Dark matter near a resonance

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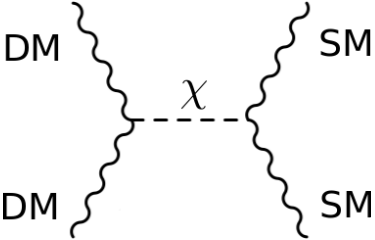
$$2M_{DM} \approx M_\chi$$



- thermally averaged annihilation cross-section strongly depend on temperature
- modified limits from indirect searches
- applications to the self-interacting dark matter

Boltzmann equation

$$\frac{dY}{dx} = -\alpha\frac{\langle\sigma v\rangle}{x^2}(Y^2 - Y_{\rm EQ}), \qquad \text{DM yield } Y = n/s, \qquad \alpha = \frac{s(m)}{H(m)}$$

- **entropy** s in coming volume is conserved, dimensionless parameter $x = m/T$

Decoupling

$$\Gamma = n_{EQ}\langle\sigma v\rangle \lesssim H(x)$$

$$x_d \approx 25 \qquad (\text{log dependence on } \langle\sigma v\rangle)$$

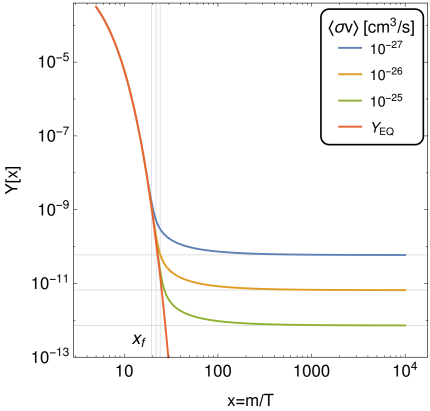
Approximate solutions

$$\frac{1}{Y_\infty} - \frac{1}{Y(x_d)} = \alpha\int_{x_d}^{\infty}\frac{\langle\sigma v\rangle}{x^2}$$

$\langle\sigma \mathbf{v}\rangle =$ **const**, $\qquad Y(x_d) \gg Y_\infty$

$$Y_\infty \approx \frac{x_d}{\alpha\langle\sigma v\rangle_0}$$

generalizations $\langle\sigma v\rangle \sim x^{-m}(1 + a x^{-n})$



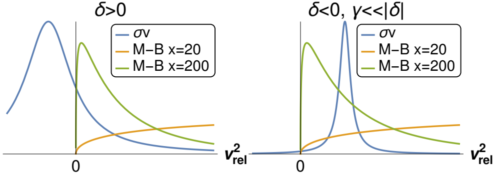
WIMP miracle $m_{\rm DM} \sim 100$ GeV, $\langle\sigma v\rangle \approx 2\times10^{-26}\ {\rm cm^3 s^{-1}} \to \Omega_{DM}h^2 \approx 0.1$

Annihilation near the resonance

$$\sigma v_{\rm rel} \sim \frac{1}{(s-M^2)^2+\Gamma^2 M^2} \approx \frac{1}{(\delta + v_{\rm rel}^2/4)^2+\gamma^2}$$

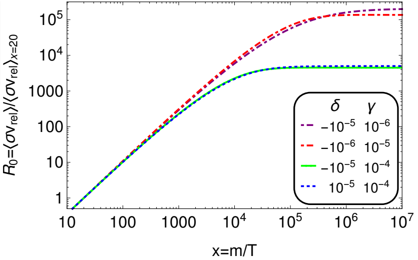
$$\delta = 4m_{DM}^2/M_R^2 - 1, \qquad \gamma = \Gamma/M_R$$

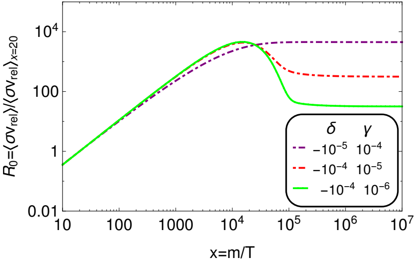
Thermal average with Maxwell-Boltzmann distribution



$$\langle \sigma v \rangle = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \sigma v_{\rm rel}$$

