi^2}\int_0^\infty du\, u^2 \sum_{n=n_{\rm be}^+, n_{\rm be}^-} [(1+n)\ln(1+n) - n\ln n]$$

$$r_{\rm be} = \frac{1}{2\pi^2}\int_0^\infty du\, u^2 \sqrt{u^2+1}(n_{\rm be}^+ + n_{\rm be}^-)$$

- Conserved quantities

  - *BE-SM in equilibrium:*

$$\frac{q_{\mathtt{be}}}{s_{\mathtt{tot}}} = \textcolor{red}{\mathrm{CONST}} \qquad \left(s_{\mathtt{tot}} = s_{\mathtt{be}} + s_{\mathtt{sm}}\right)$$

  - *BE, SM not in equilibrium:*

$$\frac{q_{\mathtt{be}}}{s_{\mathtt{be}}} = \textcolor{red}{\mathrm{CONST}} \qquad \frac{q_{\mathtt{be}}}{s_{\mathtt{sm}}} = \textcolor{red}{\mathrm{CONST}}$$

These imply q_{be}/s_{tot}*=const*

The Bose-Einstein condensate

- A BEcis present if

Charge in the excited statesc

$$q_{\mathtt{be}}^{(c)} \neq 0 \quad \text{if} \quad \frac{q_{\mathtt{be}}}{s_{\mathtt{tot}}} > \left.\frac{{m_{\mathtt{be}}}^3 \nu_{\mathtt{be}}}{s_{\mathtt{tot}}}\right|_{\varpi=1} = Y^{(e)}(\varpi = 1)$$

$\mu = m_{\mathtt{be}}$

- Always true at sufficiently early times

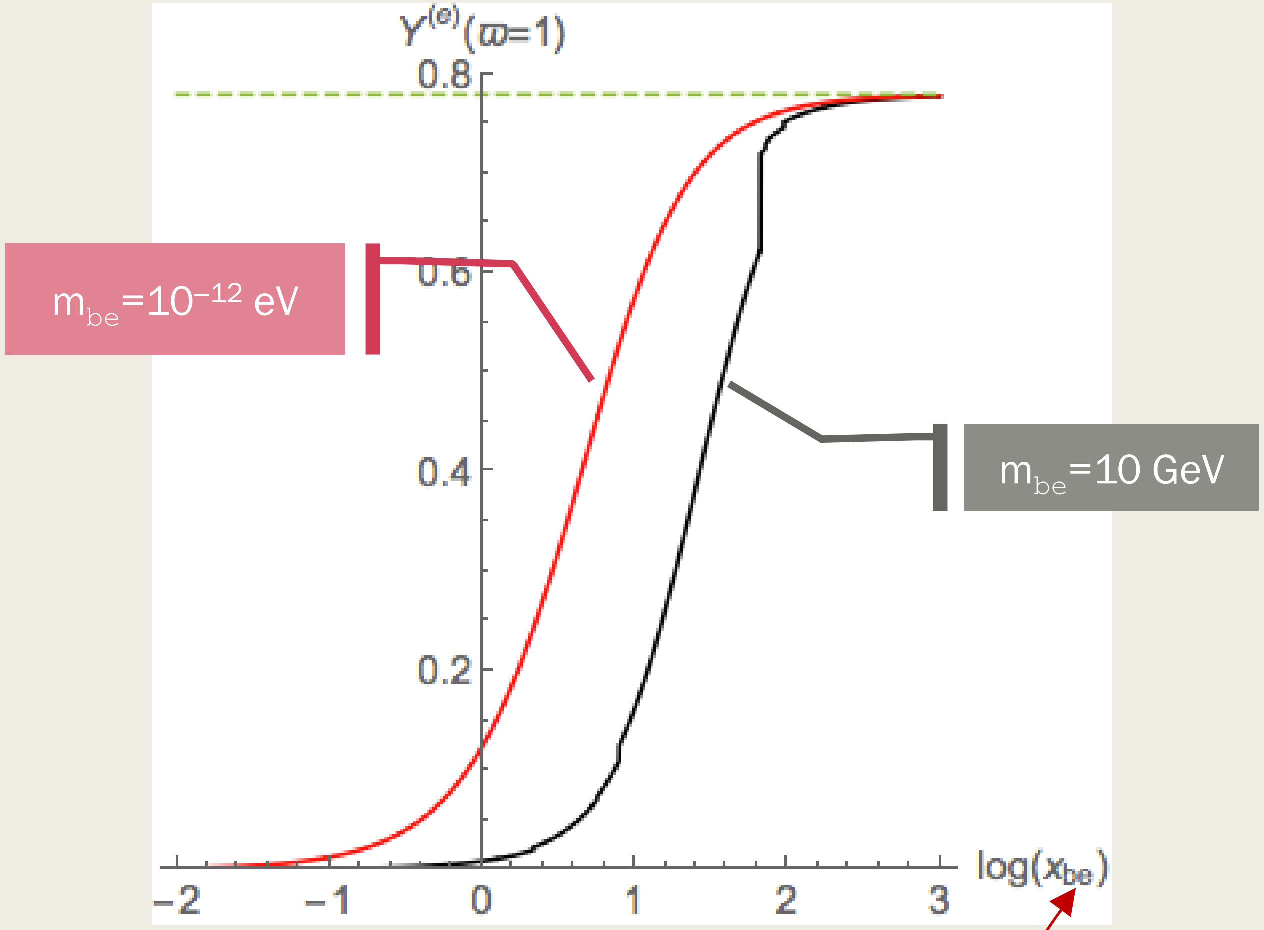
$$T \to \infty : \quad \begin{cases} {m_{\mathtt{be}}}^3 \nu_{\mathtt{be}} \sim T^2 \\ s_{\mathtt{tot}} \sim T^3 \end{cases} \quad \Rightarrow \quad \lim_{T\to\infty} Y^{(e)} = 0$$

