

# Effective particles in quantum field theory

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The concept of effective particles is introduced in the Minkowski space-time Hamiltonians in quantum field theory using a new kind of the relativistic renormalization group procedure that does not integrate out high-energy *modes* but instead integrates out the large *changes* of invariant mass. The new procedure is explained using examples of known interactions. Some applications in phenomenology, including processes measurable in colliders, are briefly presented.

# Outline

Renormalization group procedure for effective particles (**RGPEP**)  
in quantum field theory, is illustrated with a few examples

- asymptotic freedom in QCD
- jet production in TeV pion-nucleus collisions

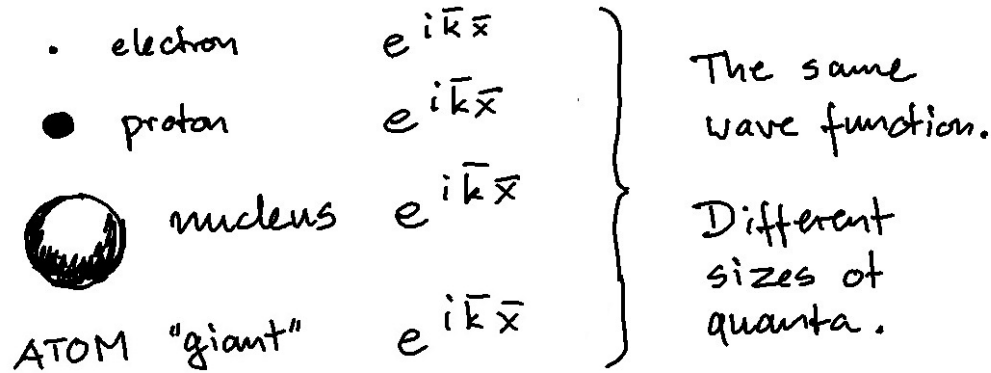
Insights mentioned concerning:  
ridge effect in pp collisions, proton radius in QED and CQM potential in AdS/QCD.

## Basic premise

QCD is a laboratory for studying problems concerning mass generation, symmetry breaking, and confinement, with potential implications in other areas of particle theory.

*Basic theoretical issues: size of quanta and RG scale*

## Drawback of the concept of a quantum field operator



$$\hat{\psi}(\vec{x}) = \int_{\mathbf{k}} \left[ u_{\mathbf{k}} e^{i\vec{k}\vec{x}} \hat{b}_{\mathbf{k}} + \dots \right]$$

FIG. 1: Quantum field operators are blind to the size of individual quanta.

## Canonical quantization of the YM field

$$\mathcal{L} = -\frac{1}{2} \text{tr} F^{\mu\nu} F_{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

$$\mathcal{T}^{\mu\nu} = -F^{a\mu\alpha} \partial^\nu A_\alpha^a + g^{\mu\nu} F^{a\alpha\beta} F_{\alpha\beta}^a / 4$$

$$P^\nu = \int d\sigma_\mu \mathcal{T}^{\mu\nu}$$

$$\mathcal{H} = \mathcal{T}^{00}$$

$$\mathcal{H}_{YM} = \mathcal{H}_{A^2} + \mathcal{H}_{A^3} + \mathcal{H}_{A^4}$$

quantization  $\hat{A}^\mu = \sum_{\sigma c} \int_k \left[ t^c \varepsilon_{k\sigma}^\mu a_{k\sigma c} e^{-ikx} + t^c \varepsilon_{k\sigma}^{\mu*} a_{k\sigma c}^\dagger e^{ikx} \right]_{on \Sigma}$

$$\hat{H}_{YM} = \int_{\Sigma} : \mathcal{H}_{YM}(\hat{A}) :$$

$$\mathcal{H}_{A^3} = g i \partial_\alpha A_\beta^a [A^\alpha, A^\beta]^a$$



$$\hat{H}_{A^3} = \int_{x \in \Sigma} : g i \partial_\alpha A_\beta^a(x) [A^\alpha(x), A^\beta(x)]^a :$$

