Lattice QCD: a numerical tool for precision physics in the Standard Model and beyond

Piotr Korcyl

Universität Regensburg / Uniwersytet Jagielloński

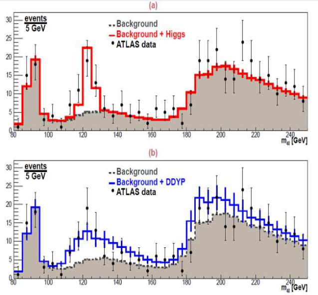
II Sympozjum Polskiego Towarzystwa Fizycznego
14 May 2016

Universität Regensburg

Interpreting data needs precise background estimation

Double Drell-Yan process

An interesting observation on the impact of lack of precise QCD background knowledge [Krasny, Placzek '15]:



Two conclusions:

- precise knowledge of QCD background crucial in correctly interpreting collider data!
- but there is not enough experimental data to constrain Double Parton Distributions ⇒ Lattice QCD!

Lattice QCD in two slides

Basics

Lattice QCD is based on analytic continuation to imaginary time:

$$it \to \tau$$

$$S = \int dx^4 (T - V) \to i \int dx_E^4 (T + V) \equiv iS_E$$

$$e^{iS} \to e^{-S_E}$$

$$\text{QFT} \to \text{statistical system}$$

Baryons

One needs combinations of field operators which have the wanted quantum numbers, e.g. for the nucleon

$$\hat{B}_\alpha(t, \vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\vec{x}} \epsilon_{ijk} \hat{u}^i_\alpha(x) \hat{u}^j_\beta(x) (C^{-1}\gamma_5)_{\beta\gamma} \hat{d}^k_\gamma(x)$$

Lattice QCD in two slides

Hadronic matrix elements

Compute correlation functions

$$\langle N(P)|\bar{\psi}_q \gamma^{\{\mu} iD^{\mu_1\}} \psi_q |N(P')\rangle =$$
$$= \bar{\psi}_q \Big\{ \sum_i \alpha_i^{\mu,\mu_1} A_{2,i}^q(Q^2) + \sum_i \beta_i^{\mu,\mu_1} B_{2,i}^q(Q^2) \Big\} \psi_q$$

which allow to estimate the Generalized Form Factors $A_{2,i}^q$ and $B_{2,i}^q$.
⇒ nucleon's total angular momentum

$$J^{u-d} = \frac{1}{2}\left(A_{2,0}^{u-d}(Q^2=0) + B_{2,0}^{u-d}(Q^2=0)\right).$$

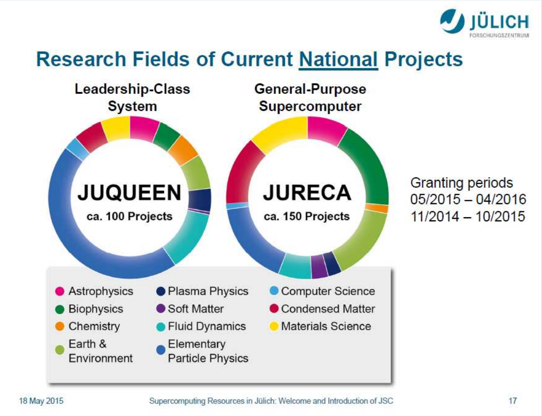
Hand on actual work to supercomputers

- use Monte Carlo methods to estimate path integrals
- use Markov chains to generate representative configurations with Boltzmann probability distribution

⇒ rough idea of the cost: 100 M core $\times$ hours $\approx$ 1 core $\times$ 11500 years

Computing resources

Overview of the usage of Europe's largest supercomputers



JUQUEEN is an IBM BlueGene/Q installation in Juelich Supercomputing Center: 458000 cores with peak performance 5.8 PFlops/s.
⇒ 11th place on TOP500 @ Nov. 2015, 3rd in Europe

