How QCD evolution kernels depend on the type of evolution variable

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Matter to the Deepest 2017
3-8 Sept. 2017, Podlesice, Poland

Outline

- Introduction
- The KrkMC projects
- Collinear factorization and evolution kernels
- Calculation of NLO kernels
- Results – kernels in various variables
- Summary

(Very) Long-term perspective

The NNLO + NLO Parton Shower for LHC

- LO Hard process + LO Shower
  Pythia, Herwig (1980-s)
- NLO Hard process + LO Shower
  MC@NLO, PowHEG (2000-s)
  KrkNLO (Jadach et.al., 2015)
- NLO Hard process + NLO Shower
  KrkMC (Jadach et.al., ongoing)

- NNLO Hard process + NLO Shower
  ??????????????????

Other developments: MINLO, Z. Nagy et.al., H. Tanaka et.al. . . .

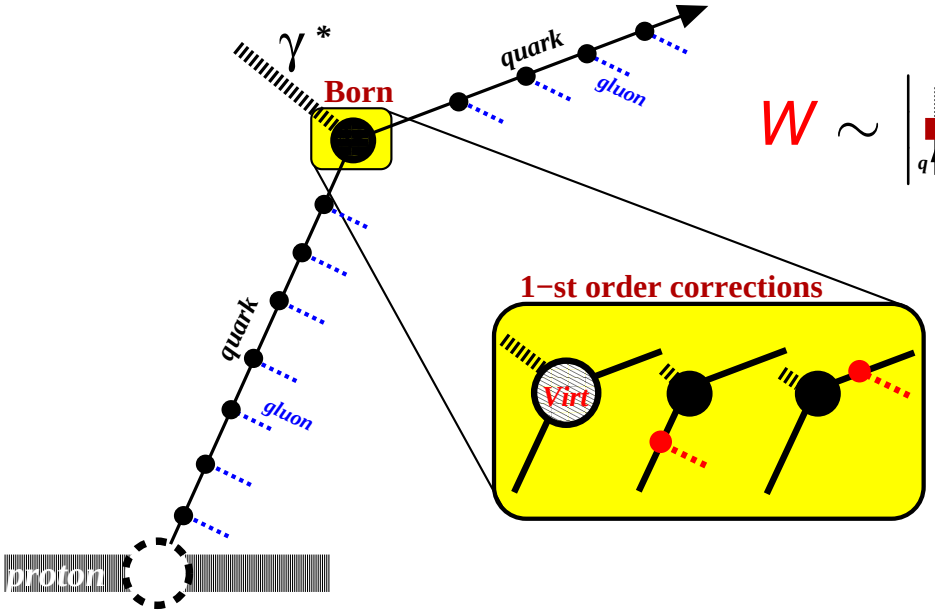
The KrkMC project

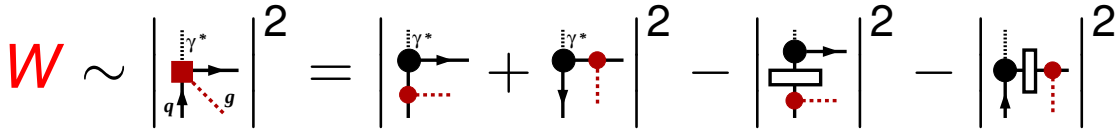
Based on collinear factorization. Requires:

- Reformulation of factorization in fully exclusive way
- Recalculation of the evolution kernels
  - exclusive
  - in four dimensions
  - well defined relation to MS-bar
- Kinematical mappings
- Reweighting procedure (positive, convergent)

