The averaging problem in cosmology

Jan Ostrowski

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General Relativity and homogeneous cosmology

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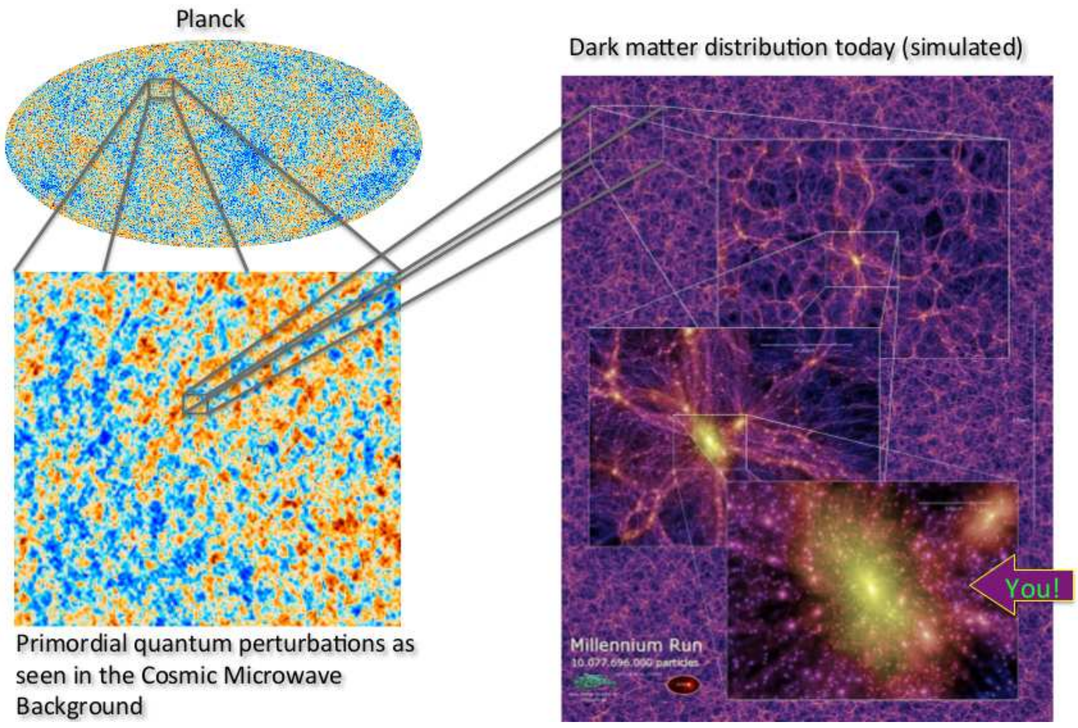