- strong temperature dependence,
- $\langle \sigma v \rangle$ grows with decreasing T – annihilation can last long after decoupling
- reaches maximum when $x \approx (\max[|\delta|, \gamma])^{-1}$





$$\frac{1}{Y_\infty} - \cancel{\frac{1}{Y(x_d)}} = -\alpha \int_{x_d}^{\infty} \frac{\langle \sigma v \rangle}{x^2} = -\alpha \int_{x_d}^{\infty} dx \frac{1}{2\sqrt{\pi x}} \int_0^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \sigma v_{\rm rel}$$

Change the order - integral over x

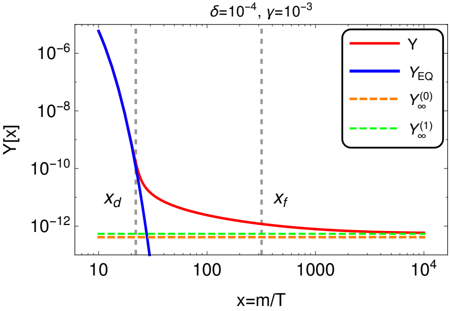
$$\frac{\mathrm{erfc}(v_{\rm rel}\sqrt{x_d}/2)}{v_{\rm rel}} \approx \frac{1}{v_{\rm rel}} - \sqrt{\frac{x_d}{\pi}}$$

Approximate solution

$$Y_\infty = \frac{x_f}{\alpha \langle \sigma v \rangle_{T=0}}$$

"Freeze-out" temperature

$$x_f^{-1} = \begin{cases} \gamma(\pi - 2\sqrt{2\pi x_d \gamma}), & \text{if } \gamma \gg |\delta|, \\ \delta(2 - 2\sqrt{\pi x_d \delta}), & \text{if } \delta \gg \gamma > 0, \\ \delta^2 \gamma^{-1}(2\pi - 4\sqrt{\pi x_d |\delta|}), & \text{if } -\delta \gg \gamma > 0. \end{cases}$$



- effective annihilation after decoupling
- at "freeze-out" temperature $\langle \sigma v \rangle$ reaches its maximal value

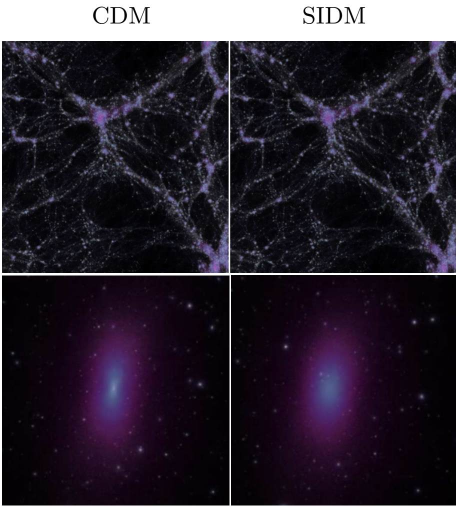
Application - self interacting dark matter

Properties of dark matter:

- electrically neutral (non luminous)
- non-baryonic (BBN)
- non-relativistic (cold) (structure formation)
- weakly interacting with ordinary matter (direct detection)
- **collisionless or self-interacting ?**

Self-interaction strength

- **no effects** on large scale structures
- **modifications** on the scales of clusters and galactic halos



Rocha+ 2013

Mean free path $l = (n\sigma)^{-1} \sim \ m/\sigma_{\rm self}$

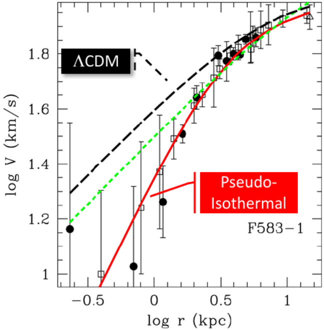
Limit on self-interaction cross section

$$\frac{\sigma_{\rm self}}{m} < \begin{cases} 1.25\,\frac{\rm cm^2}{\rm g}\,({\rm long-range}) \\ 0.7\,\frac{\rm cm^2}{\rm g}\,({\rm short-range}) \end{cases}$$

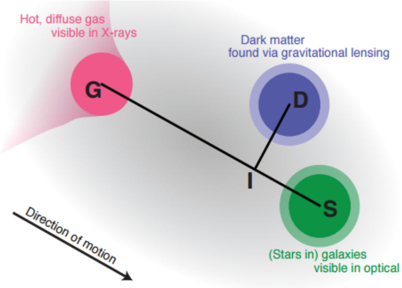
Randall+ 2008

Core DM distribution in galactic halos

$$\frac{\sigma_{\rm self}}{m} \gtrsim 0.1\,\frac{\rm cm^2}{\rm g} \sim \frac{\rm barn}{\rm GeV}$$



