On the renormalization of neutrinos in the seesaw extended 2HDM

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Neutrinos in the Grimus Neufeld model

- We have a general 2HDM and we extend the neutrino sector by one additional neutral Weyl spinor ν'_{04} that has a Majorana mass term:

  $$-\frac{1}{2}M\left(\nu'_{04}\nu'_{04}+h.c.\right),$$

  hence we have four neutrinos ν'_{0i}.
- After EWSB we get mass terms that are not diagonal. Then we go to mass eigenstates by the seesaw mechanism $\nu'_{0i} \to \nu_{0i}$.
- The seesaw mechanism generates Majorana mass terms for ν_{03} and ν_{04}, but doesn't distinguish between ν_{01} and ν_{02}.
- These can be distinguished by the mass term at a loop level that appears due to an interaction with the second Higgs doublet.
- After a loop correction we have 1 massless and three massive Majorana neutrinos, with one mass being large.

The choice of renormalization

- The on-shell (OS) renormalization works well for stable particles, however, it is known to give gauge dependent definitions of mass for unstable particles.
- Taking the complex pole as a renormalization point instead of the real part of it solves this problem[GG].
- Hence we try to employ the complex mass scheme(CMS) for renormalizing four Majorana neutrinos at loop level.

[GG] Gambino, Paolo and Grassi, Pietro Antonio, "The Nielsen identities of the SM and the definition of mass", Phys. Rev. D62:076002 (2000)

Some definitions

- The renormalized Green's functions:

$$\langle\phi_1 ... \phi_n\rangle_{1PI}^{[loop]} = \frac{\delta^n \hat{\Gamma}^{[loop]}}{\delta\phi_1 .. \delta\phi_n} \equiv \hat{\Gamma}^{[loop]}_{\phi_1 ... \phi_n}\,, \quad \hat{\Gamma}^{[loop]}_{\phi_1 ... \phi_n} \equiv \Gamma^{[loop]}_{\phi_1 ... \phi_n} + \delta\Gamma^{[loop]}_{\phi_1 ... \phi_n}\,,$$

  where $\hat{\Gamma}$ is the renormalized effective action and $\delta\Gamma$ stands for counterterms.

- ν_i is a left handed Weyl spinor with real Majorana mass m_i. Then (with $p\sigma \equiv p^\mu \sigma_\mu\,,\ p\bar{\sigma} \equiv p^\mu \bar{\sigma}_\mu$):

$$\hat{\Gamma}^{[0]}_{\nu_i \nu_i} = -m_i\,,\ \hat{\Gamma}^{[0]}_{\nu_i^\dagger \nu_i^\dagger} = -m_i\,,\ \hat{\Gamma}^{[0]}_{\nu_i^\dagger \nu_i} = p\bar{\sigma}\,,\ \hat{\Gamma}^{[0]}_{\nu_i \nu_i^\dagger} = p\sigma$$

- The corrected two point functions can be written in terms of scalar functions:

$$\hat{\Gamma}_{\nu_i \nu_i} = m_i \hat{\Sigma}_{\nu_i \nu_i}\,,\ \hat{\Gamma}_{\nu_i^\dagger \nu_i^\dagger} = m_i \hat{\Sigma}_{\nu_i^\dagger \nu_i^\dagger}\,,\ \hat{\Gamma}_{\nu_i \nu_j^\dagger} = p\sigma \hat{\Sigma}_{\nu_i \nu_j^\dagger}\,, \hat{\Gamma}_{\nu_i^\dagger \nu_j} = p\bar{\sigma} \hat{\Sigma}_{\nu_i^\dagger \nu_j}$$

- The multiplicative constants:

$$v_{0i} = Z_{ij}^{\frac{1}{2}} v_j \,,\; v_{0i}^{\dagger} = Z_{ij}^{\frac{1}{2}\dagger} v_j^{\dagger} \,,\; m_{0i} = m_i Z_{m_i} \,,\; Z_{m_i} = 1 + \delta_{m_i} \,,\; Z_{ij}^{\frac{1}{2}} = 1_{ij} + \frac{1}{2}\delta_{ij} \,,$$

where 1_{ij} is Kronecker delta function.