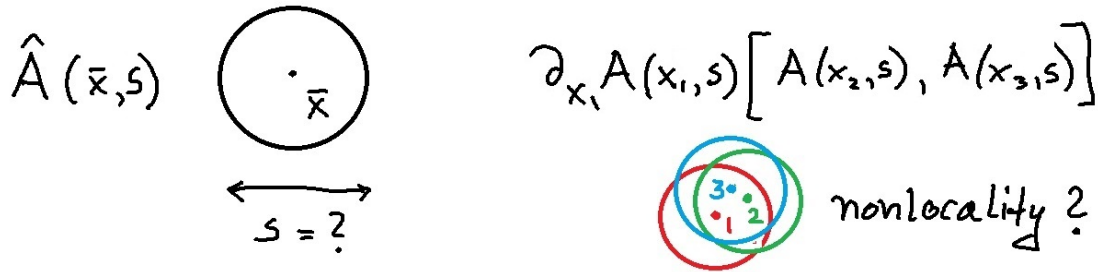


FIG. 2: Nonlocality is approximated by a local interaction for wavelengths greater than  $s$ . The particle size  $s$  is the scale parameter in the RGPEP.



$$\hat{H}_{A^3,s} = \int_{x_i \in \Sigma} g_s(x_1, x_2, x_3) : i\partial_\alpha A_\beta^a(x_1, s) [A^\alpha(x_2, s), A^\beta(x_3, s)]^a :$$

$$g_s(x_1, x_2, x_3) = ?$$

SDG, K. G. Wilson, Phys. Rev. D **48**, 5863 (1993)

K. G. Wilson *et al.*, Phys. Rev. D **49**, 6720 (1994)

F. J. Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

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SDG, Acta Phys. Pol. B **43**, 1843 (2012)

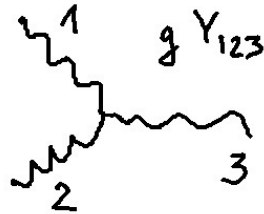
M. Gomez-Rocha, SDG, Phys. Rev. D **92**, 065005 (2015)

## RGPEP initial condition at $s = 0$ for local QCD

$$\hat{H}_{A^3, s=0} = \int_{x \in \Sigma} : g i \partial_\alpha A_\beta^a(x, 0) [A^\alpha(x, 0), A^\beta(x, 0)]^a :$$

$$\hat{A}^\mu(x, s=0) = \sum_{\sigma c} \int_k \left[ t^c \varepsilon_{k\sigma}^\mu a_{k\sigma c, s=0} e^{-ikx} + t^c \varepsilon_{k\sigma}^{\mu*} a_{k\sigma c, s=0}^\dagger e^{ikx} \right]_{on \Sigma}$$

$$\hat{H}_{A^3, s=0} = \sum_{123} \int_{123} \delta_{12,3} \left[ g Y_{123} a_{1,0}^\dagger a_{2,0}^\dagger a_{3,0} + g Y_{123}^* a_{3,0}^\dagger a_{2,0} a_{1,0} \right]$$



# RGPEP

$$H_0(a_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \dots, i_n} c_0(i_1, \dots, i_n) a_{i_1, 0}^\dagger \cdots a_{i_n, 0}$$

$c_0 = \text{the initial condition, including } CT_R$

$$H_s(a_s) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \dots, i_n} c_s(i_1, \dots, i_n) a_{i_1, s}^\dagger \cdots a_{i_n, s}$$

$c_s = ?$

$g_s(i_1, i_2, i_3)$ , among others

# RGPEP generator and non-perturbative QCD

$$H_s(a_s) = H_0(a_0)$$

$$a_s = U_s a_0 U_s^\dagger$$

$$H_s = H_f + H_I$$

$$H'_s = [G_s, H_s]$$

$$G_s = [H_f, \tilde{H}_s]$$

$$\text{NP QCD} \rightarrow H'_s = [[H_f, \tilde{H}_s], H_s]$$

$$\text{AF pQCD} \rightarrow H_s = H_f + gH_{1s} + g^2H_{2s} + g^3H_{3s} + \dots$$

