Interference effects in very precise measurement of muon charge asymmetry at FCCee

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INTRODUCTION: – if you have not heard about FCCee:)

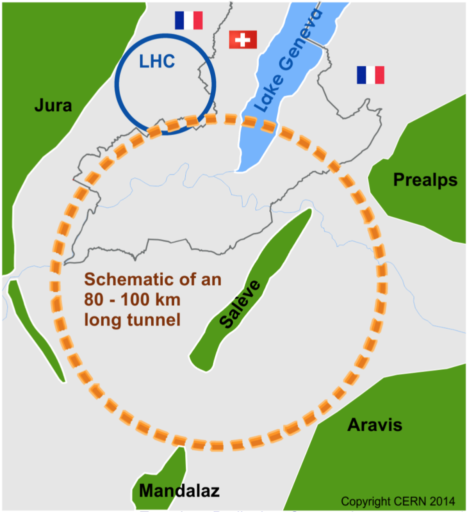
Future Circular Collider Study
GOAL: CDR and cost review for the next ESU (2018)

International FCC collaboration (CERN as host lab) to study:

- ***pp*-collider (*FCC-hh*)**
  → main emphasis, defining infrastructure requirements

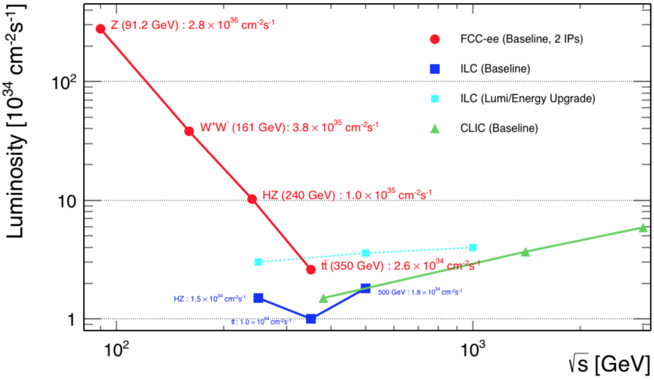
~16 T ⇒ 100 TeV *pp* in 100 km

- **80-100 km infrastructure** in Geneva area
- **e^+e^- collider (*FCC-ee*) as potential intermediate step / as a possible first step**
- *p-e* (*FCC-he*) option, HE-LHC …



FCCee as Z-factory: 1.5×10^{12} Z's/year! $10^5 \times$ LEP

Luminosities and centre-of mass energies



LEP record at the Z
$2.3 \cdot 10^{31}$ cm^{-2}s^{-1}

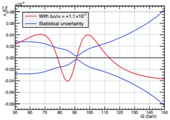
LEP2 record
$\approx 10^{32}$ cm^{-2}s^{-1}

INTRODUCTION

- $M_Z, G_F, \alpha_{QED}(0)$ outweigh other data in the "testing power" in the SM overall fit to experimental data
- However, $\alpha_{QED}(Q^2 = 0)$ is ported to $\alpha_{QED}(Q^2 = M_Z^2)$ using low energy QCD data -> this limits its usefulness beyond LEP precision.
- Patrick Janot has proposed (arxiv:1512.05544) another observable, $A_{FB}(e^+e^- \to \mu^+\mu^-)$ at $\sqrt{s_\pm} = M_Z \pm 3.5 GeV$, with a similar "testing profile" in the SM overall fit as $\alpha_{QED}(M_Z^2)$, but could be measured at high luminosity FCCee very precisely. (It is advertised as "determining $\alpha_{QED}(M_Z^2)$" from $A_{FB}(\sqrt{s_\pm})$".)
- However, A_{FB} near $\sqrt{s_\pm}$ is varying very strongly, hence is prone to large QED corrections (for instance ISR).
- In particular A_{FB} away from Z peak gets also a direct sizable contributions from **QED initial-final state interference, nickname IFI**.
- It is therefore necessary to re-discuss how efficiently these trivial but large QED effects in A_{FB} can be controlled and/or eliminated.

The aim is to reduce QED uncert. to $\delta A_{FB}(e^+e^- \to \mu^+\mu^-) < 4 \times 10^{-5}$

- Presently $\Delta\alpha_{QED}(M_Z)/\alpha_{QED} \simeq 1.1 \times 10^{-4}$ (using low energy e^+e^- data).
- Recent studies using the same method of dispersion relations are quoting possible improvements down to $\Delta\alpha/\alpha \simeq (0.5 - 0.2) \times 10^{-4}$.
- To be competitive A_{FB} has to provide $\Delta\alpha/\alpha < 10^{-4}$
- Using Fig.4 of arxiv:1512.05544 paper by Patrick Janot



$\Delta\alpha/\alpha < 10^{-4}$ translates into $\Delta A_{FB} < 4 \times 10^{-5}$

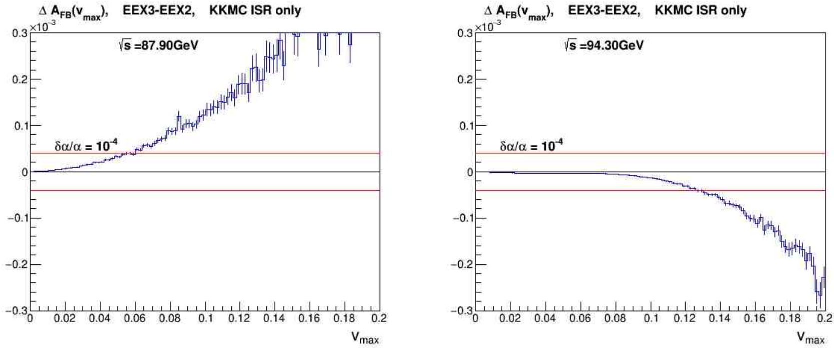
- LEP era estimate of QED uncertainty in A_{FB} outside Z peak was $\sim 2.5 \times 10^{-3}$, see "The LEP-2 MC Workshop 2000", arxiv:0007180.
- Its improvement by at least factor 200 sounds as a very ambitious goal!
- Encouraging precedent: for QED photonic corrs. to Z-lineshape ($\sim 30\%$), its uncertainty reduced down to $\delta\sigma/\sigma \simeq 3 \times 10^{-4}$, (Jadach,Skrzypek,Martinez, Phys.Lett.B280(1992)129)!

