Matching fixed order QCD with parton shower for Drell-Yan and Higgs production

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Outline and motivation

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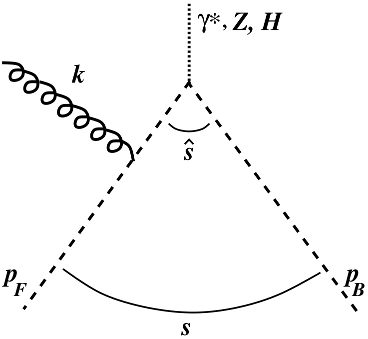
Why do we develop a new method?

- By departing from $\overline{\text{MS}}$, the NLO+PS matching becomes very simple $\rightarrow$ just multiplying by a positive MC weight.
- If is so simple at NLO+LO PS, there is a hope that pushing it to NNLO+NLO PS will be possible.

Production of a colour-neutral object



$$s = (p_F + p_B)^2$$

$$z = \frac{\hat{s}}{s}$$

Sudakov variables:

$$\alpha = \frac{2k \cdot p_B}{\sqrt{s}} = \frac{2k^+}{\sqrt{s}}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{s}} = \frac{2k^-}{\sqrt{s}}$$

$$z = 1 - \alpha - \beta$$

$$k_T^2 = s\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

Precision in QCD

Ideal world

NLO	α_s	$\alpha_s L^2$	$\alpha_s L$					$\mathrm{Virt}(\mathcal{O}(\alpha_s))$
$NNLO$	α_s^2	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$			$\mathrm{Virt}(\mathcal{O}(\alpha_s^2))$
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$			
$N^n LO$	α_s^n	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\alpha_s^n L^{2n-3}$	$\dots$	$\alpha_s^n L$	$\mathrm{Virt}(\mathcal{O}(\alpha_s^n))$
		LL	NLL	$NNLL$			$N^n LL$	

Precision in QCD

This talk: NLO matched to LL Parton Shower

NLO	α_s	$\alpha_s L^2$	$\alpha_s L$					$\text{Virt}(\mathcal{O}(\alpha_s))$
NNLO	α_s^2	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$			$\text{Virt}(\mathcal{O}(\alpha_s^2))$
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$			
$N^n LO$	α_s^n	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\alpha_s^n L^{2n-3}$	$\dots$	$\alpha_s^n L$	$\text{Virt}(\mathcal{O}(\alpha_s^n))$
		LL	*NLL*	*NNLL*			$N^n LL$	

Future: *NNLO matched to NLL Parton Shower*

Benefits of matching fixed order results with parton shower

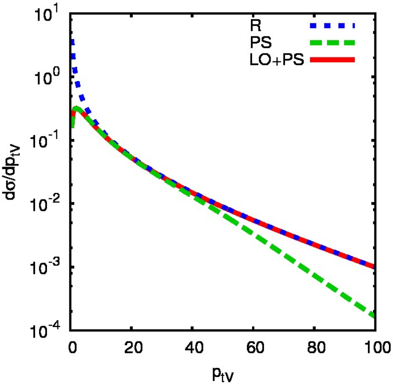
PS gives correct behaviour at low p_T and only approximate at high p_T.
The production of a gluon with p_{Tg} is given by

$$d\sigma_1^{\mathrm{PS}} = B \cdot K(p_{Tg})\, \Delta(Q, p_{Tg})\, d\phi_B d\phi_1$$

where $K \simeq R$ and the Sudakov $\Delta(Q, p_{Tg})$ suppresses emissions between scales Q and p_{Tg}.

LO+PS can be achieved by upgrading $B \cdot K$ to the exact R

$$d\sigma_1^{\mathrm{LO+PS}} = R(p_{Tg})\Delta(Q, p_{Tg})\, d\phi_B d\phi_1$$



Upgrade to NLO + PS

Naive addition of PS on top of a NLO event leads to double counting since PS will generate contributions already present at NLO!

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- MC@NLO [Frixione & Webber '02] and POWHEG [Nason '04]
  - Generate the hardest radiation based on the NLO cross section adjusted for subsequent shower emissions.
  - Pass the event to parton shower and let it produce further emissions.

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  - Generate the hardest radiation based on the NLO cross section adjusted for subsequent shower emissions.
  - Pass the event to parton shower and let it produce further emissions.
- KrkNLO [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13; Jadach, Płaczek, Sapeta, Siódmok & Skrzypek '15]
  - Run PS in a standard way.
  - Reweight the event with real$\times$virtual NLO correction.
  - Redefine PDFs to account for "collinear" part of NLO contribution.

Important subtlety

NLO cross section in $\overline{\text{MS}}$ factorization scheme (DY in $q\bar{q}$ channel)

$$d\sigma_{\text{DY}}^{\alpha_s} \quad = \quad \sigma_{\text{DY}}^{B}\, f_q^{\overline{\text{MS}}}(x_1, \hat{s}) \otimes \frac{\alpha_s}{2\pi}\, C_{q\bar{q}}^{\overline{\text{MS}}}(z) \otimes f_{\bar{q}}^{\overline{\text{MS}}}(x_2, \hat{s})\,,$$

where

$$C_{q\bar{q}}^{\overline{\text{MS}}}(z) = C_F \left[4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3}\pi^2 - 8 \right) \right].$$

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- We want to reproduce this with Monte Carlo, in a fully exclusive way.

