Maximum Tension Principle and Entropic Force

Hussain Gohar

Institute of Physics

University of Szczecin

Poland

"Collider Physics"

2nd Symposium of the Division for Physics of

Fundamental Interactions of the Polish Physical Society

13-15 May 2016

Outline

- **Maximum Tension Principle**
- **Abolishing the Maximum Tension Principle**
- **Summary**
- **References**

This talk is based on our recently published papers

1. M.P. Dabrowski and **H. G**, "Abolishing the maximum tension principle" , Phys. Lett. B 748, 428-431 (2015)

2. M.P. Dabrowski, **H. G** and V. Salzano, "Varying constants entropic-ΛCDM cosmology", Entropy 18(2), 60 (2016)

Maximum Tension Principle

- In the context of recent gravity waves detection (Abbott et al. 2016) it is interesting to look what are the limitations on the phenomenon of black holes mergers.
- As shown by **Gibbons (2002) and Schiller (2005)** due to the phenomenon of gravitational collapse and black hole formation, there exists a maximum force or maximum tension limit

$$F_{max} = c^4/4G$$

  in general relativity (c - the velocity of light, G - Newton gravitational constant). The fact is known as "The Principle of Maximum Tension". The maximum power (Dyson luminosity) which can maximally be emitted as gravity waves is $P_{max} = cF_{max} = c^5/4G$.
- This is unlike in Newton's gravity, where the two point masses may approach each other arbitrarily close and so the force between them may reach infinity. The limit can nicely be derived by the application of the cosmic string deficit angle $\phi = (8\pi G/c^4)F$ not to exceed 2π **(Gibbons; 2002)**
- It is interesting that the maximum tension limit holds also in string theory, where the tension T is given by the Regge slope parameter α', i.e. $F_{max} \propto T = 1/2\pi\alpha'$.

Maximum Tension Principle

- It is interesting to note that the factor c^4/G appears in the Einstein field equations and is $\sim 1.3 \times 10^{44}$ Newtons. If the field equations are presented in the form

$$T_{\mu\nu} = \frac{1}{8\pi}\frac{c^4}{G}G_{\mu\nu} \quad ,$$

  where $T_{\mu\nu}$ is the stress tensor and $G_{\mu\nu}$ is the (geometrical) Einstein tensor, then we can consider their analogy with the elastic force equation:

$$F = kx \quad ,$$

  where k is an elastic constant, and x is the displacement.

- In this analogy, we can think of gravitational waves being some perturbations of spacetime and the ratio c^4/G which appears in Einstein field equation plays the role of an elastic constant. Its large value means that the spacetime is extremely rigid or, in other words, it is extremely difficult to make it vibrate **(Poisson and Will; 2014)**

Maximum Tension Principle

- We make an observation that similar ratio c^4/G appears in the expression for the entropic force within the framework of entropic cosmology **(Easson et al.; 2011)**. In order to calculate this force one has to apply the Hawking temperature

$$T = \frac{\gamma \hbar c}{2\pi k_B r_h} \quad ,$$

and the Bekenstein entropy

$$S = \frac{k_B c^3 A}{4\hbar G} = \frac{\pi k_B c^3}{G\hbar} r_h^2,$$

The entropic force is defined as

$$F_r = -T\frac{dS}{dr_h}$$

and by the application of above definitions, one gets

$$F_r = -\gamma \frac{c^4}{G},$$

where the minus sign means that the force points in the direction of increasing entropy. It emerges that up to a numerical factor $\gamma/4$ and the sign, this is the maximum force limit in general relativity.

Abolishing the Maximum Tension Principle

0.1 Varying constants cosmology

In the varying constants (c, G) theories **(Moffat; 1993, Albrecht, Magueijo and Barrow; 1999)** the entropic force is given by **(Dąbrowski and Gohar; 2016)**

$$F = -T\frac{dS}{dr_h} = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{3\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} + 2\frac{\dot{r}_h}{r_h}}{\frac{\dot{r_h}}{r_h}}\right],$$

where we have applied the Hubble horizon

$$r_h(t) = \frac{c(t)}{H(t)}$$

and get

$$F = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right].$$

The following conclusions are in order. Namely, if the fundamental constants c and G are really constant, then the the entropic force reduces to a constant value $-\gamma c^4/G$. However, the variability of c and G modifies this claim in a way that the maximum force also varies in time. In particular, it seems to be infinite for a constant horizon value $\dot{r}_h = 0$ which corresponds to a model with $c(t) \propto H(t)$. The entropic force can also become infinite, if the derivatives of c and G are infinite. So the Maximum Force/Tension conjecture does not necessarily hold for varying constants theories.

