The one-loop improved Lagrangian

of the Grimus-Neufeld model

(seesaw + radiative neutrino masses)

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Our model: the 312-νSM (i.e. the Grimus-Neufeld model)

Standard Model (SM) + one fermionic singlet + two Higgs doublets

- is not a new idea: [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

The bare Lagrangian of the 312-νSM

- Gauge sector $\mathcal{L}_{\mathsf{G}}$ and Fermion-Gauge sector of the SM:
  - gauge group $U(1)_{\mathsf{Y}} \otimes SU(2)_{\mathsf{L}} \otimes SU(3)_{\mathsf{color}}$
  - gauge covariant derivative $D_\mu \psi$
  - and the Lagrangian $\mathcal{L}_{\mathsf{G\text{-}F}} = \sum_\psi \bar{\psi}\, i \not{D}\, \psi$

- Gauge-Higgs sector with the gauge covariant derivative $D_\mu \phi_a$ and the Lagrangian $\mathcal{L}_{\mathsf{G\text{-}H}} = (D^\mu \phi_a)^\dagger (D_\mu \phi_a) - V(\phi_a)$

- Higgs sector: two Higgs doublets ϕ_a in the Higgs potential $V(\phi_a)$

  [H-ON] H. E. Haber and D. O'Neil, Phys. Rev. D **83** (2011) 055017 [arXiv:1011.6188 [hep-ph]]

- Fermion-Higgs sector with the Yukawa couplings (ignoring quarks)

$$\mathcal{L}_{\mathsf{F\text{-}H}} = -\bar{\ell}^0_{Lj}\, \phi_a\, Y^{\bar{a}}_{Ljk}\, e^0_{Rk} - \bar{\ell}^0_{Lj}\, \tilde{\phi}_{\bar{a}}\, \tilde{Y}^a_{Lj} N^0 + h.c.$$

with the adjoint Higgs doublet $\tilde{\phi}_{\bar{a}} = \epsilon \phi^*_a = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} (\phi^+_a)^* \\ (\phi^0_a)^* \end{pmatrix} =: \begin{pmatrix} \phi^{0*}_{\bar{a}} \\ -\phi^-_{\bar{a}} \end{pmatrix}$

- Majorana sector with the Majorana singlet N^0: $D_\mu N^0 = \partial_\mu N^0$

The 312-νSM has parameters additionally to the "original" SM

- the singlet Majorana mass term M_R
- the neutrino Yukawa coupling of the first Higgs doublet

  $(Y_N^{(1)})_j \;:= \tilde{Y}^1_{Lj} = \frac{\sqrt{2}}{v}(M_D)_j \;\ldots$ the "Dirac mass" term

- the Yukawa couplings of the second Higgs doublet

  $(Y_E^{(2)})_{jk} := Y^2_{Ljk}$ to lepton doublets and charged lepton singlets ℓ_{Rj}

  $(Y_N^{(2)})_j \;:= \tilde{Y}^2_{Lj}$ to lepton doublets and neutral fermionic singlet N_R

- additional parameters in the Higgs sector see [H-ON]
  - $m^2_{H_2}$, $m^2_{H_3}$, $m^2_{H^\pm}$ masses of the additional Higgs bosons
  - θ_{12}, θ_{13} mixing angles between the neutral Higgs fields

    CP conservation forces the mixing to the pseudo-scalar A^0 to zero: $\theta_{13} = 0$
  - Z_2, Z_3, Z_7 ... parameters of the Higgs potential,

    not fixed by tree level mass relations

312-νSM tree level for the neutral fermions

- the Yukawa coupling $(Y_N^{(1)})_j$ mixes the neutral leptons ν_j with N_R
- the mixing gives a $(3+1)\times(3+1)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} \quad \text{with} \quad \begin{aligned} M_L &= 0_{3\times 3} \\ M_D &= (m_{De}, m_{D\mu}, m_{D\tau}) = \frac{v}{\sqrt{2}} Y_N^{(1)\top} \end{aligned}$$

