Gravitational Lensing:

A Window into the Fundamental Properties fo Nature

Aleksandra Piórkowska-Kurpas

Matter To The Deepest Recent Developments In Physics Of Fundamental Interactions XLVI International Conference of Theoretical Physics

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SPIN PLACE Katowice

Centre for Creativity and Coworking

Matter To The Deepest 2025

University of Silesia, August Chełkowski Institute of Physics

Division for Physics of Fundamental Interactions
Polish Physical Society

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W KATOWICACH

POLSKIE TOWARZYSTWO FIZYCZNE
SOCIETAS PHYSICORUM POLONIAE
Experimento et mente
1920

Deflection of the light path near massive body can be calculated within Newtonian theory of gravity:

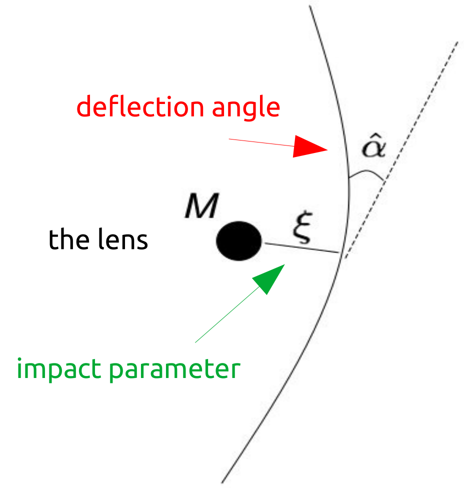
test particle with velocity *v* moving past an object of mass *M* is deflected by

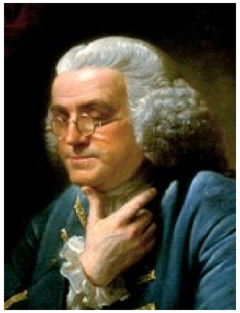
$$\hat{\alpha} = 2GM/(v^2\xi)$$

if light treated as particles

$$\hat{\alpha_{\rm N}} = 2GM/(c^2\xi)$$

[Mitchell 1784; Soldner 1804]





John Michell 1724-1793

Johann Georg von Soldner

after S. Suyu; lectures XXIV Canary Islands Winter School of Astrophysics 2012

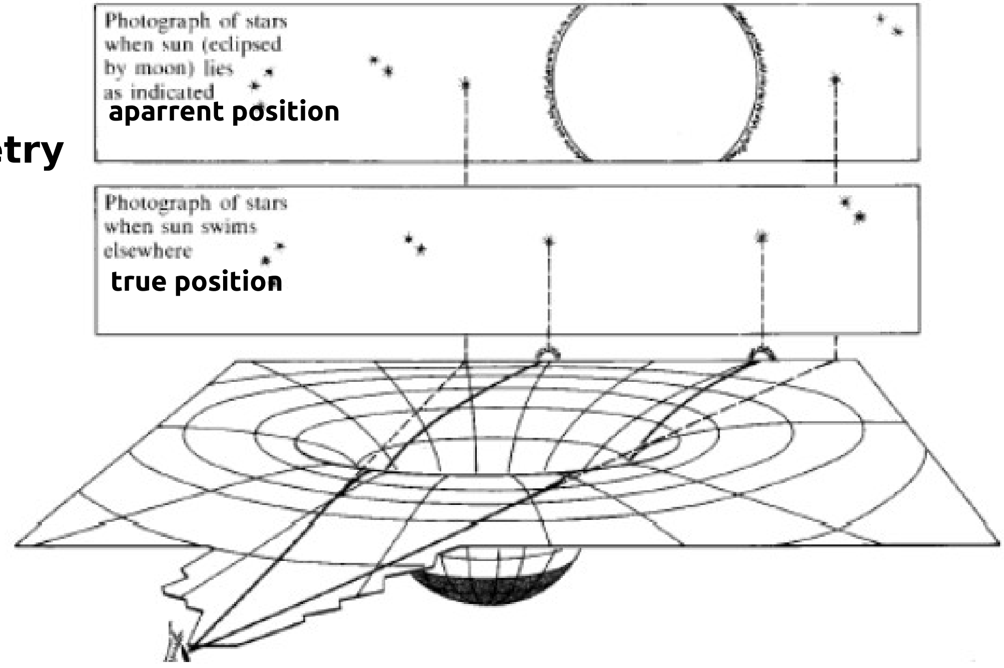
But the deflection angle drived from GR is as twice as it !

$$\hat{\alpha_{\rm E}} = 4GM/(c^2\xi) = 2\hat{\alpha_{\rm N}}$$

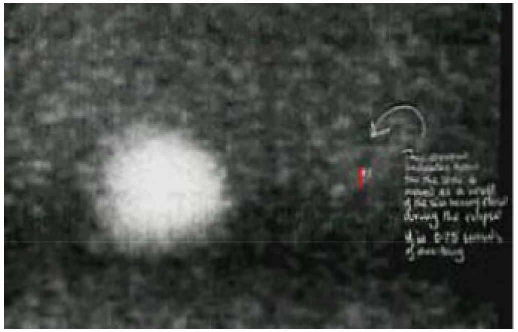
1.75 arc seconds for light nearly grazing the outside of the sun

[A. Einstein (1915); proved by A. Eddington in 1919]

problem of photon orbits in Schwarzschild geometry



Sir Arthur Eddington



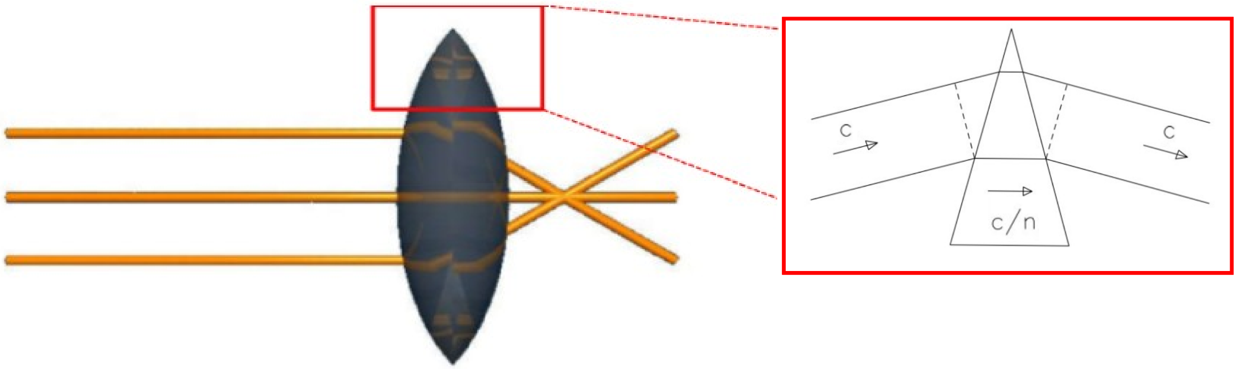
National Maritime Museum, Greenwich

Albert Einstein

Solar eclipse, 29 May 1919 [Sobral/Principle Island]