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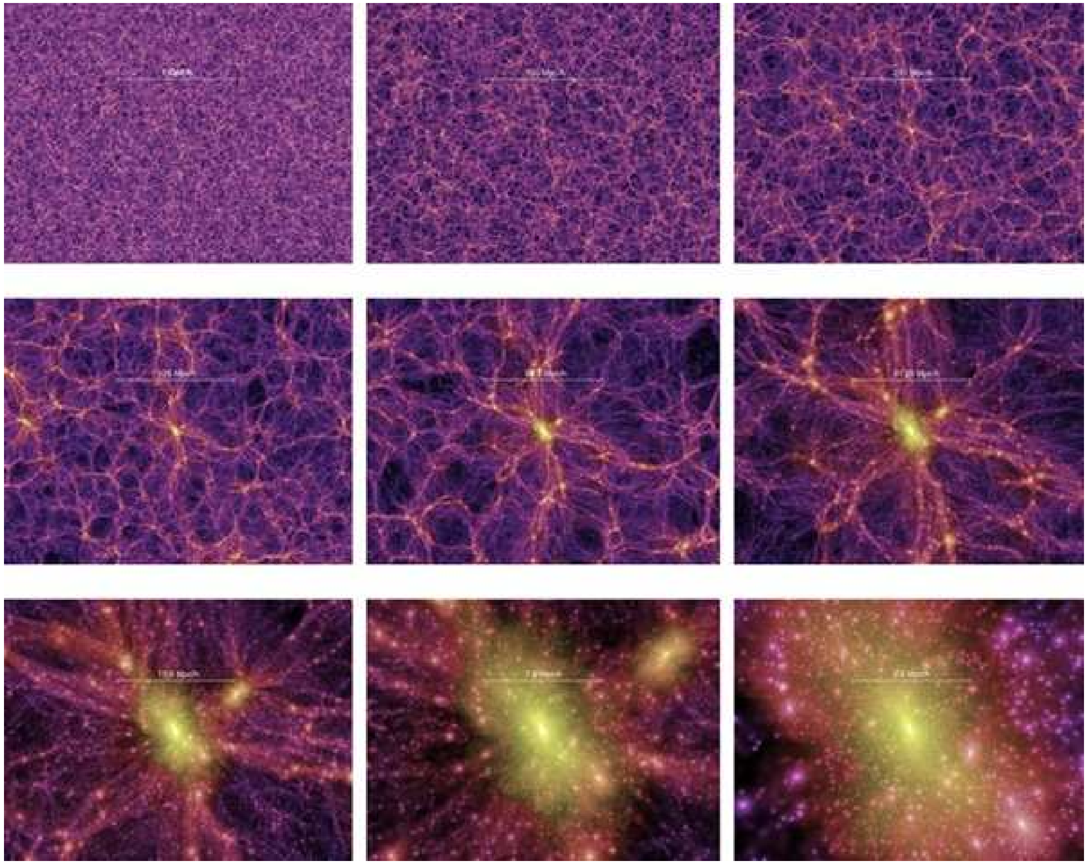
- several observational 'tensions' e.g. lithium abundance, BAO peak shift, CMB large-angle anomalies

Universe is inhomogeneous

Inhomogeneous Universe: sheets, filaments, clusters, voids



Millennium simulation, Springel et al.

Inhomogeneous cosmology: challenges

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- **Each of these problems may require different approach**

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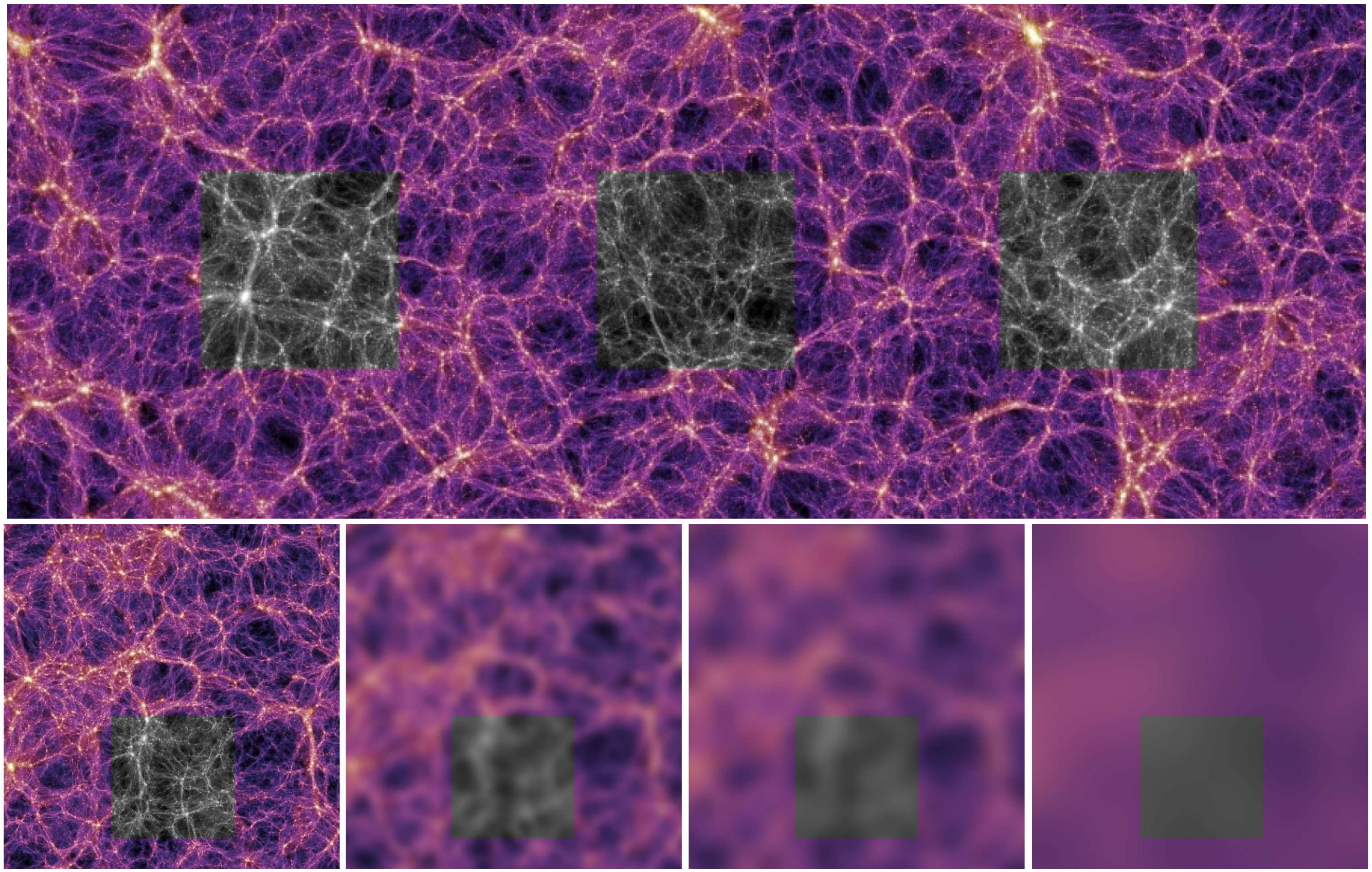
- modelling the light propagation: Dyer-Roeder approximation
- proper N-body simulations: work in progress