QED (photonic) correction effects in $A_{FB}(e^+e^- \to \mu^+\mu^-)$

General features

- Pure ISR (initial state radiation) indirect influence due to reduction of $\sqrt{s}$. Non-soft h.o. missing corrs. under very good control, see next slide.
- Pure FSR (final state radiation) for sufficiently inclusive event selection (cut-offs) generally small, but cut-off dependence has to be controlled with high quality MC.
- Direct contribution of IFI (initial-final state interference) is suppressed at the peak but sizable off-peak.
- IFI effect comes from non-trivial matrix-element, even in the soft-photon approximation.
- KKMC Monte-Carlo program (J.S., Ward, Wąs, Phys.Rev. D63 (2000)) is the most sophisticated tool to calculate all the above effects.

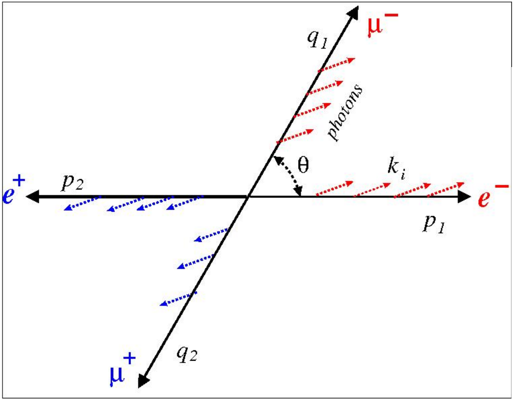
Pure ISR in A_{FB} at $\sqrt{s} \sim M_Z \pm 3\text{GeV}$

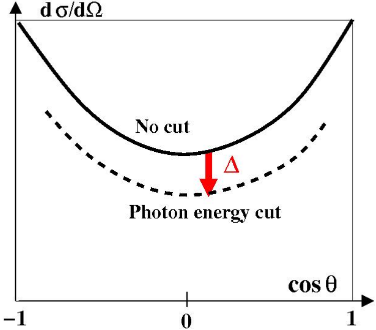


- Cut on energy of all photons $v < v_{\max}$, $v \equiv 1 - \frac{M^2_{\mu\mu}}{s} \simeq \sum_i \frac{2E_i^\gamma}{\sqrt{s}}$
- Examine downgrade non-soft of QED M.E. from EEX3 to EEX2
- For photon cut-off below $v_{\max} = 0.03$ we get $\delta A_{FB} < 4 \cdot 10^{-4}$.
- Looks good, but to be x-checked using semianalytical *KKsem*.
- Important contribution from e^+e^- soft pairs not included!!!

A general understanding of the IFI

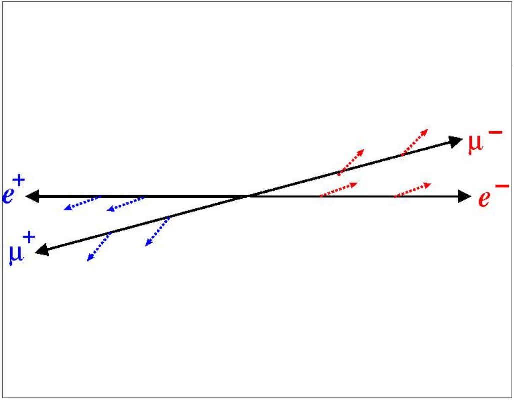
- In $e^- e^+ \to \mu^- \mu^+$ not only e^- gets annihilated, but also its accompanying elmgt. field of charge -1. New elmg. field of charge -1 is created along μ^-.
- At **wide angles** these two processes are independent sources of real photos. The effect of cut on photon energy is essentially θ-independent.

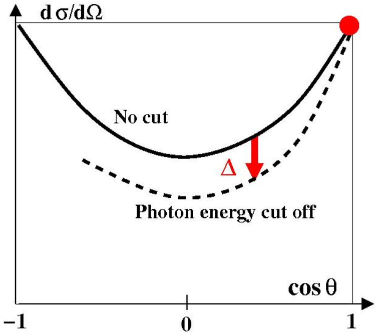




A general understanding of the IFI

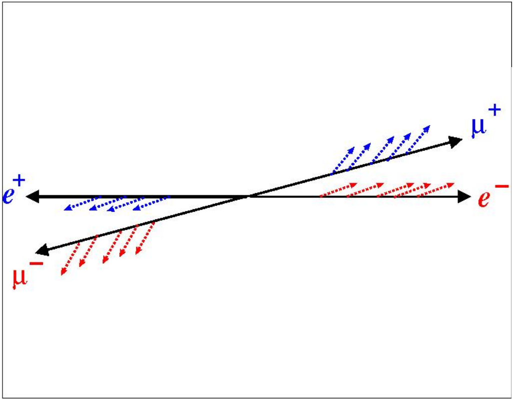
- In $e^- e^+ \to \mu^- \mu^+$ not only e^- gets annihilated, but also its accompanying elmgt. field of charge -1. New elmg. field of charge -1 is created along μ^-.
- μ^- close to initial e^- inherits part of e^- elmg. field $\to$ bremsstrahlung is weaker. Hence for $\theta \to 0$ zero effect due to cut on real photons!

