A new approach to ttbar @ NNLO

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Matter to the deepest

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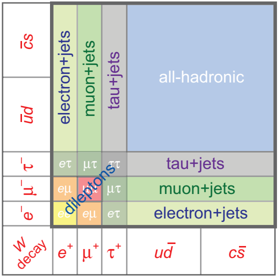
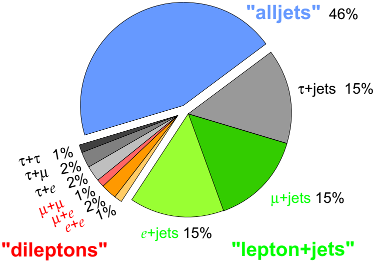
$t\bar{t}$ production at the LHC

LHC	$\sigma(tt)$ $[pb]$	L $[fb^{-1}]$	N$_{\text{event}}$
7 TeV	172.676	5	$8{,}6 \times 10^5$
8 TeV	246.652	19.7	$4{,}8 \times 10^6$
13 TeV	807.296	2.3	$1{,}8 \times 10^6$

Top Pair Decay Channels



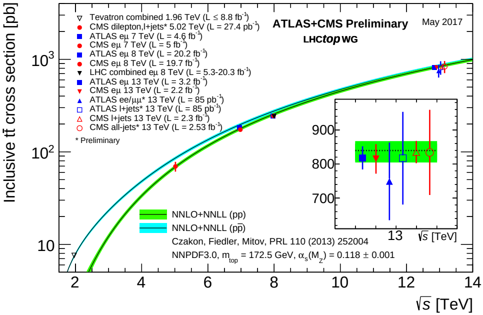
Top Pair Branching Fractions

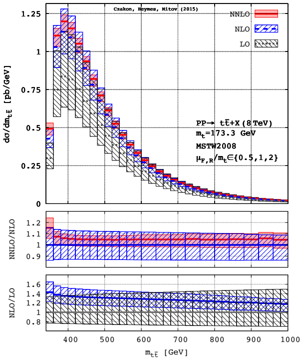


This give us a strong motivation to test, develop and improve pQCD for this process.

Status of pQCD for $t\bar{t}$ production

Only one complete NNLO calculation of inclusive and differential cross section, improved by NNLL resummation,





Overall good agreement. Scale uncertainty varies with kinematics: within 5 % for regions of interest for run I and II, JHEP 04 (17) 071. Other theoretical uncertainties for the total cross-section are: PDF $\sim 2-3\,\%, \alpha_s \sim 1{,}5\,\%, m \sim 3\,\%$ for the total XS.

Status of pQCD for $t\bar{t}$ production

There are other results for top production at $\mathcal{O}(\alpha_s^2)$ and beyond but they are partial or approximate:

- Abelof et. al. 2015. Leading N_c total cross-section NNLO.
- Catani , et. al, 2015. Partial results for NNLO +resummation. Small-q_T resummation + qT subtraction. Missing piece: soft NNLO evolution.
- Broggio et. al. 2014, Ahrens et.al, 2010 (SCET). Threshold resummation + RG.
- Kidonakis 2015. Approximate NNNLO, soft gluon corrections to single top production. See also 2012, NNLL threshold resummation.

Making comparisons is highly non-trivial.

In this talk: New approach to $t\bar{t}$ production @ NNLO in the small $q_\perp (= p_t + p_{\bar{t}})_\perp$ region. Our approach is numerical and highly automated (graph independent) and has the potential to be extend to other processes (e.g. $gg \to H$ @ NNNLO).

Why to look at the small q_T region?

Top pair production with $q_T = 0$ only occurs for Born kinematics and this implies that (Catani & Grazzini 2007, 2015)

$$\mathrm{d}\sigma^{t\bar{t}}_{\text{NNLO}}\Big|_{q_\perp = q_{0\perp} > 0} = \mathrm{d}\sigma^{t\bar{t}+\text{jet}}_{\text{NLO}}\Big|_{q_\perp = q_{0\perp}}$$