If we use $\overline{\text{MS}}$ PDFs, we need to generate terms like $\sim \left(\frac{\ln(1-z)}{1-z}\right)_+$ which are technical artefacts of $\overline{\text{MS}}$ scheme (coming from ϵ/ϵ contributions).

- If we think of parton shower as a procedure that unfolds PDFs, then, obviously, these are not $\overline{\text{MS}}$ PDFs!

The KrkNLO method

The KrkNLO method

Two essential elements

1. **Change the factorization scheme from $\overline{\text{MS}}$ to MC**
   - produce new MC PDFs
   - differences at LO
   - universality: recovering $\overline{\text{MS}}$ NLO result

2. **Reweight parton shower**
   - correct hardest emission by “real” weight
   - upgrade the cross section/distributions to NLO by multiplicative, constant “virtual” weight

PDFs in MC scheme

Definition of LO PDFs in MC factorization scheme

Rotation in flavour space:

$$\begin{bmatrix} q(x,Q^2) \\ \bar{q}(x,Q^2) \\ g(x,Q^2) \end{bmatrix}_{\mathrm{MC}} = \begin{bmatrix} q \\ \bar{q} \\ g \end{bmatrix}_{\overline{\mathrm{MS}}} + \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \begin{bmatrix} K^{\mathrm{MC}}_{qq}(z) & 0 & K^{\mathrm{MC}}_{qg}(z) \\ 0 & K^{\mathrm{MC}}_{\bar{q}\bar{q}}(z) & K^{\mathrm{MC}}_{\bar{q}g}(z) \\ K^{\mathrm{MC}}_{gq}(z) & K^{\mathrm{MC}}_{g\bar{q}}(z) & K^{\mathrm{MC}}_{gg}(z) \end{bmatrix} \begin{bmatrix} q(\frac{x}{z},Q^2) \\ \bar{q}(\frac{x}{z},Q^2) \\ g(\frac{x}{z},Q^2) \end{bmatrix}_{\overline{\mathrm{MS}}}$$

where

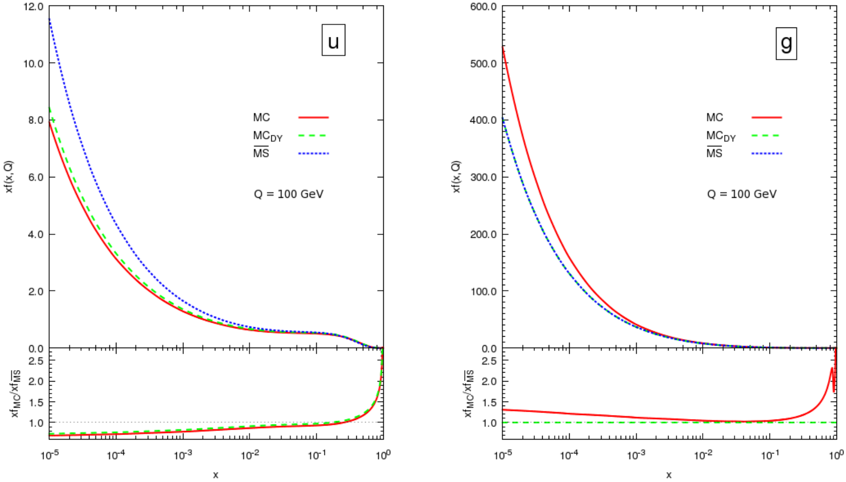
$$K^{\mathrm{MC}}_{gq}(z) = C_F \left\{ \frac{1+(1-z)^2}{z} \ln \frac{(1-z)^2}{z} + z \right\}$$

$$K^{\mathrm{MC}}_{gg}(z) = C_A \left\{ 4 \left[\frac{\ln(1-z)}{1-z} \right]_+ + 2 \left[\frac{1}{z} - 2 + z(1-z) \right] \ln \frac{(1-z)^2}{z} - 2 \frac{\ln z}{1-z} \right.$$
$$\left. - \delta(1-z) \left(\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36} \frac{T_f}{C_A} \right) \right\}$$

$$K^{\mathrm{MC}}_{qq}(z) = C_F \left\{ 4 \left[\frac{\ln(1-z)}{1-z} \right]_+ - (1+z) \ln \frac{(1-z)^2}{z} - 2 \frac{\ln z}{1-z} + 1 - z - \delta(1-z) \left(\frac{\pi^2}{3} + \frac{17}{4} \right) \right\}$$

$$K^{\mathrm{MC}}_{qg}(z) = T_R \left\{ \left[z^2 + (1-z)^2 \right] \ln \frac{(1-z)^2}{z} + 2z(1-z) \right\}$$

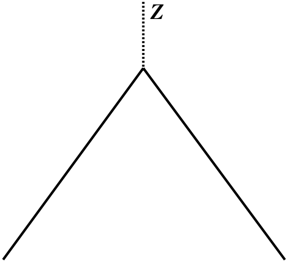
MC PDFs



- More gluons and less quarks at low x: momentum sum rules preserved!
- We checked directly the scheme independence of NLO cross sections!