WIMP-like masses:

$$T_{\mathrm{BEC}} \simeq m_{\mathrm{be}}{}^2 \frac{1.9\,\mathrm{eV}^{-1}}{g_{\star s}(T_{\mathrm{BEC}})+2}$$

Such a BEc carries little energy:

$$\left.\frac{m_{\mathrm{be}}q_{\mathrm{be}}}{\rho_{\mathrm{be}}}\right|_{T>T_{\mathrm{BEC}}} = \frac{O(100\,\mathrm{eV})}{m_{\mathrm{be}}}$$



$$x = \frac{m_{\mathrm{be}}}{T}$$

- BEc at decoupling (non-relativistic gas)

$$\underbrace{\frac{q_{\mathsf{be}}}{s_{\mathrm{sm}}} \simeq \frac{1}{m_{\mathsf{be}}}\frac{\rho_{\mathrm{DM}}}{s_{\mathrm{sm}}} = \frac{0.4\,\mathrm{eV}}{m_{\mathsf{be}}}}_{\text{Relic abundance}}\,, \qquad \underbrace{\frac{q_{\mathsf{be}}(T_d)}{(m_{\mathsf{be}}T_d)^{3/2}} > \frac{\zeta_{3/2}}{(2\pi)^{3/2}}}_{\text{Presence of a BEc}} \quad \Rightarrow \quad m_{\mathsf{be}} < 1.3\mathrm{keV}$$

- BEc now: similar arguments: $m_{\mathsf{be}} < 88\,\mathrm{eV}$

- For such light masses there are additional constraints (discussed later)

Relic abundance

- Assume: non-relativistic gas @ decoupling ($T = T_d$). Relic abundance requires:

$$\frac{q_{\mathsf{be}}}{s_{\rm sm}} \simeq \frac{1}{m_{\mathsf{be}}} \frac{\rho_{\rm DM}}{s_{\rm sm}} = \frac{0.4\,{\rm eV}}{m_{\mathsf{be}}}$$