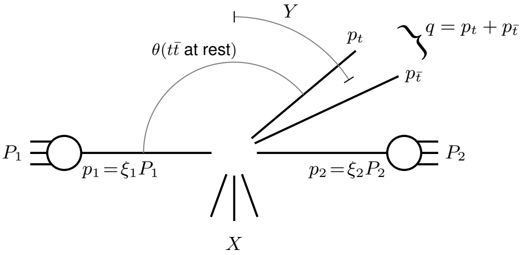
Hence, cross-sections (distributions) integrated over $q_\perp$ can be written as

$$\sigma^{t\bar{t}}_{\text{NNLO}} = \int^{q_{0\perp}} \mathrm{d}q_\perp \mathrm{d}\sigma^{t\bar{t}}_{\text{NNLO}} + \int_{q_{0\perp}} \mathrm{d}q_\perp \mathrm{d}\sigma^{t\bar{t}+\text{jet}}_{\text{NLO}}$$

The NLO results exists (e.g. Catani et. al. 2002, Czakon 2010) for $tt +$ jet. The first part is missing and we aim for it!

Soft collinear Effective Theory for $t\bar{t}$ at small $q_\perp$

$$\Lambda_{QCD} \ll q_\perp^2 \ll m^2,$$
$$s = (p_1 + p_2)^2, q^2,$$
$$(p_1 - p_t)^2 - m^2,$$
$$(p_1 - p_{\bar{t}})^2 - m^2$$



Leading radiation (X) factorises as (Xing Zhu, et. al PRD 88 (13) 074004)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2\mathrm{d}q_\perp^2\,\mathrm{d}\theta\mathrm{d}Y} = \sum_X B_1 \otimes B_2 \otimes H \otimes S + O\left(\frac{\Lambda_{QCD}^2}{q_\perp^2}, \frac{\Lambda_{QCD}^2}{q^2}\right)$$

i.e. covers wide range of differential observables.

B_i and H known at NNLO (Gehrmann et. al. ('14) and Czakon et. al. ('13)). We aim for S.

Advatages of this approach: recycles the most and it is generable!

SCET factorisation

In the small $q_\perp$ limit, four regions are not power suppressed by $\lambda = \sqrt{q_\perp^2/q^2}$ are $(k^+, k^-, k_\perp)$

- Hard $(1, 1, 1)$
- Collinear $(1, \lambda^2, \lambda)$
- Anti-collinear $(\lambda, 1, \lambda)$
- Soft $(\lambda, \lambda, \lambda 1_\perp)$

After azimuthal averaging the cross section factorises as

$$\frac{d^4\sigma}{\mathrm{d}q_\perp^2\, \mathrm{d}y\, \mathrm{d}q^2\, \mathrm{d}\cos\theta} \sim \int \mathrm{d}\xi_1\, \mathrm{d}\xi_2\, \mathrm{d}x_\perp (\ldots) \times \sum_{i=\mathrm{q},\bar{\mathrm{q}},\mathrm{g}}$$

$$B_i^{LT}(\xi_1, x_\perp^2)\, B_{\bar i}^{LT}(\xi_2, x_\perp^2) \cdot \mathrm{Tr}\left[\mathbf{H}_{i\bar i}^{LT}(q^2, m, \vec{v}_t) \int \mathrm{d}\Omega_{x_T}^{d-3} \mathbf{S}_{i\bar i}(\vec{x}_\perp, \vec{v}_t)\right] + \mathcal{O}(\alpha_s^3)$$

where $\vec{v}_t$ is the top momenta in the $t\bar{t}$ rest frame, i.e.

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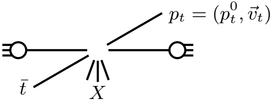
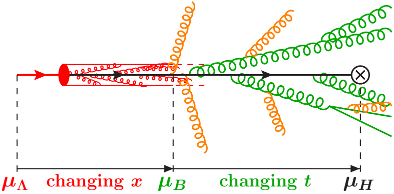


Beam functions

Generalisation of PDF, characteristic of measurements involving two scales $\mu_{\Lambda_{QCD}} \ll \mu_B \ll \mu_H$, where μ_B constrains the energy in forward direction

$$B_i(x'_\perp, x, \mu_B) = \sum_j \int_x^1 \frac{\mathrm{d}\xi}{\xi} \mathcal{I}_{ij}(x'_\perp, \frac{x}{\xi}, \mu_B) \overbrace{f_j(\xi, \mu_B)}^{\text{PDF}}$$



JHEP 1009 (2010) 005

$\mathcal{I}_{ij}$ accounts for nearly collinear radiation with wide-spread $x_\perp \leq \dfrac{1}{f(q_\perp, q^*)}$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} B_i(x'_\perp, x, \mu) = \int \mathrm{d}x'_\perp \gamma_B^i(x_\perp - x'_\perp, \mu) B_i(x'_\perp, x, \mu)$$