Axial gauge instrumental – allows for physical interpretation

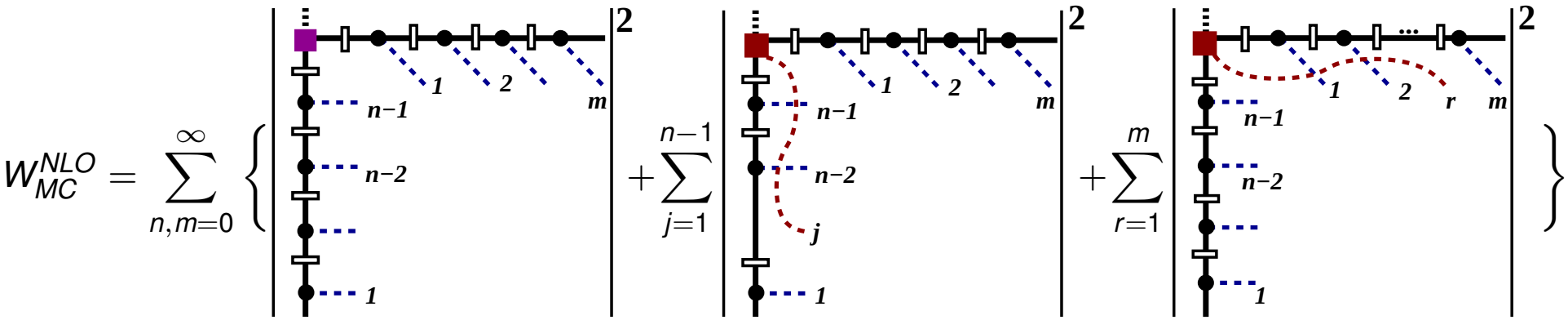
NLO-corrected Hard process





The KrkNLO project

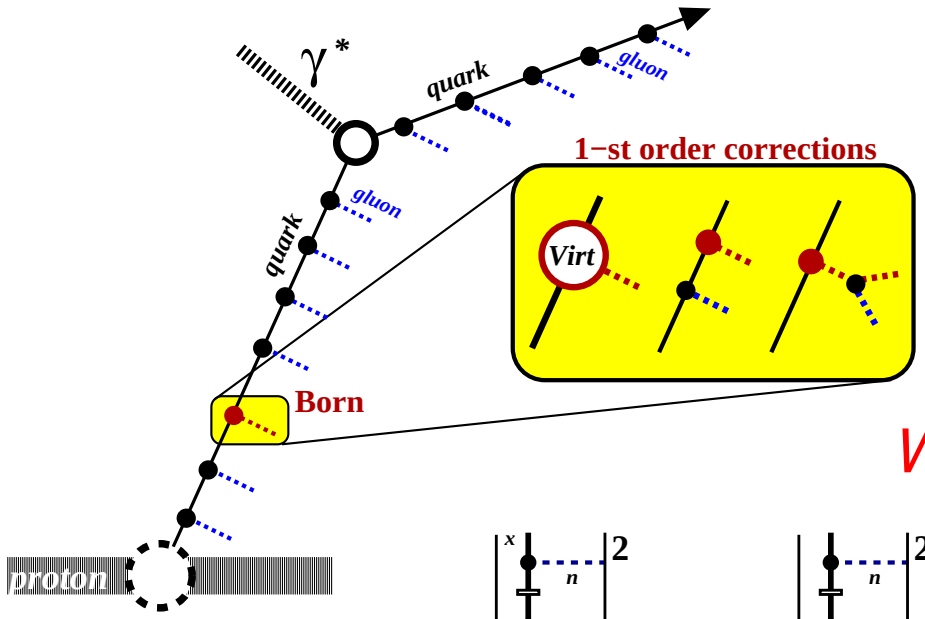
Alternative to
MC@NLO and POWHEG

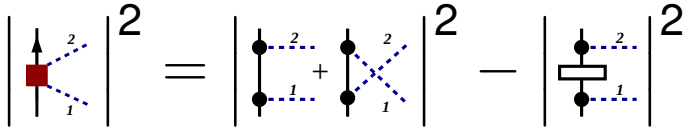


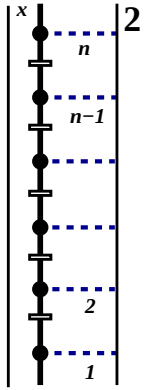
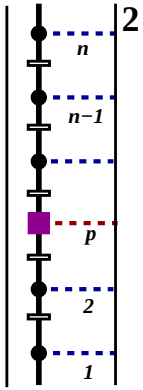
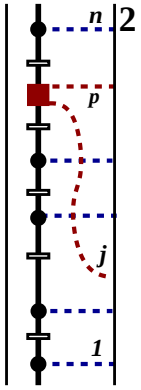
$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega},$$