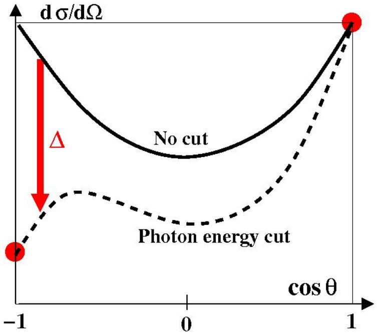




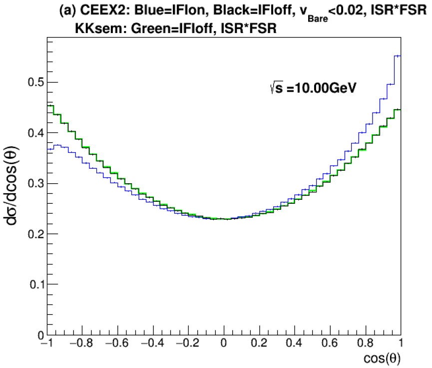
A general understanding of the IFI

- In $e^- e^+ \to \mu^- \mu^+$ not only e^- gets annihilated, but also its accompanying elmgt. field of charge -1. New elmg. field of charge -1 is created along μ^-.
- In the **backward** direction, replacing field of charge -1 with that of $+1$ is "more violent", more real photons $\to$ stronger effect of the cut, dip in $d\sigma/d\Omega$.





IFI effect in the muon angular distri. at $\sqrt{s} = 10GeV,\ M_Z \pm 3.5GeV$ for total photon energy cut $v = 1 - M^2_{\mu\mu}/s < v_{\max} = 0.02$ (KKMC)



- **A few percent effect** seen in the angular distribution.
- Good agreement of KKMC and semianalytical KKsem when IFI is off.
- (Inclusion of IFI in semianalytical KKsem is quite urgent!)

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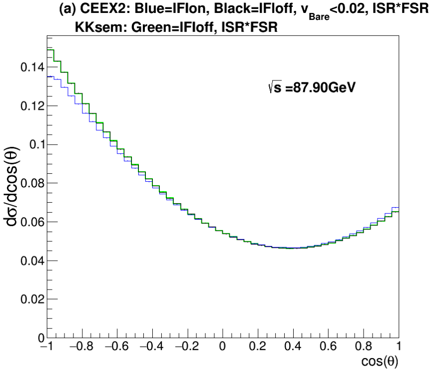

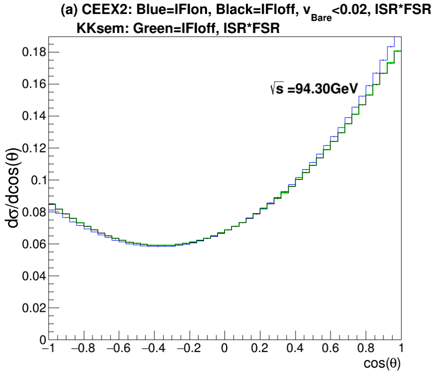

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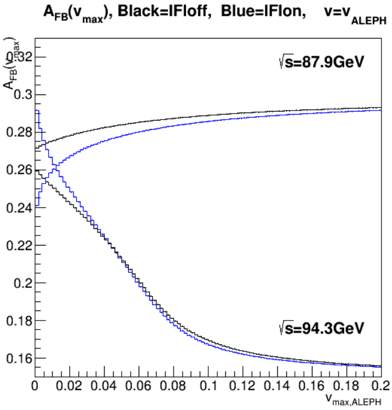
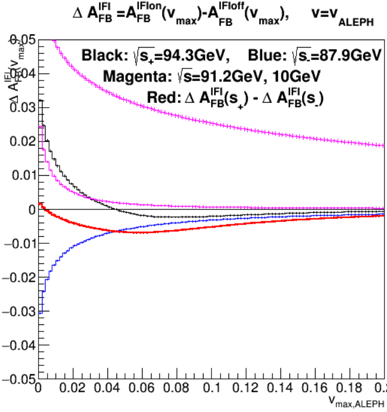
Direct influence of IFI in $A_{FB}(e^+e^- \to \mu^+\mu^-)$ at $\sqrt{s} \sim M_Z \pm 3$GeV

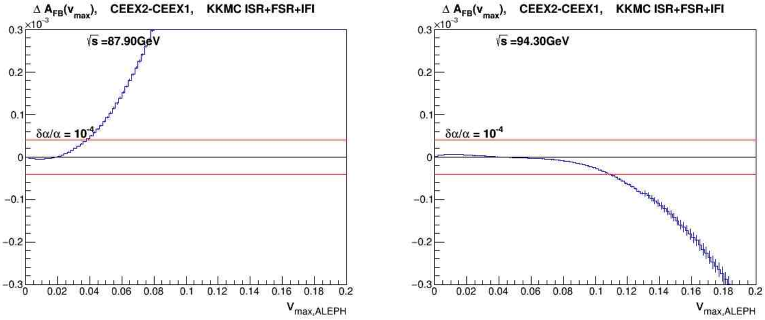
Sign of $A_{FB}(87.9GeV)$ flipped in order to better fit into plot





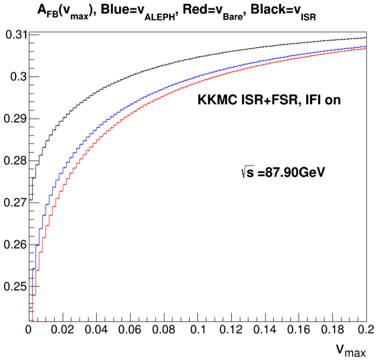
- IFI suppression by $\sim \Gamma/M$ seen comparing $\sqrt{s} = 10$GeV and 91GeV results.
- IFI effect is $\sim 3\%$ at $s_\pm$ ($\sim 1\%$ when combined).
- IFI is huge, compared to the aimed precision $\delta A_{FB} \sim 10^{-5}$
- $\sim \Gamma/M$ suppression dies out for $v_{max} < 0.04$.