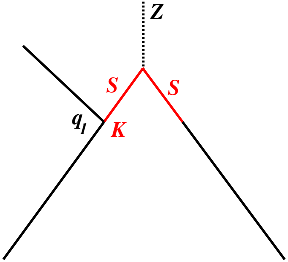
Reweighting the parton shower

Upgrading to NLO: the hardest emission



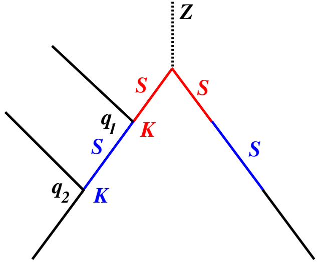
$$\sigma^{\text{LO}} = \sigma_B \otimes f_{\oplus}(Q^2, x_{\oplus}) \otimes f_{\ominus}(Q^2, x_{\ominus})$$

Upgrading to NLO: the hardest emission



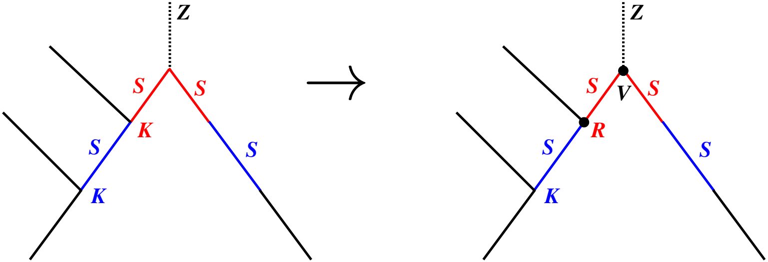
$$\sigma_{1+}^{\mathrm{PS}} = \sigma_B \otimes f_\oplus(Q^2, x_\oplus) \otimes f_\ominus(Q^2, x_\ominus)$$
$$\otimes \Big\{ S_\oplus(q_1^2, Q^2) K_\oplus(q_1^2, z_1) S_\ominus(q_1^2, Q^2) + S_\ominus(q_1^2, Q^2) K_\ominus(q_1^2, z_1) S_\oplus(q_1^2, Q^2) \Big\}$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{PS}} \quad &= \quad \sigma_B \otimes f_{\oplus}(Q^2, x_{\oplus}) \otimes f_{\ominus}(Q^2, x_{\ominus}) \\
&\otimes \Big\{ S_{\oplus}(q_1^2, Q^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(q_1^2, Q^2) \\
&\quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \\
&\quad + S_{\ominus}(q_1^2, Q^2) \otimes K_{\ominus}(q_1^2, z_1) \otimes S_{\oplus}(q_1^2, Q^2) \\
&\quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \Big\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\text{NLO+PS}} \quad = \quad & \sigma_B \, (1+V) \otimes f_\oplus(Q^2, x_\oplus) \otimes f_\ominus(Q^2, x_\ominus) \\
& \otimes \Big\{ S_\oplus(q_1^2, Q^2) K_\oplus(q_1^2, z_1) S_\ominus(q_1^2, Q^2) \, R_\oplus(q_1^2, z_1)/K_\oplus(q_1^2, z_1) \\
& \quad \otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\} \\
& \quad + S_\ominus(q_1^2, Q^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(q_1^2, Q^2) \, R_\ominus(q_1^2, z_1)/K_\ominus(q_1^2, z_1) \\
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\end{aligned}
$$

The MC weights

Real:

$$W_R^{q\bar{q}} = 1 - \frac{2\alpha\beta}{1+z^2} \qquad W_R^{qg} = 1 + \frac{\beta(\beta+2z)}{(1-z)^2+z^2}$$

$$W_R^{gg} = \frac{1+z^4+\alpha^4+\beta^4}{1+z^4+(1-z)^4} \qquad W_R^{gq} = \frac{1+\beta^2}{1+(1-z)^2}$$

Virtual:

$$W_V^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 + \frac{1}{2}\right] \qquad W_V^{qg} = 0$$

$$W_V^{gg} = \frac{\alpha_s}{2\pi} C_A \left[\frac{4}{3}\pi^2 + \frac{473}{36} + \frac{59}{18}\frac{T_f}{C_A}\right] \qquad W_V^{gq} = 0$$

- **Real weight are simple functions of kinematic variables**
  One can compute it on the fly, inside an MC, or outside, using information from event record.
- **Virtual+soft weight are constant**

Results

Drell-Yan: NLO+PS results

KrkNLO

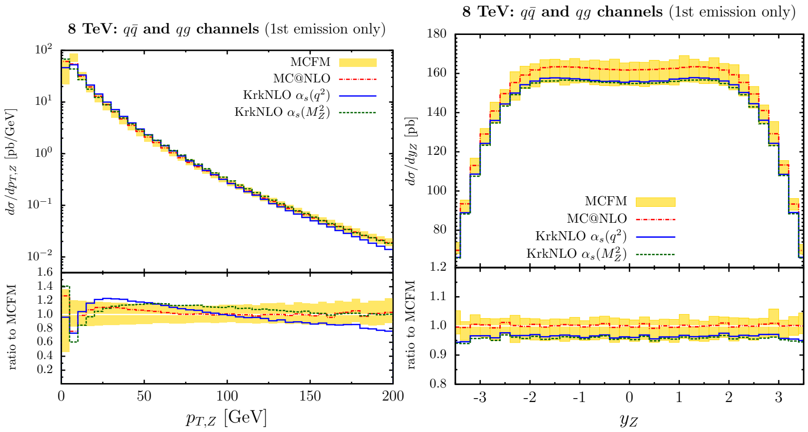
- Virtual: $\mu^2 = \mu_F^2 = \mu_R^2 = m_Z^2$
- Real: two choices
  - $\mu^2 = m_Z^2$
  - $\mu^2 = q^2$, where $q \simeq k_T$ is the PS evolution variable