Abolishing the Maximum Tension Principle

0.2 Modified entropy models

Komatsu and Kimura (2013, 2014) used nonadditive entropy **(Tsallis; 1988 and Tasallis and Cirto; 2013)** or nonextensive Tsallis entropy, given by **(Komatsu et al., PRD 87, 043531; PRD 88, 083534).**

$$S_3 = \zeta \frac{\pi k_B c^3}{\hbar G} r_h^3,$$

which as applied to the entropic force definition together with the Hawking temperature gives

$$F_{r3} = -T \frac{dS_3}{dr_h} = -\frac{3}{2} \gamma \zeta \frac{c^4}{G} r_h,$$

where ζ is a dimensional constant. The authors showed that this entropic force is responsible for the current acceleration of the universe. Since $r_h = r_h(t)$, then the entropic force F_{r3} may reach infinity again, when the horizon size becomes infinitely large. Of course the same happens, if one of the conditions $c \to \infty$ or $G \to 0$ holds (opposite to Carrol's limit $c \to 0$ or $G \to \infty$. This, again allows to avoid the maximum tension principle.

Abolishing the Maximum Tension Principle

Another example is the quartic entropy defined as **(Komatsu et al., PRD 87, 043531; PRD 88, 083534).**

$$S_4 = \xi \frac{\pi k_B c^3}{\hbar G} r_h^4,$$

where ξ is a dimensional constant. It gives an entropic force in the form

$$F_{r4} = -T \frac{dS_4}{dr_h} = -2\gamma \xi \frac{c^4}{G} r_h^2.$$

Here again $r_h = r_h(t)$, and this force F_{r4} may reach infinity when the horizon size becomes infinitely large and this happens much faster than for the volume entropy entropic force F_{r3}. Hence this also potentially violates the maximum tension principle.

Abolishing the Maximum Tension Principle

0.3 Black Holes

Similar considerations about the entropic force can be performed for black holes whose Hawking temperature and Bekenstein entropy are given by **(Bekenstein, PRD 7, 2333 ; PRD 12, 3077; Hawking, Nature 248, 30).**

$$T = \frac{\hbar\kappa}{2\pi k_B c},$$
$$S_{bh} = \frac{\pi k_B c^3}{G\hbar} r_+^2,$$

where κ and r_+ are the surface gravity and the event horizon of a black hole, respectively. In this way, the volume entropy (Tsallis entropy) and quartic entropy for black holes can be written as

$$S_3 = \lambda \frac{\pi k_B c^3}{4G\hbar} r_+^3,$$
$$S_4 = \beta \frac{\pi k_B c^3}{4G\hbar} r_+^4,$$

where λ and β are some dimensional constants.

Abolishing the Maximum Tension Principle

Since we have defined r_+ and as a general event horizon, then we start our discussion with charged Reissner-Nordström black holes for which the surface gravity is given by

$$\kappa = \frac{c^2}{r_+^2}\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}},$$

and the event horizon by

$$r_+ = \frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}}$$

where M is the mass, Q is the charge, ε_0 is the permittivity of space, and we consider a non-extremal case for which

$$M^2 > \frac{Q^2}{4\pi\varepsilon_0 G}.$$

Abolishing the Maximum Tension Principle

Now, we can calculate the entropic force (with constant c and G) as follows

$$F_r = -T\frac{dS_{bh}}{dr_+} = -\frac{c^4}{G}\frac{1}{r_+}\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}},$$

and

$$F_{r3} = -T\frac{dS_3}{dr_+} = -\frac{3\lambda c^4}{8G}\left(\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}}\right),$$

and

$$F_{r4} = -T\frac{dS_4}{dr_+} = -\frac{\beta c^4}{2G}\left(\sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{4\pi\varepsilon_0 c^4}}\right)r_+.$$

In the limit $Q \to 0$ the above formulas reduce to the Schwarzschild black hole case for which the surface gravity is $\kappa = c^4/4GM$ and the event horizon is equal to the Schwarzschild radius $r_+ = r_s = 2GM/c^2$. In such a case the entropic forces read as

$$\begin{aligned} F_r &= -T\frac{dS_{bh}}{dr_s} = -\frac{c^4}{2G}, \\ F_{r3} &= -T\frac{dS_3}{dr_s} = -\lambda\frac{3c^4}{16G}r_s, \\ F_{r4} &= -T\frac{dS_4}{dr_s} = -\beta\frac{c^4}{4G}r_s^2. \end{aligned}$$

Abolishing the Maximum Tension Principle

- From above equations, we conclude that the entropic force is divergent when the mass of a black hole (proportional to the horizon radius $r_s \propto M$) tends to infinity for the case of the volume entropy and the quartic entropy which contradicts the Principle of Maximum Tension. However, for Bekenstein entropy, the principle holds.

