Nonreciprocal mutual influences of linearly coupled and unequally damped bosonic modes

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In this presentation we consider the problem of directional transmission of population and correlations in a system composed of two coupled bosonic modes with the linear beam-splitter type interaction between the modes. Using the input-output formalism to describe the Heisenberg equations of motion for the bosonic operators we find that in the case of unequally damped modes the exceptional point emerge and analyse its effect on transfer of population and creation and transfer of correlations inside and between the modes. The exceptional point is the special parameter regime at which the parameters space divides into distinct regions, each characterised by purely imaginary or purely real eigenvalue spectra, where the populations and correlations change their exponential to oscillatory behaviours [1-4]. The exceptional points are characteristic of non-Hermitian systems and our analysis demonstrate that in the Hermitian quantum system composed of linearly coupled and unequally damped modes one can construct non-Hermitian dynamics which can lead to nonreciprocal (one directional) influences of the modes on each other.

We consider two frequency degenerate radiation modes described by bosonic creation (annihilation) operators, $a^\dagger(a)$ and $b^\dagger(b)$, respectively. The modes are damped with rates respectively κ_a and κ_b, and are coupled to each other through the optical photon exchange processes determined by the interaction Hamiltonian $ga^\dagger b + g^* b^\dagger a$. The modes are simultaneously coupled to their surrounding environments which are in squeezed vacuum states. Using the input output formalism we derive analytical expressions for the populations

$$\langle a^\dagger a\rangle = n_a + \Delta n(\lambda-1)\left[\lambda^2+(\lambda^2-1)\sinh^2\psi\right] , \quad \langle b^\dagger b\rangle = n_b + \Delta n(\lambda+1)\left[\lambda^2+(\lambda^2-1)\sinh^2\psi\right] \quad (1)$$

the internal two-photon correlations

$$\langle aa\rangle = m\left\{1+(\lambda-1)\left[\lambda^2+(\lambda^2-1)\sinh^2\psi\right]\right\} , \quad \langle bb\rangle = m\left\{1-(\lambda+1)\left[\lambda^2+(\lambda^2-1)\sinh^2\psi\right]\right\} \quad (2)$$

and mutual correlations

$$\langle a^\dagger b\rangle = i\Delta n\frac{g}{\kappa}(\lambda^2-1)\cosh^2\psi , \qquad \langle ab\rangle = im\frac{g}{\kappa}(\lambda^2-1)\cosh^2\psi \quad (3)$$

where $n_i\,(i=a,b)$ is the initial population of the ith mode, m is the degree of the initial internal two-photon correlation in the modes, $\Delta n=(n_a-n_b)/2$, $\psi=\operatorname{arctanh}(\sqrt{\gamma^2-g^2}/\kappa)$, $\gamma=(\kappa_a-\kappa_b)/2$, $\kappa=(\kappa_a+\kappa_b)/2$, and $\lambda=\gamma/\kappa$. It is seen from the above equations that the populations and correlations behave differently. Setting $g=\gamma$ and $\kappa_a\gg\kappa_b\,(\lambda\to 1)$ we see that the mutual correlations vanish that the modes become uncorrelated. The population and the internal correlations of the mode a remain unchanged whereas the population of the mode b is turned to n_a and its internal correlations to $-m$. Inversely, for $\kappa_b\gg\kappa_a\,(\lambda\to-1)$ the population and the internal correlations of the mode b remain unchanged while the population of the mode a is turned from n_a to n_b and the internal correlation is turned from m to $-m$. If one of the modes is in vacuum state and the other in the thermal state then the interaction will turn the occupied mode into the vacuum state. Thus, both mode will be found in the vacuum state despite the fact that one of them is in continuous contact with a thermal reservoir.

References

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