The effect of spacetime curvature on the light paths can be expressed in terms of an effective index of refraction n:

[Schneider et al.1992]



in classical optics lens bends light rays due to the difference in refractive index between lens material and surrounding medium

but in vacuum n=1 !

Deflection is the integral along the light path of the gradient of n perpendicular to the light path

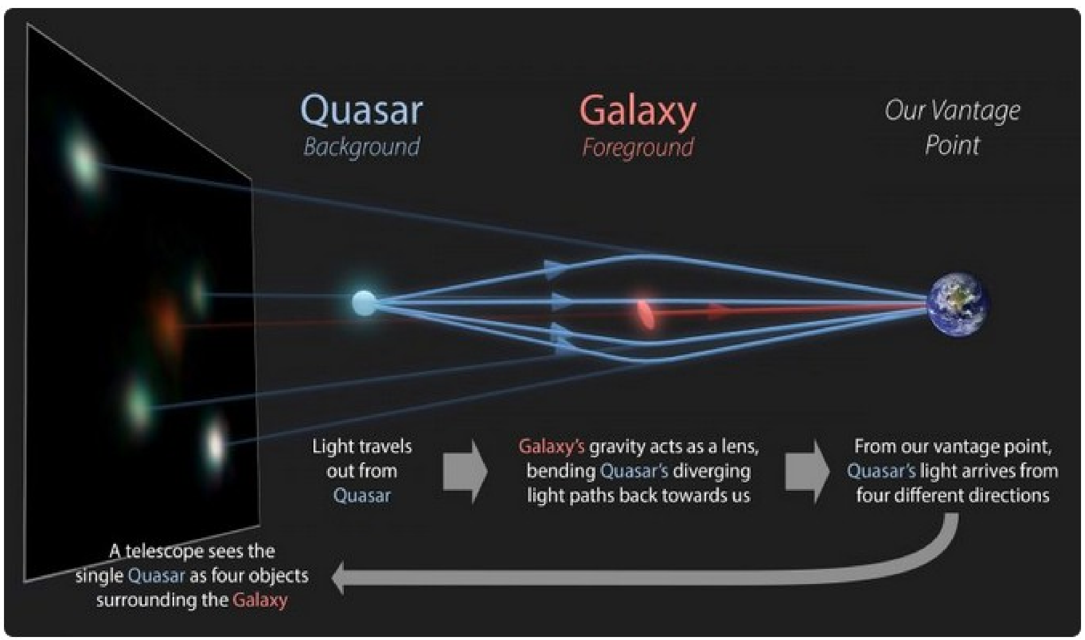


Image via R. Hurt (IPAC/ Caltech)/ The GraL Collaboration/ ESA.

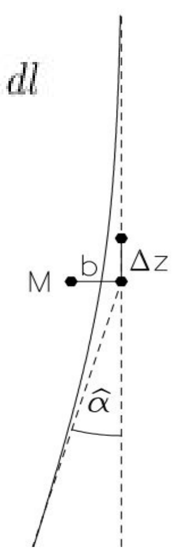
$$n = 1 - \frac{2}{c^2}\Phi = 1 + \frac{2}{c^2}|\Phi|$$

$$\vec{\hat{\alpha}} = -\int \vec{\nabla}_\perp n \, dl = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi \, dl$$

Newtonian potential of the point mass lens:

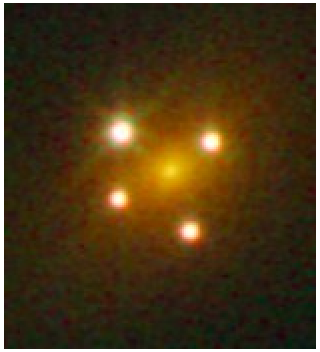
$$\Phi(b, z) = -\frac{GM}{(b^2 + z^2)^{1/2}}$$

$$\hat{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi \, dz = \frac{4GM}{c^2 b}$$

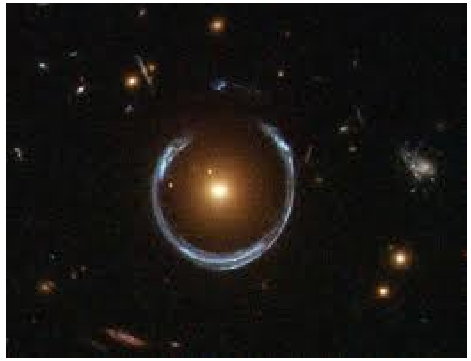


[M. Bartelmann 1996]

Different regimes of gravitational lensing:

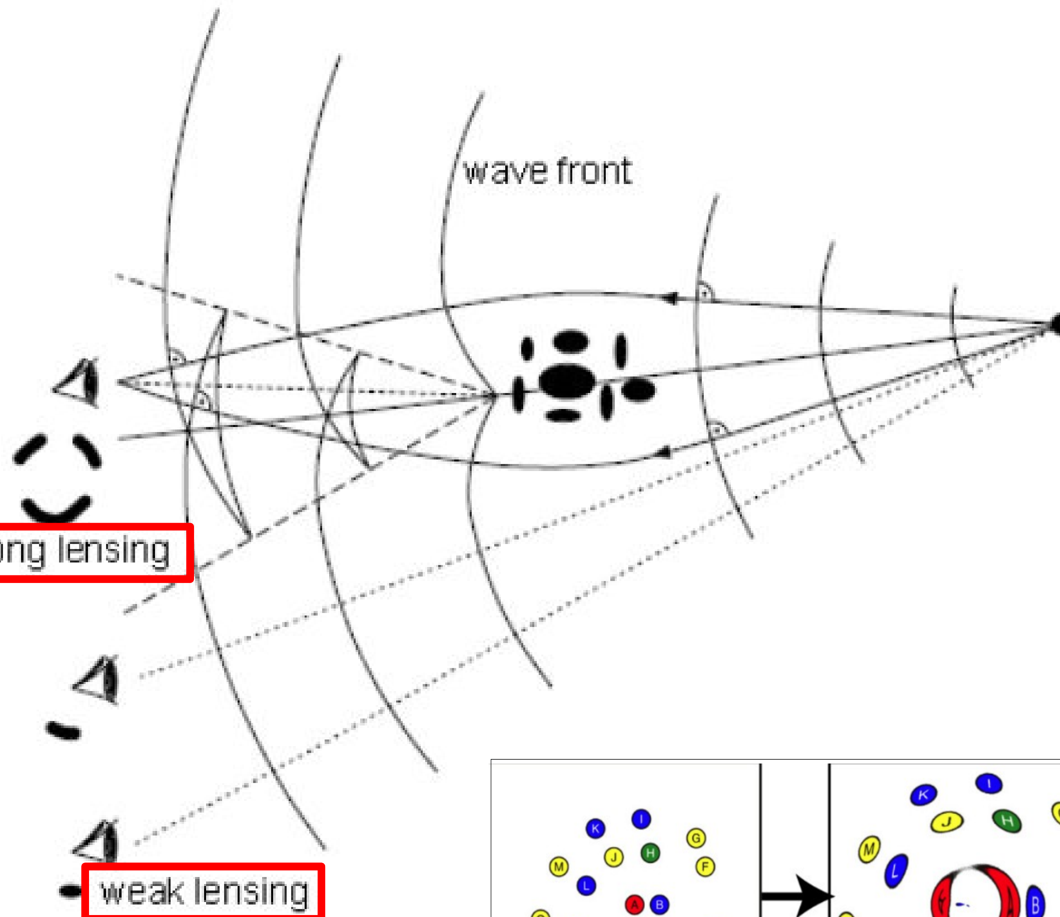


Einstein Cross (Q2237+030), HST



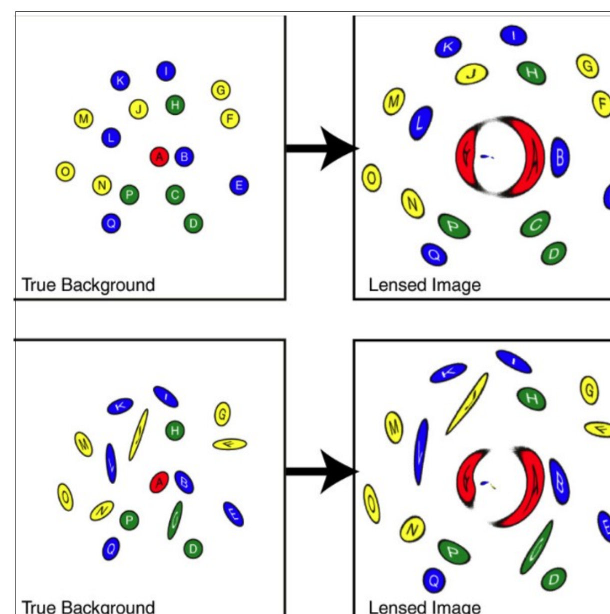
Cosmic Horseshue (SDSS J1148+1930), HST

- multiple images
- time delays between images
- images distorted into rings/arcs



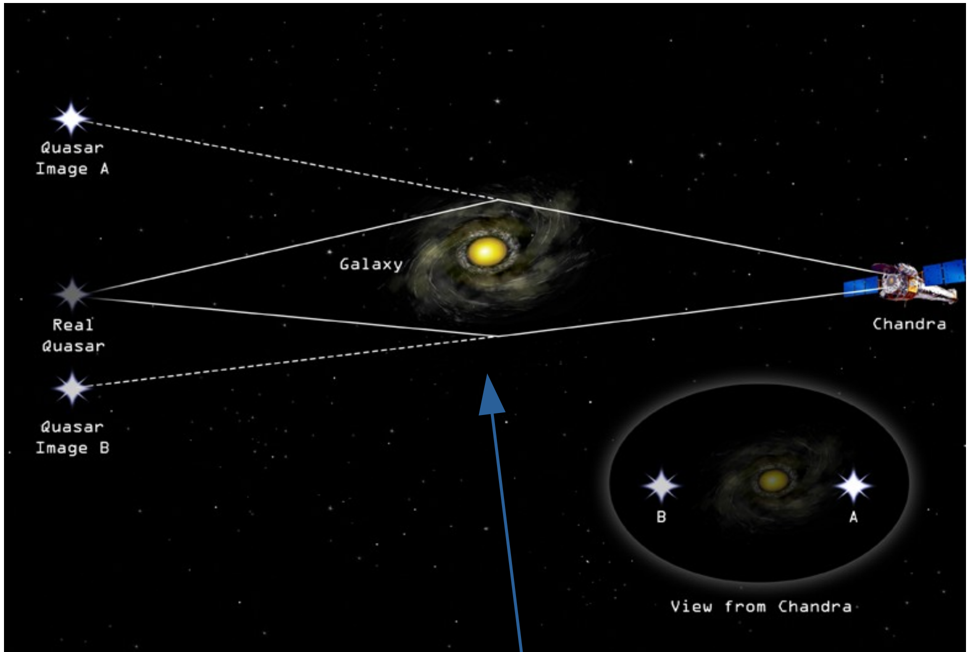
This Webb image shows the massive galaxy cluster Abell S1063. Image credit: NASA / ESA / CSA / Webb / H. Atek & M. Zamani, ESA & Webb / R. Endsley.