Hard function

Roughly speaking, the hard function is the finite loop corrections to the born process $\mathcal{M}$. More precisely, the matching condition is (Ahrens et. al. 1003.5827)

$$\mathbf{H}(q^2, m, \vec{v}_3, \mu) = \frac{12}{8(4\pi)^2 d_R} \sum_{i=q,\bar{q},g} \boldsymbol{Z}^{-1}(\epsilon,\mu) \left|\mathcal{M}^{\text{ren}}\right\rangle \left\langle\mathcal{M}^{\text{ren}}\right| \boldsymbol{Z}^{-1\,\dagger}(\epsilon,\mu)$$

where $\boldsymbol{Z}^{-1}$ is an operator that removes the poles ($d = 4 - 2\epsilon$) the infrared part of on-shell scatterings.

It has a perturbative expansion and for $m = 0$ it can be easily related to the Catani operators that factorises infrared poles

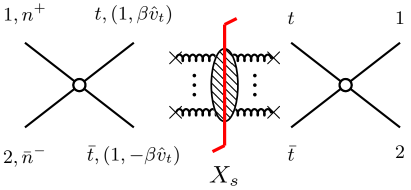
$$\boldsymbol{Z}^{(1)} = 2\mathbf{I}^{(1)}\text{finite}, \qquad \boldsymbol{Z}^{(2)} = \mathbf{I}^{(2)} - 2\mathbf{I}^{(1)}\boldsymbol{Z}^{(1)} + \text{finite}.$$

Soft function in momentum space

Soft radiation with at fixed q_T for the on-shell (crossing the red cut):

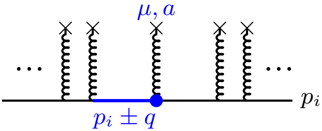
$$\boldsymbol{S}(\vec{q}_\perp, \vec{v}_t, \mu) =$$

$$\sum_{X_s} \int_{\{k_i\}} \Bigg[$$



$$\Bigg] \delta^{d-2}\left(\vec{q}_\perp - \sum_i \vec{k}_{i\perp}\right) \prod_i \frac{\delta^+(k_i)}{(k^+)^\alpha}$$

Feynman rules for the blob are exact, and this is connect to the hard subprocess in the Eikonal approximation:



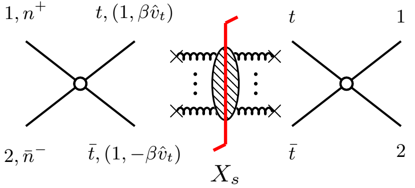
$$\frac{g_s \, \mathbf{T}_i^a \, p_i^\mu}{p_i \cdot q \pm (i0, 0)}$$

Azimuthally averaged soft function

The q_T dependence factorises in general, i.e.

$$\overline{\boldsymbol{S}}(\vec{q}_\perp,\vec{v}_t,\mu)=\int\frac{\mathrm{d}\Omega_q^{d-3}}{\Omega_q^{d-3}}\boldsymbol{S}(\vec{q}_\perp,\vec{v}_t,\mu)=$$

$$\sum_{X_s}2(q_\perp^2)^{G_{x_s}}\left[\int_{\{k_i\}}\right.$$



$$\left.\right]\delta\left(1-\left|\sum_i\vec{k}_{i\perp}\right|^2\right)\prod_i\frac{\delta^+(k_i)}{(k^+)^\alpha}$$

where G_{x_s} power and ($\hat{v}_t=(\cos\theta,\vec{v}_{t\perp})$).

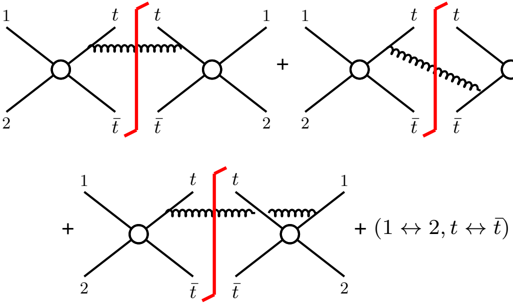
The integrand is now dimensionless, and Lorentz invariance implies that

$$\overline{\boldsymbol{S}}(\vec{q}_\perp,\vec{v}_t,\mu)=\overline{\boldsymbol{S}}(q_\perp^2,\beta,\cos\theta,\mu)$$