  - M_ν has rank 2 $\Rightarrow$ only two masses are non-zero

- diagonalizing M_ν

$$U_{(\nu)}{}^* M_\nu = \mathrm{diag}(m_1, m_2, m_3, m_4) U_{(\nu)} =: \hat{m} U_{(\nu)} \quad \text{with} \quad m_1 = m_2 = 0$$

by the unitary matrix

$$U_{(\nu)} = \begin{pmatrix} u_{1e} & u_{1\mu} & u_{1\tau} & 0 \\ u_{2e} & u_{2\mu} & u_{2\tau} & 0 \\ c u_{3e} & c u_{3\mu} & c u_{3\tau} & is \\ is u_{3e} & is u_{3\mu} & is u_{3\tau} & c \end{pmatrix} \quad \text{where} \quad \begin{aligned} c^2 &= \frac{m_4}{m_4+m_3} \\ s^2 &= \frac{m_3}{m_4+m_3} \end{aligned}$$

  - $u_{\alpha k}$ is the neutrino mixing matrix (with $\alpha, k = 1\ldots 3$, α mass, k flavour)

312-νSM tree level for the neutral fermions

- from ${U_{(\nu)}}^* M_\nu = \hat{m} U_{(\nu)}$ and $(Y_N^{(1)})_k = \frac{\sqrt{2}}{v}(M_D)_k$ we get

$$u_{1k}^*(Y_N^{(1)})_k = u_{2k}^*(Y_N^{(1)})_k = 0$$

- the two tree-level massless "neutrinos" $\zeta_{1,2}^M$ are degenerate
- use the second Higgs coupling $(Y_N^{(2)})_k$ to distinguish them:

$$u_{1k}^*(Y_N^{(2)})_k = 0 \qquad \text{and} \qquad u_{2k}^*(Y_N^{(2)})_k =: d \neq 0$$

$\Rightarrow$ parametrize the Yukawa couplings as

$$(Y_N^{(1)})_k = \frac{\sqrt{2} m_D}{v} u_{3k} \qquad (Y_N^{(2)})_k := d\, u_{2k} + d' u_{3k}$$

@ loop level these parameters

- generate a loop induced mass $m_2 \propto d^2$
  - **this is the Grimus-Neufeld model**
- and a loop correction δm_3 for the seesaw mass $m_3^0 \propto \frac{m_D^2}{M_R}$

Improving by including one-loop predictions: using $\tilde{m}_i$ and δm_i

How can we do that?

- when renormalizing the Lagrangian expressed in the mass eigenstates
  - one gets a counter term $\delta^{\text{ct}} m$ for each non vanishing mass m
    - we have $m_3 > 0$ already at tree level ...

"Trick" of Grimus and Lavoura

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

- renormalize the Lagrangian expressed in interaction eigenstates
  - the counter term for the mass matrix

$$\delta^{\text{ct}} M_\nu = \begin{pmatrix} \delta^{\text{ct}} M_L & (\delta^{\text{ct}} M_D)^\top \\ \delta^{\text{ct}} M_D & \delta^{\text{ct}} M_R \end{pmatrix} \qquad \text{has} \qquad \delta^{\text{ct}} M_L = 0_{3\times 3}$$

    - since $M_L^{\text{tree}} = 0_{3\times 3}$
  - the counter term $\delta^{\text{ct}}(M_D)_k = \frac{1}{\sqrt{2}}[(\delta^{\text{ct}} v)(Y_N^{(1)})_k + v(\delta^{\text{ct}} Y_N^{(1)})_k]$
    - is fixed by the vacuum and the Higgs coupling
  - $\delta^{\text{ct}} M_R$ is "fixed" by the not measured heavy singlet ... and ignored

The Grimus-Lavoura procedure

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

reducing the problem to the "light" neutrinos $\zeta^M_{1,2,3}$:

- staying in the interaction eigenstates basis $\quad M_\nu = \begin{pmatrix} 0 & M_D^\top \\ M_D & M_R \end{pmatrix}$

  leads to an effective 3×3-neutrino mass matrix $\mathcal{M}_\nu$

  – at tree level $\quad \mathcal{M}_\nu^{\text{tree}} = -M_D^\top \, M_R^{-1} \, M_D \;,$

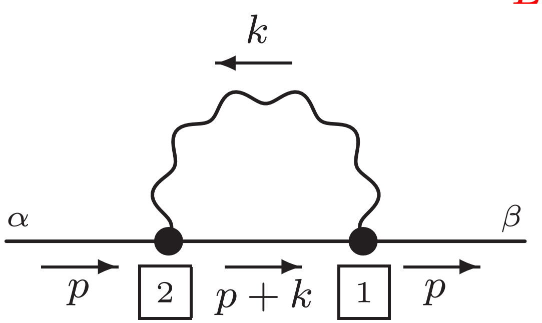
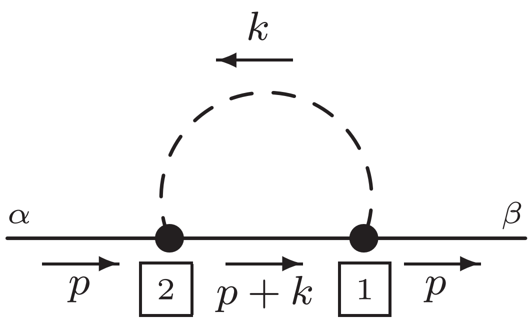
  – and at one-loop level $\quad \mathcal{M}_\nu^{\text{1-loop}} = \mathcal{M}_\nu^{\text{tree}} + \delta\mathcal{M}_\nu \;,$

  with $\quad \delta\mathcal{M}_\nu = \delta M_L - \delta M_D^\top \, M_R^{-1} \, M_D - M_D^\top \, M_R^{-1} \, \delta M_D + M_D^\top \, M_R^{-1} \, \delta M_R \, M_R^{-1} \, M_D$

- assuming δM_R to be irrelevant (as M_R is a free unmeasurable parameter)

- assuming corrections with δM_D to be subdominant $\propto Y^2 m_{\ell^\pm} \frac{m_D}{M_R}$ or $g^2 m_{\ell^\pm} \frac{m_D}{M_R}$

$\Rightarrow$ loop corrections come from δM_L, calculated from

$\Sigma_V^{[2]}(p^2) =$  $\qquad \Sigma_S^{[2]}(p^2) =$ 

The Grimus-Lavoura procedure

calculating $\delta^{\text{loop}} M_L$ from the selfenergy $\Sigma^{[2]}_{(p^2)}$

- at vanishing external momentum $p^2 = \tilde{m}^2 = m^2_{\text{phys}} \sim 0$
  - for the mass term only the neutral bosons contribute
  - Z^0 and G^0 combine to a gauge invariant contribution

$\Rightarrow$ we get an effective 1-loop improved 3×3-mass matrix

$$(\mathcal{M}_\nu^{\text{1-loop}})_{jk} = u_{2j}u_{2k}A + (u_{2j}u_{3k} + u_{3j}u_{2k})B + u_{3j}u_{3k}C$$

$*$ which is obviously only rank 2 :

$$u^*_{\alpha j}(\mathcal{M}_\nu^{\text{1-loop}})_{jk}u^*_{\beta k} = \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B & C \end{array}\right)_{\alpha\beta} ,$$

depending on SM parameters and on

- the neutral Higgs masses m_h^2, m_H^2, m_A^2, and mixing θ_{12}
- the heavy singlet mass $M_R \sim m_4$
- and the Yukawa coupling parameters d, d', and m_D

- the tree-level seesaw contribution $\frac{v^2|Y_N^{(1)}|^2}{2m_4} = \frac{m_D^2}{m_4}$ is included in C