- slight distortions of background sources
- deflection angles not generally observable
- statistical approach higly needed → intrinsic galaxy shapes → noise



Joshua A. Frieman, Lectures, University of Chicago

Strong gravitational lensing:



https://chandra.harvard.edu/

Newtonian potential at lens plane

effective lesing (Fermat) potential

$$\phi(\boldsymbol{\theta}) = \frac{D_{ls}}{D_l D_s}\frac{2}{c^2}\int \Phi(D_l\boldsymbol{\theta}, z)dz$$

Travel time of light rays from images → time delay:

$$\Delta t = \frac{1+z_l}{c}\frac{D_{ol}D_{os}}{D_{ls}}\left[\frac{(\boldsymbol{\theta}-\boldsymbol{\beta})^2}{2} - \phi(\boldsymbol{\theta})\right]$$

t_{geom} t_{grav}

Massimo Meneghetti, Introduction to Gravitational Lensing; Lecture scripts

Schneider, Ehlers, Falco, 'Gravitational Lenses'

Schneider, Kochanek, Wambsganss, 'Gravitational Lensing: Strong, Weak and Micro'

in the light ray formalism:

(thin screen approximation)

$$\boldsymbol{\eta} = \frac{D_s}{D_d}\boldsymbol{\xi} - D_{ds}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

In terms of angular coord.:

$$\boldsymbol{\eta} = D_s\boldsymbol{\beta}$$

$$\boldsymbol{\xi} = D_d\boldsymbol{\theta}$$

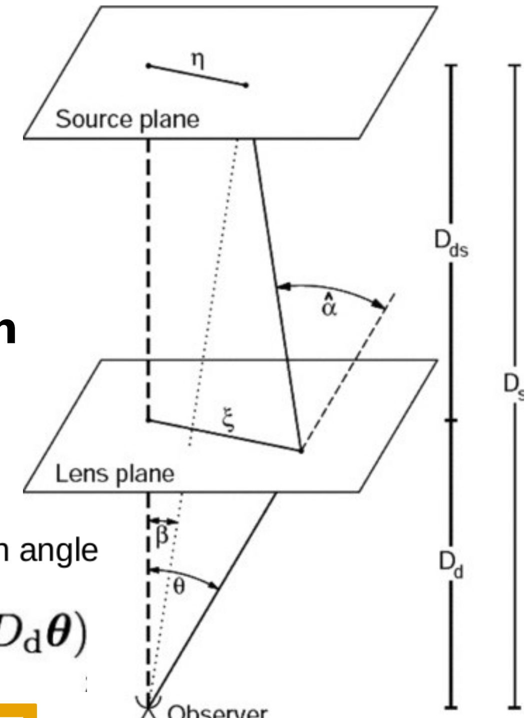
lens equation

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

reduced deflection angle

$$\frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}}(D_d\boldsymbol{\theta})$$

$$\boldsymbol{\alpha} = \nabla_{\boldsymbol{\theta}}\phi$$



$$\boldsymbol{\theta} - \boldsymbol{\beta} - \nabla_{\boldsymbol{\theta}}\phi = 0$$

Bartelmann& Schneider, 2001
Schneider, 2006

S. Suyu; lectures XXIV Canary Islands Winter School of Astrophysics 2012

$$\vec{\nabla}_\theta\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi\right] = 0$$

Fermat potential

Fermat Principle

$$\nabla_{\boldsymbol{\theta}}\Delta t = 0$$

Pierre de Fermat

Images are located at points where the total time delay function is stationary

Magnification and distortion:

main features of gravitational lensing !

Liouville's Theorem

Joseph Liouville

Lensing conserves surface brightness

Flux F = surface brightness x solid angle

Magnification = $F_{observed}$ / $F_{intrinsic}$ = $d\Omega_{observed}$ / $d\Omega_{intrinsic}$

after S. Suyu; lectures XXIV Canary Islands Winter School of Astrophysics 2012

Jacobian matrix for gravitational lensing:

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \left(\delta_{ij} - \frac{\partial^2 \phi(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j}\right) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

shear

$$\gamma \equiv \gamma_1 + \mathrm{i}\gamma_2 = |\gamma| \mathrm{e}^{2\mathrm{i}\varphi}$$

$$\gamma_1 \equiv \frac{\partial \phi}{2\partial\theta_1 \partial\theta_1} - \frac{\partial \phi}{2\partial\theta_2 \partial\theta_2}$$

$$\gamma_2 \equiv \frac{\partial \phi}{\partial\theta_1 \partial\theta_2}$$

stretches source image tangentially around the lens

convergence

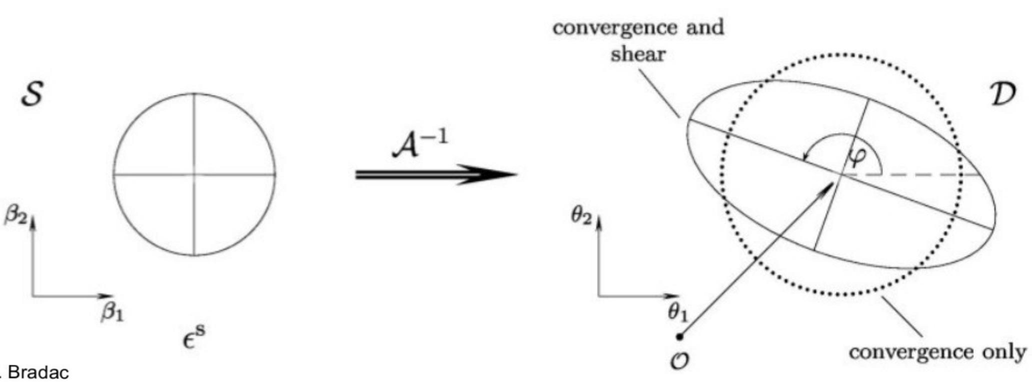
$$\kappa = \frac{\partial \phi}{2\partial\theta_1 \partial\theta_1} + \frac{\partial \phi}{2\partial\theta_2 \partial\theta_2}$$

magnifies source image by increasing its size

Magnification in terms of κ and γ is:

$$\mu = \frac{1}{\det \mathcal{A}} = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