- Use μ to fix the relic abundance

$$\frac{0.4\,{\rm eV}}{m_{\mathsf{be}}} s_{\rm sm}(T_d) \simeq 2(m_{\mathsf{be}} T_d/2\pi)^{3/2} \cosh(\mu/T_d)\, e^{-m_{\mathsf{be}}/T_d}$$

- Decoupling:
  - *Evolution equation for energy transfer* $\dot{\vartheta} + 4\mathbb{H}\vartheta = -\Gamma\vartheta\,, \quad \vartheta = T_{\tt be} - T_{\tt sm}$
  - *Decoupling condition*

Hubble parameterc

Heat capacities

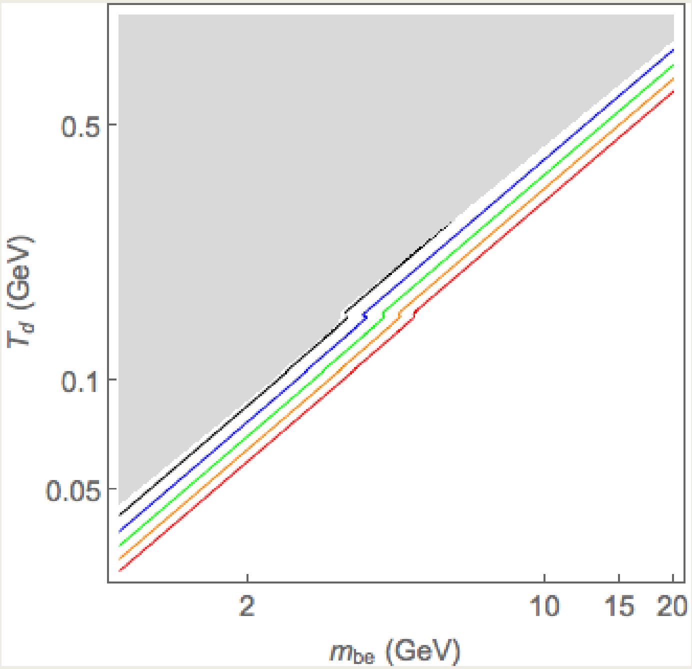
Higgs portal coupling

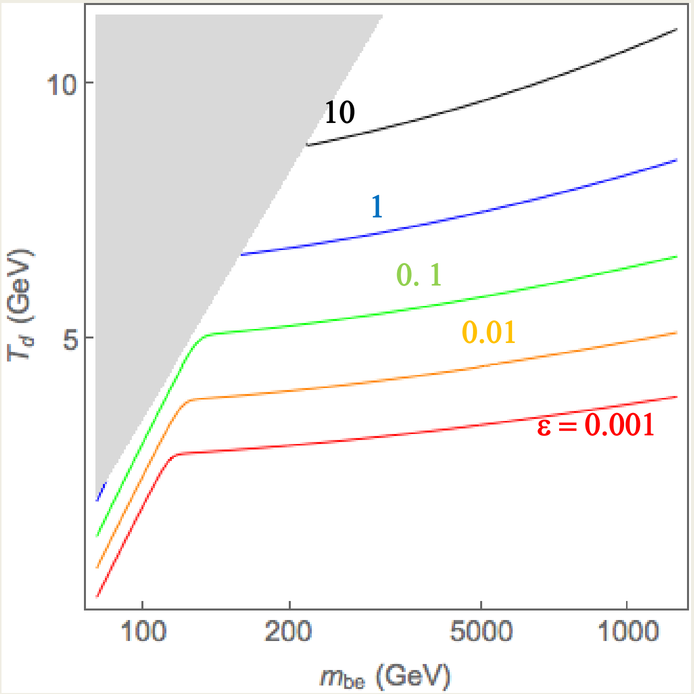
$$T = T_d: \quad \mathbb{H} = \Gamma = \underbrace{\left(\frac{1}{c_{\tt be}} + \frac{1}{c_{\tt sm}}\right)} \frac{\epsilon^2\, \mathbf{G}}{T}$$