Ingredients of precision

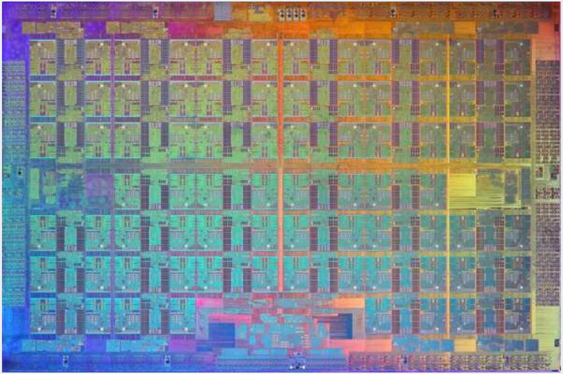
Lattice QCD became a precision tool for QCD, since today we have at our disposal:

- better computer architectures
- better algorithms
- better observables
- better control over the systematics

$\Rightarrow$ LQCD provides precise results for QCD processes with all systematics under control!

Ingredients of precision

New computer architectures



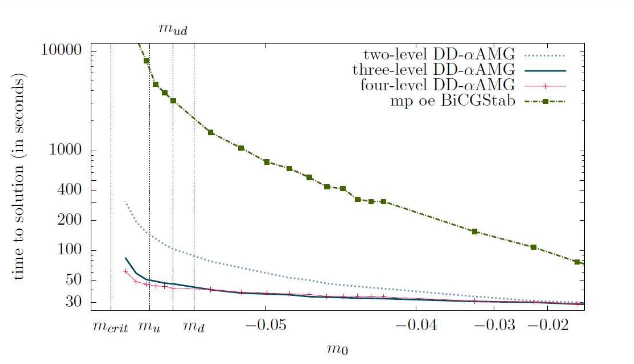
Intel Xeon Phi processor (codename Knights Landing):

64 cores, 256 hardware threads, shared L2 cache, AVX512 instruction set, 2 vector units/core, 16 GB MCDRAM $\Rightarrow > 3$ TFlops/s double precision

Ingredients of precision

New algorithms

Invertions of the Dirac operator contribute significantly to the total numerical cost.



Domain Decomposition Adaptive Algebraic Multigrid Solver (DD-αAMG) is the state-of-the-art solver developed and implemented by mathematicians and physicists from Univ. of Wuppertal and Regensburg.

Ingredients of precision

New observables

Observables defined with Wilson flow [Lüscher '13-'15] provide excellent statistical precision. Wilson flow is defined by introducing a fictitious time dimension and along which the gauge fields are evolved according to

$$\frac{d}{dt} B_\mu(x,t) = D_\nu G_{\mu\nu}(x,t) = -\frac{\delta S_{\mathrm{YM}}[B]}{\delta B_\mu(x,t)}, \qquad B_\mu(x,0) = A_\mu(x)$$

For example one can define the renormalized coupling constant in a finite volume scheme [ALPHA '16] and compute the Λ parameter in a theory with 3 dynamical quarks with a 2% precision

$$L_0 \Lambda = 0.0303(8)$$

The scale L_0 will be linked to dimensionfull quantities in the upcoming simulations.

Control over the systematics

In order to get the final number:

Translating a hadron matrix element obtained from a numerical simulation to the continuum $\overline{\text{MS}}$ scheme involves the following systematic effects:

List of systematics

- finite volume, nowadays $m_\pi L > 4$
- chiral extrapolation, nowadays $m_{u,d} = m_{u,d}^{\text{physical}}$
- continuum extrapolation, nowadays $a \approx 0.045\text{fm} - 0.05\text{fm}$
- renormalization, nowadays fully non-perturbative

$\Rightarrow$ perturbative estimates of the renormalization constants contribute the most significant part to the final result's uncertainty!