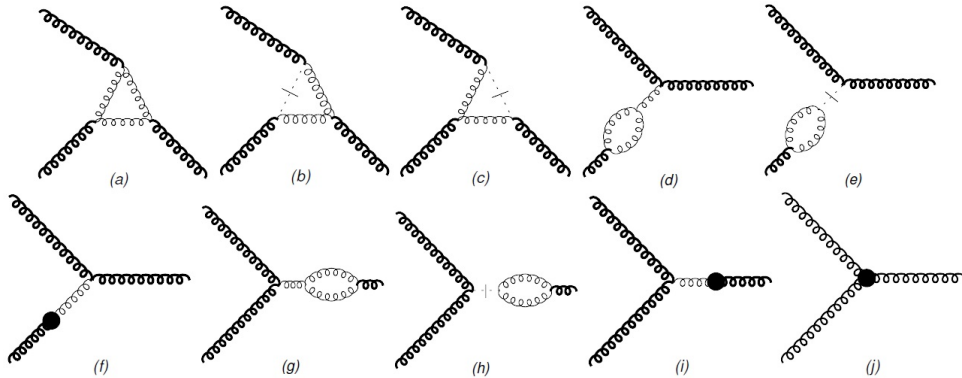


FIG. 3: Third-order contributions to the three-gluon vertex

$$H_{A^3(1+3)_s} = \sum_{123} \int_{123} \delta_{12.3} e^{-s^4 \mathcal{M}_{12}^4} \left[ V_{s123} a_{1,s}^\dagger a_{2,s}^\dagger a_{3,s} + V_{s123}^* a_{3,s}^\dagger a_{2,s} a_{1,s} \right]$$

$$\lim_{\kappa^\perp \rightarrow 0} V_{s123} = g_s Y_{123} \qquad g_s = g_0 + \frac{g_0^3}{48\pi^2} N_c 11 \ln \frac{s}{s_0}$$

RGPEP Hamiltonian  $\beta(s)$   $\leftrightarrow$  Gross-Wilczek-Politzer  $\beta(\lambda)$