$\hookrightarrow$ differences formally beyond NLO, indicative of missing higher orders

Compared to:

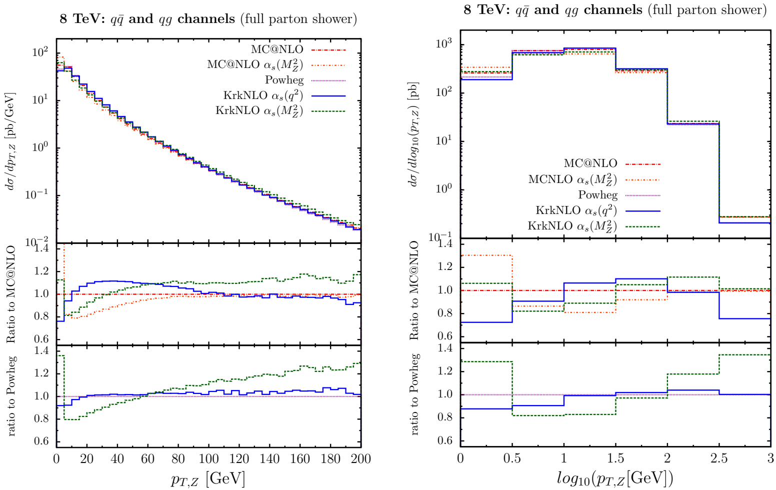
- **MCFM**: pure NLO, $\mu^2 = m_Z^2$
- **MC@NLO**: from Sherpa/Herwig 7, with the evolution var. $q^2 \simeq k_T^2$
- **POWHEG**: from Herwig 7 with the evolution variable k_T^2

Drell-Yan: Matched results, botch channels, 1st emission



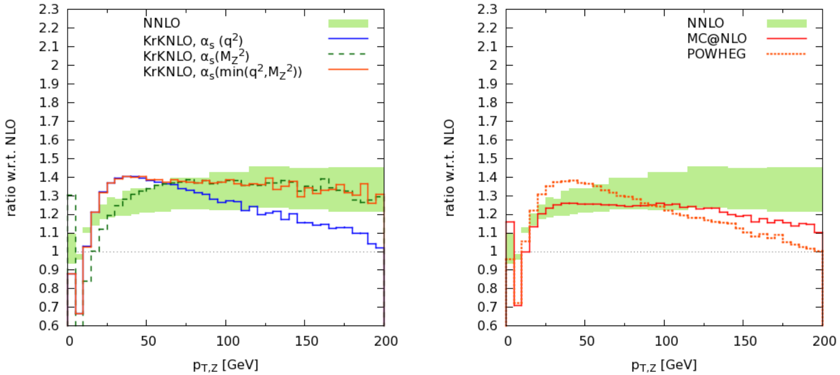
- Moderate differences between KrkNLO $\alpha_s(q^2)$ and MC@NLO in the region below M_Z and between KrkNLO $\alpha_s(M_Z^2)$ and MC@NLO in the region above M_Z

Drell-Yan: Matched results, both channels, full PS



- KrkNLO $\alpha_s(q^2)$ stays overall very close to MC@NLO
- KrkNLO $\alpha_s(q^2)$ almost coincides with POWHEG $p_{T,Z}$ distributions

Drell-Yan: Comparison to NNLO



- KrkNLO with $\alpha_s(\min(q^2, M_z^2))$ nicely follows full NNLO at high $p_{T,Z}$

Higgs production in gluon fusion

- The KrkNLO method for Drell-Yan and Higgs production is now implemented in Herwig 7
- It will be available with next release of the program

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$gg \to H$

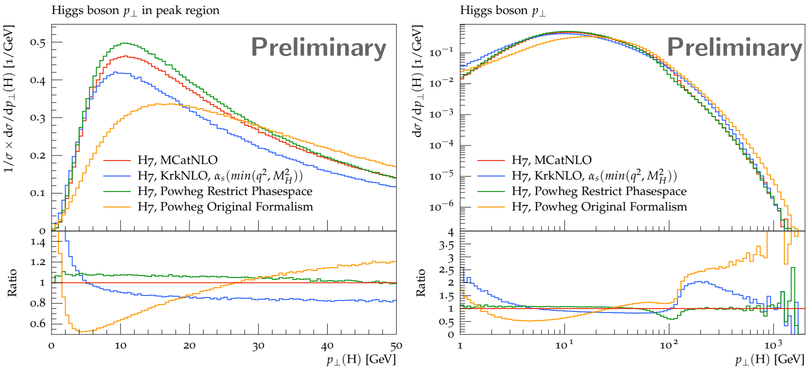
- All results obtained with KrkNLO/Powheg/MC@NLO implementations in Herwig 7
- $\mu_F = \mu_R = m_H$

Total cross section

MC@NLO	18.857 ± 0.006 pb
Powheg	18.870 ± 0.007 pb
KrkNLO	17.170 ± 0.004 pb

Preliminary

Higgs production in gluon fusion



- 10-20% differences between KrkNLO and Powheg/MC@NLO at lower $p_{T,H}$

Conclusions

- KrkNLO: a new method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
  - Virtual correction is just a constant and does not depend on Born kinematics.
- The method has been implemented for Drell-Yan and Higgs production on top of Catani-Seymour shower in Sherpa 2. 0 and Herwig 7 event generators.
- A range of comparisons to MCFM, DYNNLO, MC@NLO and POWHEG.
- The results of KrkNLO matching procedure at NLO+LL level come out consistent with fixed order NLO and other matching methods.
  $\hookrightarrow$ Still, 20-30% difference between various methods are common.