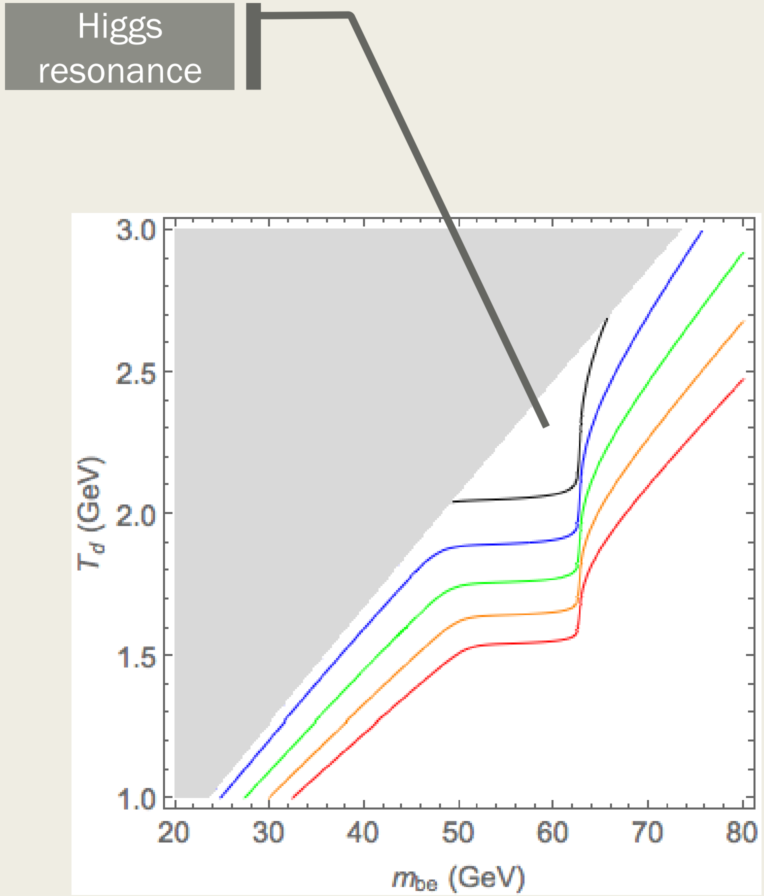
$$\mathbf{G} = \int_0^\beta ds \int_0^\infty dt \int d^3\mathbf{x} \left\langle \mathcal{O}_{\tt be}(-is,\mathbf{x})\dot{\mathcal{O}}_{\tt be}(t,\mathbf{0})\right\rangle \left\langle \mathcal{O}_{\tt sm}(-is,\mathbf{x})\dot{\mathcal{O}}_{\tt sm}(t,\mathbf{0})\right\rangle$$

$$\mathcal{O}_{\tt sm} = |\phi|^2\,,\ \mathcal{O}_{\tt be} = |\chi|^2$$

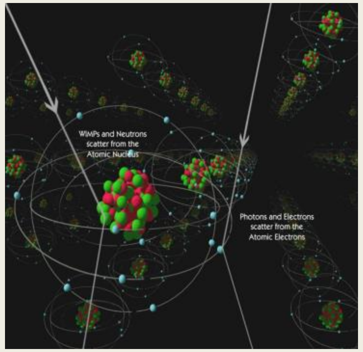
- Results for T_d:







Direct detection



- The cross-section:

Exclude forward region

$$\sigma = \frac{1}{2q_{\mathtt{be}}|\mathbf{p}|}\left(\frac{1}{\mathcal{V}}\int' \frac{d^3\mathbf{q}}{2E_{\mathbf{q}}\,(2\pi)^3}\,\langle W_{i\to f}\rangle_\beta\right)\,,$$