Minkowskian  $s$   $\leftrightarrow$   $1/\lambda$  Euclidean

three-dimensional length  $\leftrightarrow$  four-dimensional length

M. Gomez-Rocha, SDG, Phys. Rev. D **92**, 065005 (2015)

# Effective-particle picture of nucleons

proton in the RGPEP

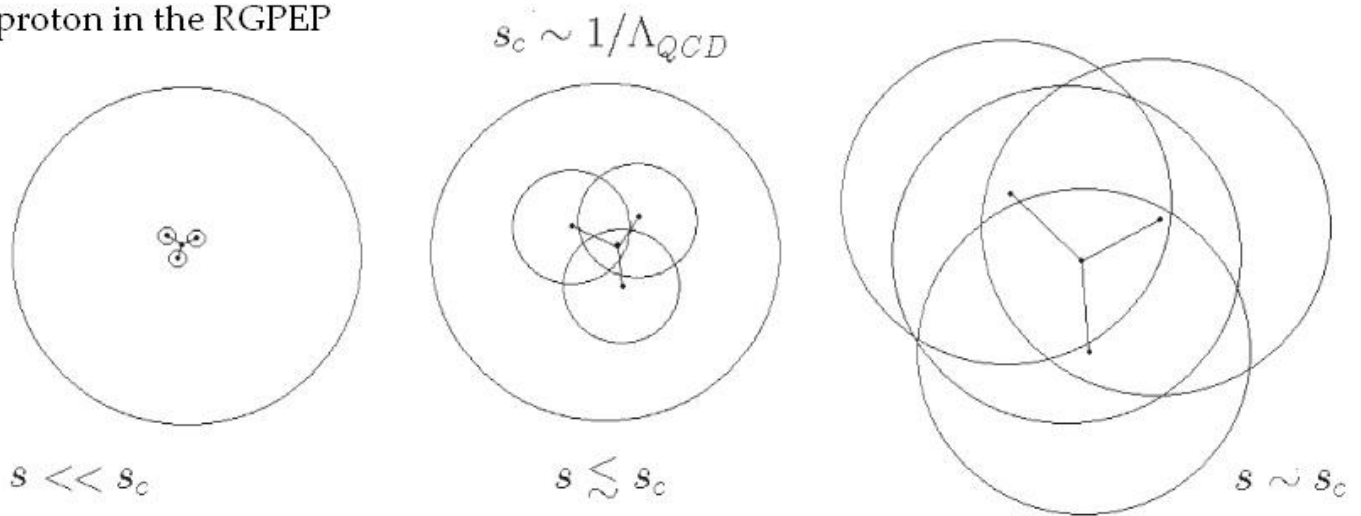


FIG. 4: The RGPEP scale-dependent proton picture

## Effective-particle color arrangement

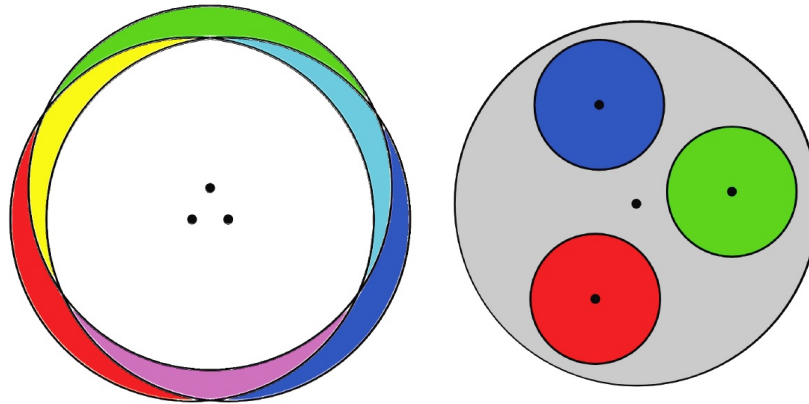


FIG. 5: Nucleon white interior with colored boundary coupling to mesons

**Ridge effect in  $pp$  collisions** from 7 TeV to 13 TeV

P. Kubiczek, SDG, Lith. J. Phys. **55**, 155 (2015)



## Proton radius in QED

How do we go from QFT to the Schrödinger equation

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\cancel{\partial} - e\cancel{A} - m)\psi$$

$$\mathcal{L} \rightarrow \mathcal{H} \rightarrow \hat{H} = \int d^3x \hat{\mathcal{H}} \rightarrow \hat{H}_R = \int d^3x \hat{\mathcal{H}}_R \rightarrow \hat{H}_R + \hat{H}_{CT} \rightarrow \hat{H}_s$$

Family of effective Hamiltonians:  $\hat{H}_{QEDs}$

$s = 0$  in canonical theory

$$\hat{H}_{QEDs}|\psi_s\rangle = E|\psi_s\rangle \rightarrow \hat{H}_{Sch} = ?$$

How does the parameter  $s$  enter in  $\hat{H}_{Sch}$ ?

# The RGPEP interpretation of atomic valence constituents

$$\begin{array}{c}
 \left[ \begin{array}{c} \dots \\ \dots \\ e\gamma e\bar{e}p\gamma \\ e\gamma p\gamma \\ e\gamma p \\ \boxed{ep} \end{array} \right] \\
 \psi_0(e_0, p_0) + \infty
 \end{array}
 \rightarrow
 \begin{array}{c}
 \left[ \begin{array}{c} \dots \\ \dots \\ e\gamma e\bar{e} \\ e\gamma\gamma \\ e\gamma \\ e \end{array} \right]
 \times
 \left[ \begin{array}{c} \dots \\ \dots \\ p\bar{e} \\ p\gamma\gamma \\ p\gamma \\ p \end{array} \right]
 + \dots
 \sim
 |e_s p_s\rangle + \dots \\
 \psi_s(e_s, p_s) + \text{corrections}
 \end{array}$$

“True” Schrödinger equation in QED  $\hat{H}_{QEDs}|\psi_s\rangle = E|\psi_s\rangle$

$$\frac{\vec{p}^2}{2\mu} \psi_s(\vec{p}) + \int \frac{d^3k}{(2\pi)^3} V_s(\vec{p}, \vec{k}) \psi_s(\vec{k}) = -E_B \psi_s(\vec{p})$$

$$V_s(\vec{p}, \vec{k}) = e^{-s^4(\vec{p}^2 - \vec{k}^2)^2/c^4} \frac{-4\pi}{(\vec{p} - \vec{k})^2} G_E(\vec{q}^2) \quad \mathbf{4\% !}$$

definition of  $\vec{p}$  in atoms matches **AdS/QFT** SDG, Acta Phys. Pol. B **42**, 1933 (2011)