BACKUP

Fixed order calculations in QCD

General structure of NLO cross sections:

$$d\sigma = \Big[B + V(\alpha_s) + C(\alpha_s)\Big] d\phi_B + R(\alpha_s)\, d\phi_B d\phi_1$$

- B, R, V - Born, real and virtual part
- C - collinear subtraction counterterm (for initial state radiation case)

Each part: V, C and $\int R d\phi_1$ is separately divergent (soft and collinear). Divergences cancel in the sum.

Calculation possible e.g. by means of subtraction procedure

$$\begin{aligned} d\sigma &= \Big[B + V(\alpha_s) + \int_1 A(\alpha_s) d\phi_1 + C(\alpha_s)\Big] d\phi_B + \\ & \int_1 \Big[R(\alpha_s) - A(\alpha_s)\Big]\, d\phi_1 d\phi_B\,, \end{aligned}$$

where $A \simeq R$, such that it reproduces collinear and soft singularities.

- Good for inclusive observables or distributions at high-p_T.

Parton shower

In the collinear region, fixed order calculation becomes unreliable because each α_s^n is accompanied by a large, logarithmic coefficient, $\ln^n$, and

$$(\alpha_s \ln)^n \sim 1 \text{ for all } n\,.$$

These terms must be summed to all orders and this is what the Parton Shower (PS) is aiming at. In the collinear limit

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s(q^2)}{2\pi} \frac{dq^2}{q^2} P(z) dz\,.$$

This can be iterated and used to resum all leading log contributions. In particular, non-emission probability (Sudakov form factor) is given by

$$\Delta(q_1, q_2) = \exp\left[-\int_{q_1}^{q_2} \frac{\alpha_s(q^2)}{2\pi} \frac{dq^2}{q^2} \int_{z_0}^{1} P(z)\, dz \right].$$

In Monte Carlo event generators, the scale of i$^{\text{th}}$ emission, q_i, is found by solving

$$\Delta(q_{i-1}, q_i) = R_i\,,$$

where $R_i \in [0,1]$ is a random number and q_{i-1} is a scale of previous emission.

MC@NLO [Frixione & Webber '02]

- Naive addition of PS on top of NLO event leads to double counting since PS will generate contributions already present at NLO.
- These affects both resolvable and non-resolvable emissions.
- MC@NLO fixes that by modifying NLO subtraction procedure.

The first emission is generated according to:

$$d\sigma = \mathbb{S}\, d\phi_B + \mathbb{H}\, d\phi_B d\phi_1\,,$$

where

$$\mathbb{S} = B + V + C + \int K\, d\phi_1\,, \qquad\qquad \mathbb{H} = R - K\,.$$

This is then followed by the emissions from parton shower.

- NLO accuracy of the above is manifest.
- $-K$ in $\mathbb{H}$ cancels resolvable and $\int K$ in $\mathbb{S}$ unresolvable (from Sudakov expansion) emissions of the parton shower

POWHEG [Nason '04]

- Generate the hardest radiation at NLO accuracy.
- Pass the event, with its positive, NLO weight, to PS for further generation of soft radiation.

The formula for generation of NLO accurate hardest emission:

$$d\sigma = \bar{B}^S \left[\Delta_S(Q_0) + \Delta_S(p_T) \frac{R^S}{B} d\phi_1 \right] d\phi_B + R^F d\phi_R \,,$$

where

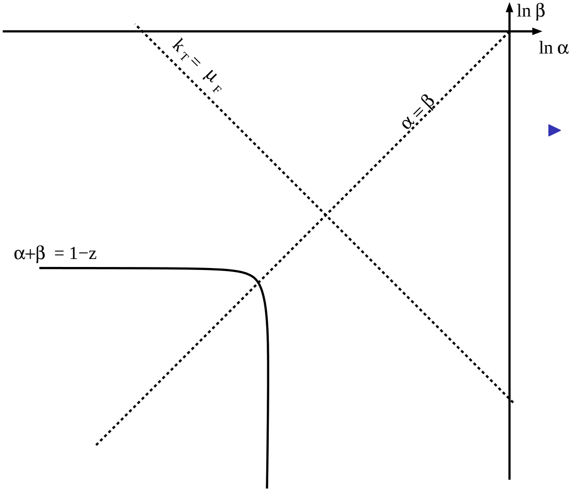
$$\bar{B}^S = B + V + \int R^S d\phi_1 \,, \qquad R = R^S + R^F \,,$$

and

$$\Delta_S(p_T) = \exp\left[- \int \frac{R^S}{B} d\phi_1 \Theta(k_T(\phi_1) - p_T) \right] .$$

- One can show that the above formula yields NLO accuracy.