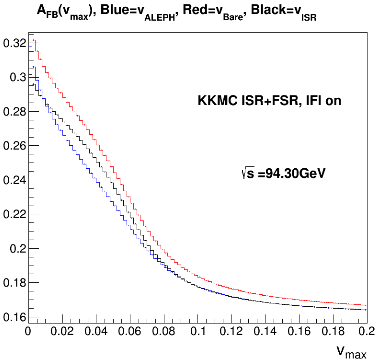
Attempt of estimating total QED uncert. δA_{FB} at $\sqrt{s} \sim M_Z \pm 3$GeV



- Examined CEEX2 $\rightarrow$ CEEX1 downgrade of M.E. in KKMC for ISR+FSR+IFI.
- Energy cut-ff on all photons using FSR-inclusive $v = v_{\max,ALEPH}$.
- Naively, we get $\delta A_{FB} < 4 \cdot 10^{-4}$ for photon cut-off $v_{\max} \leq 0.03$ as wanted...
- However, this test does not quantify QED uncertainty in IFI in a reliable way, because IFI remains in exactly the same soft-photon resumation scheme.
- Quality of the soft-photon resumation of IFI has to be examined separately – it was not done in a systematic way at so high precision level.

How important is the type of kinematic cuts in A_{FB} ?





- v_{ALEPH} is FSR-inclusive, $v_{bare} = 1 - M^2_{\mu\mu}/s$ is FSR-sensitive and v_{ISR} from $M^2_{\mu\mu}$ after ISR before FSR (from MC).
- It matters a lot, $> 1\%$, especially above Z!
- It does not seem to cancel between s_+ and s_-.
- MC like KKMC is mandatory to control/eliminate this effect.
- N.B. Effect of changing definition of muon $\cos\theta$ is completely negligible!

Theoretical uncertainty of soft-resummed IFI contribution to resonant matrix element implemented in KKMC

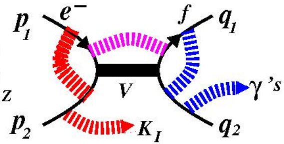
- Basicaly, soft-resumed M.E. in KKMC looks perfect, but all resummed calculation are to some extent non-unique.
- Pioneering works in the soft-photon resummation for resonant $e + e^-$ annihilation including IFI were done by Frascati group, (Greco et.at. Phys. Lett. B101 (1975) 234, Phys. Lett. B171 (1980) 118.)
- KKMC implements and extends this technique, see ref. [JWW-2001], Jadach,Ward,Wąs, Phys.Rev. D63(2001)113009
- Possible source of uncertainty: virtual formfactor.

Multiphoton matrix element in KKMC

Neglecting for clarity non-soft parts it reads (see [JWW-2001]):

$$\sigma(s) = \frac{1}{flux(s)} \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_{n+2} \prod_{i=1}^{n} \int \frac{d^3 k_i}{2k_i^0} \mathfrak{M}^{\mu_1,\mu_2,\dots,\mu_n}(k_1,\dots,k_n) \left[\mathfrak{M}_{\mu_1,\mu_2,\dots,\mu_n}(k_1,\dots,k_n)\right]^*$$

$$\mathcal{M} = \sum_{V=\gamma,Z}$$



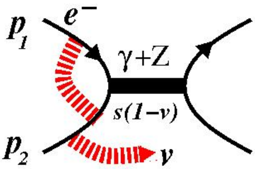
$$\mathfrak{M}^{\mu_1,\dots,\mu_n}(k_1,\dots,k_n) = \sum_{V=\gamma,Z} e^{\alpha B_4(p_i,q_i)+\alpha\Delta B_4^V(P-K_I)} \sum_{\{I,F\}} \prod_{i\in I} j_I^{\mu_i}(k_i) \prod_{r\in F} j_F^{\mu_r}(k_r)\, \mathcal{M}_V^{(0)}(P-K_I)$$

$$j_I^{\mu}(k) = \frac{e}{4\pi^{3/2}}\left(\frac{p_1^\mu}{p_1\cdot k} - \frac{p_2^\mu}{p_2\cdot k}\right),\quad j_F^{\mu}(k) = \frac{e}{4\pi^{3/2}}\left(\frac{q_1^\mu}{q_1\cdot k} - \frac{q_2^\mu}{q_2\cdot k}\right),\quad P = p_1+p_2,\quad K_I = \sum_{i\in I} k_j.$$

- $B_4(p_i,q_i)$ is YFS virtual formfactor. The additional $\alpha\Delta B_4^Z(P) = -2\frac{\alpha}{\pi}\ln\frac{-t}{s}\ln\frac{M_Z^2 - iM\Gamma_Z - (P-K_I)^2}{M_Z^2 - iM_Z\Gamma_Z}$, $\Delta B_4^\gamma = 0$, (Greco et.al. 1974) is mandatory for real/virtual cancellations of $\sim \frac{\alpha}{\pi}\ln\frac{\Gamma_Z}{M_Z}$. (To be improve further?).
- Almost complete $\mathcal{O}(\alpha^2)$ (except penta-boxes) QED virtual and real corrs. and EW $\mathcal{O}(\alpha^1)$ (DIZET) are also included in KKMC.

High precision Z-lineshape QED ISR formula used at LEP

decades of work by: Yennie, Frautschi, Suura, Gribov Lipatov, Kuraev, Fadin, Greco, Pancherini, Srivastava, Jackson, Martin, Berends, Burgers, Jadach, Skrzypek, Ward,...



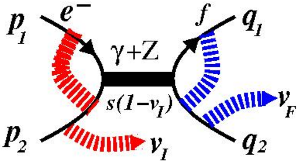
$$\sigma(s, v_{\max}) = \int_0^{v_{\max}} dv\; F(\gamma_I)\gamma_I v^{\gamma_I - 1}\; \sigma_B\big(s(1-v)\big)\; \big[1 + \mathrm{NIR}(\mathrm{v})\big],$$

$$F(\gamma) \equiv \frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}, \quad \gamma_I = 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

- Non-infrared perturbative function NIR(v), for $\delta\sigma/\sigma \simeq 2\times 10^{-4}$ precision, to be found in J.S.+Skrzypek+Pietrzyk Phys.Lett.B280(1992)129.
- One can add Electroweak corrections to σ_B, 1st order FSR, generalize to $d\sigma/d\Omega$ etc. as it was done in ZFITTER.