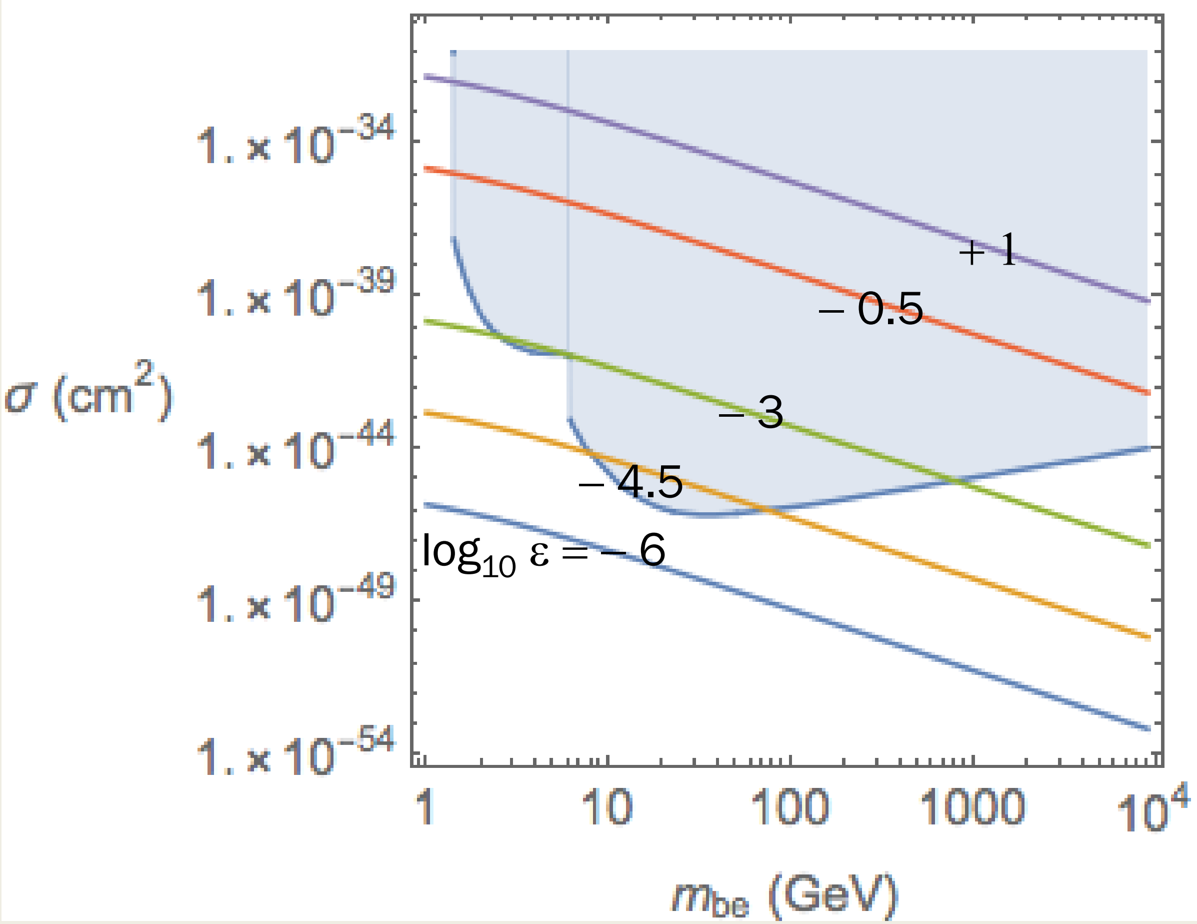
Space-time volume

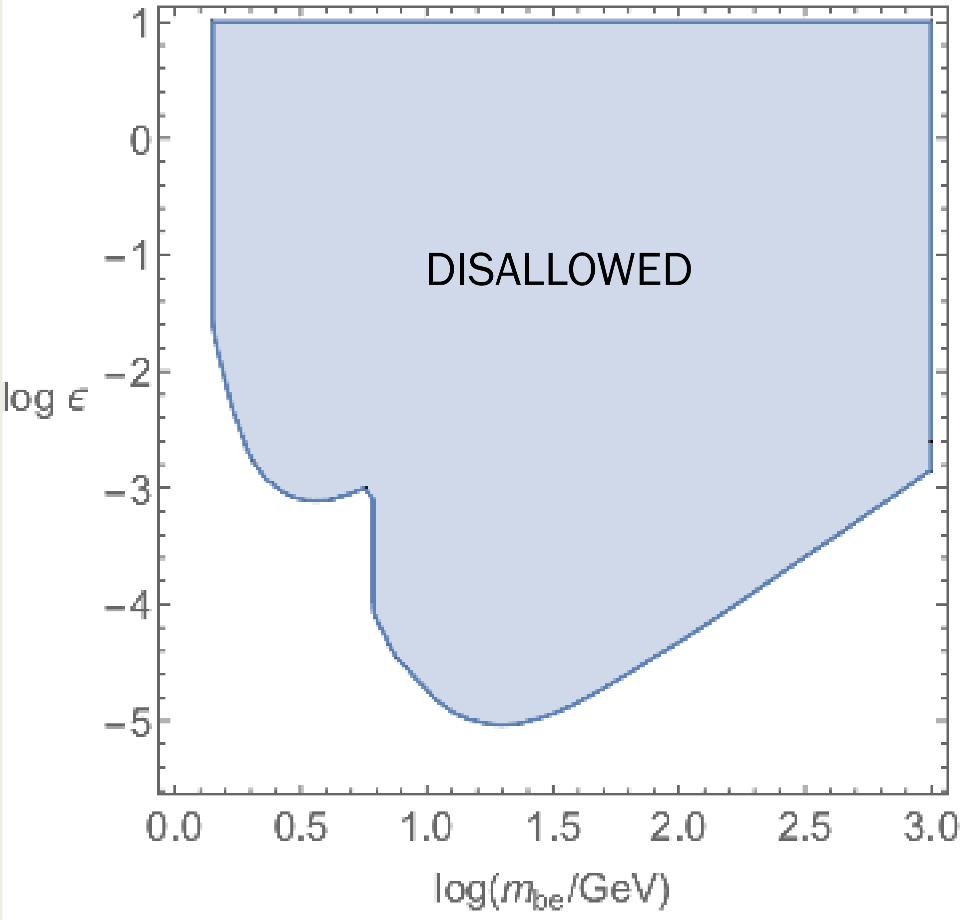
States of the Bose gas

$$W_{i\to f} = \left|{}_{\mathtt{out}}\langle f\,|i\rangle_{\mathtt{in}}\right|^2\,, \qquad |i\rangle_{\mathtt{in}} = a_{\mathbf{p}}^{\mathtt{in}\,\dagger}\,|0;X\rangle \quad |f\rangle_{\mathtt{out}} = a_{\mathbf{q}}^{\mathtt{out}\,\dagger}\,|0;Y\rangle$$