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\mathrm{MS}}$



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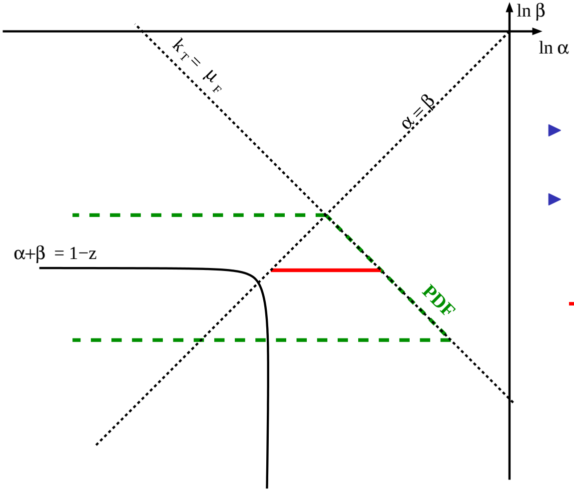
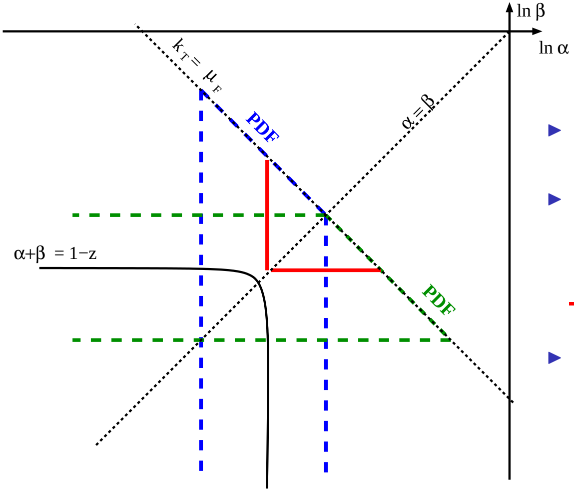


- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get

$$\int \frac{1}{1-z}\frac{d\beta}{\beta} = 2\frac{\ln(1-z)}{1-z}$$

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



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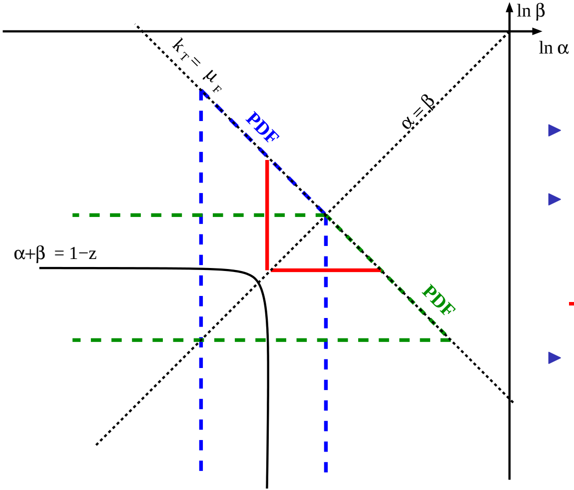

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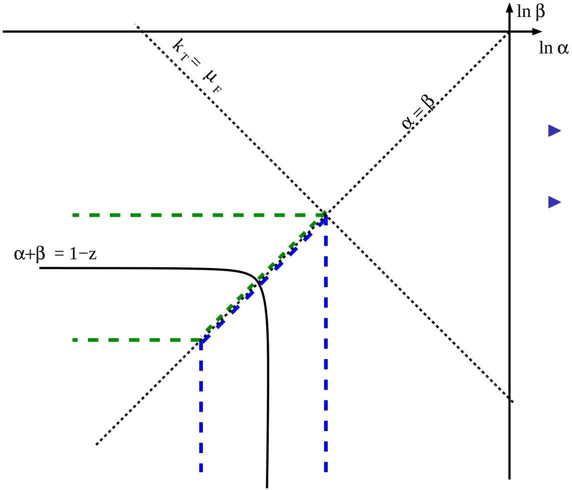
Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.

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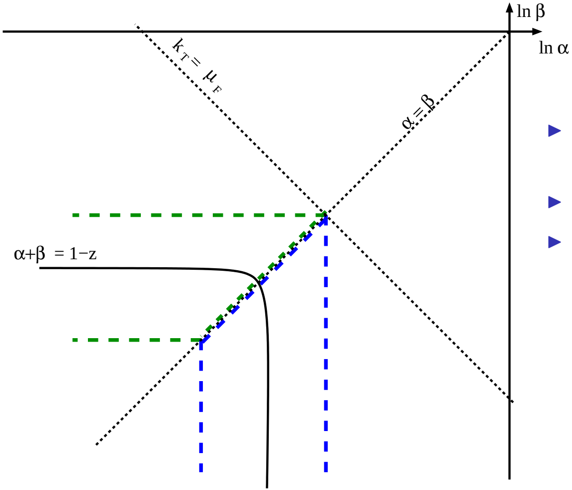