KKMC extensively tested with ISR+FSR (IFI off) formula

implemented in semianalytical program KKsem, part of KKMC distribution



$$\frac{d\sigma}{d\Omega}(s,\theta,v_{\max}) = \int dv_I\, dv_F\, \delta(v-v_I-v_F)\theta(v<v_{\max})$$

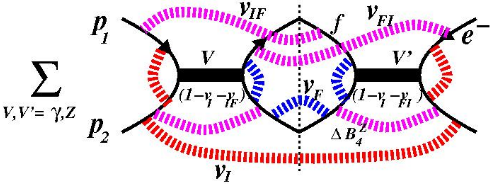
$$\times\, F(\gamma_I)\gamma_I v_I^{\gamma_I-1}\; F(\gamma_F)\gamma_I v_F^{\gamma_F-1}\; \frac{d\sigma_0}{d\Omega}(s(1-v_I),\theta)\; \left[1+\mathrm{NIR}(\mathrm{v_I},\mathrm{v_F})\right],$$

$$v = 1-(q_1+q_2)^2/s, \quad \gamma_F = 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_f^2}-1\right)$$

- In KKsem $d\sigma_0/d\Omega$ is decorated with EW corrections
- For $v_{\max} < 0.2$ definition of θ is not essential.
- Non-IR function NIR(v_I, v_F) from analytical integration of the MC distributions.
- $\delta(v-v_I-v_F) \to \delta(1-v-(1-v_I)(1-v_F))$ more realistic for hard emissions.

NEW formula for precision calibration of ISR+FSR+IFI

Eq.(90) in [JWW2001] and in older Frascati works, implemented recently in KKsem

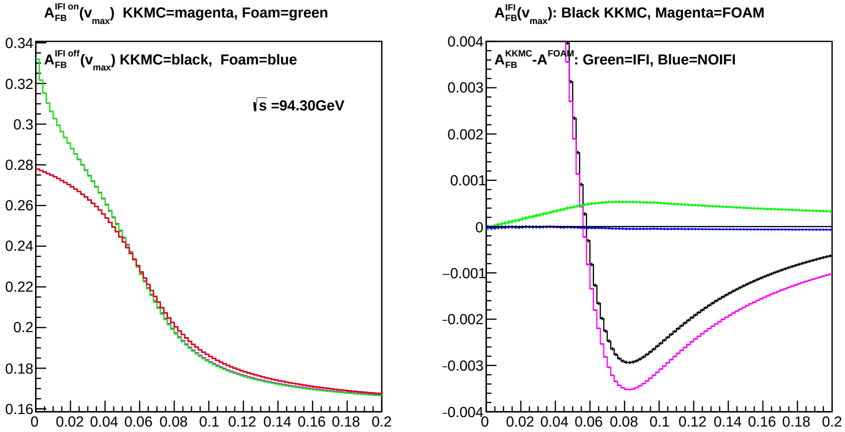


$$\frac{d\sigma}{d\Omega}(s,\theta,v_{\max}) = \sum_{V,V'=\gamma,Z} \int dv\, dv_I\, dv_F\, dv_{IF}\, dv_{FI}\, \delta(v-v_I-v_F-v_{IF}-v_{FI})\theta(v<v_{\max})$$
$$\times F(\gamma_I)\gamma_I v_I^{\gamma_I-1}\; F(\gamma_F)\gamma_I v_F^{\gamma_F-1}\; F(\gamma_{IF})\gamma_{IF} v_{IF}^{\gamma_{IF}-1}\; F(\gamma_{FI})\gamma_{FI} v_{IF}^{\gamma_{FI}-1}$$
$$\times e^{2\alpha\Delta B_4^V}\mathcal{M}_V^{(0)}(s(1-v_I-v_{IF}),\theta)\; [e^{2\alpha\Delta B_4^{V'}}\mathcal{M}_{V'}^{(0)}(s(1-v_I-v_{FI}),\theta)]^*\; \left[1+\mathrm{NIR}(v_I,v_F)\right],$$

- Convolution of **four** radiator functions (instead of two)!
- Extra virtual formfactor ΔB_4^Z due to IFI for resonant contrib.
- $\gamma_I = Q_e^2\frac{\alpha}{\pi}[\frac{s}{m_e^2}-1]$, $\gamma_{IF}=\gamma_{FI}=Q_eQ_f\frac{\alpha}{\pi}\ln\frac{1-\cos\theta}{1+\cos\theta}$, $F(\gamma)=\frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}$

New KKsem versus KKMC test at the $\delta A_{FB} \sim 10^{-4}$ level

$v_{\max}$ = cutoff on total photon energy in units of the beam energy



Small difference between KKMC and 5-dim. KKsem/Foam (green curve) disappears at soft limit $v_{\max} \to 0$, and is still to be explained.

Both calculations include IFI within soft photon resummation.

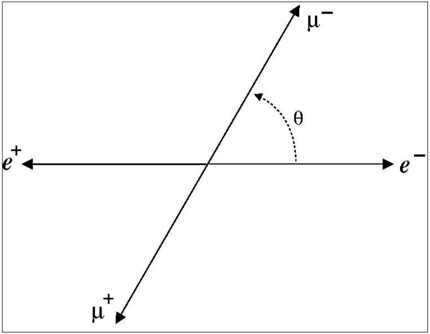
Summary

- The influence of IFI on A_{FB} is huge, as compared to precision scale aimed at FCCee.
- Strong $\sqrt{s}$ dependence of A_{FB} near $M_Z \pm 3.5 GeV$ matters (ISR).
- However, IFI could be calculated in perturbative QED very precisely, thanks to power of the soft photon resummation, similarly as huge QED correction to Z lineshape.
- IFI effect is strongly dependent on the type and strength of kinematic cuts, hence good quality MC implementation is mandatory, to take them out from the data.
- KKMC simulates soft (hard) real photons including IFI in an almost perfect way (virtual form-factor to be improved?).
- Main work needed to crosscheck KKMC and get more/better quantitative results with $\delta A_{FB} \sim 10^{-5}$ or better “reso;ution”.
- Encouragement and feedback from Patrick Janot is appreciated!