Kuzio de Narray+ 2008



Harvey+ 2015

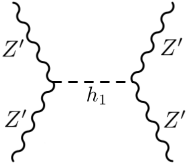
Self-interaction from Higgs resonance

Abelian vector dark matter

- extra complex scalar S charged under $U(1)_X$, VEV $\langle S \rangle = v_X$
- scalar mixing angle α, two mass eigenstates h_1, h_2
- **dark matter** candidate $U(1)_X$ **vector boson**, $M_{Z'} = g_x v_x \leftarrow$ Higgs mechanism

Resonance $2M_{DM} \approx M_{h_1} = 125$ GeV

- decay width $\gamma = \Gamma_{h_1}/M_{h_1} = 3.2 \times 10^{-5}$
- no invisible Higgs decays $2M_{Z'} > M_{h_1}$,
- fine-tuning $\delta = 4M_{Z'}^2/M_{h_1} - 1 \ll \gamma$



$g_x < 4\pi$(petrubativity)

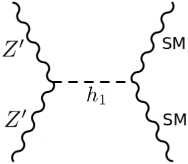
$|\sin\alpha| < 0.36$ (ATLAS+CMS)

$$\frac{\sigma_{\rm self}}{M_{Z'}} < 1.1\ \mathbf{cm}^2\mathrm{g}^{-1}$$

DM abundance $\Omega_{DM} h^2 \sim x_f / \langle \sigma v \rangle_0$

non-resonant case $\langle \sigma v \rangle_0 \approx 2 \times 10^{-26}$ cm^{3}s^{-1}, $x_f = 20$

Higgs resonance $\langle \sigma v \rangle_0 \approx 10^{-19}$ cm^{3}s^{-1}, $x_f = 1/(\pi\gamma) = 10^4$



$\sigma_{\rm self}/M_{Z'} \sim (g_x \sin\alpha)^4$

$\langle \sigma v \rangle_0 \sim (g_x \sin\alpha)^2$

$$\sigma_{\rm self}/M_{Z'} \lesssim 10^{-8}\ \mathbf{cm}^2\mathrm{g}^{-1}$$

Resonance with the second scalar and bounds from indirect searches

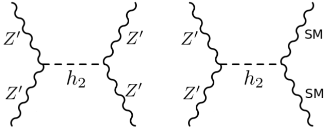
Annihilation cross section

$\langle \sigma v \rangle \propto (g_x \cos\alpha \sin\alpha)^2$

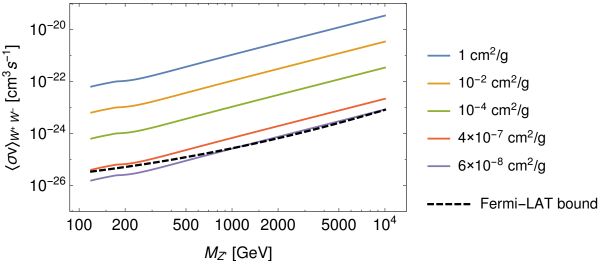
It can be supressed by $\alpha \ll 1$.

Self-interaction

$\sigma_{\text{self}} \propto (g_x \cos\alpha)^4$



Fermi-LAT constraints



Lower bound on annihilation rate $\langle \sigma v \rangle$

$$\langle \sigma v \rangle_0 \gtrsim \frac{2.2 \cdot 10^2}{\epsilon \eta} \left(\frac{M_{DM}}{100 \text{ GeV}} \right)^{3/2} \left(\frac{\sigma_{\text{self}}/M_{DM}}{1 \text{ cm}^2/\text{g}} \right)^{1/2} \left(\frac{100}{g_*} \right)^{1/2} \left(\frac{0.12}{\Omega_{DM} h^2} \right) 2\times 10^{-26} \text{ cm}^3\text{s}^{-1}$$

$\epsilon \in \{2, \pi\}$ – depends on the parameters of the resonance δ, γ

$\eta < \eta_{\max}$ – depends on the couplings, limited by perturbativity (VDM $\eta_{\max} = 1/48$)

Summary

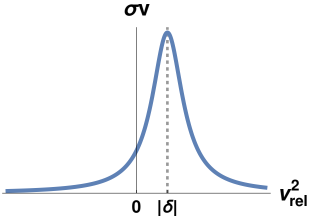
- Thermally averaged **cross-sections** for dark matter annihilation near the resonance **strongly depend on temperature**.
- There exist **approximate formulas for relic density** in terms of annihilation cross-section at low tempretures and parameters of the resonance
- **Self-interaction** rates are **limited** by the **indirect searches**.