NLO-corrected middle-of-the-ladder kernel, C_F^2

The KrkMC project



$W \sim$ 

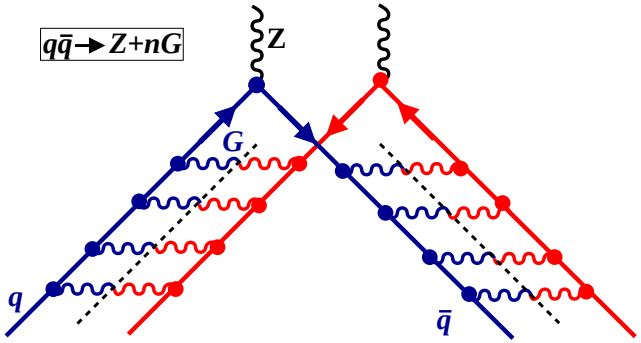
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$  $$+ \sum_{p=1}^{n}$$  $$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$  $$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}.$$

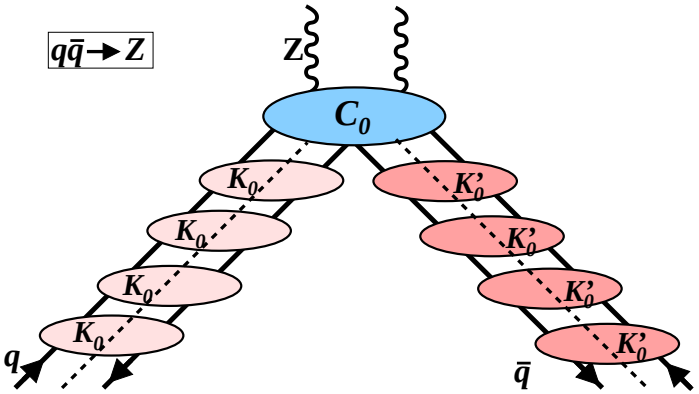
Prototype MC code based on the KrkMC scheme works!

Collinear factorization
and construction of the evolution kernel

LO cascade and
construction of the ladder

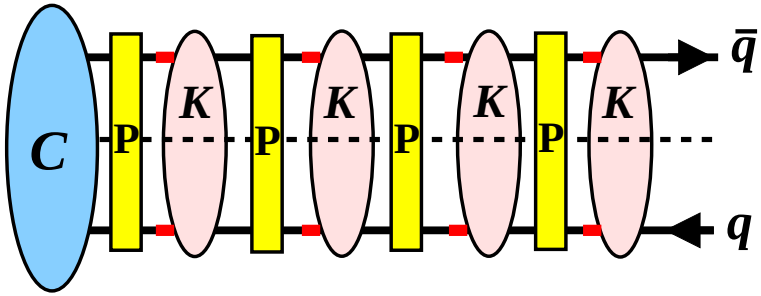


Include NLO and
group graphs in the ladder into
"two-particle-ireducible" sets K



Use "projection operators" P to
split the ladder and extract kernels

$$\Gamma_{qq} = \mathrm{Tr}\left[\frac{\hat{n}}{4nq} K \hat{p}\right]$$



Projection operators, kernels

The projection operators work as follows

- Close fermionic or gluonic lines
- Put incoming parton on-shell
- Extract pole parts of the expression

The kernel P_{qq} is defined as the residue of Γ:

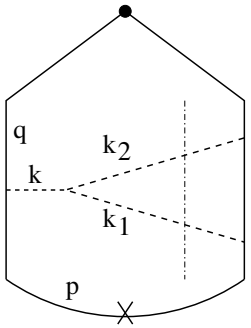
$$\Gamma_{qq} = \delta_{1-x} + \frac{1}{\epsilon}\left[\left(\frac{\alpha}{2\pi}\right)P^{(1)} + \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 P^{(2)} + \dots\right] + \dots$$

$$P_{qq} = \left(\frac{\alpha}{2\pi}\right)P^{(1)} + \left(\frac{\alpha}{2\pi}\right)^2 P^{(2)} + \dots$$

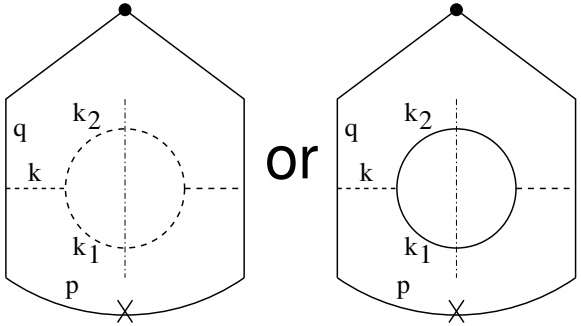
Graphs to be considered for P_{qq} kernel