APPENDIX A: Understanding the essence IFI

within a simple toy model

- Consider $e^- e^+ \to \gamma^* \to \mu^- \mu^+ + n\gamma$,
- with flat CMS energy $\sqrt{s}$ dependence,
- in the high energy regime $\sqrt{s} >> m_e, m_\mu$
- first for wide muon scattering angle θ



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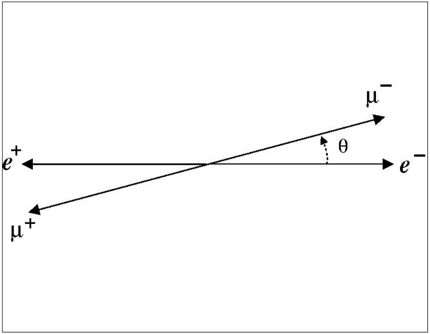
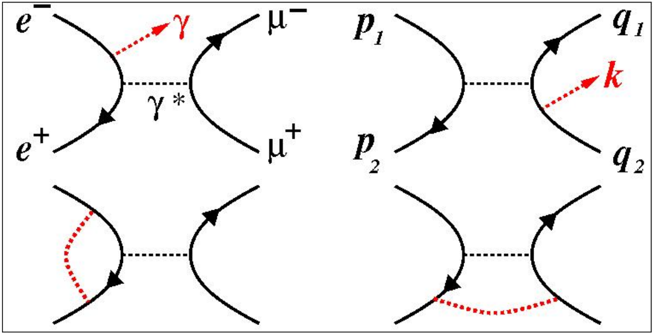
- with flat CMS energy $\sqrt{s}$ dependence,
- in the high energy regime $\sqrt{s} >> m_e, m_\mu$
- and next for small scattering angle $\theta \to 0$



Understanding IFI – simple toy model

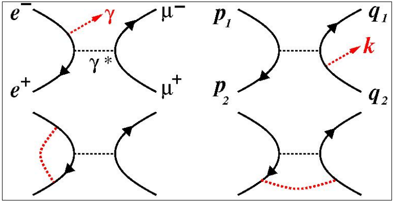
Diagrams and kinematics

- Our final goal is IFI in multiphoton case,
- but the essence of the IFI can be grasped analyzing single real/virtual photon emission case, with (generic) Feynman diagrams:



$$s = 2p_1 \cdot p_2, \quad t = -2p_1 \cdot q_1 = -s\frac{1-\cos\theta}{2}, \quad u = -2p_1 \cdot q_2 = -s\frac{1+\cos\theta}{2},$$

IFI in a simple toy model



Photon emissions M.E. is $\mathcal{M}^\mu(k) \simeq \mathcal{M}^\mu_{Born} J^\mu(k),\ J^\mu(k) = J^\mu_I(k) - J^\mu_F(k)$ and

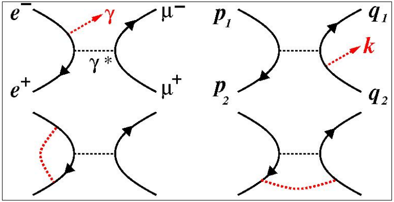
$$J^\mu_I(k) = \frac{eQ_q}{4\pi^{3/2}} \Big(\frac{p_1^\mu - k^\mu}{p_1 \cdot k} - \frac{p_2^\mu - k^\mu}{p_2 \cdot k} \Big), \quad J^\mu_F(k) = \frac{eQ_\mu}{4\pi^{3/2}} \Big(\frac{q_1^\mu - k^\mu}{q_1 \cdot k} - \frac{q_2^\mu - k^\mu}{q_2 \cdot k} \Big).$$

Adding vertex and box to real photon integrated with cut K:

$$\frac{d\sigma}{d\Omega}(c, K) \simeq \frac{d\sigma_{Born}}{d\Omega} \Big[1 - \int d^4k \Big(J^\mu_I(k) \cdot J^\mu_I(k) + J^\mu_F(k) \cdot J^\mu_F(k) - 2J^\mu_I(k) \cdot J^\mu_F(k) \Big)_{virt.}$$
$$+ \int \frac{d^3K}{k^0} \theta(K - k^0) \Big(J^\mu_I(k) \cdot J^\mu_I(k) + J^\mu_F(k) \cdot J^\mu_F(k) - 2J^\mu_I(k) \cdot J^\mu_F(k) \Big)_{real} \Big]$$

where $c = \cos\theta$, $E = \sqrt{s}/2$, and $\varepsilon << 1$ is IR regulator.

IFI in a simple toy model



Photon emissions M.E. is $\mathcal{M}^\mu(k) \simeq \mathcal{M}^\mu_{Born} J^\mu(k)$, $J^\mu(k) = J^\mu_I(k) - J^\mu_F(k)$ and

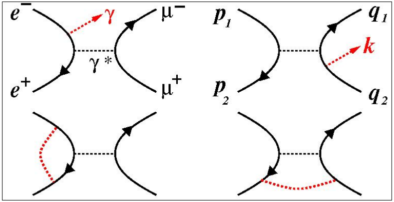
$$J^\mu_I(k) = \frac{eQ_q}{4\pi^{3/2}}\Big(\frac{p_1^\mu - k^\mu}{p_1\cdot k} - \frac{p_2^\mu - k^\mu}{p_2 \cdot k}\Big), \quad J^\mu_F(k) = \frac{eQ_\mu}{4\pi^{3/2}}\Big(\frac{q_1^\mu - k^\mu}{q_1\cdot k} - \frac{q_2^\mu - k^\mu}{q_2 \cdot k}\Big).$$

After integrating over photon angle:

$$\frac{d\sigma}{d\Omega}(c,K) \simeq \frac{d\sigma_{Born}}{d\Omega}\Big[1 - \int_{\varepsilon E}^{E} \frac{dk^0}{k_0}\Big(2\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} + 2\frac{\alpha}{\pi}\ln\frac{s}{m_\mu^2} - 4\frac{\alpha}{\pi}\ln\frac{|t|}{|u|}\Big)_{virt.}$$
$$+ \int_{\varepsilon E}^{K} \frac{dk^0}{k^0}\Big(2\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} + 2\frac{\alpha}{\pi}\ln\frac{s}{m_\mu^2} - 4\frac{\alpha}{\pi}\ln\frac{|t|}{|u|}\Big)_{real}\Big]$$

where $c = \cos\theta$, $E = \sqrt{s}/2$, and $\varepsilon << 1$ is IR regulator.