Non-perturbative renormalization of lattice QCD

To define QCD as a QFT beyond perturbation theory it is not enough to write down its classical Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}}(x) = \frac{1}{2g^2}\mathrm{tr}\{F_{\mu\nu}(x)F_{\mu\nu}(x)\} + \sum_{i=1}^{N_f} \bar{\psi}_i(\not{D} + m_i)\psi_i(x)$$

One needs to define the functional integral:

- Introduce a Euclidean space-time lattice and discretise the continuum action such that the doubling problem is solved
- Consider a finite space-time volume $\Rightarrow$ the functional integral becomes a finite dimensional ordinary or Grassmann integral
- Take the infinite volume limit $L \to 1$
- Take the continuum limit $a \to 0$

From asymptotic freedom expect

$$g_0^2 = g_0^2(a) \overset{a\to 0}{\sim} \frac{-1}{2b_0 \ln a}$$

Non-perturbative renormalization of lattice QCD

$$\mathcal{L}_{\mathrm{QCD}}(x) = \frac{1}{2g^2}\mathrm{tr}\{F_{\mu\nu}(x)F_{\mu\nu}(x)\} + \sum_{i=1}^{N_f}\bar{\psi}_i(\not{D} + m_i)\psi_i(x)$$

- The basic parameters of QCD are g_0 and $m_{0,i}$, $i = u, d, \dots$
- To renormalise QCD one must impose a corresponding number of renormalisation conditions
- If we only consider gauge invariant observables $\Rightarrow$ no need to renormalize quark, gluon, ghost field and gauge parameter.
- All physical information (particle masses and energies, particle interactions) is contained in the (Euclidean) correlation functions of gauge invariant composite, local fields $\phi_i(x)$

$$\langle \phi_1(x_1) \dots \phi_n(x_n) \rangle$$

- a priori each ϕ_i requires renormalisation, and thus further renormalisation conditions.

Non-perturbative renormalization of lattice QCD

How to renormalize QCD in a hadronic scheme?

Sketch of the procedure, using e.g. hadronic observables F_π, m_π, m_K, m_D:

1. Choose a value of the bare coupling $g_0^2 = 6/\beta$; this determines the lattice spacing (i.e. mass independent scheme); choose some inital values for the bare quark mass parameters and a spatial lattice volume $(L/a)^3$ that is large enough to contain the hadrons;
2. tune the bare quark mass parameters such that m_π/F_π, m_K/F_π, m_D/F_π take their desired values (e.g. experimental)
3. the lattice spacing is obtained from $a(\beta) = (aF_\pi)(\beta)/F_\pi|_{\mathrm{exp.}}$
4. reduce the value of g_0^2 (i.e. increase β) and increase L/a accordingly

Non-perturbative renormalization of lattice QCD

- for g_0^2 (or β) in some interval one obtains:

$$F_\pi, m_\pi, m_K, m_D \Rightarrow g_0, am_{0,l}(g_0), am_{0,s}(g_0), \ldots$$

- these are bare parameters
- SM for energies $\ll m_W$ reduces to QCD + QED + tower of effective weak interaction vertices (4-quark operators, 6-quark operators, ...)
- the structure of this effective "weak hamiltonian" is obtained perturbatively e.g. in the $\overline{\mathrm{MS}}$ scheme
- How can we relate the bare lattice parameters to the renormalized ones in, say, the $\overline{\mathrm{MS}}$ scheme
- $\Rightarrow$ introduce an intermediate renormalization scheme which can be evaluated both perturbatively and non-perturbatively

Non-perturbative renormalization of lattice QCD

We have a hierarchy of scales

- The renormalization scale μ must reach the perturbative regime: $\mu \gg \Lambda_{QCD}$
- the lattice cutoff must still be larger: $\mu \ll a^{-1}$
- The volume must be large enough to contain pions: $L \gg 1/m_\pi$
- $$L/a \gg \mu L \gg m_\pi L \gg 1 \Rightarrow L/a \approx \mathcal{O}(10^3)$$

  $\Rightarrow$ widely different scales cannot be resolved simultaneously on a finite lattice! $\Rightarrow$ window problem

Method

Consider the correlation functions of flavor non-singlet bilinear quark operators of the form $\langle \mathcal{O}_\Gamma(x)\mathcal{O}_\Gamma(0)\rangle$, where:

$$\mathcal{O}_\Gamma(x) = \bar{\psi}(x)\Gamma\psi(x), \qquad \Gamma = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\}.$$