- For charged Reissner-Nordström black holes, the entropic force for the Bekenstein entropy approaches to $c^4/2G$ while M goes to infinity, so the Principle of Maximum Tension holds as well. However, for the case of the volume and the quartic entropies, the entropic force diverges to infinity when mass tends to infinity, which is again an example of the Maximum Tension Principle violation

Abolishing the Maximum Tension Principle

0.4 Generalized Uncertaintity Principle

The generalized uncertainty principle (GUP) modifies the Heisenberg principle at the Planck energies into **(Tawfik and Diab, IJMPD 23 1430025 (2014); Tawfik and Diab, arXiv:1502.04562)**

$$\Delta x \Delta p = \frac{\hbar}{2}\left[1 + \alpha^2 (\Delta p)^2\right],$$

where x is the position and p the momentum, and

$$\alpha = \alpha_0 \frac{l_{pl}}{\hbar}$$

(α_0 is a dimensionless constant). GUP corrects the Bekenstein entropy and the Hawking temperature of black holes into ($C = \text{const.}$)

$$S_{GUP} = S + \frac{\alpha^2 \pi}{4} \ln S - \frac{(\alpha^2 \pi)^2}{8} \frac{1}{S} + ... + C,$$

and

$$T_{GUP} = T - \frac{\alpha^2 \pi}{2} T^2 - 4(\alpha^2 \pi)^4 T^4.$$

Abolishing the Maximum Tension Principle

The above GUP corrected entropy and temperature can be derived by using the quadratic form of GUP. One can also use the linear GUP and modified dispersion relations for possible other modifications of the Bekenstein entropy and the Hawking temperature but we will not be investigating such a case here. By using the above definitions, we can write the GUP corrected entropic force

$$F_{rGUP} = -T_{GUP}\frac{dS_{GUP}}{dr}$$

or

$$F_{rGUP} = -[F_r + \frac{\alpha^2\pi}{4}\frac{F_r}{S} - \frac{\alpha^2\pi}{2}TF_r + ...].$$

From above equations one can conclude that the GUP force can be influenced by the Hawking temperature and the Bekenstein entropy and possibly through their dependence on the running fundamental constants c and G they may cause it to diverge then abolishing the Principle of Maximum Tension in this GUP case.

Summary

- We have studied the Principle of Maximum Tension in the context of different theories of gravity. Firstly, we noticed that the entropic force, applied recently to cosmology which can be the reason of global acceleration of the universe is just (up to a factor) the maximum force between the two relativistic bodies surrounded by their horizons.
- We have further explored the issue of abolishing the Principle of Maximum Tension in a couple of physical cases. For example: varying constants cosmologies, generalised uncertainty principle etc.
- It is also possible to abolish the principle if one applies other definitions of entropy than the Bekenstein area entropy such as volume and quartic entropies.

References

- G.W. Gibbons, Found. Phys. 32, 1891 (2002).
- C. Schiller, Int. Journ. Theor. Phys. 44, 1629 (2005).
- J.D. Barrow and G.W. Gibbons, Mon. Not. Royal Astron. Soc. 446, 3874 (2014).
- E. Poisson and C.M. Will, Gravity, (Cambridge University Press, Cambridge, 2014).
- A. Easson, P. H. Frampton, and G. F. Smoot, Phys. Lett. B 696, 273 (2011); Int. J. Mod. Phys. A 27, 125066 (2012); Y. F. Cai, J. Liu, and H. Li, Phys. Lett. B 690, 213 (2010); Y. F. Cai and E. N. Saridakis, Phys. Lett. B 697, 280 (2011); T. Qiu and E. N. Saridakis, Phys. Rev. D 85, 043504 (2012).
- C. Tsallis and L.J.L. Cirto, Eur. Phys. Journ. C73,103520 (2013).
- C. Tsallis, J. Stat. Phys. 52, 479 (1988).
- D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B 216, 41 (1989); K. Konishi,G. Paffuti and P. Provero, Phys. Lett. B 234-276 (1990); M. Maggiore, Phys. Lett. B 304, 65 (1993); Phys. Rev. D 49, 5182-5187 (1994); A. N. Tawfik and A. M. Diab, Int. J. Mod. Phys. D 23 1430025 (2014); "Black Hole Corrections due to Minimal Length and Modified Dispersion Relation" arXiv:1502.04562