- The OS condition for mass counterterm reads (here and now, Re stands for taking the real part only of the loop functions and not couplings):

$$\mathrm{Re}\left(\hat{\Sigma}_{v_i v_i} + \hat{\Sigma}_{v_i^{\dagger} v_i^{\dagger}} + \hat{\Sigma}_{v_i v_i^{\dagger}} + \hat{\Sigma}_{v_i^{\dagger} v_i}\right)\Big|_{p^2 = m_i^2} = 0 \,.$$

- The mass counterterm is then:

$$\delta_{m_i} = \frac{1}{2}\mathrm{Re}\left(\Sigma_{v_i v_i} + \Sigma_{v_i^{\dagger} v_i^{\dagger}} + \Sigma_{v_i v_i^{\dagger}} + \Sigma_{v_i^{\dagger} v_i}\right)\Big|_{p^2 = m_i^2}$$

- If we fix the phases of the fields, that $m_{0i} \in \mathbb{R}$, we can just drop the Re to have $\delta_{m_i}, m_i^2 \in \mathbb{C}$. At one loop level, we evaluate at $p^2 = m_{0i}^2 + O\left(g^2\right)$.

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- If we fix the phases of the fields, that $m_{0i} \in \mathbb{R}$, we can just drop the Re to have $\delta_{m_i}, m_i^2 \in \mathbb{C}$. At one loop level, we evaluate at $p^2 = m_{0i}^2 + O\left(g^2\right)$.

- The residue equal to one and particles stay in the same basis:

$$\left[\mathrm{Re}\hat{\Sigma}_{\nu_i^\dagger\nu_i} + m_i^2\frac{\partial}{\partial p^2}\mathrm{Re}\left(\Sigma_{\nu_i\nu_i} + \Sigma_{\nu_i^\dagger\nu_i^\dagger} + \Sigma_{\nu_i\nu_i^\dagger} + \Sigma_{\nu_i^\dagger\nu_i}\right)\right]\Big|_{p^2=m_i^2} = 0\,,$$

$$\mathrm{Re}\hat{\Sigma}_{\nu_i^\dagger\nu_i}\Big|_{p^2=m_i^2} = \mathrm{Re}\hat{\Sigma}_{\nu_i^\dagger\nu_i}\Big|_{p^2=m_i^2} = -\mathrm{Re}\hat{\Sigma}_{\nu_i^\dagger\nu_i^\dagger}\Big|_{p^2=m_i^2} = -\mathrm{Re}\hat{\Sigma}_{\nu_i\nu_i}\Big|_{p^2=m_i^2}$$

$$\mathrm{Re}\left(\hat{\Gamma}_{\nu_i\nu_j} + m_j\hat{\Sigma}_{\nu_i\nu_j^\dagger}\right)\Big|_{p^2=m_j^2} = 0\,, \quad \mathrm{Re}\left(\hat{\Gamma}_{\nu_i^\dagger\nu_j^\dagger} + m_j\hat{\Sigma}_{\nu_i^\dagger\nu_j}\right)\Big|_{p^2=m_j^2} = 0$$

- The counterterms:

$$\frac{1}{2}\left(\delta_{ii}^\dagger + \delta_{ii}\right) = -\mathrm{Re}\Sigma_{\nu_i^\dagger\nu_i} - m_i^2\frac{\partial}{\partial p^2}\mathrm{Re}\left(\Sigma_{\nu_i\nu_i} + \Sigma_{\nu_i^\dagger\nu_i^\dagger} + \Sigma_{\nu_i\nu_i^\dagger} + \Sigma_{\nu_i^\dagger\nu_i}\right)\Big|_{p^2=m_i^2}$$

$$\delta_{ii}^\dagger - \delta_{ii} = \mathrm{Re}\Sigma_{\nu_i\nu_i}\left(m_i^2\right) - \mathrm{Re}\Sigma_{\nu_i^\dagger\nu_i^\dagger}\left(m_i^2\right)$$

$$\delta_{ij}^\dagger = \frac{2}{m_i^2 - m_j^2}\mathrm{Re}\left(m_j\Gamma_{\nu_i\nu_j}\left(m_j^2\right) + m_j^2\Sigma_{\nu_i\nu_j^\dagger}\left(m_j^2\right) + m_i\Gamma_{\nu_i^\dagger\nu_j^\dagger}\left(m_j^2\right) + m_i m_j\Sigma_{\nu_i^\dagger\nu_j}\left(m_j^2\right)\right)$$

$$\delta_{ij} = \frac{2}{m_i^2 - m_j^2}\mathrm{Re}\left(m_i\Gamma_{\nu_i\nu_j}\left(m_j^2\right) + m_i m_j\Sigma_{\nu_i\nu_j^\dagger}\left(m_j^2\right) + m_j\Gamma_{\nu_i^\dagger\nu_j^\dagger}\left(m_j^2\right) + m_j^2\Sigma_{\nu_i^\dagger\nu_j}\left(m_j^2\right)\right)$$

- We cannot simply drop the reality requirement, since the Im part cannot be absorbed into $\delta_{ii}^\dagger + \delta_{ii}$.