Impose the following coordinate space conditions in the chiral limit:

$$\lim_{a\to 0}\langle \mathcal{O}_\Gamma^X(x)\mathcal{O}_\Gamma^X(0)\rangle\big|_{x^2=x_0^2} = \langle \mathcal{O}_\Gamma(x_0)\mathcal{O}_\Gamma(0)\rangle_{\text{cont}}^{\text{free,massless}}.$$

The renormalized operator is

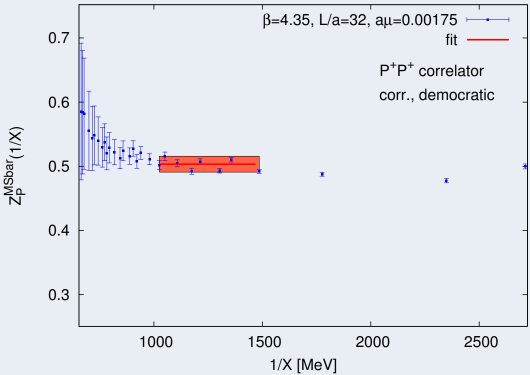
$$\mathcal{O}_\Gamma^X(x, x_0) = Z_\Gamma^X(x_0)\mathcal{O}_\Gamma(x),$$

x_0 is the renormalization point, which must satisfy:

$$a \ll x_0 \ll \Lambda_{\text{QCD}}^{-1}$$

to keep the discretization & non-perturbative effects under control.

$Z_P^{\overline{\rm MS}}(\mu = 2\,{\rm GeV}) = 0.503(1)(3)(2) = 0.503(6)$



[P.K., K. Cichy, K. Jansen, '13]

Step scaling

The running of the renormalization constants can be estimated non-perturbatively on the lattice by calculating the step scaling function, i.e. the ratio

$$\Sigma_\Gamma(\mu, 2\mu) = \lim_{a\to 0} \frac{Z_\Gamma(2\mu, a)}{Z_\Gamma(\mu, a)}$$

Example:

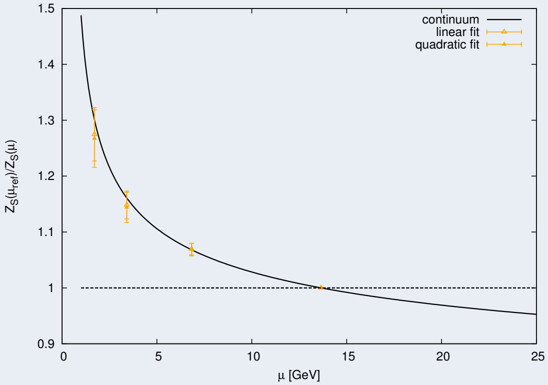
$$\frac{Z_\Gamma(\sqrt{X^2} = 0.0196\mathrm{fm})}{Z_\Gamma(\sqrt{X^2} = 0.0392\mathrm{fm})} = \frac{Z_\Gamma(\mu = 10\mathrm{GeV})}{Z_\Gamma(\mu = 5\mathrm{GeV})}\Big|_{\beta=7.90}, \Big|_{\beta=8.62}, \Big|_{\beta=9.00}, \Big|_{\beta=9.50}$$

On the lattice this corresponds to the ratios:

$$\frac{Z_\Gamma(X = \{1,1,1,1\})}{Z_\Gamma(X = \{2,2,2,2\})}\Big|_{\beta=7.90}, \quad \frac{Z_\Gamma(X = \{2,2,2,2\})}{Z_\Gamma(X = \{4,4,4,4\})}\Big|_{\beta=8.62},$$

$$\frac{Z_\Gamma(X = \{3,3,3,3\})}{Z_\Gamma(X = \{6,6,6,6\})}\Big|_{\beta=9.00}, \quad \frac{Z_\Gamma(X = \{4,4,4,4\})}{Z_\Gamma(X = \{8,8,8,8\})}\Big|_{\beta=9.50}$$

