

## 4 jet production in High Energy Factorization

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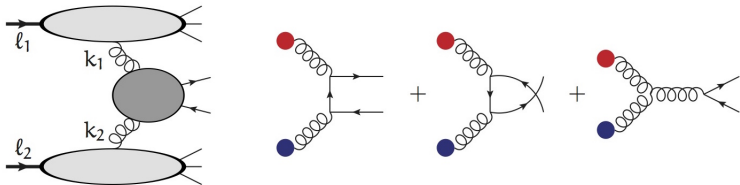
Work in collaboration with  
Krzysztof Kutak, Rafal Maciula, Antoni Szczurek and Andreas van Hameren,

Supported by NCN grant DEC-2013/10/E/ST2/00656  
of Krzysztof Kutak

- 1 The framework: off-shell amplitudes and PDFs
- 2 Test of HE factorisation for hard central 4-jet production
- 3 Collinear-factorisation vs. HEF in DPS for central 4-jet production
- 4 Summary and perspectives
- 5 Backup

## High-Energy-factorisation: original formulation

High-Energy-factorisation (*Catani,Ciafaloni,Hautmann, 1991 / Collins,Ellis, 1991*)



$$\sigma_{h_1, h_2 \rightarrow q\bar{q}} = \int d^2 k_{1\perp} d^2 k_{2\perp} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \mathcal{F}_g(x_1, k_{1\perp}) \mathcal{F}_g(x_2, k_{2\perp}) \hat{\sigma}_{gg} \left( \frac{m^2}{x_1 x_2 s}, \frac{k_{1\perp}}{m}, \frac{k_{2\perp}}{m} \right)$$

where the  $\mathcal{F}_g$ 's are the gluon densities, obeying BFKL, BK, CCFM evolution equations.

Non negligible transverse momentum is associated to small- $x$  physics.

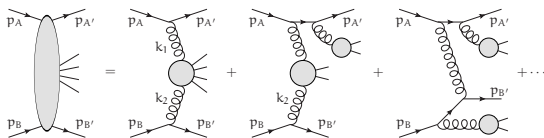
Momentum parameterisation:

$$k_1^\mu = x_1 p_1^\mu + k_{1\perp}^\mu, \quad k_2^\mu = x_2 p_2^\mu + k_{2\perp}^\mu \quad \text{for} \quad p_i \cdot k_i = 0 \quad k_i^2 = -k_{i\perp}^2 \quad i = 1, 2$$

## Off-shell amplitudes

Problem: general partonic processes must be described by gauge invariant amplitudes  
 (⇒ See A. van Hameren's Talk)

Off-shell gauge-invariant amplitudes obtained by embedding them into on-shell processes. For off-shell gluons: represent  $g^*$  as coming from a  $\bar{q}qg$  vertex, with the quarks taken to be on-shell

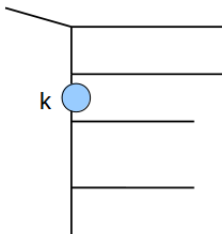


Prescriptions: *K. Kutak, P. Kotko, A. van Hameren, T. Salwa (2013)*

Any legs via recursion relations: *P. Kotko (2014), A. van Hameren (2014)*

Applications:  $\left\{ \begin{array}{l} \text{production of forward dijets initiated with gluons : } gg^* \rightarrow gg \\ \text{production of forward dijets initiated with quarks : } q\bar{q}^* \rightarrow gg \\ \text{Test of TMDs in multi-jet production : } pp \rightarrow n \text{ ( = 4 in this talk ) jets} \end{array} \right.$

## Our PDFs: the prescription



Survival probability without emissions

Kimber, Martin, Ryskin prescription, '01 :

$$T_s(\mu^2, k^2) = \exp\left(-\int_{\mu^2}^{k^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi}\right) \times \sum_{a'} \int_0^{1-\Delta} dz' P_{aa'}(z')$$

$$\Delta = \frac{\mu}{\mu + k}, \quad \mu = \text{hard scale}$$

$$\mathcal{F}(x, k^2, \mu^2) \sim \partial_{\lambda^2} (T_s(\lambda^2, \mu^2) \times g(x, \lambda^2)) \Big|_{\lambda^2=k^2}$$

DLC 2016 (Double Log Coherence)