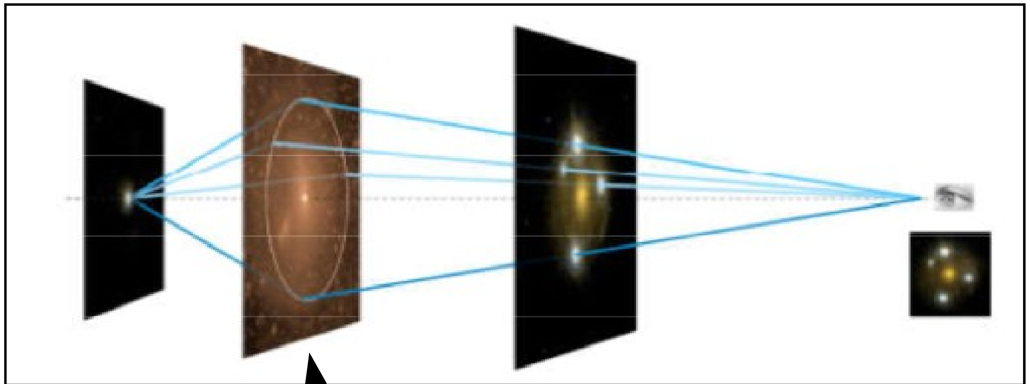
Galaxy Cluster RCS2 032727-132623 (HST image/NASA)



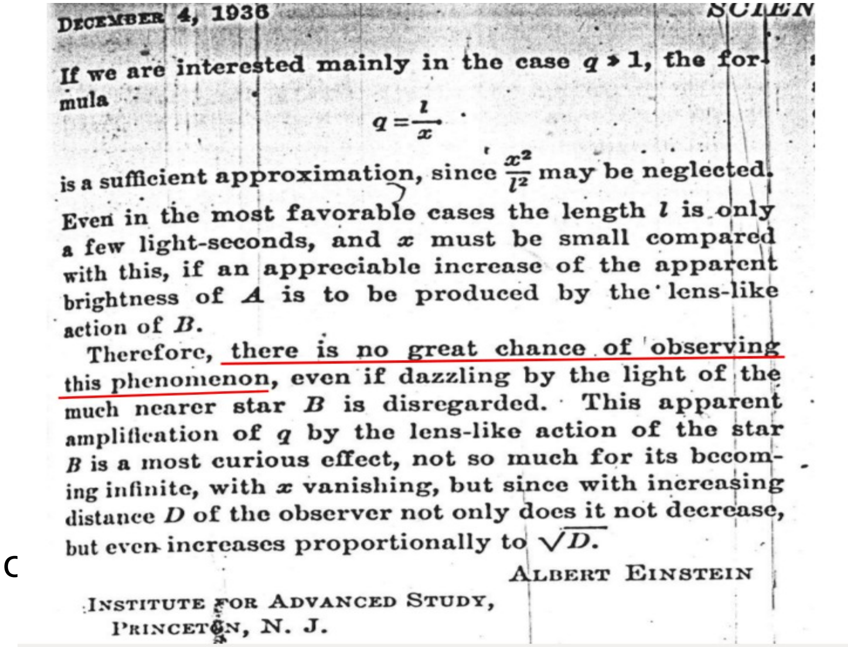
Credit: M. Bradac

magnification

$$\mu(\boldsymbol{\theta}) = \frac{1}{\det A(\boldsymbol{\theta})}$$



Einstein ring -
introduces angular scale characteristic of a given lensing c



DECEMBER 4, 1936 SCIEN

If we are interested mainly in the case $q \gg 1$, the formula

$$q = \frac{l}{x}$$

is a sufficient approximation, since $\frac{x^2}{l^2}$ may be neglected.

Even in the most favorable cases the length l is only a few light-seconds, and x must be small compared with this, if an appreciable increase of the apparent brightness of A is to be produced by the lens-like action of B.

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star B is disregarded. This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming infinite, with x vanishing, but since with increasing distance D of the observer not only does it not decrease, but even increases proportionally to $\sqrt{D}$.

ALBERT EINSTEIN

INSTITUTE FOR ADVANCED STUDY,
PRINCETON, N. J.

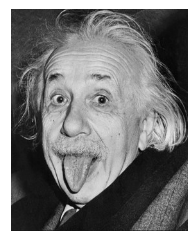
Einstein radius

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}}$$

for the lens of 1 $M_\odot$
and for typical galactic
distances of 10 kpc

$$\theta_E \sim 0''.001$$

unobservable!

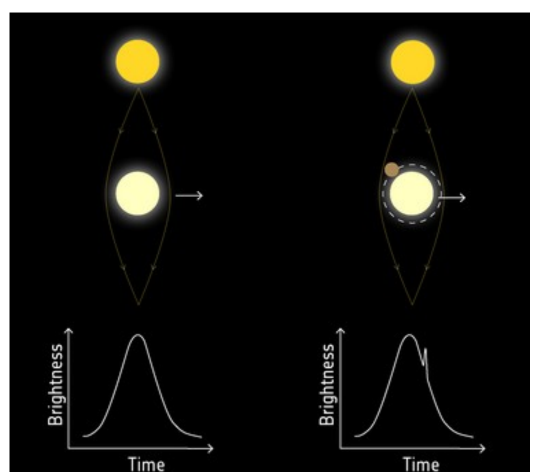


strong lensing of stars by a star:

- small value of deflection angle
- unlikely alignment requirement for lensing

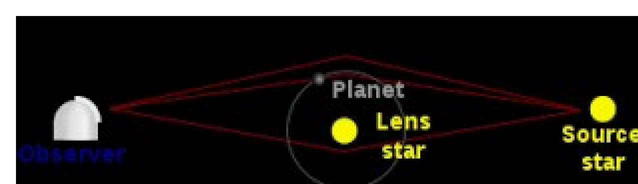
"there is no great chance of observing this phenomenon"

[Einstein, 1936]



prof. Bohdan Paczyński

MICROLENSING!