Soft function @ NLO

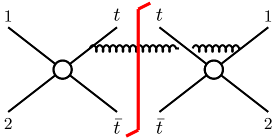
Its calculation neglects recoil $\Rightarrow$ many integrals become scaleless. Only graphs involving massive partons contribute

$$\overline{\mathbf{S}}(q_\perp, \theta, \beta, \mu)\Big|_{\alpha_s}$$



Rapidity divergences

The price paid for neglecting recoil is introducing spurious singularities at intermediate stages



$$\propto \int \mathrm{d}^d k \frac{\delta^+(k)}{(k^+)^\alpha} \frac{\delta(1-k_\perp^2)}{k^+(1-\vec{k}\cdot\vec{v}_t)}$$

Over the region $(k^+ \sim \lambda, k^- \sim \lambda^{-1}, k_\perp \sim \lambda^0)$ this integral has a rapidity divergence

$$\sim \int^\infty \frac{\mathrm{d}k_+}{(k^+)^{1+\alpha}} \tag{1}$$

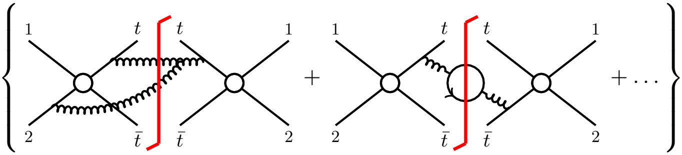
This is a consequence of using soft approximations for high energy modes ($k_0 \sim \lambda^{-1}$), but such regions cancel at cross section level.

A complex α regulate divergences preserving gauge invariance and keep scaleless integrals scaleless.

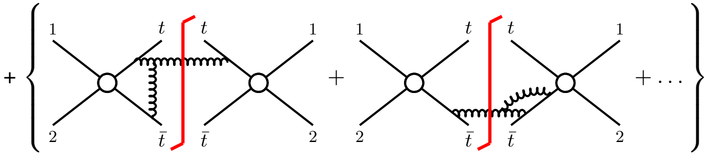
Soft function @ NNLO

The same regularisation procedure can be used and only graphs that involve at least one top quark contribute

$$\overline{\mathbf{S}}(q_\perp, \theta, \beta, \mu)\Big|_{\alpha_s^2} =$$



Double cuts



Single cuts

The loop has been integrated out analytically (Bierenbaum et. al. 2011). We focus on the double cuts (higher dimensional integrals).

Integration strategy

We designed, and automated, a method which can be applied to all double cuts.

Non-trivially, this is possible because all graphs share a common structure that can be algorithmically exploited:

1. Identify ALL divergences.
2. Map integration variables to hypercubes
3. Splittings/Power counting
4. Sector decomposition
5. Outside of the boundary: weighing the boundary
6. Numerical integration, send to Cuba (Hahn, Comput. Phys. C. 168 (05) 78)

Preliminaries

In general, every double cut graph G, of the NNLO soft function, can be written as a product of a infrared part and a weight part:

$$G \propto \int \underbrace{\mathrm{d}^d k_1 \mathrm{d}^d k_2 \; \frac{\delta^+(k_1)\delta^+(k_2)}{(k_1^+ k_2^+)^\alpha} \delta\left(1 - |k_{1\perp} + k_{2\perp}|^2\right)}_{\text{common}} \; \underbrace{\mathcal{I}_G \times \mathcal{W}_G}_{\text{graph part}},$$

the defining property of $\mathcal{W}$ is that remains finite no matter the kinematics top pair.

Boundary and its divergences

We define as boundary the region in particular kinematics where both top quarks are produced at rest $p_t = p_{\bar{t}} = (m, \vec{0})$ and this implies $\mathcal{W}_G = cte$

$$G \propto \int \underbrace{\mathrm{d}^d k_1 \mathrm{d}^d k_2 \; (\ldots)}_{common} \; \mathcal{I}_G$$

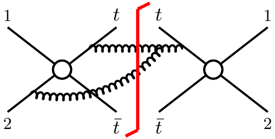
For any graph divergences appear when $(k^{\pm}, k^{\mp}, k_{\perp})$:

- Soft $\sim (\lambda, \lambda, \lambda)$,
- Initial state collinear $\sim (\lambda^2, 1, \lambda)$,
- Final state collinear $k_1 \cdot k_2 = 0$,
- Rapidity divergence $(\lambda, \lambda^{-1}, 1)$,
- Azimuthal integrable singularity ($\sim \lambda^{-1/2}$) of the measure $\vec{k}_{2\perp} \cdot \vec{k}_{2\perp}/(k_{1\perp} k_{2\perp}) \to \lambda$,

Divergent regions overlap!