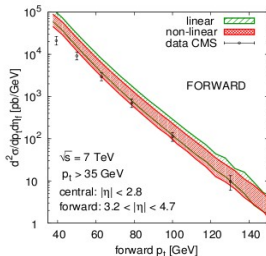
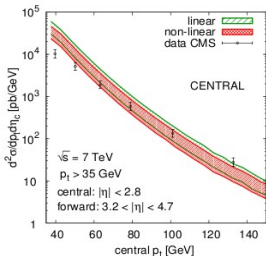
K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren,  
JHEP 1604 (2016) 175 (arXiv:1602.06814)Available on request to [krzysztof.kutak@ifj.edu.pl](mailto:krzysztof.kutak@ifj.edu.pl)

## Example: central-forward dijets production

Hybrid factorization, (Deak, Hautmann, Jung, Kutak, '09):

$$\sigma_{h_1, h_2 \rightarrow q\bar{q}} = \int d^2 k_{1\perp} dx_1 dx_2 \mathcal{F}(x_1, k_{1\perp}, \mu) f(x_2, \mu) \hat{\sigma}(x_1, x_2, k_{1\perp}, \mu)$$

Kutak, Sapeta, '12:



- Reasonable agreement with data
- No traditional parton showers: the Unintegrated PDF as a parton shower.
- Hybrid factorization formula for dijet production (fully differential) can be derived from Color-Glass-Condensate P. Kotko, K. Kutak, C. Marquet, E. Petreska, A. van Hameren, JHEP 1509 (2015) 106

## Conjectured formula for 4 jets production:

Following content based on

K. Kutak, R. Maciula, M. S., A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175

&

K. Kutak, R. Maciula, M. S., A. Szczurek, A. van Hameren, *in preparation*

$$\begin{aligned} \sigma_{4\text{-jets}} &= \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2k_{T1} d^2k_{T2} \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F) \\ &\times \frac{1}{2\hat{s}} \prod_{l=i}^4 \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4\text{-jet}} (2\pi)^4 \delta \left( P - \sum_{l=1}^4 k_l \right) \overline{|\mathcal{M}(i^*, j^* \rightarrow 4 \text{ part.})|^2} \end{aligned}$$

- Ansatz motivated by  $2 \rightarrow 2$  case
- PDFs and matrix elements well defined.
- No proof à la Collins-Soper-Sterman around (not yet...)
- Reasonable description of data justifies this formula *a posteriori*

## Our framework

**AVHLIB** (A. van Hameren) : <https://bitbucket.org/hameren/avhlib>

- complete Monte Carlo program for tree-level calculations
- any process within the Standard Model
- any initial-state partons on-shell or off-shell
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization
  
- **Flavour scheme:**  $N_f = 5$
- **Running**  $\alpha_s$  from the MSTW68cl PDF sets
- **Massless quarks approximation**  $E_{cm} = 7/8 TeV \Rightarrow m_{q/\bar{q}} = 0$ .
- **Scale**  $\mu_R = \mu_F \equiv \mu = \frac{H_T}{2} \equiv \frac{1}{2} \sum_i p_T^i$ , (sum over final state particles)

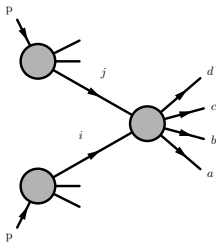
**We don't take into account correlations in DPS:**  $D(x_1, x_2, \mu) = f(x_1, \mu) f(x_2, \mu)$ .

There are attempts to go beyond this approximation:

Golec-Biernat, Lewandowska, Snyder, M.S., Stasto, *Phys.Lett.* B750 (2015) 559-564  
 Rinaldi, Scopetta, Traini, Vento, *JHEP* 1412 (2014) 028



## 4-jet production: Single Parton Scattering ( SPS )



We take into account all the ( according to our conventions ) 20 channels.

Here  $u$  and  $d$  stand for different quark flavours in the initial ( final ) state.

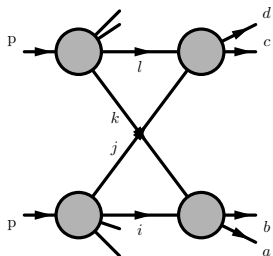
We do not introduce K factors, amplitudes@LO.