Mechanisms of cancellation

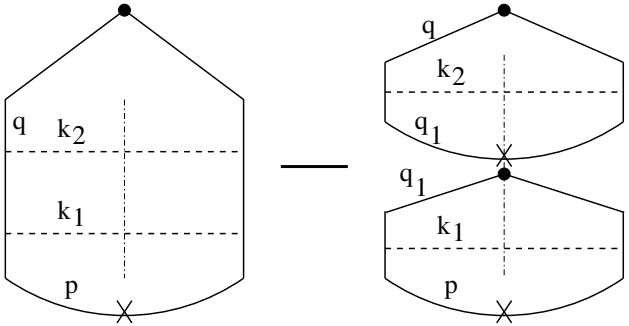
- Single pole $1/\epsilon$ graphs – invariant
- Triple pole $1/\epsilon^3$ graph – needs New PV prescription:



- Double pole – real-virtual cancellation:
  Partial cancellation only



- Double pole – cancelled by counter-term:

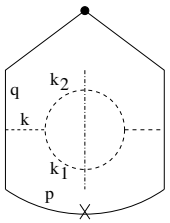
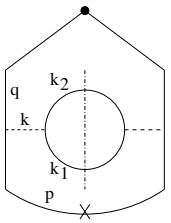


Cut-offs to be considered

Various ordering variables

- Standard one i.e. virtuality: $-q^2 < Q^2$
- Transverse momentum of the emitted partons: $\max\{k_{1\perp}, k_{2\perp}\} < Q$ or $k_{1\perp} + k_{2\perp} < Q$
- Rapidity (angle) of the emitted partons: $\max\{k_{1\perp}/\alpha_1, k_{2\perp}/\alpha_2\} < Q$ or $|\vec{k}_{1\perp} + \vec{k}_{2\perp}|/(\alpha_1 + \alpha_2) < Q$ because $k_\perp^2 = 2pn\alpha\beta \to k_\perp/\alpha \sim \sqrt{\beta/\alpha}$

We define $k_{i\perp} \equiv |\vec{k}_{i\perp}|$ and $k_i = \alpha_i p + \beta_i n + k_{i\perp}^{(m)}$

Calculation of  and 

The starting formula reads

$$\Gamma_G = c_G^V g^4 \; x \; \mathrm{PP}\left[\frac{1}{\mu^{4\epsilon}} \int d\Psi \delta\left(x - \frac{qn}{pn}\right) \frac{1}{q^4} W_G\right],$$

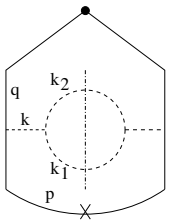
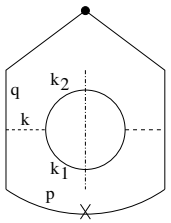
$$d\Psi = \frac{d^m k_1}{(2\pi)^m} 2\pi\delta^+(k_1^2) \frac{d^m k_2}{(2\pi)^m} 2\pi\delta^+(k_2^2)$$

$$= (2\pi)^{-2m+2} \frac{1}{4} \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} d^{m-2}\vec{k}_{1\perp} d^{m-2}\vec{k}_{2\perp},$$

$$c_G^V = \frac{1}{2} C_G C_F,$$

$$W_G = \frac{1}{4qn} \frac{1}{k^4} \mathrm{Tr}\left(\hat{n}\hat{q}\gamma^\mu \hat{p}\gamma^\lambda \hat{q}\right) d_{\nu''\nu'}(k_2) d_{\mu\mu''}(k_1 + k_2) d_{\lambda''\mu'}(k_1) d_{\mu'\lambda}(k_1 + k_2)$$

$$\times \; V(k_1^{\mu''} + k_2^{\mu''}, -k_2^{\nu''}, -k_1^{\lambda''}) V(k_1^{\mu'}, k_2^{\nu'}, -k_1^{\lambda'} - k_2^{\lambda'}).$$

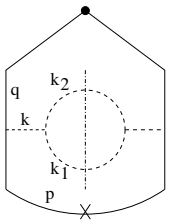
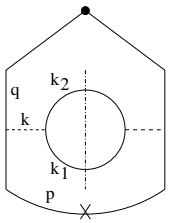
Technicalities for  and 

- These graphs have double poles: from $1/q^2$ and $1/k^2$
- Trick: calculate the difference w.r.t. the standard case $-q^2 < Q^2$

$$\Delta\Gamma^{X-q^2} = \Gamma(X < Q^2) - \Gamma(-q^2 < Q^2)$$

  Singularity at $q^2 = 0$ gets excluded, i.e. double pole is eliminated
- We look for a coefficient of single pole,
  so the other singular integral can be simplified to

$$dk^2 (k^2)^{-1+\epsilon} \to dk^2 \frac{1}{\epsilon} \delta(k^2 = 0)$$

Technicalities for  and  , cont.