Non-perturbative running of Z_S



[P.K., K. Cichy, K. Jansen, in prep]

Suppose we have renormalised lattice QCD non-perturbatively, how is the the continuuum limit approached?
Symanzik's effective continuum theory:

- purpose: render the a-dependence of lattice correlation functions explicit. $\Rightarrow$ structural insight into the nature of cutoff effects
- at scales far below the cutoff a^{-1}, the lattice theory is effectively continuum like, the influence of cutoff effects is expanded in powers of a:

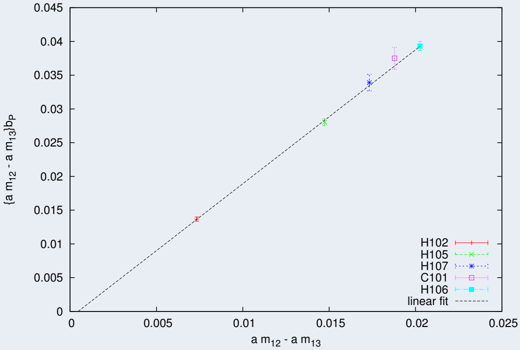
$$S_{\mathrm{eff}} = S_0 + aS_1 + a^2 S_2 + \ldots, S_0 = S_{\mathrm{QCD}}^{\mathrm{cont}}$$

$$S_k = \int d^4 x \mathcal{L}_k(x)$$

At small distances one expects that there is neither explicit nor spontaneous chiral symmetry breaking: the dominant effects are the discretization errors $\Rightarrow$ difference between the massless and massive correlation functions are the mass dependent discretization errors.

b_P coefficient

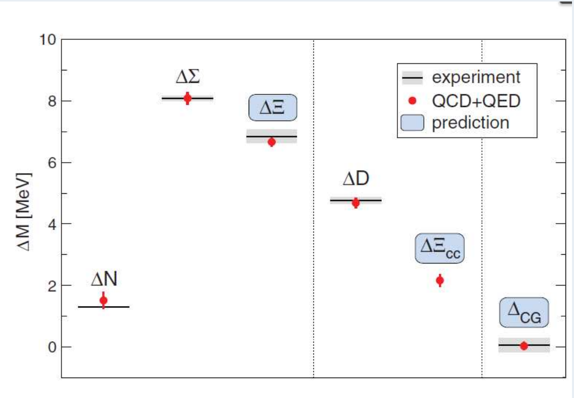
$$\left(\bar{\psi}(x)\gamma_5\psi(x)\right)^{\mathrm{R,\ impr}} = Z_P\left(1 + b_P m\right)\bar{\psi}(x)\gamma_5\psi(x)$$



[P.K., G. Bali, in prep]

Applications

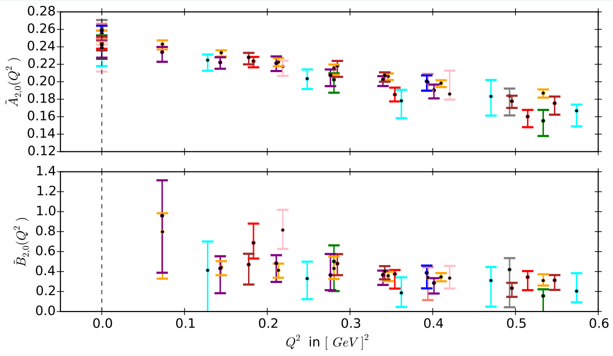
Nucleons mass splittings [BMW collab. '16]



⇒ simulations of QCD + QED!

Applications

Generalized Form Factors [RQCD collab. '16]



$\Rightarrow J^{u-d}(m_\pi^{\text{physical}}) = 0.238(8)$

Summary

Lattice QCD is a precision tool!

- Lattice QCD allows to compute hadronic matrix elements with all systematic errors under control
- With more computer resources and improved algorithms more challenging problems can be now addressed

Stay tuned! Even more interesting results will come!
$\Rightarrow$ LATTICE'16 @ Southampton

Thank you for your attention!