~ 95 % of the total cross section

There are 19 different channels contributing to the cross section at the parton-level:

$$\begin{aligned}
 & gg \rightarrow 4g, gg \rightarrow q\bar{q}2g, qg \rightarrow q3g, q\bar{q} \rightarrow q\bar{q}2g, qq \rightarrow qq2g, qq' \rightarrow qq'2g, \\
 & gg \rightarrow q\bar{q}q\bar{q}, gg \rightarrow q\bar{q}q'\bar{q}', qg \rightarrow qgq\bar{q}, qg \rightarrow qgq'\bar{q}', \\
 & q\bar{q} \rightarrow 4g, q\bar{q} \rightarrow q'\bar{q}'2g, q\bar{q} \rightarrow q\bar{q}q\bar{q}, q\bar{q} \rightarrow q\bar{q}q'\bar{q}', q\bar{q} \rightarrow q'\bar{q}'q'\bar{q}', \\
 & q\bar{q} \rightarrow q'\bar{q}'q''\bar{q}'', qq \rightarrow qq\bar{q}, qq \rightarrow qq'q', qq' \rightarrow qq'q\bar{q},
 \end{aligned}$$

## 4-jet production: Double parton scattering ( DPS )



$$\sigma = \sum_{i,j,a,b;k,l,c,d} \frac{S}{\sigma_{\text{eff}}} \sigma(i,j \rightarrow a,b) \sigma(k,l \rightarrow c,d)$$

$$S = \begin{cases} 1/2 & \text{if } ij = kl \text{ and } ab = cd \\ 1 & \text{if } ij \neq kl \text{ or } ab \neq cd \end{cases}$$

$$\sigma_{\text{eff}} = 15 \text{ mb},$$

Experimental data may hint at different values of  $\sigma_{\text{eff}}$ ; main conclusions not affected

In our conventions, 9 channels from  $2 \rightarrow 2$  SPS events,

$$\begin{aligned} \#1 &= gg \rightarrow gg, & \#6 &= u\bar{u} \rightarrow d\bar{d} \\ \#2 &= gg \rightarrow u\bar{u}, & \#7 &= u\bar{u} \rightarrow gg \\ \#3 &= ug \rightarrow ug, & \#8 &= uu \rightarrow uu \\ \#4 &= gu \rightarrow ug, & \#9 &= ud \rightarrow ud \\ \#5 &= u\bar{u} \rightarrow u\bar{u} \end{aligned}$$

$\Rightarrow$  45 channels for the DPS; only 14 contribute to  $\geq 95\%$  of the cross section :

$$\begin{aligned} &(1, 1), (1, 2), (1, 3), (1, 4), (1, 8), (1, 9), (3, 3) \\ &(3, 4), (3, 8), (3, 9), (4, 4), (4, 8), (4, 9), (9, 9) \end{aligned}$$

## Hard jets

We reproduce all the LO results (only SPS) for  $pp \rightarrow n \text{ jets}$ ,  $n = 2, 3, 4$  published in  
 BlackHat collaboration, Phys.Rev.Lett. 109 (2012) 042001  
 S. Badger et al., Phys.Lett. B718 (2013) 965-978

Asymmetric cuts for hard central jets

$$p_T \geq 80 \text{ GeV}, \quad \text{for leading jet}$$

$$p_T \geq 60 \text{ GeV}, \quad \text{for non leading jets}$$

$$|\eta| \leq 2.8, \quad R = 0.4$$

PDFs set: MSTW2008LO@68cl

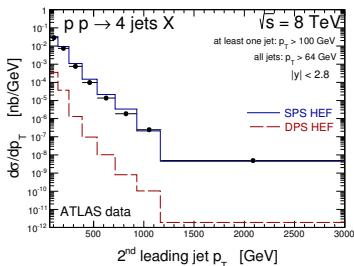
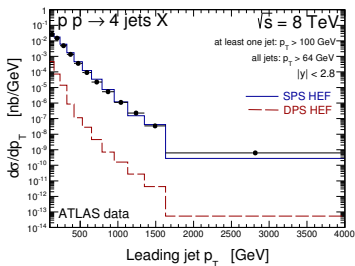
$$\sigma(\geq 2 \text{ jets}) = 958_{-221}^{+316} \quad \sigma(\geq 3 \text{ jets}) = 93.4_{-30.3}^{+50.4} \quad \sigma(\geq 4 \text{ jets}) = 9.98_{-3.95}^{+7.40}$$

Cuts are too hard to pin down DPS and/or benefit from HEF: 4-jet case

Collinear case	{	$9.98_{-3.95}^{+7.40}$ SPS $0.094_{-0.036}^{+0.06}$ DPS	HEF case	{	$10.0_{-5.3}^{+6.9}$ SPS $0.05_{-0.029}^{+0.054}$ DPS
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## Differential cross section

Most recent ATLAS paper on 4-jet production in proton-proton collision:  
**ATLAS, JHEP 1512 (2015) 105**



- All channels included and running  $\alpha_s$  @ NLO
- Good agreement with data
- DPS effects are manifestly too small for such hard cuts: this could be expected.