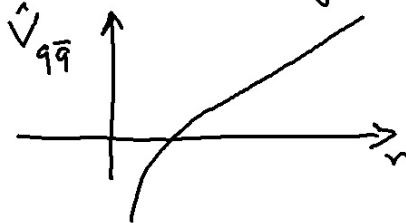
$$\beta = m_l/(m_p + m_l) , \quad c = \sqrt{m_l m_p}/(m_l + m_p) , \quad x = p_l^+/(p_l^+ + p_p^+)$$

$$p^\perp = c [(1-x) p_l^\perp - x p_p^\perp] / \sqrt{x(1-x)} \quad \text{Brodsky – de Teramond}$$

$$p^z = c (m_l + m_p) (x - \beta) / \sqrt{x(1-x)} \quad \text{holography}$$

# CQM potential in AdS/QCD

Instant form of dynamics



$$\hat{E} = \hat{E}_q + \hat{E}_{\bar{q}} + \hat{V}_{q\bar{q}}$$

$$\hat{V}_{q\bar{q}} \sim \sigma r \Rightarrow M \sim \sigma r$$

Front form of dynamics

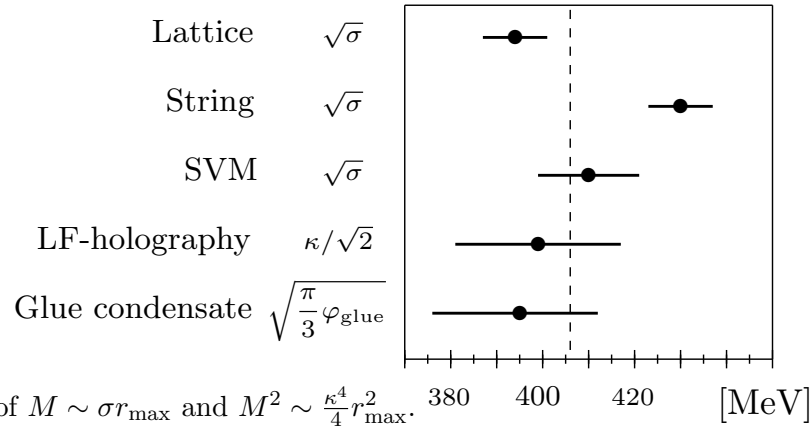


$$\hat{M}^2 = (\hat{p}_q^- + \hat{p}_{\bar{q}}^- + \hat{U}_{q\bar{q}}) \hat{p}^+ - \hat{p}_\perp^2$$

$$\hat{U}_{q\bar{q}} \sim \frac{1}{4} \kappa^4 r^2 \Rightarrow M^2 \sim \frac{1}{4} \kappa^4 r^2$$