Example

Integrating out deltas and irrelevant angles, one end up with a four dimensional integral, e.g.



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$$\int \frac{\mathrm{d}k_{+1}\mathrm{d}k_{+2}\mathrm{d}k_{1T}\mathrm{d}k_{2T}}{k_{1+}^{1+\alpha}\, k_{2+}^{\alpha-1}[k_{2+}^2 + k_{2T}^2]} \frac{k_{1T}k_{2T}\left[\left[1-(k_{1T}-k_{2T})^2\right]\left[1-(k_{1T}+k_{2T})^2\right]\right]^{-\frac{1}{2}-\epsilon}}{\left[k_{1T}^2 k_{1+}k_{2+} + k_{2T}^2 k_{1+}k_{2+} + k_{1T}^2 k_{2+}^2 + k_{2T}^2 k_{1+}^2 - k_{1+}k_{2+}\right]}$$

The integrand diverges when:

- $k_{1+} \to 0$,
- $k_{2+} \to 0$ and $k_{1+} \to 0$,
- $k_{1+} \to 0$ and $k_{1\perp} \to 0$,
- $k_1 \cdot k_2 \to 0$.

Mappings

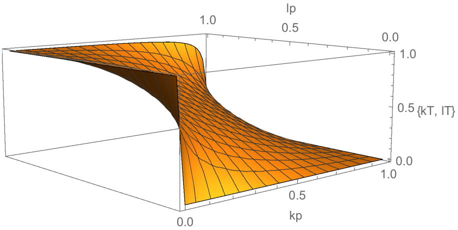
Before SD can be applied, integration variables should be mapped to hypercubes,

$$\int_{[0,\infty)^4]} \mathrm{d}k_{1+}\mathrm{d}k_{2+}\mathrm{d}k_{1\perp}\mathrm{d}k_{2\perp}\,(...) = \sum_{n=0}^{N} \int_{[0,1]^4} \mathrm{d}x_1 \ldots \mathrm{d}x_4 \widetilde{\mathcal{I}}_G,$$

with the additional constraint that $\widetilde{\mathcal{I}}_G$ should have at most end-point singularities.

This is not trivial since there are singularities occurring on a manifold,

$$\frac{1}{k_1 \cdot k_2} \to \infty \Leftrightarrow$$



Not unique way, but power counting methods allow us to keep $N \leq 4$.

Sector decomposition

Method to factorise singularities of divergent $(k_i^{-\alpha} \mathrm{d}^{4-2\epsilon} k_i)$,

$$\int_{[0,1]^n} \mathrm{d}^n\vec{x}\ \mathcal{I}(\vec{x},\epsilon,\alpha) = \sum_{r,s} \frac{1}{\alpha^r \epsilon^s} \underbrace{\int \mathrm{d}^n\vec{x}\ \mathcal{F}_{rs}(\vec{x})}_{\substack{\text{finite} \\ \text{Analytically / Numerically}}} ,$$

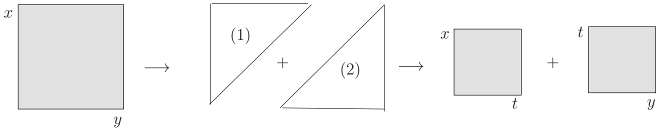
The concept existed for some time (Hepp 1966) but the quest for efficiency and automation continues (e.g. Borowka et. al. '16).

We use our on implementation based on Binoth & Heinrich '00.

Example

1.- Disentangle overlapping singularities

$$\int_0^1 dx \int_0^1 dy \, \frac{\overbrace{\mathcal{W}(x,y)}^{\text{Weight part}}}{\underbrace{(x+y)^{2+\epsilon}}_{\text{Infrared part}}} = \int_0^1 dx \int_0^1 dt \, \frac{\mathcal{W}(x, t\,x)}{(1+t)^{2+\epsilon} x^{1+\epsilon}} + \int_0^1 dt \int_0^1 dy \, \frac{\mathcal{W}(y\,t, y)}{(1+t)^{2+\epsilon} y^{1+\epsilon}}$$



The algorithm is independent of $\mathcal{W}$!!!