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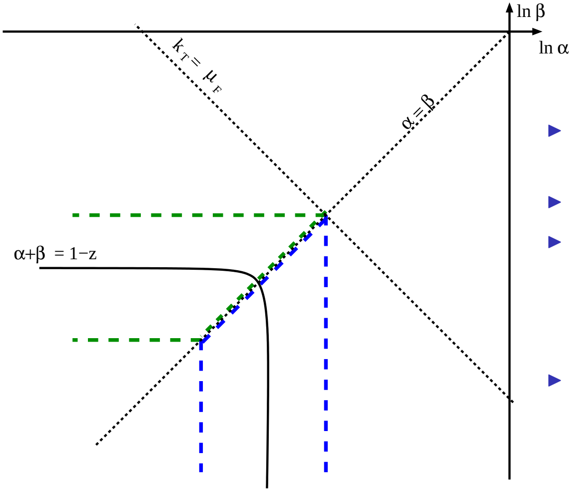

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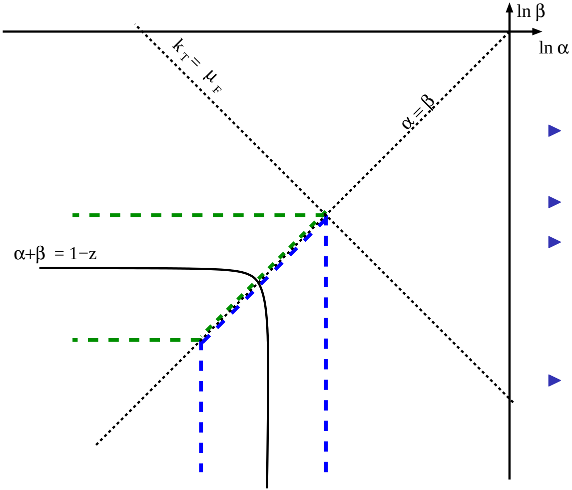

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Could the change of factorization scheme help us to simplify NLO+PS matching?

Implementation on top of the Catani-Seymour shower

↪ We used Sherpa 2.0.0 implementation of the CS shower.

Phase space measure of emitted gluon

$$\frac{d\alpha}{\alpha}\frac{d\beta}{\beta} = \frac{d\alpha d\beta}{\beta(\alpha+\beta)} + \frac{d\alpha d\beta}{\alpha(\alpha+\beta)}$$

- The evolution variable:

$$q_F^2 = s(\alpha+\beta)\beta, \qquad\qquad q_B^2 = s(\alpha+\beta)\alpha\,,$$

hence

$$\frac{d\alpha d\beta}{\alpha\beta} = \frac{dq_F^2}{q_F^2}\frac{dz}{1-z} + \frac{dq_B^2}{q_B^2}\frac{dz}{1-z}.$$

- The CS shower covers all space of (α, β).

$$\alpha+\beta \leq 1 \qquad \Rightarrow \qquad z \geq 0 \quad \text{and} \quad q_{F,B}^2 \leq s$$

$$\alpha, \beta > 0 \qquad \Rightarrow \quad (1-z)^2 > q_F^2/s \quad \text{or} \quad (1-z)^2 > q_B^2/s$$

Implementation on top of the Catani-Seymour shower

↪ It turns out that coefficient functions of the CS shower equal to those from the MC scheme of Jadach et al. arXiv:1103.5015. Hence, CS $\equiv$ MC.

The $C_2(z)$ function:

$$C_2^{\rm MC}(z)\Big|_{\rm real} = \int (R-K)$$

- For the $q\bar{q}$ channel:

$$C_{2q}^{\rm MC}(z)\Big|_{\rm real} = \frac{\alpha_s}{2\pi} C_F\left[-2(1-z)\right]$$

- For the qg channel:

$$C_{2g}^{\rm MC}(z)\Big|_{\rm real} = \frac{\alpha_s}{2\pi}\, T_R\, \frac{1}{2}(1-z)(1+3z)$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $C_{2q}^{\rm MC}(z)$ and $C_{2g}^{\rm MC}(z)$ from $\overline{\rm MS}$ PDFs
  - absorbing all z-dependent terms from $\overline{\rm MS}$ coefficient functions
- The virtual correction:

$$C_{2q}\Big|_{\rm virt} = \delta(1-z)\left(\frac{4}{3}\pi^2 - \frac{5}{2}\right)$$

is applied multiplicatively.

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Simple form of the coefficient functions with no singular terms!

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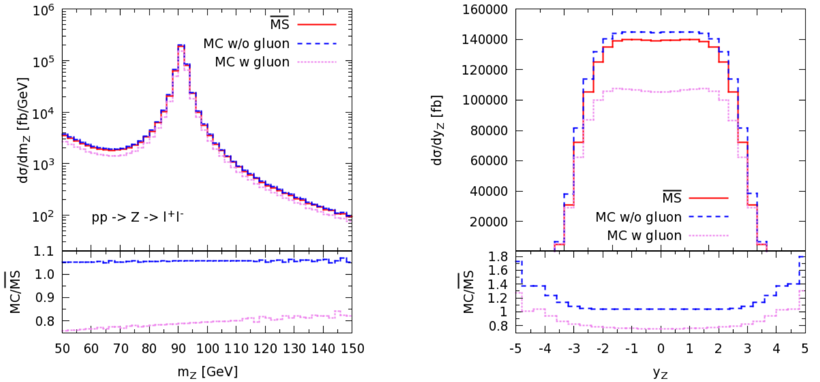
$\overline{\text{MS}}$ vs MC at LO