- It is convenient to change variables

$$\vec{\kappa}_1 = \frac{\alpha_1}{\alpha_1+\alpha_2}\left(\vec{k}_{1\perp}+\vec{k}_{2\perp}\right), \quad \vec{\kappa}_2 = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\left(\frac{\vec{k}_{2\perp}}{\alpha_2}-\frac{\vec{k}_{1\perp}}{\alpha_1}\right)$$

so that the singular variables k^2 and q^2 become diagonal

$$k^2 = \frac{(1-x)^2}{\alpha_1\alpha_2}\kappa_2^2, \quad -q^2 = \frac{1-x}{\alpha_1}\left(\kappa_1^2\frac{1}{\alpha_1}+\kappa_2^2\frac{x}{\alpha_2}\right)$$

The case of $k_{1\perp} + k_{2\perp} < Q$

The real emission graphs Vf and Vg give

$$\Delta\Gamma^{k_\perp - q}_{Vf+Vg} = C_F \left(\frac{\alpha_S}{2\pi}\right)^2 \frac{1}{2\epsilon} \frac{1+x^2}{1-x} \ln \frac{1}{1-x} \Big[-\beta_0 + 4C_A \Big(l_0 + \ln(1-x) \Big) \Big]$$

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F.$$

and the virtual partners give just the opposite

$$\Delta\Gamma^{k_\perp - q}_{virtual} = -\Delta\Gamma^{k_\perp - q}_{Vf+Vg}$$

so we get complete cancellation – kernel remains unchanged

The same invariance of the kernel holds for rapidity (angular) variables $\max\{k_{1\perp}/\alpha_1, k_{2\perp}/\alpha_2\} < Q$ and $|\vec{k}_{1\perp} + \vec{k}_{2\perp}|/(\alpha_1 + \alpha_2) < Q$.

The case of $\max\{k_{1\perp} + k_{2\perp}\} < Q$

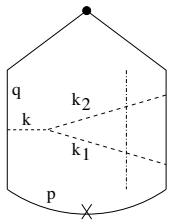
The real emission graphs Vf and Vg give this time

$$\Delta\Gamma^{\max k_\perp - q}_{Vf+Vg} = \Delta\Gamma^{k_\perp - q}_{Vf+Vg} + C_F\left(\frac{\alpha_S}{2\pi}\right)^2 \frac{1}{\epsilon}\frac{1+x^2}{1-x}\left[C_A\left(\frac{\pi^2}{3} - \frac{1}{8}\right) + \beta_0\left(\ln 2 - \frac{23}{24}\right)\right]$$

and we do not have complete cancellation.
This leads to the modification of the kernel

$$P_{qq}(\max\{k_{1\perp}, k_{2\perp}\} < Q) - P_{qq}(-q^2 < Q^2) = \\ = C_F\left(\frac{\alpha_S}{2\pi}\right)^2 \frac{1+x^2}{1-x}\left[C_A\left(\frac{2\pi^2}{3} - \frac{1}{4}\right) + \beta_0\left(2\ln 2 - \frac{23}{12}\right)\right]$$

This is the new result

Triple/Single pole graph 

(Yg)

Single pole graphs do not contribute to $\Gamma(X < Q^2) - \Gamma(-q^2 < Q^2)$

Yg has triple pole unless one modifies PV regularisation prescription

gluon propagator (axial gauge): $\frac{1}{l^2}\left(g^{\mu\nu} - \frac{l^\mu n^\nu + n^\mu l^\nu}{nl}\right)$

Spurious singul. $1/nl$ cancel in full set of diags, but need regulariz.