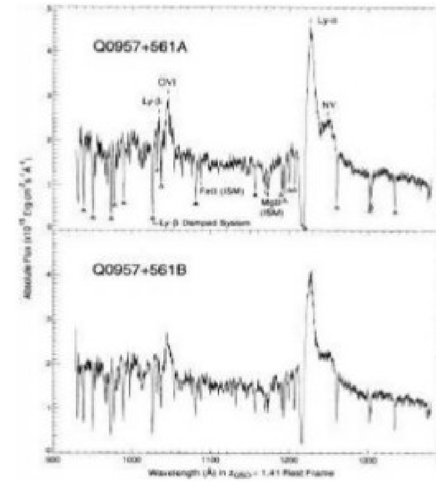


F.Zwicky (1937): multiple images can be detected if one consider deflector as more massive than stars, e.g. galaxies

The first observation:

double quasar QSO-0957+561A,B

Walsh, Carswell & Weynmann 1979

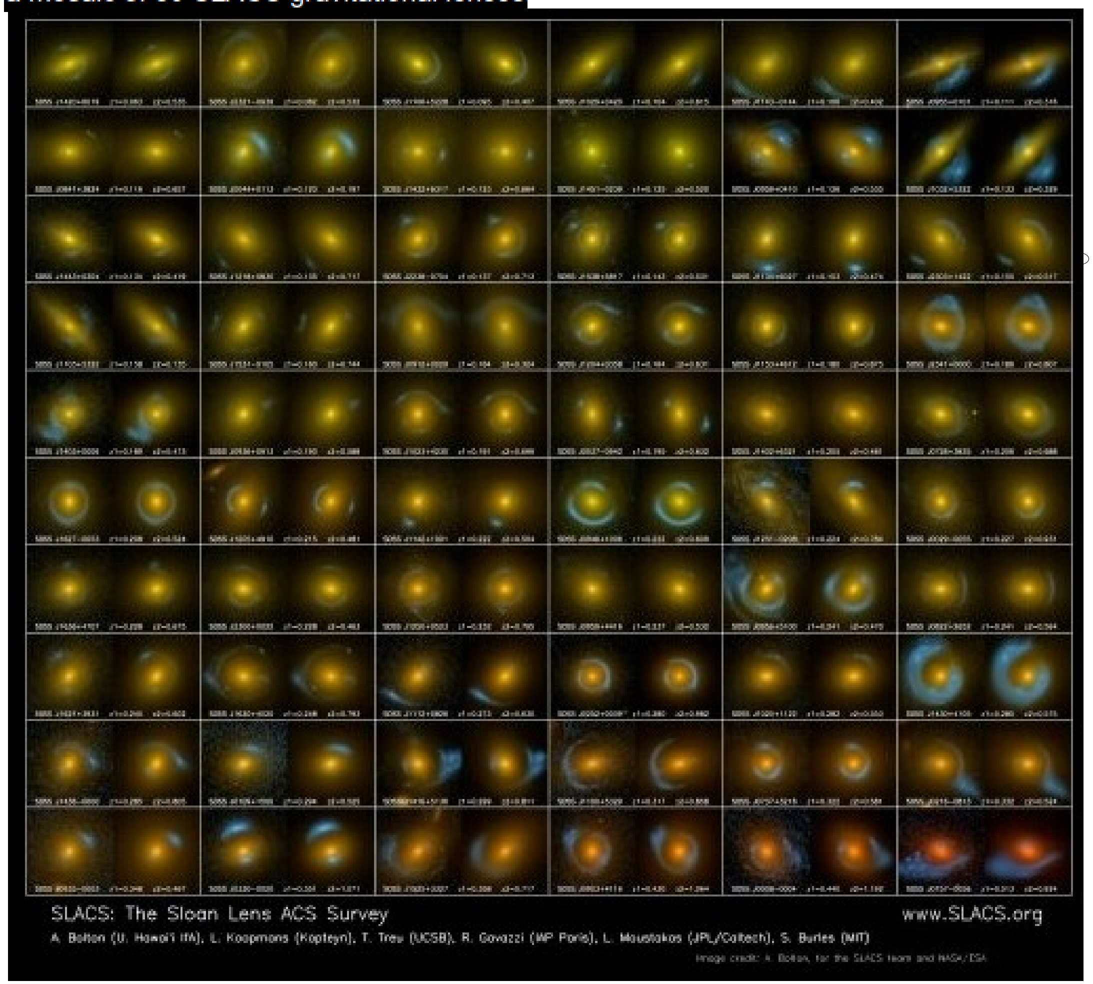


[Michalitsianos et al. 1997]

identical spectra!

- 1978-1992 – only 11 strong lensing systems was discovered

a mosaic of 60 SLACS gravitational lenses



https://web.physics.utah.edu/~bolton/slacs/Images.html

- Era of massive galaxy surveys:

now we know **hundreds of strong lensing systems !**

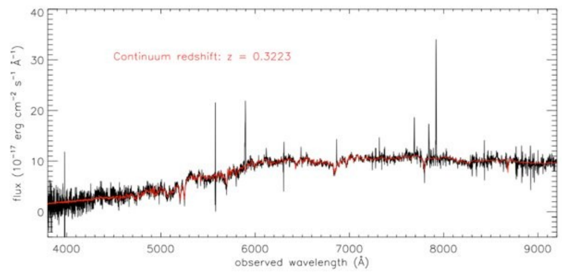
SLACS, BELLS, CFHT – SL2S, CLASS, SQLS, HAGGLeS, AEGIS, COSMOS, CASSOWARY

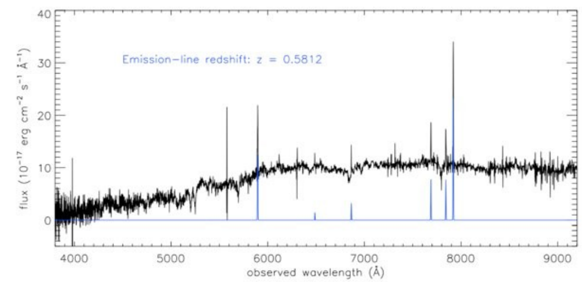
spectroscopic searches concentrated on sources!

Two main observational startegies for lens detection:

- looking for the presence of emission lines at redshifts higher than that of the target galaxy

- + HST ACS follow-up imaging

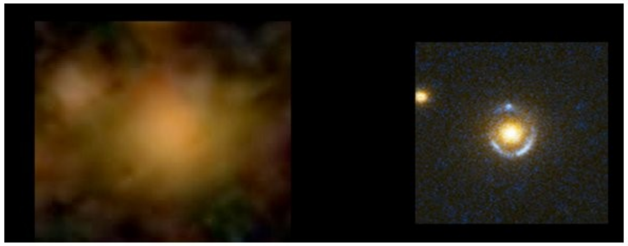




lens candidates selected from SDSS/BOSS data

Sloan Lens ASC Survey **SLACS**

BOSS emission-line lens survey **BELLS**



http://www.physics.utah.edu/~bolton/slacs/What_is_SLACS.html

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early-type galaxies more likely serve as intervening galaxies

they contain most of the stellar mass of the Universe which affects gravitational lensing statistics

homogeneous sample!