Diagonalizing $(\mathcal{M}_\nu^{\text{1-loop}})_{jk}$ gives the masses $\tilde{m}_2$ and $\tilde{m}_3 = m_3 + \delta m_3$

- using $u^*_{\alpha j}$ the mass $\tilde{m}_1 = 0$ can be factored out

$\Rightarrow$ the diagonalization can be simplified to a 2×2 seesaw relation

$$R^* \cdot \begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \tilde{m}_2 & 0 \\ 0 & \tilde{m}_3 \end{pmatrix} \cdot R \quad \text{with} \quad R = e^{i\frac{\alpha}{2}} \begin{pmatrix} \bar{c} & \bar{s} \\ -\bar{s}^* & \bar{c}^* \end{pmatrix}$$

- with $\bar{c} = e^{\frac{i}{2}(\gamma+\delta)} \cos\beta$ and $\bar{s} = e^{\frac{i}{2}(\gamma-\delta)} \sin\beta$
- R is needed as it describes the mixing between ζ_2^M and ζ_3^M

- we can determine the masses $\tilde{m}_2$ and $\tilde{m}_3$ from A, B, and C alone :

$$\tilde{m}_2^2 + \tilde{m}_3^2 = \mathrm{Tr}\left[R \begin{pmatrix} A & B \\ B & C \end{pmatrix}^\dagger \begin{pmatrix} A & B \\ B & C \end{pmatrix} R^\dagger\right] = |A|^2 + 2|B|^2 + |C|^2$$

$$\text{and} \quad \tilde{m}_2 \tilde{m}_3 = \det[R^*] \det \begin{bmatrix} A & B \\ B & C \end{bmatrix} \det[R^\dagger] = e^{-2i\alpha}[AC - B^2]$$

- using measured masses $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{sol}}}$ and $\tilde{m}_3 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}}$

$\Rightarrow$ solving above equations we express the two parameters d and $|d'|$

- by the masses and the other parameters (including m_D^2 and $\phi' = \arg[d']$)

Determining d and $|d'|$

- with $A = d^2 f_1$, $B = d' d f_1 + i d \frac{m_D}{v} f_2$, and $C = d'^2 f_1 + 2 i d' \frac{m_D}{v} f_2 + \frac{m_D^2}{v^2} f_3$, the f_i depending on the parameters of the Higgs sector (and the SM)

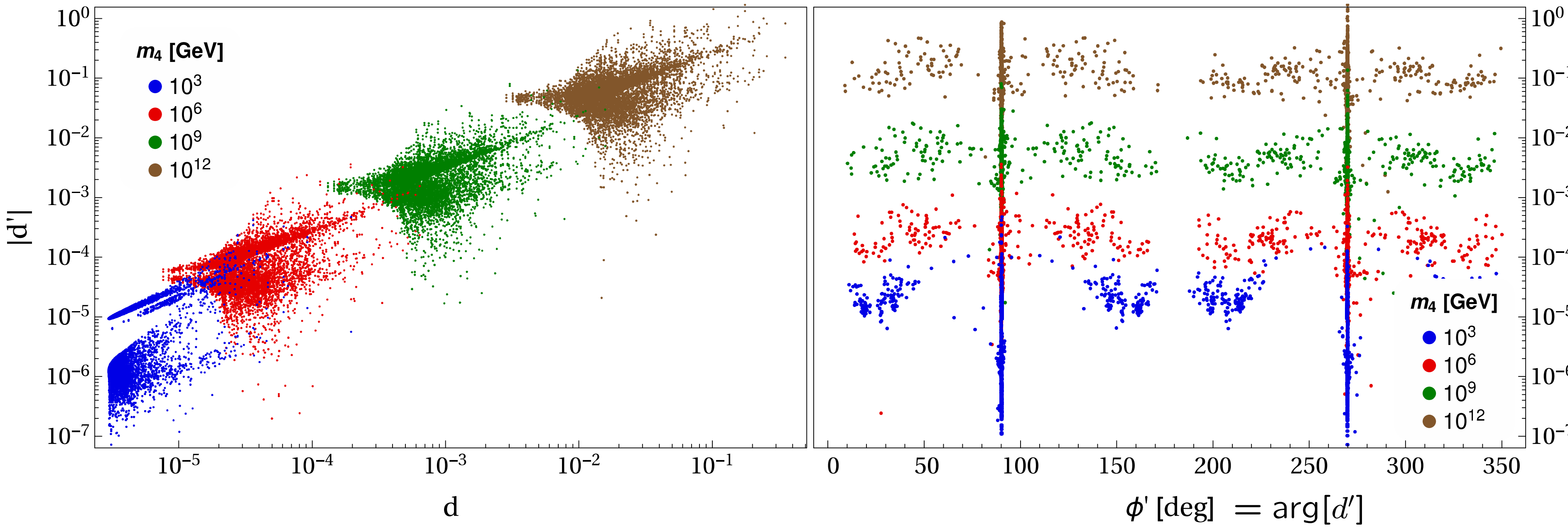
  $\Rightarrow$ determining d is simple:

  $$\tilde{m}_2 \tilde{m}_3 = |AC - B^2| = d^2 \frac{m_D^2}{v^2} |f_1 f_3 - f_2^2| \quad \Rightarrow \quad d^2 = \frac{v^2}{m_D^2} \frac{\tilde{m}_2 \tilde{m}_3}{|f_1 f_3 - f_2^2|}$$

  * since the equation does not depend on $|d'|$

- the other equation involves a fourth order polynomial in $|d'|$

  $\Rightarrow$ $|d'| = |d'|[v^2; m_h, m_H, m_A, s_{12}; \tilde{m}_2, \tilde{m}_3, \tilde{m}_4; m_D^2; \phi']$ and $d' = |d'| e^{i\phi'}$



Determining the rotation matrix R

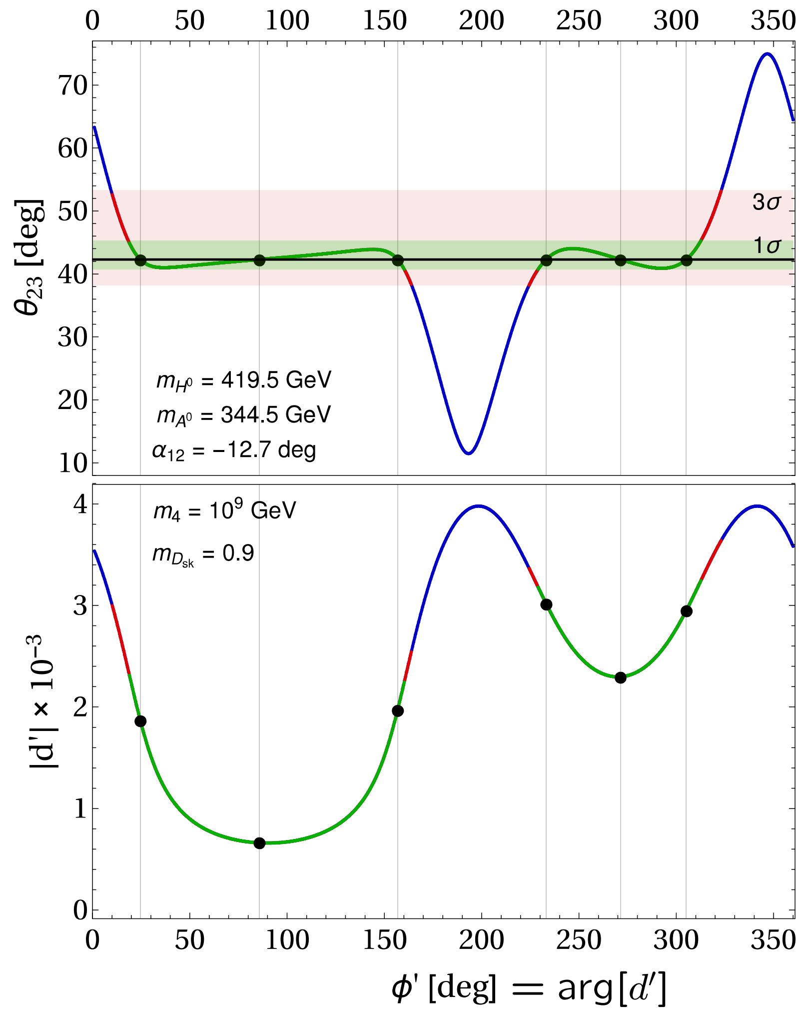
- the 2×2 seesaw relation gives

$$\begin{aligned} \tan^2\beta &= \frac{|A|^2 + |B|^2 - \tilde{m}_2^2}{\tilde{m}_3^2 - |A|^2 - |B|^2} \\ e^{2i\alpha} &= \frac{\tilde{m}_2\,\tilde{m}_3}{AC - B^2} \\ e^{i\delta} &= -\frac{(A^*B + B^*C)\tan\beta}{|A|^2 + |B|^2 - \tilde{m}_2^2} = \ldots \\ e^{i\gamma} &= \frac{e^{i\alpha}}{\tilde{m}_3}(Ce^{-i\delta} - B\tan\beta) = \ldots \end{aligned}$$

... indicate different possible analytic expressions

- R rotates to the 1-loop mass eigenstates

$$\tilde{\zeta}_1^M = \zeta_1^M \quad , \qquad \begin{pmatrix} \tilde{\zeta}_2^M \\ \tilde{\zeta}_3^M \end{pmatrix} = R \cdot \begin{pmatrix} \zeta_2^M \\ \zeta_3^M \end{pmatrix}$$



- the W^+ coupling of $\tilde{\zeta}_i^M$ to ℓ_k^- is the neutrino mixing matrix U_{PMNS}

  $\Rightarrow$ we can express the $u_{\alpha k}$ in $(Y_N^{(i)})_k$ by R and U_{PMNS}

$$(Y_N^{(1)})_k = \frac{\sqrt{2}m_D}{v} u_{3k} \qquad (Y_N^{(2)})_k := d\,u_{2k} + d'\,u_{3k}$$

Summarizing the 312-νSM (i.e. the Grimus-Neufeld model)