IFI in a simple toy model



Photon emissions M.E. is $\mathcal{M}^{\mu}(k) \simeq \mathcal{M}^{\mu}_{Born} J^{\mu}(k),\ J^{\mu}(k) = J^{\mu}_{I}(k) - J^{\mu}_{F}(k)$ and

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Finally, the remnant of the virtual not eaten up by real is:

$$\frac{d\sigma}{d\Omega}(c, K) \simeq \frac{d\sigma_{Born}}{d\Omega}\left[1 - \int_K^E \frac{dk^0}{k_0}\left(2\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} + 2\frac{\alpha}{\pi}\ln\frac{s}{m_\mu^2} - 4\frac{\alpha}{\pi}\ln\frac{|t|}{|u|}\right)_{virt.}\right]$$

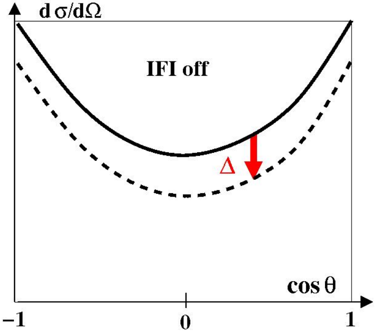
where $c = \cos\theta$, $E = \sqrt{s}/2$, and $\varepsilon << 1$ is IR regulator.

Let us now analyze the above result!

IFI in a simple toy model

First switch IFI off, make cutoff K stronger starting from $K = E$ ($\Delta = 0$):

$$\frac{d\sigma}{d\Omega}(c, K) \simeq \frac{d\sigma_{Born}}{d\Omega}\left[1 - \int_K^E \frac{dk^0}{k_0}\left(2\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} + 2\frac{\alpha}{\pi}\ln\frac{s}{m_\mu^2}\right)_{virt.} = \frac{d\sigma_{Born}}{d\Omega}(1 - \Delta(K/E))\right]$$



IFI in a simple toy model

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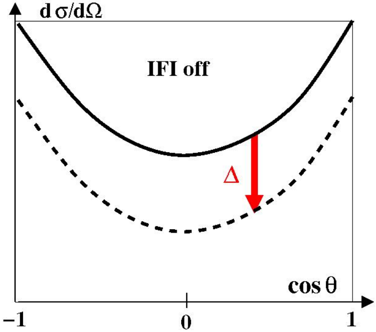
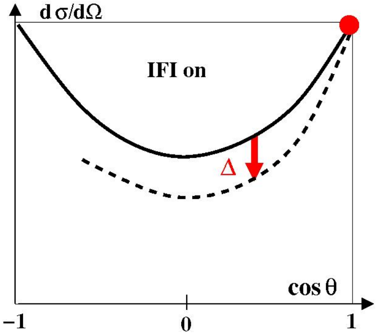


IFI is the king at $c = cos\theta = \pm 1$ ends

Now switch IFI on and look at $t \to 0$ ($c \to 1$) side, $s - |t| - |u| = 0$, $|u| \to s$.
IFI kills bot ISR and FSR $\to$ QED dies out in the forward scat.:

$$\Delta = \int_K^E \frac{dk^0}{k_0} \Big(2\frac{\alpha}{\pi} \ln\frac{s}{m_e^2} + 2\frac{\alpha}{\pi} \ln\frac{s}{m_\mu^2} - 4\frac{\alpha}{\pi} \ln\frac{|t|}{|u|} \Big) \to \int_K^E \frac{dk^0}{k_0} \Big(2\frac{\alpha}{\pi} \ln\frac{t}{m_e^2} + 2\frac{\alpha}{\pi} \ln\frac{t}{m_\mu^2} \Big) \simeq 0.$$

i.e. $d\sigma/d\Omega$ in the forward direct. $c = 1$, $t = 0$, stays unchanged!



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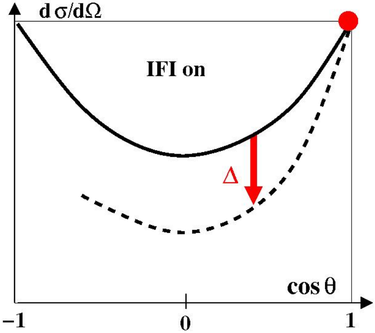
i.e. $d\sigma/d\Omega$ in the forward direct. $c = 1$, $t = 0$, stays unchanged!

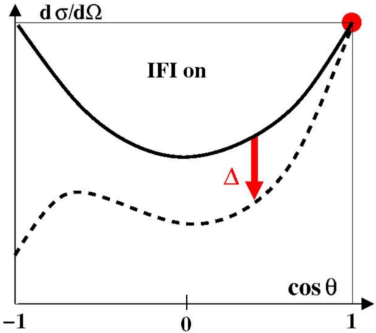


IFI is the king at $c = \cos\theta = \pm 1$

In the backward scattering: $u \to 0$ ($c \to -1$ side),
$s - |t| - |u| = 0$, $|t| \to s$, IFI enhances total QED corr. by factor 2:

$$\Delta = \int_K^E \frac{dk^0}{k_0}\left(2\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} + 2\frac{\alpha}{\pi}\ln\frac{s}{m_\mu^2} - 4\frac{\alpha}{\pi}\ln\frac{|t|}{|u|}\right) \to \int_K^E \frac{dk^0}{k_0}\left(4\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} + 4\frac{\alpha}{\pi}\ln\frac{s}{m_\mu^2}\right),$$

creating a dip in the muon angular distribution in backward direction.