SM

- Constraints from XENON & CDMSLite

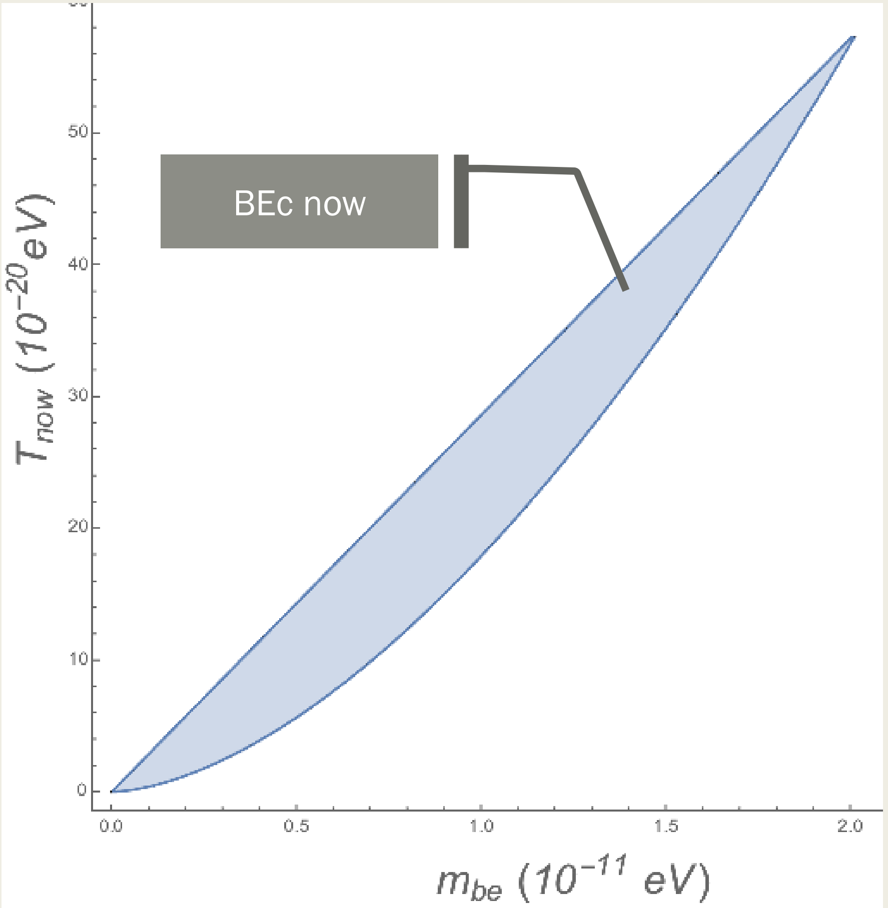




BEC now: tiny masses

- LSS: Bose gas non-relativistic at z_{LSS}~3400

$$a^3 s_{be} = \text{const} \;\Rightarrow\; x_{now} > 3.5 \times 10^7$$



- BBN:

Effective number of ν species

$$\rho_{\texttt{be}}\big|_{\texttt{BBN}} = \frac{3}{\pi^2}\frac{7}{4}\left(\frac{4}{11}\right)^{4/3}\Delta N_\nu\, T_\gamma^4 \qquad T_\gamma \simeq 0.06\,\mathrm{MeV}$$

$$\frac{s_{\rm be}}{s_{\rm sm}} = \mathrm{const} \quad \Rightarrow \quad \mathtt{x}_{\texttt{BBN}} \simeq \sqrt{\frac{\mathtt{x}_{\texttt{now}}}{2.3\times 10^{16}}}\,,$$

$$\Delta N_\nu \lesssim 7.2\times 10^{-5}\,.$$

- No equilibrium with the SM: would require a very large portal coupling ε

- For such small masses the gas is axion-like (except that the BEc does not require it to be non-relativistic).

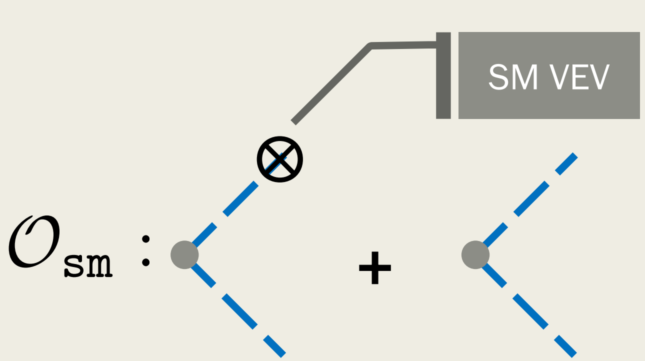
Comments

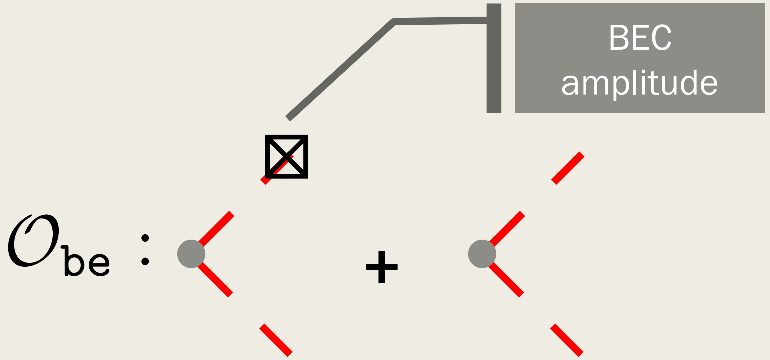
- Indirect detection: like usual Higgs portal
- WIMP-like restrictions: from direct detection (μ “fixes” the relic abundance)
- BEc:
  - *WIMP-like: only very early (most energy in the excited states; most charge in the BEc)*
  - *Light masses: can have BEc now, very hard to detect*
- Small-mass BEc can have significant effect in galactic dynamics