A. Trawiński, SDG, *Model of the AdS/QFT duality*, Phys. Rev. D **88**, 105025 (2013)

# Hadron masses at maximal separation of effective $q\bar{q}$ pair



A. Trawiński, S. Głazek, S. Brodsky, G. de Teramond, H. Dosch, *Phys. Rev. D* **90**, 074017 (2014).

# Jet production in pion-nucleus collisions

A. P. Trawiński, PhD Thesis, UW 2016

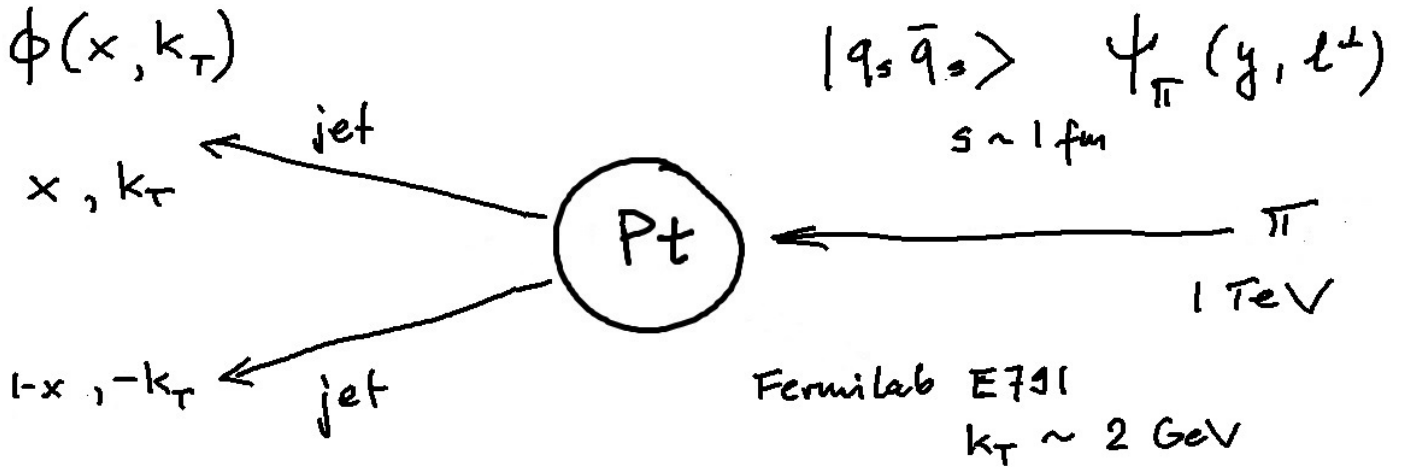


FIG. 6: Pion is wedged into two quark jets by a nucleus.

The RGPEP pion with  $U_{\text{eff}}$  from AdS/QCD

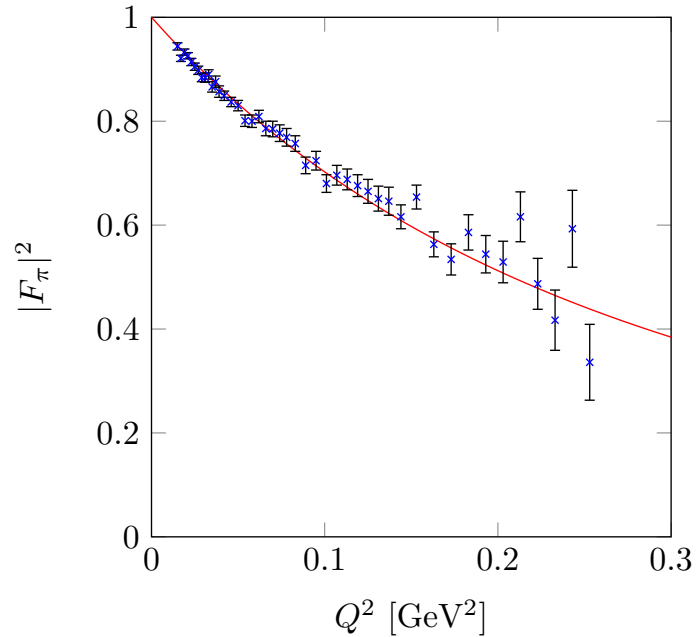
the RGPEP scale  $s_c \sim \frac{1}{\Lambda_{\text{QCD}}}$



$$\psi_{\pi} \sim e^{-s_c^2 \gamma M_{q\bar{q}}^2}$$

$$\gamma M_{q\bar{q}}^2 = \frac{k_{\perp}^2 + m_{s_c}^2}{x(1-x)}$$

# The RGPEP pion of $\sim 1$ fm quarks with $U_{\text{eff}}$ from AdS/QCD



Data: A. R. Amendolia, Nucl. Phys. B **277**, 168 (1986) Theory: A. P. Trawiński, PhD Thesis, UW 2016



$|\pi\rangle = |q_{s_c} \bar{q}_{s_c}\rangle$  in terms of quarks and gluons with  $1/s \sim 100 \text{ GeV}$

$$q_s = U_s q_0 U_s^\dagger$$

$$q_{s_1} = U_{s_1} U_{s_2}^\dagger q_{s_2} U_{s_2} U_{s_1}^\dagger$$

$$W = U_{s_1} U_{s_2}^\dagger$$

$$|\pi\rangle = |q_{s_c} \bar{q}_{s_c}\rangle$$

$$= W |q_s \bar{q}_s\rangle$$

$$s \sim 1/100 \text{ GeV}$$

$$\text{thick line with circle} = \text{thin line with circle} + \text{thin line with wavy loop} + \text{thin line with double wavy loop} + \dots$$