2.- Use *plus prescription* (add a clever zero)

$$\int_0^1 \int_0^1 \mathrm{d}x\, \mathrm{d}t \, \frac{\mathcal{W}(x, t\,x)}{x^{1+\epsilon}(1+t)^{2+\epsilon}} = \mathcal{W}(0,0) \int_0^1 \int_0^1 \frac{\mathrm{d}x \mathrm{d}t}{x^{1+\epsilon}} + \underbrace{\int_0^1 \int_0^1 \mathrm{d}x\, \mathrm{d}t \, \frac{[\mathcal{W}(x, t\,x) - \mathcal{W}(0,0)]}{x^{1+\epsilon}(1+t)^{2+\epsilon}}}_{c_0 + \epsilon\, c_1 + \epsilon^2\, c_2 + \dots}$$

Key steps of sector decomposition

1.- Disentangle and factorise singularities ($k_i^{-\alpha}\mathrm{d}^{4-2\epsilon}k_i$)

$$\int\limits_{[0,1]^n} \mathrm{d}^n\vec{x}\, f(\vec{x},\epsilon,\alpha) = \sum_i \int\limits_{[0,1]^n} \mathrm{d}^n\vec{x}\; \mathcal{F}_i(\vec{x},\epsilon,\alpha) \prod_j \frac{1}{x_j^{r_i+s_i\alpha+t_i\epsilon}}$$

with $r_i, s_i, t_i \in \mathbb{R}$ and $\mathcal{F}_i(\vec{x},\epsilon,\alpha)$ integrable.

2.-Plus prescription

$$\int_0^1 \mathrm{d}\,x_j \frac{\mathcal{F}_i(\ldots,x_j,\epsilon,\alpha)}{x_j^{r_{ij}+s_{ij}\alpha+t_{ij}\epsilon}} = \sum_{n=0}^{|r_{ij}|-1} \frac{\mathcal{F}_i^{(n)}(\ldots,0,\epsilon,\alpha)}{n!} \overbrace{\int_0^1 \mathrm{d}\,x_j\, x_j^{n-r_{ij}-s_{ij}\alpha-t_{ij}\epsilon}}^{=\frac{1}{n-r_{ij}-s_{ij}\alpha-t_{ij}\epsilon+1}}$$

$$+\int_0^1 \mathrm{d}\,x_j \frac{\mathcal{F}_i(\ldots,x_j,\epsilon,\alpha) - \sum_{n=0}^{|r_{ij}|-1} \frac{\mathcal{F}_i^{(n)}(\ldots,0,\epsilon,\alpha)x_j^n}{n!}}{x_j^{r_{ij}+s_{ij}\alpha+t_{ij}\epsilon}}$$

After this is done for all $\{x_i\}$ one can expand in (α,ϵ).

Re-weighting

All the previous steps (mapping, power counting, sector decomposition and ϵ and α expansion) are valid for any weight!

$$G \propto \int \underbrace{\mathrm{d}^d k_1 \mathrm{d}^d k_2 \; \frac{\delta^+(k_1)\delta^+(k_2)}{k_1^+ k_2^+} \delta\left(1 - |k_{1\perp} + k_{2\perp}|^2\right)}_{\text{common}} \underbrace{\mathcal{I}_G \times \mathcal{W}_G}_{\text{graph part}},$$

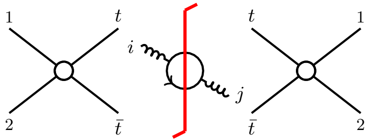
but, in principle the situation is far more difficult because of the angles between $\{\vec{v}_{t\perp}, \vec{k}_{1\perp}, \vec{k}_{2\perp}\}$.