## DPS effects in collinear and HEF

Inspired by [Maciula, Szczurek, Phys.Lett. B749 \(2015\) 57-62](#)

DPS effects are expected to become significant for lower  $p_T$  cuts, like the ones of the CMS collaboration, [Phys.Rev. D89 \(2014\) no.9, 092010](#)

$$p_T(1,2) \geq 50 \text{ GeV}, \quad p_T(3,4) \geq 20 \text{ GeV}, \quad |\eta| \leq 4.7, \quad R = 0.5$$

CMS collaboration :  $\sigma_{tot} = 330 \pm 5 \text{ (stat.)} \pm 45 \text{ (syst.) nb}$

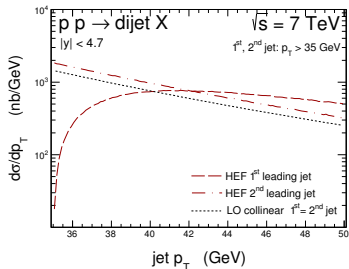
LO collinear factorization :  $\sigma_{SPS} = 697 \text{ nb}, \quad \sigma_{DPS} = \mathbf{125 \text{ nb}}, \quad \sigma_{tot} = 822 \text{ nb}$

LO HEF  $k_T$ -factorization :  $\sigma_{SPS} = 548 \text{ nb}, \quad \sigma_{DPS} = \mathbf{33 \text{ nb}}, \quad \sigma_{tot} = 581 \text{ nb}$

**In HE factorization DPS gets suppressed and does not dominate at low  $p_T$**

Counterintuitive result from well-tested perturbative framework  $\Rightarrow$  phase space effect ?

## An old problem: higher order corrections to 2-jet production



**Figure:** The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF.

NLO corrections to 2-jet production suffer from instability problem when using symmetric cuts: [Frixione, Ridolfi, Nucl.Phys. B507 \(1997\) 315-333](#)

Symmetric cuts rule out from integration final states in which the momentum imbalance due to the initial state non vanishing transverse momenta gives to one of the jets a lower transverse momentum than the threshold.

ATLAS data vs. theory (nb) @ LHC7 for 2,3,4 jets. Cuts are defined in [Eur.Phys.J. C71 \(2011\) 1763](#); theoretical predictions from [Phys.Rev.Lett. 109 \(2012\) 042001](#)

#jets	ATLAS	LO	NLO
2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$1193(3)^{+130}_{-135}$
3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$54.5(0.5)^{+2.2}_{-19.9}$
4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$5.54(0.12)^{+0.08}_{-2.44}$

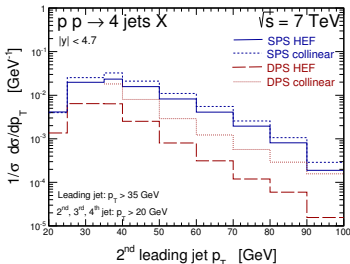
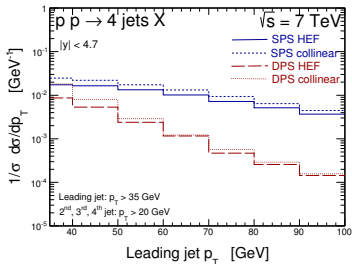
Reconciling HE and collinear factorisation: asymmetric  $p_T$  cuts

In order to open up wider region of soft final states and thereof expected that the DPS contribution increases

$$p_T(1) \geq 35 \text{ GeV}, \quad p_T(2, 3, 4) \geq 20 \text{ GeV}, \quad |\eta| < 4.7, \quad \Delta R > 0.5$$

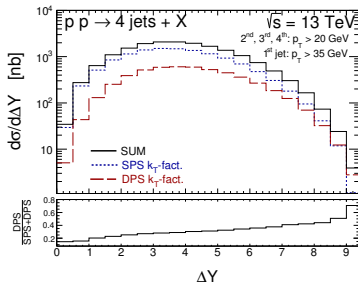
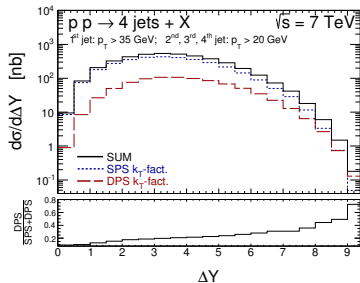
LO collinear factorization :  $\sigma_{SPS} = 1969 \text{ nb}$ ,  $\sigma_{DPS} = 514 \text{ nb}$ ,  $\sigma_{tot} = 2309 \text{ nb}$