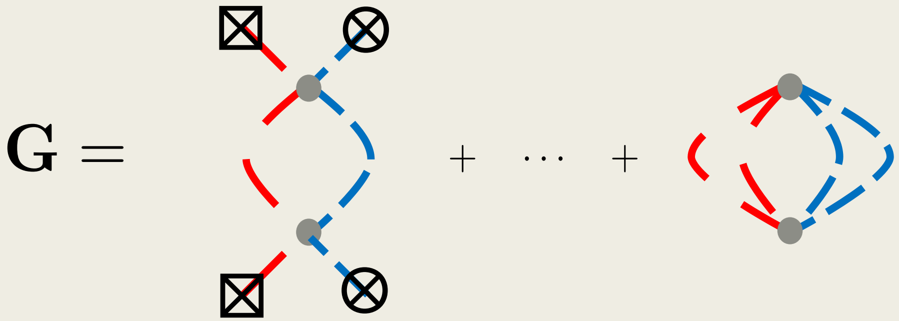

THANK YOU

Extra slides

- Calculate G

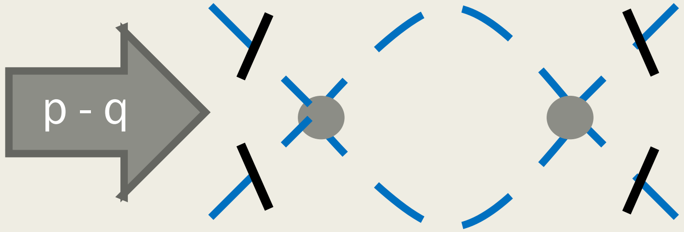

$\mathcal{O}_{\mathtt{sm}}$:
+
SM VEV


$\mathcal{O}_{\mathtt{be}}$:
+
BEC amplitude


$\mathbf{G} =$
$+ \cdots +$

- **Calculate σ:**
  - *Relevant only for WIMP like masses ⇒ no BEc*
  - *Thermal average over initial gas states* ***X,*** *sum over final gas states* ***Y***

$$\langle W_{i\to f}\rangle_\beta = \int d^4x\, d^4x'\, d^4y\, d^4y' e^{i(p\cdot y - q\cdot y' - p\cdot x + q\cdot x')}(\partial_x^2 + m^2)(\partial_{x'}^2 + m^2)$$
$$\times\, (\partial_y^2 + m^2)(\partial_{y'}^2 + m^2)\,\langle \mathbf{T}\left[\eta(x^0 - i\beta, \mathbf{x})\eta(x'^0 - i\beta, \mathbf{x}')\eta(y^0, \mathbf{y})\eta(y'^0, \mathbf{y}')\right]\rangle_\beta\ ,$$



$$\sigma = \left[\frac{1}{\sqrt{\pi}\,u} e^{-u^2} + \left(1 + \frac{1}{2u^2}\right)\mathrm{Erf}(u)\right]\sigma_0$$

$$u = \frac{|\mathbf{p}|}{m_{\mathtt{H}}}\sqrt{\frac{m_{\mathtt{be}}}{2T}} \quad (\text{usually } \ll 1)$$

$$\sigma_0 = \frac{1}{8\pi m_{\mathtt{be}}^2}\left[\frac{m_{\mathtt{be}} m_N}{m_{\mathtt{be}} + m_N}\,\frac{\epsilon\, g_{\mathtt{N-H}} v}{m_{\mathtt{H}}^2}\right]^2$$