Narrow resonance changes pattern of QED cancellations a lot...

In particular the role for IFI changes

Let us analyze simpler/cleaner example of $e^- e^+ \to R \to \mu^- \mu^+$,
at the resonance position $\sqrt{s} = M_R$:

- ISR: Virtual $\sim -\frac{2\alpha}{\pi} \ln \frac{s}{m_e^2} \ln \frac{E}{\lambda}$, as without resonance;
  Real $\sim +\frac{2\alpha}{\pi} \ln \frac{s}{m_e^2} \ln \frac{\Gamma_R}{\lambda}$ cut by resonance;
  $\sigma(K)$ suppressed by $\left[1 - \frac{2\alpha}{\pi} \ln \frac{M_R}{\Gamma_R}\right]$ for any cut $K > \Gamma_R$.
- FSR as without resonance: $\sigma(K)$ suppressed by $1 - \frac{2\alpha}{\pi} \ln \frac{s}{m_\mu^2} \ln \frac{E}{K}$
- IFI: Virtual $\sim -\frac{4\alpha}{\pi} \ln \frac{t}{u} \ln \frac{\Gamma_R}{\lambda}$ cut by resonance!!!
  Real $\sim +\frac{4\alpha}{\pi} \ln \frac{t}{u} \ln \frac{\Gamma_R}{\lambda}$ cut by resonance;
  $d\sigma(K)/d\Omega$ suppressed strongly by Γ_R/M_R for any cut $K > \Gamma_R$!
  And by milder $\left[1 - \frac{2\alpha}{\pi} \ln \frac{t}{u} \ln \frac{\Gamma_R}{K}\right]$ for $K < \Gamma_R$.

Away from resonance one goes gradually to the previous non-resonant case, and the QED calculation with the photon resummation at the amplitude level is mandatory.

It is not trivial but feasible, because soft photon approximation can be exploited.

Appendix B

Definition of $v_{ALEPH} = 1 - z_{ALEPH}$ deduced from muon angles (acollinearity) according to 1996 ALEPH note:

$$z_{ALEPH} = \frac{\sin\theta_1 + \sin\theta_2 - |\sin\theta_1 + \theta_2|}{\sin\theta_1 + \sin\theta_2 + |\sin\theta_1 + \theta_2|}.$$

Definition of muon scattering angle according to Phys.Lett. B219, 103 (1989):

$$\cos\theta_{PL} = (E_1\cos\theta_1 - E_2\cos\theta_2)/(E_1 + E_2)$$

Definition of muon scattering angle according to Phys.Rev. D41, 1425 (1990):

$$y_1 = \sin\theta_2/(\sin\theta_1 + \sin\theta_2), \quad y_2 = \sin\theta_1/(\sin\theta_1 + \sin\theta_2),$$

$$\cos\Theta_{PRD} = y_1\cos\theta_1 - y_2\cos\theta_2.$$

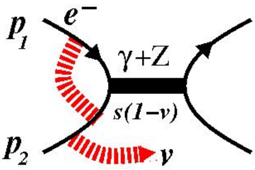
Appendix C
Semianalytical formulas for MC (KKMC) calibration

Step by step:

ISR, ISR+FSR and ISR+FSR+IFI.

High precision Z-lineshape QED ISR formula used at LEP

decades of work by: Yennie, Frautschi, Suura, Gribov Lipatov, Kuraev, Fadin, Greco, Pancherini, Srivastava, Jackson, Martin, Berends, Burgers, Jadach, Skrzypek, Ward,...



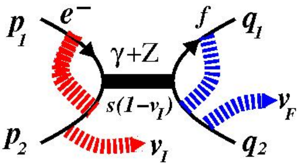
$$\sigma(s, v_{\max}) = \int_0^{v_{\max}} dv\; F(\gamma_I)\gamma_I v^{\gamma_I - 1}\; \sigma_B\big(s(1-v)\big)\; \big[1 + \mathrm{NIR}(\mathrm{v})\big],$$

$$F(\gamma) \equiv \frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}, \quad \gamma_I = 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

- Non-infrared perturbative function NIR(v), for $\delta\sigma/\sigma \simeq 2\times 10^{-4}$ precision, to be found in J.S.+Skrzypek+Pietrzyk Phys.Lett.B280(1992)129.
- One can add Electroweak corrections to σ_B, 1st order FSR, generalize to $d\sigma/d\Omega$ etc. as it was done in ZFITTER.

KKMC extensively tested with ISR+FSR (IFI off) formula

implemented in semianalytical program KKsem, part of KKMC distribution



$$
\frac{d\sigma}{d\Omega}(s,\theta,v_{\max}) = \int dv_I\, dv_F\, \delta(v - v_I - v_F)\theta(v < v_{\max})
$$

$$
\times\, F(\gamma_I)\gamma_I v_I^{\gamma_I - 1}\; F(\gamma_F)\gamma_I v_F^{\gamma_F - 1}\; \frac{d\sigma_0}{d\Omega}(s(1-v_I),\theta)\, \left[1 + \mathrm{NIR}(\mathrm{v_I}, \mathrm{v_F})\right],
$$

$$
v = 1 - (q_1 + q_2)^2/s, \quad \gamma_F = 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_f^2} - 1\right)
$$

- In KKsem $d\sigma_0/d\Omega$ is decorated with EW corrections
- For $v_{\max} < 0.2$ definition of θ is not essential.
- Non-IR function $\mathrm{NIR}(v_I, v_F)$ from analytical integration of the MC distributions.
- $\delta(v - v_I - v_F) \to \delta(1 - v - (1-v_I)(1-v_F))$ more realistic for hard emissions.