- $+5\%$ effect at central rapidities in $q\bar{q}$ and -20% for both channels
- pronounced difference at large y coming from the $x \sim 1$ region

$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

Validation: $\overline{\text{MS}}$ scheme vs MC scheme at NLO

Cross section, truncated at given $\mathcal{O}(\alpha_s)$, cannot depend on factorization scheme

$$\sigma_{\text{tot}}^{\overline{\text{MS}}} \stackrel{!}{=} \sigma_{\text{tot}}^{\text{MC}}$$

At $\mathcal{O}(\alpha_s)$:

$$C_q^{\overline{\text{MS}}} f_q f_{\bar{q}} = \Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}}$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

$\hookrightarrow$ Identical validation performed with both $q\bar{q}$ and qg channels.

Reweighting procedure

The "Sudakov" form factor for he CS shower

$$S(Q^2, \Lambda^2, x) = \int_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \; \frac{D(q^2, x/z)/z}{D(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $D(q^2, x)$ - parton distribution functions

The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Drell-Yan: Matched results, total cross section

$q\bar{q}$ channel

	$\sigma_{\text{tot}}^{q\bar{q}}$ [pb]
MCFM	1273.4 ± 0.1
MC@NLO	1273.4 ± 0.1
POWHEG	1272.1 ± 0.7
KrkNLO $\alpha_s(q^2)$	1282.6 ± 0.2
KrkNLO $\alpha_s(M_Z^2)$	1285.3 ± 0.2

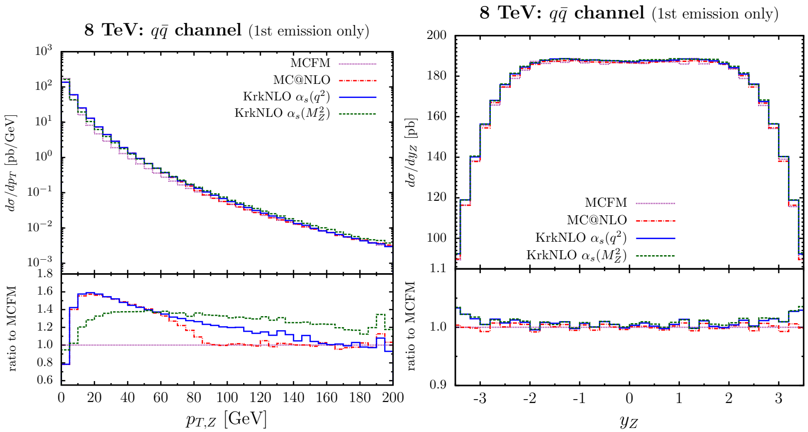
- sub-percent differences from beyond-NLO terms in the KrkNLO result (MC PDFs, mixed real-virtual)
- negligible difference between fixed and running coupling

$q\bar{q} + qg$ channels

	$\sigma_{\text{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_s(q^2)$	1045.4 ± 0.1
KrkNLO $\alpha_s(M_Z^2)$	1039.0 ± 0.1

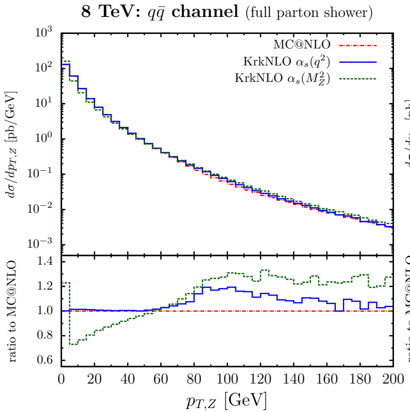
- beyond-NLO terms reach up to 4% in the KrkNLO result
  $\hookrightarrow$ resulting from large gluon luminosity leading to $f^{\text{MC}}/f^{\overline{\text{MS}}} < 1$
- small differences between fixed and running coupling choices

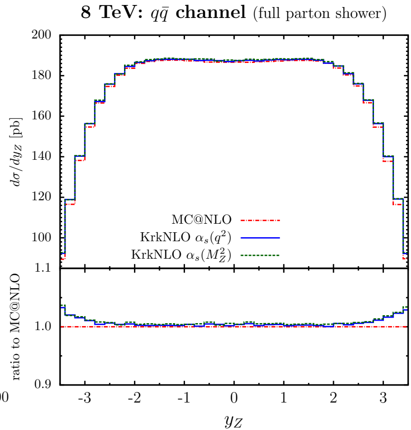
Drell-Yan: Matched results, $q\bar{q}$, 1st emission



- Reproduction of y_Z distribution at NLO.
- Agreement of KrkNLO $\alpha_s(q^2)$ with MC@NLO at low $p_{T,Z}$: PS domination
- KrkNLO results above MC@NLO and MCFM at higher $p_{T,Z}$: $\mathcal{O}(\alpha_s^2)$ terms

Drell-Yan: Matched results, $q\bar{q}$, full PS





- Low $p_{T,Z}$ part of the spectrum changes but KrkNLO $\alpha_s(q^2)$ with MC@NLO agree there because of shower domination
- KrkNLO results above pure NLO at high $p_{T,Z}$: admixture of NNLO terms
- Diffs between two KrkNLO result at high $p_{T,Z}$: running coupling effects