BACKUP SLIDES

Annihilation near the resonance

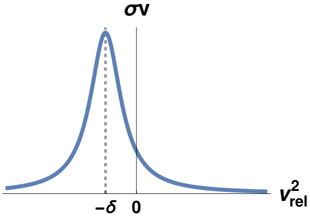
$$\sigma v_{\rm rel} = \frac{\omega}{s}\beta_f \frac{4M^2\bar{\Gamma}^2 B_i B_f}{\bar{\beta}_f\bar{\beta}_i}\frac{1}{(s-M^2)^2+\Gamma^2 M^2} \approx \frac{4\omega}{M^2\bar{\beta}_i}\frac{\bar{\gamma}^2 B_i B_f}{(\delta+v_{\rm rel}^2/4)^2+\gamma^2}$$

- initial states m_i, final states m_f, resonance M
- statistical factor $\omega = (2S_R+1)/(2S_i+1)^2$
- resonance decay branching ratios B_i, B_f
- phase space $\beta = \frac{1}{8\pi}\sqrt{1-4m^2/s}$, $\bar{\beta} = \beta|_{s=m}$
- small parameters $\delta = 4m_i^2/M^2 - 1$, $\gamma = \Gamma/M$



Resonance peak in physical region

$2m_i < M$, $\delta < 0$,
$\bar{\Gamma} = \Gamma$ - physical width
peak is kinematically accesible



Resonance peak in unphysical region

$2m_i > M$, $\delta > 0$,
$\bar{\Gamma}B_i/\bar{\beta} \sim g_i$ - coupling constant

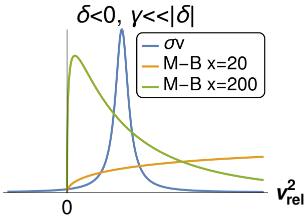
Thermally-averaged cross-sections - another case

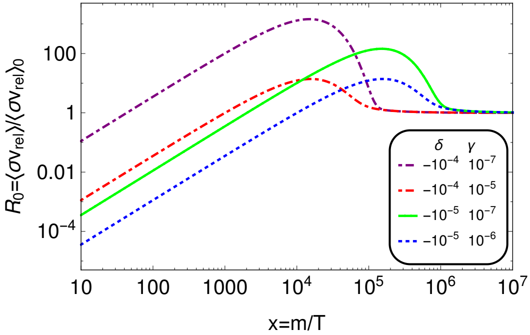
$$\frac{4\omega}{M^2\bar{\beta}_i}\frac{\bar{\gamma}^2 B_i B_f}{(\delta + v_{\rm rel}^2/4)^2+\gamma^2}$$

$\langle v_{\rm rel}^2\rangle = 6/x$

Narrow resonance in physical region $\delta < 0$, $\gamma \ll |\delta|$

- maximum of $\langle \sigma v\rangle$ at $x \approx |\delta|^{-1}$,
- $\langle \sigma v\rangle_{\rm max} = \delta/\gamma \langle \sigma v\rangle_0$





$$\frac{dY}{dx} = -\alpha\frac{\langle\sigma v\rangle}{x^2}(Y^2 - \cancel{Y_{\rm EQ}^2})$$

$x > x_d \Rightarrow Y \gg Y_{\rm EQ}$

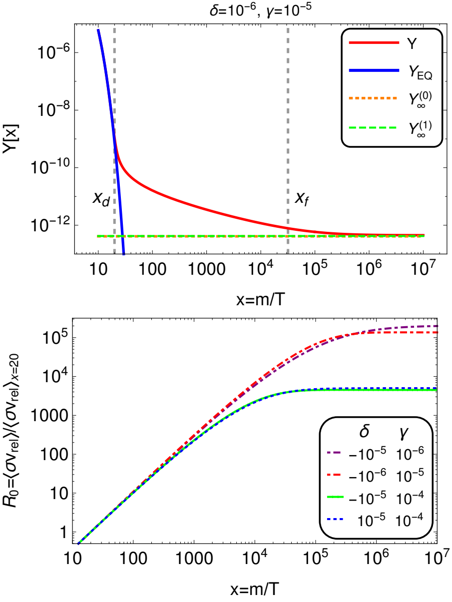
$$\frac{1}{Y_\infty} - \cancel{\frac{1}{Y(x)}} = -\alpha\int_x^\infty \frac{\langle\sigma v\rangle}{x^2}$$