There is way round this, we can recycle the angles chosen over the boundary using that:

$$\mathcal{I}_G(\neq v_{t\perp}), \qquad G(v_{t\perp}^2) = \frac{1}{\Omega^{d-3}} \int \mathrm{d}\Omega^{d-3} G(v_{t\perp}^2).$$

Cross-Check 1: Cancellation of alpha poles

Only graphs with a fermion loop are proportional to n_f, hence cancellation of α poles must occur within this subset:

$\mathsf{F}_{ij} \equiv$ 

$$\mathsf{F}_{1t} = n_f \mathbf{T}_1 \cdot \mathbf{T}_t \left(\frac{c}{\alpha} + \ldots \right), \qquad \mathsf{F}_{2t} = -n_f \mathbf{T}_1 \cdot \mathbf{T}_t \left(\frac{c}{\alpha} + \ldots \right)$$

$$\mathsf{F}_{1\bar{t}} = n_f \mathbf{T}_1 \cdot \mathbf{T}_{\bar{t}} \left(\frac{c}{\alpha} + \ldots \right), \qquad \mathsf{F}_{2\bar{t}} = -n_f \mathbf{T}_1 \cdot \mathbf{T}_{\bar{t}} \left(\frac{c}{\alpha} + \ldots \right),$$

$$c(\text{analytically}) = -\frac{8}{3\alpha\epsilon} - \frac{8(3\gamma + 5 - 3\log(2))}{9\alpha}$$

$$c(\text{numerically}) = -\frac{4{,}13}{\alpha} - \frac{2{,}66}{\alpha\epsilon} + \mathcal{O}(10^{-3})$$

The sum of α poles cancels due to colour conservation and the fact initial state parton has the same flavour:

$$(\mathbf{T}_1 - \mathbf{T}_2) \cdot (\mathbf{T}_t + \mathbf{T}_{\bar{t}}) = (\mathbf{T}_1^2 - \mathbf{T}_2^2) = 0$$

Cross-check 2: SCET renormalisation

The ϵ poles can be removed by SCET infrared renormalisation. Hence, they should be equal and opposite

$$\boldsymbol{S}(\mu) = \boldsymbol{Z}_s^\dagger(\mu,\epsilon)\boldsymbol{S}_{\text{bare}}(\epsilon)\boldsymbol{Z}_s(\mu,\epsilon)$$
$$\boldsymbol{S}(\mu) = \boldsymbol{S}^{(0)}(\mu) + \alpha_s(\mu)\,\boldsymbol{S}^{(1)}(\mu) + \mathcal{O}(\alpha_s^3)$$
$$\boldsymbol{Z}_s(\mu) = \boldsymbol{Z}_s^{(0)}(\mu) + \alpha_s(\mu)\,\boldsymbol{Z}_s^{(1)}(\mu) + \mathcal{O}(\alpha_s^3)$$

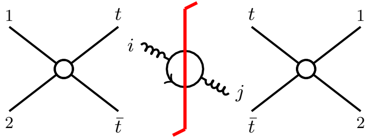
$\boldsymbol{Z}$ has an universal structure, its soft part $\boldsymbol{Z}_s(\epsilon)$ for $t\bar{t}$ production is already known (Xing Zhu et. al. PRL 110.082001).

We checked the cancellation of ϵ poles for the fermion bubble!

Cross-check 3: Analytic vs numeric fermion bubble

By means of the partial differential equations approach, we have been able to solve analytically the fermion bubble up to order $\epsilon^0\alpha^0$,

$\mathsf{F}_{ij} \equiv$

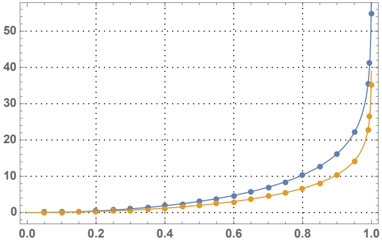


The kinematical part of graphs connecting top quarks yields

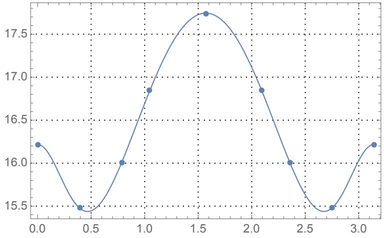
$$
\begin{aligned}
&2\mathsf{F}_{t\bar{t}} - \mathsf{F}_{tt} - \mathsf{F}_{t\bar{t}}\Big|_{\text{kinematical}} \propto \frac{-8}{\epsilon}\,\frac{\left(\left(\beta^2+1\right)\ln\left(\frac{1-\beta}{\beta+1}\right)+2\beta\right)}{3\beta} + \\
&\frac{8}{9\beta}\left[-\left(\beta^2+1\right)\mathsf{Ln}\left[\frac{2}{\beta+1}-1\right]\left(3\gamma+24\mathsf{Ln}\left[\frac{1}{256}\cos\left(\frac{\theta}{2}\right)\right]+5\right)\right. \\
&\left.+\,\beta\left[12\mathsf{Ln}\left[\frac{\sqrt{2}(1-\beta^2)}{1-\beta^2\cos^2\theta}\right]-10-6\gamma\right]+6\left[\beta^2+1\right]\left[\mathsf{Li}_2\left[\frac{(\beta-1)\tan^2\left(\frac{\theta}{2}\right)}{\beta+1}\right]-\mathsf{Li}_2\left[\frac{(\beta+1)\tan^2\left(\frac{\theta}{2}\right)}{\beta-1}\right]\right]\right]
\end{aligned}
$$

