Evolution of dark matter density considering kinetic decoupling

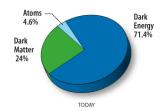
Mateusz Duch University of Warsaw



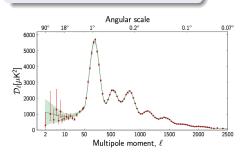
Matter To The Deepest, Podlesice 8 September 2017

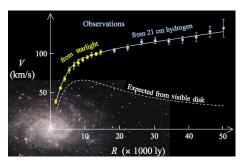
MD, Bohdan Grządkowski, 1705.10777, to appear in JHEP

Motivation – Dark Matter



Convicing evidence on various astrophysical and cosmological scales







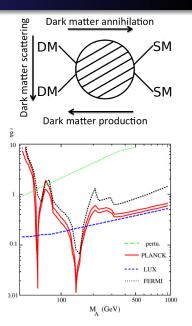
Motivation – Dark Matter

Properties of dark matter:

- electrically neutral (non luminous)
- non-relativistic (cold) (structure formation)
- stable or long-lived
- weakly interacting with ordinary matter

Dark matter interactions

- annihilation production in the early universe and indirect detection (FERMI, MAGIC, H.E.S.S, ...)
- scattering on nucleons direct detection (LUX, XENON, PANDA, ...) strong constraints
- production collider searches



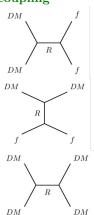
 $Gross,\ Lebedev,\ Mambrini,\ 2015$

Breit-Wigner resonance $2M_{DM} \approx M_R$ enhanced annihilation \Rightarrow suppressed coupling

• insensitive to direct detection, but constrained by indirect searches

enhancement of annihilation rate at low velocity

- kinetic decoupling $(T_{DM} \neq T_{SM})$?
- enhancement of the self-interaction cross-section ?



Standard freeze-out mechanism

Boltzmann equation for DM $\int \mathbf{L}[f_{DM}]d^3p = \int \mathbf{C}[f_{DM}]d^3p$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v_{\rm rel} \rangle (n^2 - n_{EQ}^2)$$

DM yield
$$Y = n/s$$
, $\frac{dY}{dx} = -\alpha \frac{\langle \sigma v_{\text{rel}} \rangle}{x^2} (Y^2 - Y_{\text{EQ}}^2)$, $\alpha = \frac{s(m)}{H(m)}$

- x = m/T dimensionless parameter
- s entropy density \leftarrow conserved in the comoving volume

Chemical decoupling $x = x_d$

$$\Gamma = n_{EQ} \langle \sigma v_{\rm rel} \rangle \lesssim H(x)$$

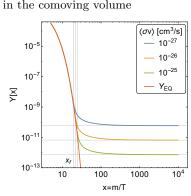
Approximate solutions

$$\langle \sigma v_{\rm rel} \rangle (T) = const$$

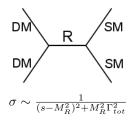
$$Y_{\infty} \approx \frac{x_d}{\alpha \langle \sigma v_{\rm rel} \rangle_0}$$

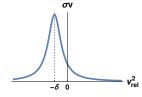
Standard assumption

 ${\rm DM}$ is kinetically coupled to ${\rm SM}$ (has the same temperature) during freeze-out



Breit-Wigner approximation





Annihilation cross-section - s-wave

$$\sigma v_{\rm rel} = \sum_{f \neq i} \frac{64\pi\omega}{M^2} \frac{\eta_i \eta_f \beta_f}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2}$$

Dimensionless parameters:

$$\begin{split} \eta_{i/f} &= \frac{\Gamma B_{i/f}}{M_R \bar{\beta}_{i/f}}, ~~ \delta = \frac{4 M_{DM}^2}{M_R^2} - 1, ~~ \gamma = \frac{\Gamma_R}{M_R} \\ &\text{couplings} &\text{position} &\text{width} \end{split}$$

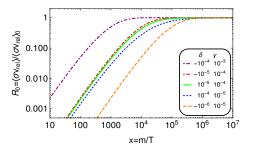
statistical spin-dependent factor

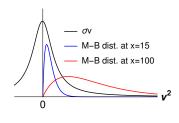
$$\omega = \frac{2s_R + 1}{(s_{DM} + 1)^2}$$

Thermally averaged annihilation cross-section

Averaged cross section
$$\langle \sigma v_{\rm rel} \rangle$$
 normalized to $\langle \sigma v_{\rm rel} \rangle_{T=0}$

$$\langle \sigma v_{\rm rel} \rangle \propto \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv \frac{v^2 e^{-xv^2/4}}{(\delta + v^2/4)^2 + \gamma^2}$$



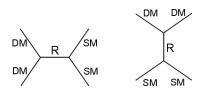


 $\langle \sigma v_{\rm rel} \rangle$ grows for smaller temperatures

Kinetic decoupling – simple picture

Condition $T_{DM} = T_{SM}$ is not always fulfilled.