Universe at 150 Mpc scales (black boxes)



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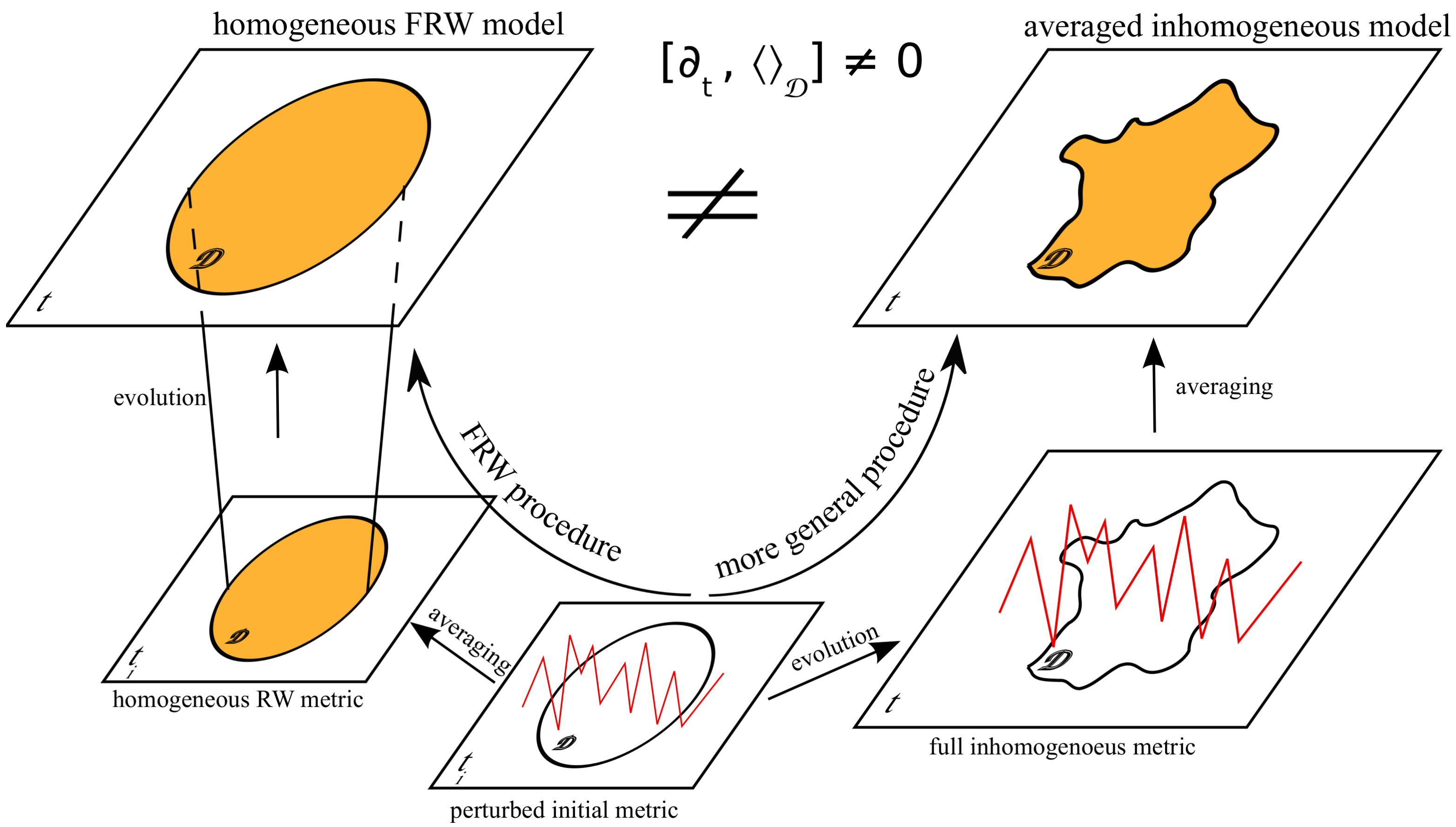
Averaging in $3+1$ foliation