LO HEF  $k_T$ -factorization :  $\sigma_{SPS} = 1506 \text{ nb}$ ,  $\sigma_{DPS} = 297 \text{ nb}$ ,  $\sigma_{tot} = 1803 \text{ nb}$



DPS dominance pushed to even lower  $p_T$  but restored in HE factorization as well

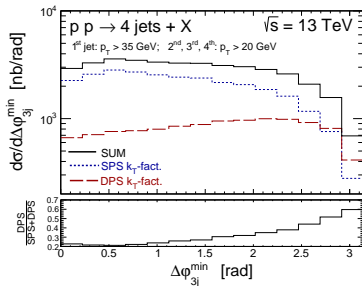
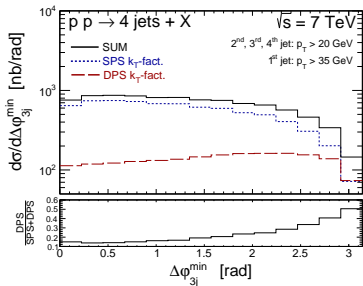
## Pinning down double parton scattering: large rapidity separation



- It is interesting to look for kinematic variables which could make DPS apparent.
- The maximum rapidity separation in the four jet sample is one such variable, especially at 13 GeV.
- for  $\Delta Y > 6$  the total cross section is dominated by DPS.



## Pinning down double parton scattering: "min3" azimuthal separation



- Definition:  $\Delta\phi_{3j}^{\text{min}} = \min_{i,j,k[1,4]} (|\phi_i - \phi_j| + |\phi_j - \phi_k|)$ ,  $i \neq j \neq k$
- Proposed by ATLAS in [JHEP 12 105 \(2015\)](#) for high  $p_T$  analysis
- High values favour configurations closer to back-to-back, i.e. DPS
- For  $\Delta\phi_{3j}^{\text{min}} \geq \pi/2$  the total cross section is dominated by DPS

## Summary and conclusions

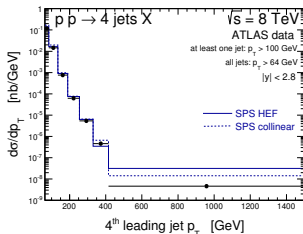
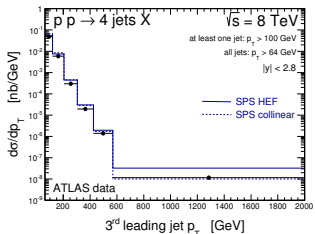
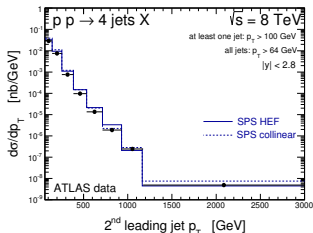
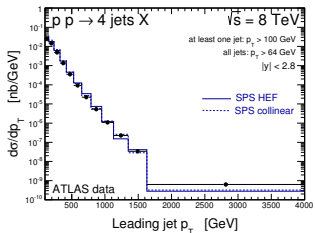
- We have a complete framework for the evaluation of cross sections from amplitudes with off-shell quarks and TMDs via KMR procedure obtained from NLO collinear PDFs
- HE factorisation reproduces well ATLAS data @ 7 and 8 TeV for hard central inclusive 4-jet production. Essential agreement with collinear predictions.
- HE factorisation smears out the DPS contribution to the cross section for less central jet, pushing the DPS-dominance region to lower  $p_T$ , but asymmetric cuts are in order: initial state transverse momentum generates asymmetries in the  $p_T$  of final state jet pairs.
- It would be interesting to have an experimental analysis with cuts which are *asymmetric and soft* ( $\Rightarrow$  Szymanowski's talk).
- Further insight into HE factorisation prediction will come with progress in NLO results and with the addition of final state parton showers. Work in progress...

## Summary and conclusions

- We have a complete framework for the evaluation of cross sections from amplitudes with off-shell quarks and TMD PDFs via KMR procedure obtained from NLO collinear PDFs
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Thank you for your attention !

## Comparing collinear factorization and HEF



Collinear factorization performs slightly better for intermediate values and HEF does a better job for the last bins, except for the 4th jet.

## One more interesting variable

$$\Delta S = \arccos \left( \frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|} \right), \quad \vec{p}_T(j_i, j_k) = p_{T,i} + p_{T,j}$$

We roughly describe the data via pQCD effects within our HEF approach which are (equally partially) described by parton-showers and soft MPIs by CMS.

For more variables to pin down DPS  $\Rightarrow$  see Maciula's talk

CMS collaboration Phys.Rev. D89 (2014) no.9, 092010