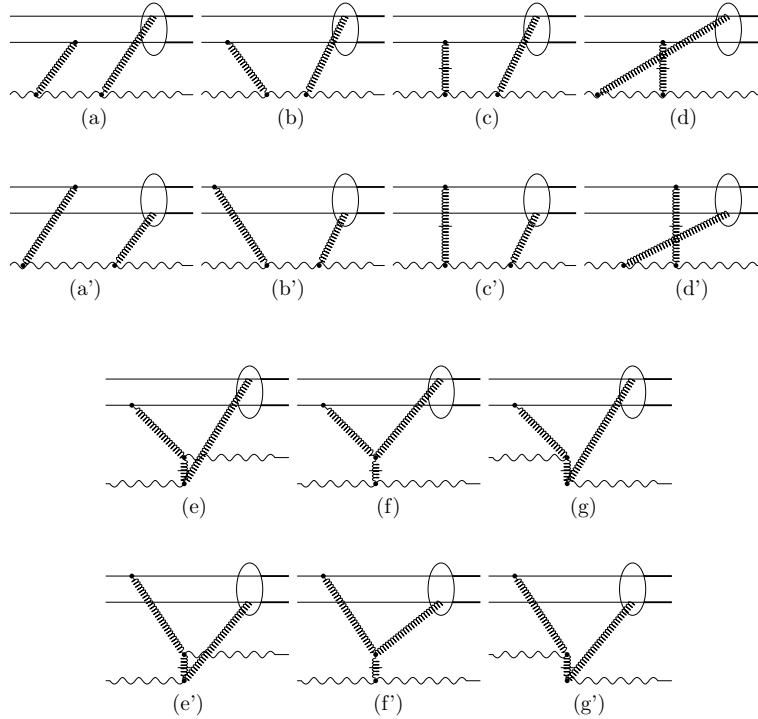
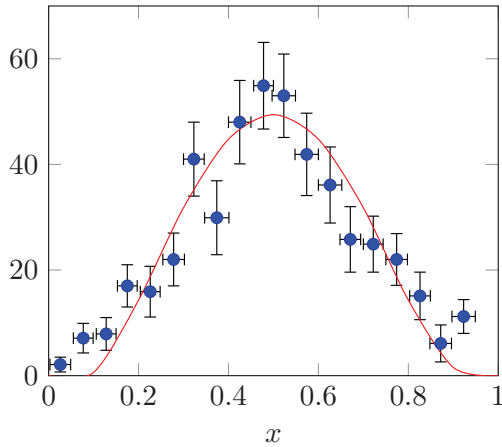
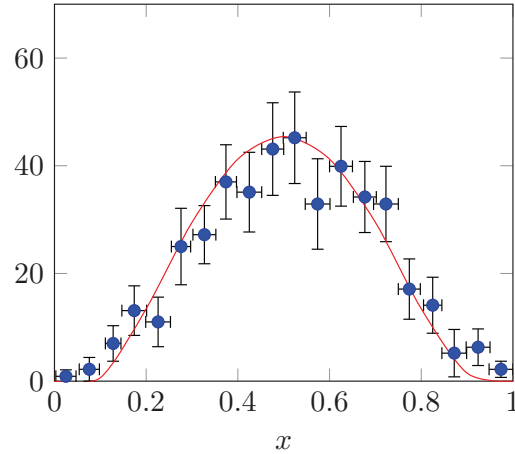


FIG. 7: Examples of pion splitting on gluons in a nucleus, described using  $H_{QCDs}$  with  $1/s \sim 100$  GeV.

$1.25 \text{ GeV} < |k^\perp| < 1.5 \text{ GeV}$



$1.5 \text{ GeV} < |k^\perp| < 2.5 \text{ GeV}$



Jet counts distribution  $\phi(x, k_T)$ , for jets induced by pions impinging on Pt, in two jet- $k_T$  bins.

Data from E791, Phys. Rev. Lett. **86**, 4768 (2001); theory from A. P. Trawiński, PhD Thesis, UW 2016.