- $x > x_f = (\epsilon\ max[|\delta|m\gamma])^{-1}$, $\epsilon \sim \mathbf{1}$
  $\langle\sigma v\rangle \approx \langle\sigma v\rangle_0 = const$
- $Y_\infty = x_f/(\alpha\langle\sigma v\rangle_0)$

$$Y_\infty^{(0)} = \frac{1}{\alpha\epsilon\max[|\delta|,\gamma]\langle\sigma v\rangle_0}$$

No dependence on x_d and $\langle\sigma v\rangle$ for $x < x_f$

How to find ϵ?



Relic abundance - approximate formulas

$$\frac{1}{Y_\infty} - \cancel{\frac{1}{Y(x_d)}} = -\alpha \int_{x_d}^{\infty} \frac{\langle \sigma v \rangle}{x^2} = -\alpha \int_{x_d}^{\infty} dx \frac{1}{2\sqrt{\pi x}} \int_0^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \sigma v_{\rm rel}$$

Change of the integration order

$$\int_{x_d}^{\infty} \frac{\exp(-x v_{\rm rel}^2/4)}{2\sqrt{\pi x}} dx = \frac{\mathrm{erfc}(v_{\rm rel}\sqrt{x_d}/2)}{v_{\rm rel}} \approx \frac{\mathbf{1}}{\mathbf{v}_{\rm rel}} - \sqrt{\frac{x_d}{\pi}}$$

First approximation - how to find ϵ?

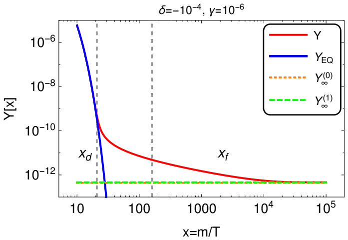
$$\frac{1}{Y_\infty^{(0)}} = \int_0^\infty \alpha \langle \sigma v \rangle_0 v_{\rm rel} \frac{\delta^2 + \gamma^2}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2} =$$

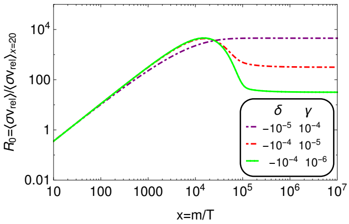
$$= \alpha \langle \sigma v \rangle_0 (\delta^2 + \gamma^2) \frac{\pi - 2\arctan(\delta/\gamma)}{\gamma} \approx$$

$$\approx \alpha \langle \sigma v \rangle_0 \times \begin{cases} \pi\gamma, & \text{if } \gamma \gg |\delta|, \\ 2\delta, & \text{if } \delta \gg \gamma > 0, \\ 2\pi\delta^2/\gamma, & \text{if } -\delta \gg \gamma > 0. \end{cases}$$

No dependence on x_d

Non-resonant $1/Y_\infty = \alpha \langle \sigma v \rangle_0 / x_f$



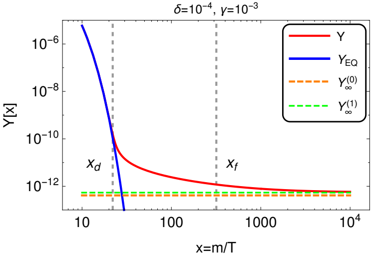


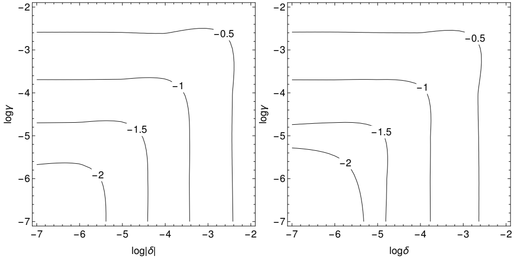
Relic abundance - approximate formulas

$$\int_{x_d}^{\infty} \frac{\exp(-x v_{\rm rel}^2/4)}{2\sqrt{\pi x}} dx = \frac{\mathrm{erfc}(v_{\rm rel}\sqrt{x_d}/2)}{v_{\rm rel}} \approx \frac{1}{v_{\rm rel}} - \sqrt{\frac{\mathbf{x_d}}{\pi}}$$