- In order to absorb Im part, we are led to introduce another constant:

$$\frac{1}{2}\left(\bar{\delta}_{ii}+\delta_{ii}\right)=-\Sigma_{\bar{\nu}_i\nu_i}-m_i^2\frac{\partial}{\partial p^2}\left(\Sigma_{\nu_i\nu_i}+\Sigma_{\bar{\nu}_i\bar{\nu}_i}+\Sigma_{\nu_i\bar{\nu}_i}+\Sigma_{\bar{\nu}_i\nu_i}\right)_{m_i^2},$$

$$\bar{\delta}_{ii}-\delta_{ii}=\Sigma_{\nu_i\nu_i}\left(m_i^2\right)-\Sigma_{\bar{\nu}_i\bar{\nu}_i}\left(m_i^2\right)$$

$$\bar{\delta}_{ij}=\frac{2}{m_i^2-m_j^2}\left(m_j\Gamma_{\nu_i\nu_j}\left(m_j^2\right)+m_j^2\Sigma_{\nu_i\bar{\nu}_j}\left(m_j^2\right)+m_i\Gamma_{\bar{\nu}_i\bar{\nu}_j}\left(m_j^2\right)+m_im_j\Sigma_{\bar{\nu}_i\nu_j}\left(m_j^2\right)\right)$$

$$\delta_{ij}=\frac{2}{m_i^2-m_j^2}\left(m_i\Gamma_{\nu_i\nu_j}\left(m_j^2\right)+m_im_j\Sigma_{\nu_i\bar{\nu}_j}\left(m_j^2\right)+m_j\Gamma_{\bar{\nu}_i\bar{\nu}_j}\left(m_j^2\right)+m_j^2\Sigma_{\bar{\nu}_i\nu_j}\left(m_j^2\right)\right)$$

- Which leads to a difference between particle and antiparticle:

$$\nu_{0i}=Z_{ij}^{\frac{1}{2}}\nu_j\,,\ \nu_{0i}^{\dagger}=\bar{Z}_{ij}^{\frac{1}{2}}\bar{\nu}_j\Rightarrow\left(Z_{ij}^{\frac{1}{2}}\nu_j\right)^{\dagger}=\bar{Z}_{ij}^{\frac{1}{2}}\bar{\nu}_j\,.$$

- At a loop level:

$$\nu_i^{\dagger}=\left(1-\frac{1}{2}\bar{\delta}_{ii}+\frac{1}{2}\delta_{ii}^{\dagger}\right)\bar{\nu}_i\,,$$

Field renormalization in CMS

- A stable Majorana particle has:

$$\bar{\delta}_{ij} = \delta_{ij}^{\dagger}\,,\; \bar{\delta}_{ii} = \delta_{ii}^{\dagger} \Rightarrow v_i^{\dagger} = \bar{v}_i\,,\; \delta_{m_i} \in \mathbb{R}$$

- As an example, consider that all couplings are real, then:

$$\bar{\delta}_{ii} - \delta_{ii} = \Sigma_{v_i v_i}\left(m_i^2\right) - \Sigma_{\bar{v}_i \bar{v}_i}\left(m_i^2\right) = 0 \Rightarrow \bar{\delta}_{ii} = \delta_{ii}\,.$$

  Meaning that:

$$v_i^{\dagger} = \left(1 - \frac{1}{2}\delta_{ii} + \frac{1}{2}\delta_{ii}^{\dagger}\right)\bar{v}_i = \left(1 - i\mathrm{Im}\delta_{ii}\right)\bar{v}_i = e^{i\mathrm{Im}\delta_{ii}}\,\bar{v}_i + O\left(\delta^2\right).$$

- So the imaginary part in loop functions give the phase difference for fields:

$$\mathrm{Im}\delta_{ii} = -\mathrm{Im}\Sigma_{\bar{v}_i v_i}(m_i^2) - 2m_i^2\mathrm{Im}\frac{\partial}{\partial p^2}\left(\Sigma_{v_i v_i} + \Sigma_{\bar{v}_i v_i}\right)\Big|_{p^2=m_i^2}$$

The model

- Starting from flavor and the Higgs basis, we can rotate the neutrinos in such a way, that ν'_{01} has no interactions with neutral scalars at all:

$$\begin{aligned}\mathscr{L}_\nu &= -\frac{1}{\sqrt{2}} y_3 \nu'_{03} \left(v + h' + i\chi^0\right) \nu'_{04} - \frac{1}{\sqrt{2}} d\, \nu'_{02} \left(H' + iA'\right) \nu'_{04} \\ &\quad - \frac{1}{\sqrt{2}} d' \nu'_{03} \left(H' + iA'\right) \nu'_{04} + \frac{1}{2} M\, \nu'_{04} \nu'_{04} + h.c.\end{aligned}$$