In the $3+1$ setting, averaging and time derivatives do not commute: $\partial_t \langle A \rangle \neq \langle \partial_t A \rangle$



Wiegand, Buchert; Journal of Cosmology 2011

Kinematical decomposition, averaging

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  $\rightarrow$ shear: $\sigma^i{}_j := -K^i{}_j - \frac{1}{3} \Theta \delta^i{}_j$; rate of shear: $\sigma^2 = \frac{1}{2} \sigma^i{}_j \sigma^j{}_i$

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- Spatial averaging: $\langle\ \rangle_{\mathcal{D}} = \frac{1}{V}\int_{\mathcal{D}} d\mu_g$

Averaged equations

- We define the domain dependent scale factor

$$a_{\mathcal{D}}(t) := \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_{\mathbf{i}}}} \right)^{1/3}$$

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- We apply the following commutation rule to the Raychaudhuri and Hamilton equations

$$\partial_t \langle \Psi(t, X^k) \rangle_{\mathcal{D}} - \langle \partial_t \Psi(t, X^k) \rangle_{\mathcal{D}} = \langle \Theta \Psi \rangle_{\mathcal{D}} - \langle \Theta \rangle_{\mathcal{D}} \langle \Psi \rangle_{\mathcal{D}}$$

- We obtain the generalised Friedmann equations for inhomogeneous fluids:

  → the averaged Raychaudhuri equation:

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \frac{M_{\mathcal{D}_{\mathbf{i}}}}{V_{\mathcal{D}_{\mathbf{i}}} a_{\mathcal{D}}^3} = \mathcal{Q}_{\mathcal{D}}$$

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where the kinematical backreaction term is given by (for irrotational dust):

$$\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\langle (\Theta - \langle \Theta \rangle_{\mathcal{D}})^2 \rangle_{\mathcal{D}} - 2\langle \sigma^2 \rangle_{\mathcal{D}}$$

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- **fence-sitters**: the backreaction is small, but as we obtain the more and more accurate observations it can not be ignored when interpreting the data, especially on the small scales i.e. in the local Universe.