Thermal equilibrium is maintained by the scattering of DM on the abundant light SM states.



- proper relic abundance requires small coupling of DM to SM
- scattering process is not resonantly enhanced

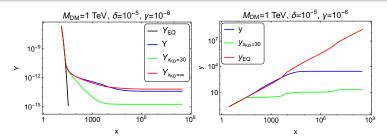
Comparision of the Hubble rate to the scattering rate

Bi at al, 2011

$$H(T_{kd}) \sim \Gamma_{\mathrm{scat}}(T_{kd}) \Rightarrow x_{kd} \lesssim \left(\frac{\max[\delta, \gamma]^{3/2}}{10^{-6}}\right)^{\frac{1}{4}} \Longrightarrow \mathbf{T_{kd}} \sim \mathbf{T_d}.$$

Kinetic and chemical decoupling temperatures are comparable

Kinetic decoupling – detailed description



Temperature parameter

$$T_{DM} \propto \int p^2 f(p) d^3 p$$
 $y \equiv \frac{M_{DM} T_{DM}}{s^{2/3}}$

before decoupling: $y \propto T_{SM}^{-1} \propto x$ after decoupling: $y \approx const$

 $Bringmann,\ 0903.0189$

 $s \sim T_{SM}^3$

Second moment of Boltzmann equation

$$\int p^2 \mathbf{L}[f_{DM}] d^3 p = \int p^2 \mathbf{C}[f_{DM}] d^3 p$$

Kinetic decoupling – detailed description

Coupled Boltzmann equation

$$\begin{split} \frac{dY}{dx} &= -\frac{1 - \frac{x}{3} \frac{g'_{*s}}{g_{*s}}}{Hx} s \left(Y^2 \langle \sigma v_{\rm rel} \rangle_{x_{DM}} - Y_{EQ}^2 \langle \sigma v_{\rm rel} \rangle_x \right) \\ \frac{dy}{dx} &= -\frac{1 - \frac{x}{3} \frac{g'_{*s}}{g_{*s}}}{Hx} \left[2M_{DM} c(T) (y - y_{EQ}) + \right. \\ &\left. - sy \left(Y \left(\langle \sigma v_{\rm rel} \rangle_{x_{DM}} - \langle \sigma v_{\rm rel} \rangle_2 |_{x_{DM}} \right) - \frac{\mathbf{Y}_{EQ}^2}{\mathbf{Y}} \left(\langle \sigma \mathbf{v}_{\rm rel} \rangle_{\mathbf{x}} - \frac{\mathbf{y}_{EQ}}{\mathbf{y}} \langle \sigma \mathbf{v}_{\rm rel} \rangle_2 |_{\mathbf{x}} \right) \right) \right] \end{split}$$

Aarssen, Bringmann, Goedecke 2012

Scattering and annihilation have both influence on temperature

scattering rate c(T)

$$c(T) = \frac{1}{12(2\pi)^3 M_{DM}^4 T} \sum_f \int dk k^5 \omega^{-1} g |\mathcal{M}_f|_{t=0; s=M_{DM}^2 + 2M_{DM}\omega + M_f^2}^2$$

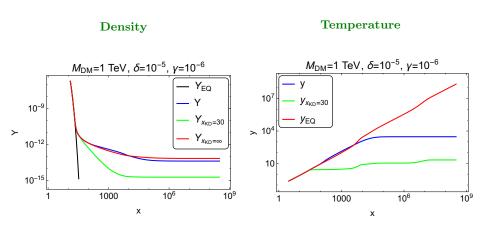
the averaged cross section $\langle \sigma v_{\rm rel} \rangle_2$

$$\langle \sigma v_{\rm rel} \rangle_2 = \int_0^\infty dv_{\rm rel} \frac{x^{3/2}}{4\sqrt{\pi}} \sigma v_{\rm rel} \left(1 + \frac{1}{6} v_{\rm rel}^2 x \right) v_{\rm rel}^2 \exp^{-v_{\rm rel}^2 x/4}.$$

Boltzmann distribution maintained by DM self-interactions

departure from Boltz. dist.- Binder, Bringmann, Gustafsson, Hryczuk 1706.07433

Kinetic decoupling – solutions of Boltzmann equation



 $\langle \sigma v_{\rm rel} \rangle$ grows for smaller velocities \Rightarrow annihilation heats up DM