Cross-check 3: Analytic vs numeric fermion bubble

Our numerical evaluation, with absolute accuracy of $(O)(10^{-3})$, shows agreement numerics/analytics of the pole part (yellow) and the finite part (blue)



Graphs ($\theta = 0$) vs β



Graphs ($\beta = 0{,}9$) vs θ

$p_t = \dfrac{m}{\sqrt{1-\beta^2}}\,(1, \beta\cos\theta, p_{tT})$ is the top momentum in the $t\bar{t}$ rest frame.

Conclusions

- Show progress on a corroboration of the tt cross sections at NNLO.
- We use a SCET and note that its evaluation can be automated.
- Integration strategy is graph independent.
- Our approach recycles a maximal number of well tested strategies in the literature.
- Validation using fermion bubble: 1) α poles cancel, 2) ϵ poles renormalised and 3) agreement with analytics (partial differential equations).
- This approach can be generalised to other processes, e.g. $gg \to H$ at NNNLO.

Acknowledgements

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In general, the beam functions associated to gluon parent partons have a non-trivial Lorentz structure, e.g.

$$B_g^{\mu\nu} = \frac{g_\perp^{\mu\nu}}{2} B_g + \left(\frac{g_\perp^{\mu\nu}}{2} + \frac{x_\perp^\mu x_\perp^\nu}{x_\perp^2} \right) B_g' \,,$$

$$B_g(z, x_\perp^2, \mu) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^i B_g^{(i)}$$

$$B_g'(z, x_\perp^2, \mu) = \sum_{i=2}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^i B_g^{\prime(i)}$$

where $g_\perp^{\mu\nu} = g_{\mu\nu} - (p_1^\mu p_2^\nu - p_2^\mu p_1^\nu)/p_1 \cdot p_2$. Moreover, up to NNLO only the transverse part contributes since

$$\int \mathrm{d}^{d-2} x_\perp \, f(x_\perp^2) \left(\frac{g_\perp^{\mu\nu}}{2} + \frac{x_\perp^\mu x_\perp^\nu}{x_\perp^2} \right) \mathbf{H}^{(0)}_{\mu\nu\alpha\beta}(q^2, m, \vec{v}_3, \mu)\, g_\perp^{\alpha\beta} = 0 \qquad (2)$$

Simplified factorisation up to NNLO

Azimuthal integration commutes past the hard and beam functions, see A. Adler Master thesis,

$$\frac{d^4\sigma}{\mathrm{d}q_\perp^2\,\mathrm{d}y\,\mathrm{d}q^2\,\mathrm{d}\cos\theta} = \frac{\Omega_{d-3}c_\epsilon\,\beta}{s\sqrt{q^2}}\int \mathrm{d}\xi_1\,\mathrm{d}\xi_2\, x_T^{d-3}\mathrm{d}x_\perp\, J_0(x_\perp q_\perp)$$

$$\sum_{i=\mathrm{q},\bar{\mathrm{q}},\mathrm{g}}\left(\frac{x_\perp^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{i\bar{i}}(x_\perp^2\mu^2)} B_i(\xi_1, x_\perp^2, \mu)\, B_{\bar{i}}(\xi_2, x_\perp^2, \mu)$$

$$\times\,\mathrm{Tr}\left[\overline{\mathbf{H}}_{i\bar{i}}(q^2, m, \vec{v}_t, \mu)\int\frac{\mathrm{d}\Omega_{d-3}}{\Omega_{d-3}}\mathbf{S}_{i\bar{i}}(\vec{x}_\perp, \vec{v}_t, \mu)\right] + \mathcal{O}\left(\alpha_s^3\right),$$

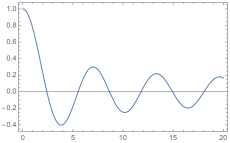

Figura: $J_0(q_\perp x_\perp)$