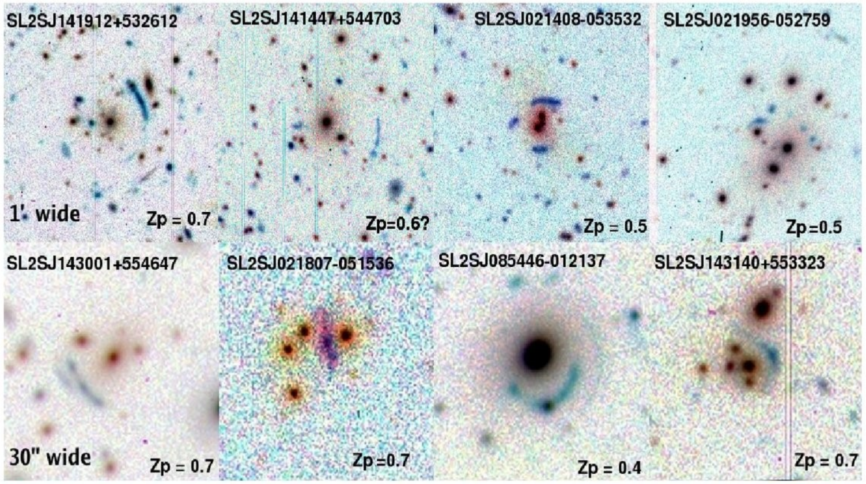
- targets: massive red galaxies

- fully automated software (*RingFinder*) looking for tangentially elongated blue features around lensing galaxy

[Gavazzi et al. 2014]

Strong Lensing Legacy Survey **SL2S**

Canada France Hawaii Telescope Legacy Survey (CFHTLS)



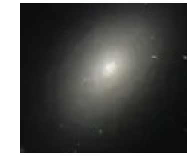
http://www-sl2s.iap.fr/

Gravitational lenses as a tool for cosmology

The idea: image separations in the system depend on angular diameter distances to the lens and to the source, which in turn are determined by background cosmology

lenses as standard(izable) rulers !

NGC 1316



NGC 4150

Singular Isothermal Sphere (SIS)
– the simplest realistic model

$$\rho(r) = \frac{\sigma_v^2}{2\pi G}\frac{1}{r^2}$$

majority of cases the lens is a late-type E/SO galaxy

Koopmans et al. 2006, 2009

$$\theta_E = 4\pi \frac{D_{ls}}{D_s}\frac{\sigma_{SIS}^2}{c^2}$$

distance from the lens to the source

from angular image separations (astrometry)

distance from observer to the source

velocity dispersion in lensing galaxy (spectroscopy)

observable: **distance ratio**

$$\frac{D_s}{D_{ls}} = \frac{4\pi\sigma_0^2}{c^2\theta_E}$$

$D^{th}(z_l, z_s, p)$ $\qquad$ D^{obs}

σ_{SIS} lens velocity dispersion is well approximated by σ_0 - central stellar velocity dispersion (see eg. Grillo et al. 2008)

possibility to constraining the cosmological model
provided that we have good knowledge of the lens model

$$D_A(z;\boldsymbol{p}) = \frac{c}{H_0}\frac{1}{1+z}\int_0^z \frac{dz'}{h(z';\boldsymbol{p})}$$

dimensionless expansion rate dependent on redshift z and cosmological model parameters

gets canceled in the distance ratio $\frac{D_s}{D_{ls}}$

Biesiada (2006)

Biesiada, AP (2008)

Biesiada, AP, Malec (2010)

Biesiada, Malec, AP (2011)

method is independent of the Hubble constant's value and is not affected by dust absorption or source evolutionary effects

!

'Hubble Tension'

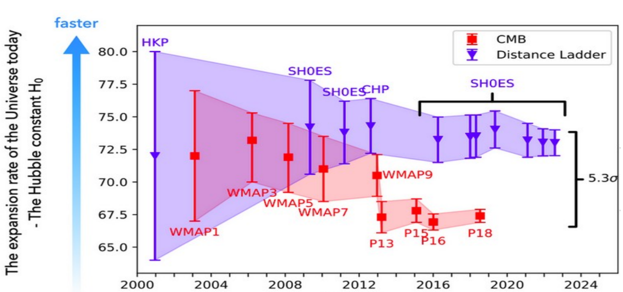
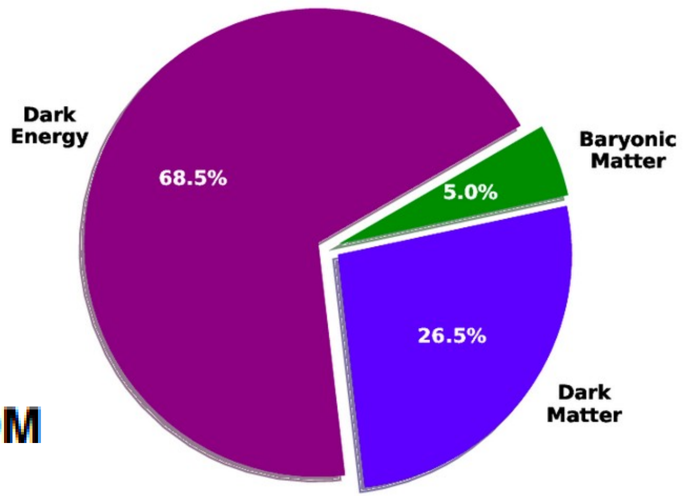


Image Credit: D'arcy Kenworthy

Cosmological consensus:

most of the energy in the Universe exists in the form of the mysterious dark energy



ΛCDM

$$\Omega_\Lambda = 0.6889 \pm 0.0056$$
$$\Omega_M = 0.3111 \pm 0.0056$$

Aghanim et al. (2021)