Illustration - the Higgs portal with vector dark matter

Additional complex scalar field S

• singlet of $U(1)_Y \times SU(2)_L \times SU(3)_c$, charged under $U(1)_X$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + (D_{\mu} S)^* D^{\mu} S + \tilde{V}(H, S)$$

$$V(H,S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |\mathbf{S}|^2 |\mathbf{H}|^2$$

Vacuum expectation values: $\langle H \rangle = \frac{v_{SM}}{\sqrt{2}}, \quad \langle S \rangle = \frac{v_x}{\sqrt{2}}$

$U(1)_X$ vector gauge boson V_μ

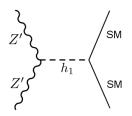
- Stability condition no mixing of $U(1)_X$ with $U(1)_Y$ $\mathcal{Z}_2: V_{\mu} \to -V_{\mu}, \qquad S \to S^*, \qquad S = \phi e^{i\sigma}: \phi \to \phi, \ \sigma \to -\sigma$
- Higgs mechanism in the hidden sector $M_{Z'} = g_x v_x$

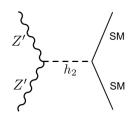
Higgs couplings – mixing angle α , $M_{h_1} = 125 \text{ GeV}$

$$\mathcal{L} \supset rac{h_1 \cos lpha + h_2 \sin lpha}{v} \left(2M_W W_\mu^+ W^{\mu-} + M_Z^2 Z_\mu Z^\mu - \sum_f m_f \bar{f}f
ight)$$

Resonance with a Higgs scalar

$\langle \sigma v_{\rm rel} \rangle \propto \sin^2 \alpha \cos^2 \alpha$





Small α required by relic abundance

Resonance with the SM-like Higgs

- $M_{Z'} \approx 125/2 \text{ GeV}$
- decay channel $h_1 \to Z'Z'$, if open suppressed by $\sin^2 \alpha$ and by phase space

$$\sqrt{1 - 4M_{Z'}^2/M_{h_1}^2} = \sqrt{\delta} \ll 1 \qquad \Gamma_{h_1 \to Z'Z'} \ll \Gamma_{SM}$$

• Breit-Wigner approximation is sufficient

Resonance with the second Higgs

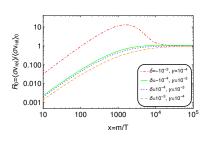
- $M_{Z'} pprox M_{h_2}/2$ GeV $h_2 o SMSM$ suppressed by $\sin^2 \alpha$, $h_2 o Z'Z'$ dominates
 - near threshold effects

Velocity dependent width

Beyond Breit-Wigner approximation - resummed propagator

Im
$$\Pi_h(s)/\sqrt{s} \approx \Gamma_{non-DM}(m_h^2) + \Gamma_{DM}(s)$$

$$\Gamma_{DM}(s) \approx \eta m_h \sqrt{1 - 4M_{Z'}^2/s} \approx \eta m_h v/2$$



$$\sigma v(s) \sim \frac{1}{(\delta + v^2)^2 + (\gamma_{non-DM} + \eta v/2)^2}$$

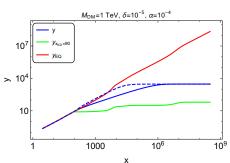
- for larger T scales like $1/v^2$
- saturates $\delta \sim \eta v/2$

$$\sigma v(s) \sim \frac{1}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2}$$

- for larger T scales like $1/(\delta v^2)$
- saturates $\max[\delta, \gamma] \sim v^2/4$

Solutions of Boltzmann equation - v-dependent width

Density



Temperature

dashed lines - Breit-Wigner approximation

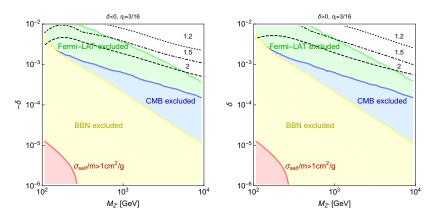
- earlier chemical decoupling
- annihilation lasts longer

• annihilation less effective in changing DM temperature

 $\langle \sigma v_{\rm rel} \rangle$ grows for smaller velocities \Rightarrow annihilation heats up DM

Bounds on the parameter space

Mixing angle α set by relic density



Effects of kinetic decoupling may change the relic density by more than order of magnitude

Summary

- When thermally averaged cross-section is temperature dependent and scattering on the SM states is suppressed kinetic decoupling effects are important
- To include the effects of **early kinetic decoupling** one has to solve the set of coupled Boltzmann equations
- If coupling between resonant mediator and DM is not suppressed then Breit-Wigner approximation is modified by **near threshold effects**.
- Large self-interactions are strongly constrained by the DM indirect searches.