**Conclusion:**

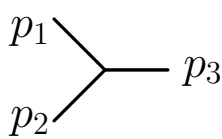
**RGPEP opens new options for developing QFT**

# Regularization

example  $\hat{H}_{A^3} \rightarrow \hat{H}_{A^3 R}$

$$\hat{H}_{A^3} = \sum_{123} \int_{123} \delta_{12.3} \left[ g Y_{123} a_1^\dagger a_2^\dagger a_3 + g Y_{123}^* a_3^\dagger a_2 a_1 \right]$$

$$\hat{H}_{A^3 R} = \sum_{123} \int_{123} \delta_{12.3} R \left[ g Y_{123} a_1^\dagger a_2^\dagger a_3 + g Y_{123}^* a_3^\dagger a_2 a_1 \right]$$



$$x_1 = p_1^+ / p_3^+ = x \quad k_1^\perp = p_1^\perp - x_1 p_3^\perp = \kappa^\perp$$

$$x_2 = p_2^+ / p_3^+ = 1 - x \quad k_2^\perp = p_2^\perp - x_2 p_3^\perp = -\kappa^\perp$$

$$r_i = x_i^\delta e^{-k_i^{\perp 2} / \Delta^2} \quad i = 1, 2, 3 \quad x_3 = 1 \quad k_3^\perp = 0^\perp$$

$$R = \prod_{i=1}^3 r_i$$

$$Y_{123} = i f^{c_1 c_2 c_3} \left[ \varepsilon_1^* \varepsilon_2^* \cdot \varepsilon_3 \kappa - \varepsilon_1^* \varepsilon_3 \cdot \varepsilon_2^* \kappa \frac{1}{x_2} - \varepsilon_2^* \varepsilon_3 \cdot \varepsilon_1^* \kappa \frac{1}{x_1} \right]$$

## Suggestion concerning glueballs

$$\begin{array}{c} \text{bare gluons of size } 0 \\ \left[ \begin{array}{c} \dots \\ |gggggg\rangle \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \end{array} \right] \end{array} = \begin{array}{c} \text{gluons of size } s \\ \left[ \begin{array}{c} \dots \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \\ |g\rangle \end{array} \right] \otimes \left[ \begin{array}{c} \dots \\ |ggggg\rangle \\ |gggg\rangle \\ |ggg\rangle \\ |gg\rangle \\ |g\rangle \end{array} \right] + \dots
 \end{array}$$

$$|gg\rangle + |ggg\rangle + \dots = |g_s g_s\rangle + |g_s g_s g_s\rangle + \dots$$

## QCD bound states and QED Hydrogen atom analogy

$$V_c = -\frac{\alpha_{\text{atom}}}{r} \sim \alpha_{\text{atom}} = \alpha(s_{\text{atom}})$$

$$\psi(\vec{r}) \sim e^{-\alpha_{\text{atom}}\mu|\vec{r}|} \quad \tilde{\psi}(\vec{k}) \sim \frac{1}{[k^2 + \alpha_{\text{atom}}^2\mu^2]^2}$$

$$\left[ P^+ \hat{H}_{QED}(s_{\text{atom}}) - P^\perp{}^2 \right] |\psi_{\text{atom}}\rangle = M_{\text{atom}}^2 |\psi_{\text{atom}}\rangle$$

$H_{QCD} \neq H_{QED}$  in the RGPEP  $m_g(s) = ? \rightarrow$  hadrons

**Eigenvalue problem  $\leftrightarrow$  Operators creating hadrons**

$$\mathbf{NP} \rightarrow H'_t = [[H_f, \tilde{H}_t], H_t]$$

$$\mathbf{AF} \rightarrow H_t = H_f + gH_{1t} + g^2 H_{2t} + g^3 H_{3t} + \dots$$

$$H'_{1t} = [[H_f, \tilde{H}_{1t}], H_f]$$

$$H'_{1t mn} = -(\mathcal{M}_m^2 - \mathcal{M}_n^2)^2 H_{10 mn}$$

$$H_{1t} = f_t H_{10}$$

$$f_t = e^{-t(\mathcal{M}_c^2 - \mathcal{M}_a^2)^2}$$

$$H_{A^3 1t} = \sum_{123} \int_{123} \delta_{12,3} e^{-t \mathcal{M}_{12}^4} \left[ Y_{123} a_{t1}^\dagger a_{t2}^\dagger a_{t3} + Y_{123}^* a_{t3}^\dagger a_{t2} a_{t1} \right]$$