We take as input

- the Higgs masses m_h, m_H, m_A, and mixings s_{12} and s_{13};
  - for the CP conserving case: $s_{13} = 0$.
- the neutrino masses: $\tilde{m}_1 = 0$, $\tilde{m}_2$, $\tilde{m}_3$, $\tilde{m}_4 \approx m_4 \approx M_R$
  - and the neutrino mixing matrix: U_{PMNS}
- the first Yukawa coupling, giving the tree-level neutrino masses: m_D^2
- the phase of the second Yukawa coupling, $\phi' = \arg[d']$
  - subject to the constraint, that the fourth order equation gives $|d'| \in (0, 1)$

We determine the Yukawa couplings $(Y_N^{(i)})_k$

- we check the selfconsistency of the one-loop corrections:
  - the input masses $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{sol}}}$ and $\tilde{m}_3 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}}$ should produce the correct mass differencies
    - inverted hierarchy could be possible, too: $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{atm}}}$ and $\tilde{m}_3 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}}$
- using scatter plots we look at the distribution of the parameters

- Our model fixes the Yukawa couplings of the neutral singlet
- We can make predictions for processes involving the predicted $(Y_N^{(i)})_k$
  - up to now we managed only one simple bachelor thesis looking at an overly simple assumption
- the full renormalization of the model is still missing
  - see next talk ...

Thank you

for discussion

and comments

and of course for the conference! ☺