- The seesaw transformation $\left(s^2 = \frac{m_{03}}{m_{04}+m_{03}},\ c^2 = \frac{m_{04}}{m_{04}+m_{03}}.\right)$ gets the neutrinos into their mass eigenstates $\nu'_{0i} \to \nu_{0i}$:

$$\begin{aligned}\mathscr{L}_\nu = &-\frac{1}{2} m_{03}\, \nu_{03} \nu_{03} - \frac{1}{2} m_{04}\, \nu_{04} \nu_{04} \\ &- \frac{1}{\sqrt{2}} \left[y\left(h' + i\chi^0\right) - d' \left(H' + iA'\right)\right] \left(cs\nu_{03}\nu_{03} + i\left(c^2 - s^2\right) \nu_{03}\nu_{04} + cs\nu_{04}\nu_{04}\right) \\ &- \frac{1}{\sqrt{2}} d \left(H' + iA'\right) \nu_{02} \left(-is\nu_{03} + c\nu_{04}\right) + h.c.\end{aligned}$$

- We fix the phases of neutrinos so that we have real bare mass parameters. To summarize, in this basis we have:

$$y, m_{03}, m_{04}, d, s \in \mathbb{R},\ d' \in \mathbb{C}.$$

Renormalization constants

- ν_1, ν_2 and ν_3 are stable at 1 loop level, so the counterterms are the same as we would have in the OS scheme:

$$\nu_j^\dagger = \bar{\nu}_j \,, \delta_{m_3} \in \mathbb{R} \,, \; \delta_{jj}^\dagger = \bar{\delta}_{jj} \,, \, \delta_{ij}^\dagger = \bar{\delta}_{ij} \,, \; i = 1,2,3,4 \,; \; j = 1,2,3.$$

- For an unstable ν_4 we have:

$$\delta_{m_4}, \delta_{i4}, \bar{\delta}_{i4}, \delta_{44}, \bar{\delta}_{44} \in \mathbb{C} \,, \; \nu_4^\dagger = \left(1 - \frac{1}{2}\bar{\delta}_{44} + \frac{1}{2}\delta_{44}^\dagger\right) \bar{\nu}_4 \,, \; i = 1,2,3.$$

- There is no bare mass, hence no mass counterterms for ν_2 and ν_1 and no counterterms for mixing:

$$\delta_{m_1} = \delta_{m_2} = \delta_{12} = \bar{\delta}_{12} = \delta_{21} = \bar{\delta}_{21} = 0.$$

- Since the ν_1 doesn't interact with the neutral scalar sector at all, we have there is no mass term possible for ν_1.

$$\Gamma_{\nu_1 \nu_1}(p^2) = 0 \,.$$

- Also, due to our choice of the basis, we have $\Gamma_{\nu_1 \nu_2} = 0$, but note that $\Gamma_{\nu_1 \nu_3} \neq 0$ and $\Gamma_{\nu_1 \nu_4} \neq 0$.

Mass term for ν_2

- Loop level mass for ν_2 is finite and gauge invariant:

$$m_2 = -\hat{\Gamma}_{\nu_2\nu_2}(0) = -\Gamma_{\nu_2\nu_2}(0)$$

- Considering CP conserving case, there is no mixing between CP even and CP odd Higgses. Then loop level mass is:

$$m_2 = -\frac{d^2}{32\pi(m_3+m_4)}\left(m_3^2\left[B_0\left(0,m_3^2,m_A^2\right) - c_{12}^2 B_0\left(0,m_3^2,m_H^2\right) - s_{12}^2 B_0\left(0,m_3^2,m_h^2\right)\right]\right.$$
$$\left. - m_4^2\left[B_0\left(0,m_4^2,m_A^2\right) - c_{12}^2 B_0\left(0,m_4^2,m_H^2\right) - s_{12}^2 B_0\left(0,m_4^2,m_h^2\right)\right]\right),$$

  where s_{12} and c_{12} describes mixing between SM Higgs h and second CP even Higgs H.
- The contribution for the SM Higgs goes away when $s_{12} \to 0$ (and $c_{12} \to 1$).
- If we also have $m_H = m_A$, $m_2 \to 0$.

Conclusion

- In the Grimus Neufeld model, assuming multiplicative renormalization constants, there is no counterterm for the mass of the second neutrino, but the loop induced mass is finite and gauge invariant:

$$m_2 \sim \frac{d^2}{m_3 + m_4} f\left(m_A, m_h, m_H, m_3, m_4, s_{12}, s_{13}\right)$$

- Generalizing the OS to the CMS renormalization scheme for fermions with Majorana mass terms leads to the introduction of an additional phase between particle and antiparticle, when the particle is unstable. This is the case for ν_4.
- Further study on gauge dependence of the neutrino 2 point functions and the effects, caused by the instability of ν_4 will follow...

Thank you for listening