Dimension 4: Quantum Origins of Spacetime Smoothness

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The large-scale smoothness of the Universe

Does it need to be standard smoothness?

- Mathematical ambiguity

Exotic $\mathcal{R}^n$ - a differentiable manifold that is homeomorphic but not diffeomorphic to standard smooth $\mathbb{R}^n$

- It doesn't exist for all $n \neq 4$.
- In fourth dimension there are uncountably many (non-isomorphic ones)!

All Exotic $\mathcal{R}^4$'s have necessarily non-vanishing Riemann curvature

A PHYSICAL MEANING?

Cosmological Constant value as a topological invariant

But: Curvature depends crucially on embeddings

Our Strategy

Embedding: (small exotic $\mathcal{R}^4$) $\to \mathbb{R}^4 \to M^4$

- ⋆ invariant part of the scalar curvature

Is it even possible that appropriate M^4 exists?

Yes!

$$K3\#\overline{CP(2)}$$

Embedding (small exotic $\mathcal{R}^4$) $\longrightarrow K3\#\overline{CP(2)}$ is allowed by two topology changes

$$\mathcal{S}^3 \longrightarrow \Sigma(2,5,7) \longrightarrow P\#P$$

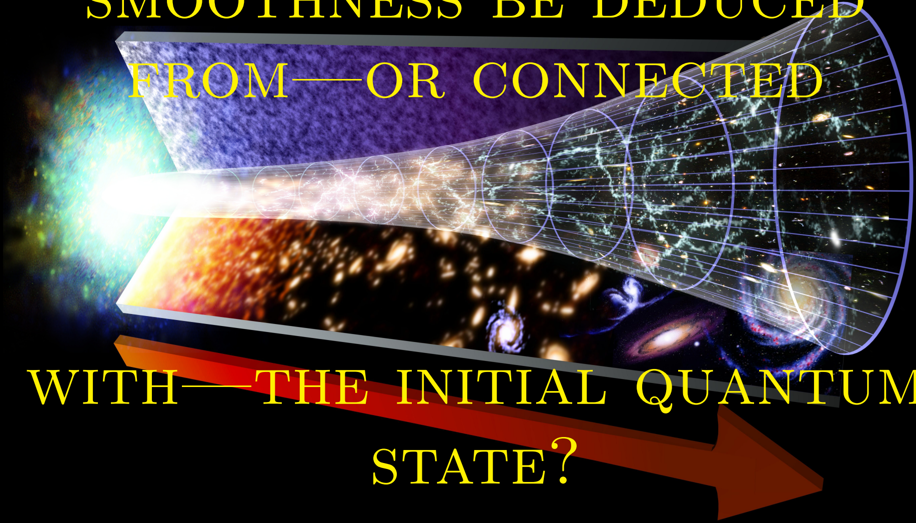
Topological calculations versus Planck data

$$\Omega_\Lambda = \frac{c^5}{3hGH_0^2} \cdot \exp\left(-\frac{3}{CS(\Sigma(2,5,7))} - \frac{3}{CS(P\#P)} - \frac{\chi(A_{\text{cork}})}{4}\right)$$

$$\Omega_\Lambda = 0.6853$$

$$(\Omega_\Lambda)_{\text{PLANCK}} = 0.683$$

Could the large-scale smoothness be deduced from—or connected with—the initial quantum state?



$\mathcal{H}$ - a separated complex Hilbert space

Lattice of Projections $\mathbb{L} = (\mathbb{L}(\mathcal{H}), \vee, \wedge)$

$\mathbb{L}(\mathcal{H})$ contains projections on closed linear subspaces of $\mathcal{H}$

$$P_1 \wedge P_2 \equiv P_1 \cap P_2$$
$$P_1 \vee P_2 \equiv \mathrm{span}(P_1 \cup P_2)$$

$\longrightarrow$ partially ordered by set inclusion

G. Takeuti: *Two applications of logic to mathematics*, lemma 1.1, p.8

For every family $\{A_i | i \in I\}$ of self-adjoint pairwise commuting operators there exists a complete Boolean algebra of projections B, such that given the spectral decompositions of each A_i:

$$A_i = \int \lambda dE^i_\lambda,$$

it holds that $\forall_{i \in I}\ dE^i_\lambda \in B$.

Let $\mathcal{H} = L^2(\mathbb{R}^n, d^n x)$ and B_Q be the complete maximal Boolean subalgebra of $\mathbb{L}$ containing position operator $\hat{Q}$ on $\mathcal{H}$. Then B_Q is atomless.

IN FACT B_Q IS ISOMORPHIC TO MEASURE ALGEBRA:

$$B_Q \simeq Bor(\mathbb{R}^n)/_{\mathcal{N}}$$

$\mathcal{N}$ - IDEAL OF MEASURE ZERO SETS

$Sh(B_Q)$

Boolean-Valued Model of ZFC | **Model of Random Forcing**

G. Takeuti: *Two applications of logic to mathematics*

Real numbers R from ZFC model $Sh(B_Q)$ are in $1-1$ correspondence with self-adjoint operators from B_Q.

EXTENSION OF THE REAL LINE IN RANDOM FORCING:

$$R \longrightarrow R[G], \ R \subset R[G]$$

PHYSICS IN LARGE SCALES:

$\mathbb{R}$ AS DEDEKIND COMPLETE ORDERED FIELD

$\star$ ALL MODELS ARE ISOMORPHIC

THE GAP

(1st order) $R, R[G] \Longrightarrow \mathbb{R}$ (2nd order)

The large-scale smoothness of universe (spacetime manifold)

It needs to be exotic smoothness on R^n

$\star$ The '1st order to 2nd order gap' is filled but the smoothness structure on R^n has to be exotic one

$\star$ they exist only for $n = 4 \longrightarrow$ 4D spacetime

Canceling of zero modes

The zero-point energy of quantum field corresponding to a particle of mass m:

$$\frac{E}{V} = \int_{R^3} \frac{d^3k}{(2\pi)^3} \frac{\sqrt{\mathbf{k}^2 + m^2}}{2}, \; m \in R[G], \; \mathbf{k} \in R^3.$$

Here all such integrals equal 0 since we integrate over the null set $R^3 \subset R^3[G]$.

QM lattice of projections:

- vanishing of zero-modes
- necessity of exotic smoothness

The Complete Picture

[1] Król, Asselmeyer-Maluga, Bielas, Klimasara, *From Quantum to Cosmological Regime. The Role of Forcing and Exotic 4-Smoothness*. Universe 2017, 3, 31.

Low-Dimensional Topology:

- value of cosmological constant
- necessity of four dimensions

~ Thank You ~ for Your Attention