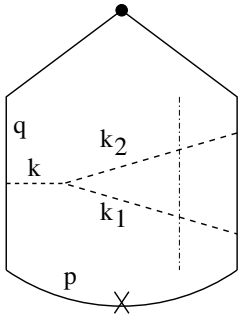
Standard CFP: regularize with PV $\boxed{\text{only the gluon propagator}}$

Principal Value: $\left[\frac{1}{nl}\right]_{PV} = \frac{nl}{(nl)^2 + \delta^2 (pn)^2}$

New proposal: regularize with PV $\boxed{\text{all (+)-singularities}}$ of the integrand

$$\frac{d^m l}{l_+^{1-\epsilon}} \to d^m l \left[\frac{1}{l_+}\right]_{PV} \left(1 + \epsilon \ln l_+ + \epsilon^2 \frac{1}{2} \ln^2 l_+ + \dots\right), l_+ = \frac{nl}{np}$$

Results for Yg graph with $-q^2 < Q^2$ cut-off



Standard PV [Heinrich, Kunszt, 1998]:

$$N(\epsilon, Q^2)\left[\frac{P_{qq}(x)}{\epsilon^3} - 2I_0\frac{P_{qq}(x)}{\epsilon^2} + \frac{p_{qq}(x)}{\epsilon}\Big(-2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\Big)\right] + \mathcal{O}\Big(\frac{1}{\epsilon}\Big)$$

$1/\epsilon^3$ poles, not invariant!

New PV prescription [Jadach et.al. 2011]:

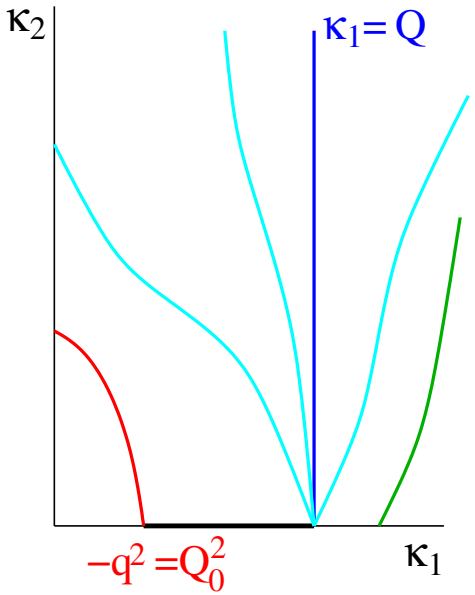
$$\frac{p_{qq}(x)}{\epsilon}\Big(2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\Big) + \mathcal{O}\Big(\frac{1}{\epsilon}\Big)$$

$1/\epsilon$ poles, INVARIANT!

$$I_0 = \int_0^1 \frac{dx}{[x]_{PV}} \sim \int_\delta^1 \frac{dx}{x} = -\ln\delta, \quad I_1 = \int_0^1 \frac{dx\ln x}{[x]_{PV}} \sim -\frac{1}{2}\ln^2\delta,$$

$$P_{qq}(x) = p_{qq} + \epsilon(1-x), \quad p_{qq} = \frac{1+x^2}{1-x}$$

General: Which evol vars lead to the same kernel



- The red $-q^2 = Q_0^2$ line marks the subtracted region below.
- The singularities lie at the origin of the frame ($q^2 = 0$) and along the line $\kappa_2^2 \sim k^2 = 0$.
- The integration path is the thick line along $\kappa_2 = 0$ between crossing points of $-q^2 = Q_0^2$ and the cut-off with the axis.
- Cut-offs (blue) are equivalent if they cross the κ_1-axis at the same point.

Some comments

- Obvious remark – the LO kernel is invariant.
- The dependence of the MS NLO kernel on the choice of the upper kinematic boundary Q is absolutely unthinkable!!!
  It contradicts the "theorem" proven in CFP paper.
- The problem is absent for gluonstrahlung (non-singlet) graphs.
- It is absent for single-pole diagrams.
- It is also absent for gluon splitting, for $Q = |\vec{k}_{1T} + \vec{k}_{2T}|$, $Q = k_{1T} + k_{2T}$ and for rapidities (angular ordering)
- The change is present only for diagrams with FSR gluon splitting into gluon pair calculated for $Q = \max\{k_{1T} + k_{2T}\}$, combined with the relevant virtual diagram counter-partner.
- Analytical inspection shows that source is the mismatch between upper kinematic limit in virtual and real diagram contributions.

Possible explanations

- CFP scheme is basically wrong?
- It is due to partial introduction of the dimensional regularization in CFP, only for collinear singularity, not for lightcone variable?
- It is due to the NPV regularisation instead of PV as in CFP?
- Is it due to the use of the axial gauge?
- Maybe it is just a different factorization scheme and it cancels when combined with NNLO hard process?
- Maybe it is absent in case of the k_T -ordering variant, which is actually used in the realistic PS MC like HERWIG or PYTHIA? Our k_T's are both with respect to incoming parton.