Second approximation - dependence on x_d

$$\frac{1}{Y_\infty^{(1)}} = \alpha \langle \sigma v \rangle_0 \times \begin{cases} \gamma(\pi - 2\sqrt{2\pi x_d \gamma}), & \text{if } \gamma \gg |\delta|, \\ \delta(2 - 2\sqrt{\pi x_d \delta}), & \text{if } \delta \gg \gamma > 0,, \\ \delta^2 \gamma^{-1}(2\pi - 4\sqrt{\pi x_d |\delta|}), & \text{if } -\delta \gg \gamma > 0. \end{cases}$$





$Y_\infty \sim x_f/\langle \sigma v \rangle_0$ $\langle \sigma v \rangle_0$ **can be many times larger than** 2×10^{-26} $\mathbf{cm^3 g^{-1}}$

Abelian vector dark matter

Additional complex scalar field S

- singlet of $U(1)_Y \times SU(2)_L \times SU(3)_c$
- charged under $U(1)_X$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + (D_\mu S)^* D^\mu S + \tilde{V}(H, S) \tag{1}$$

$$V(H, S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2 \tag{2}$$

Vacuum expectation values:

$$\langle H \rangle = \frac{v_{SM}}{\sqrt{2}}, \qquad \langle S \rangle = \frac{v_x}{\sqrt{2}} \tag{3}$$

$U(1)_X$ vector gauge boson V_μ

- $D_\mu = \partial_\mu + i g_x V_\mu$
- Stability condition - no mixing of $U(1)_X$ with $U(1)_Y$ ~~$B_{\mu\nu} V^{\mu\nu}$~~

  $\mathcal{Z}_2 : V_\mu \to -V_\mu, \qquad S \to S^*, \qquad S = \phi e^{i\sigma}$: $\phi \to \phi$, $\sigma \to -\sigma$
- V_μ acquires mass due to the Higgs mechanism in the hidden sector

$$M_{Z'} = g_x v_x \tag{4}$$

Abelian vector dark matter

Scalar mixing

$$S = \frac{1}{\sqrt{2}}(v_x + \phi_S + i\sigma_S) \quad , \quad H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H), \quad \text{where} \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{5}$$

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_x \\ \kappa v v_x & 2\lambda_S v_x^2 \end{pmatrix}, \quad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix} \tag{6}$$

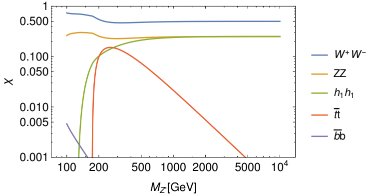
$M_{h_1} = 125$ GeV - observed Higgs particle

Higgs couplings

$$\mathcal{L} \supset \frac{h_1 \cos\alpha + h_2 \sin\alpha}{v} \left(2M_W W_\mu^+ W^{\mu-} + M_Z^2 Z_\mu Z^\mu - \sum_f m_f \bar{f} f \right) \tag{7}$$

Typical WIMP cross section $x_f \approx 25$

$$\langle \sigma v \rangle_0 = \frac{x_f}{25}\left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega_{DM}h^2}\right) 2\times 10^{-26}\ \mathrm{cm}^3\mathrm{s}^{-1}$$



$$\langle \sigma v \rangle_0 \gtrsim \frac{1.1\cdot 10^4}{\epsilon}\left(\frac{M_{DM}}{100\ \mathrm{GeV}}\right)^{3/2}\left(\frac{\sigma_{\mathrm{self}}/M_{DM}}{1\ \mathrm{cm}^2/\mathrm{g}}\right)^{1/2}\left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega_{DM}h^2}\right) 2\times 10^{-26}\ \mathrm{cm}^3\mathrm{s}^{-1}$$

$$\delta \gg \gamma \Rightarrow x_f = 1/(2\delta), \qquad \epsilon = 2$$
$$\gamma \gg |\delta| \Rightarrow x_f = 1/(\pi\gamma), \qquad \epsilon = \pi$$

$$\frac{\sigma_{\mathrm{self}}}{M_{DM}} = \frac{4\pi\omega}{M_{DM}^3}\left(\frac{\gamma B}{\bar\beta}\right)^2 (\epsilon x_f)^2$$

$\gamma B/\bar\beta$ – limited by perturbativity