Nobel Prize in Physics 2011

© The Nobel Foundation. Photo: U. Montan
Saul Perlmutter
Prize share: 1/2

© The Nobel Foundation. Photo: U. Montan
Brian P. Schmidt
Prize share: 1/4

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Adam G. Riess
Prize share: 1/4

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

Parameter	*Planck*+SNe+BAO	*Planck*+BAO/RSD+WL
w_0	-0.957 ± 0.080	-0.76 ± 0.20
w_a	$-0.29^{+0.32}_{-0.26}$	$-0.72^{+0.62}_{-0.54}$
H_0 [km s^{-1}Mpc^{-1}]	68.31 ± 0.82	66.3 ± 1.8
σ_8	0.820 ± 0.011	$0.800^{+0.015}_{-0.017}$
S_8	0.829 ± 0.011	0.832 ± 0.013
$\Delta\chi^2$	-1.4	-1.4

***Planck* 2018 results.** **Aghanim et al. (2021)**

DE pressure and density (spatially-averaged)

Dark Energy equation of state parameter : $p = w\rho$

$w = 0$ → dust
$w = 1/3$ → radiation
$w = -1$ → cosmological constant

If Dark Energy is a generic dynamical fluid, its equation of state parameter should in general be a function of time.

Time-varying EoS as a Taylor expansion over a(t) (linear order): $w(z) = w_0 + w_a \frac{z}{1+z}$

CPL parametrization

Chevalier, Polarski (2001); Linder (2003)

$$h^2(z;\mathbf{p}) = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w)} \text{ for } XCDM \text{ cosmology} \Longrightarrow \mathbf{p} = \{\Omega_m, w\}$$

$$h^2(z;\mathbf{p}) = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \exp\left(-\frac{3w_1 z}{1+z}\right) \text{ for CPL parametrization} \Longrightarrow \mathbf{p} = \{\Omega_m, w_0, w_1\}$$

lenses as standard(izable) rulers – next steps forward

- compilation of 118 lenses from SLD, SLACS, BELLS and SL2S catalogues $\rho \sim r^{-\gamma}$
- generalization of the SIS model to spherically symmetric power-law mass distribution

mass inside the Einstein radius:

$$M_{lens} = \frac{c^2}{4G}\frac{D_s D_l}{D_{ls}}\theta_E^2$$

[Schneider et al.1992]

$$M_{lens} = M_{dyn}$$

dynamical mass inside the aperture projected to lens plane (from solving the Jeans equation):

$$M_{dyn} = \frac{\pi}{G}\sigma_{ap}^2 R_E \left(\frac{R_E}{R_{ap}}\right)^{2-\gamma} f(\gamma) = \frac{\pi}{G}\sigma_{ap}^2 D_l \theta_E \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

$$\theta_E = 4\pi \frac{\sigma_{ap}^2}{c^2}\frac{D_{ls}}{D_s}\left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

$$f(\gamma) = -\frac{1}{\sqrt{\pi}}\frac{(5-2\gamma)(1-\gamma)}{3-\gamma}\frac{\Gamma(\gamma-1)}{\Gamma(\gamma-3/2)} \times \left[\frac{\Gamma(\gamma/2-1/2)}{\Gamma(\gamma/2)}\right]^2$$

[Koopmans et al. 2005]

- making sample more uniform – velocity dispersion correction

new observable:

$$\mathcal{D}^{obs} = \frac{c^2\theta_E}{4\pi\sigma_{ap}^2}\left(\frac{\theta_{ap}}{\theta_E}\right)^{2-\gamma} f^{-1}(\gamma)$$

$$\sigma_0 = \sigma_{ap}(\theta_{eff}/(2\theta_{ap}))^{-0.04}$$

[Jorgensen et al.1995]

we need to transform all velocity dispersions measured within an aperture to those, measured within circular aperture of radius *Reff / 2*

uncertainties of the effective radius contribute less than 1% to the uncertainty of σ_0

this operation makes our observable more homogeneous for the sample of lenses located at different redshifts

- taking into account possible evolution of the power-law index γ with redshift

[Ruff et al. 2011]
[Brownstein et al. 2012]
[Sonnenfeld et al. 2013]

$$\gamma(z_l) = \gamma_0 + \gamma_1 z_l$$

S. Cao, M. Biesiada, R. Gavazzi, AP & Z.-H. Zhu (2015)

Monte Carlo (CosmoMC package) simulations of the posterior likelihood

$$\mathcal{L} \sim \exp(-\chi^2/2)$$

$$\chi^2 = \sum_{i=1}^{118} \left(\frac{\mathcal{D}^{th}(z_{l,i}, z_{s,i}; \mathbf{p}, \gamma) - \mathcal{D}^{obs}(\sigma_{0,i}, \theta_{E,i})}{\Delta \mathcal{D}_i^{obs}} \right)^2$$

γ taken as a free parameter !

5%

SLACS Team

$$\delta \mathcal{D} = \frac{\Delta \mathcal{D}}{\mathcal{D}} = \sqrt{4(\delta\sigma_{ap})^2 + (1-\gamma)^2(\delta\theta_E)^2}$$

TABLE 2

DARK ENERGY ($XCDM$ MODEL AND CPL PARAMETRIZATION) CONSTRAINTS OBTAINED ON THE FULL 118 STRONG LENSING (SL) SAMPLE.

Cosmology (Sample)	w_0	w_1	γ_0	γ_1
XCDM1 (SL; σ_{ap})	$w_0 = -1.45^{+0.54}_{-0.95}$	$w_1 = 0$	$\gamma_0 = 2.03 \pm 0.06$	$\gamma_1 = 0$
XCDM1 (SL; σ_0)	$w_0 = -1.15^{+0.56}_{-1.20}$	$w_1 = 0$	$\gamma_0 = 2.07 \pm 0.07$	$\gamma_1 = 0$
XCDM2 (SL; σ_{ap})	$w_0 = -1.48^{+0.54}_{-0.94}$	$w_1 = 0$	$\gamma_0 = 2.06 \pm 0.09$	$\gamma_1 = -0.09 \pm 0.16$
XCDM2 (SL; σ_0)	$w_0 = -1.35^{+0.67}_{-1.50}$	$w_1 = 0$	$\gamma_0 = 2.13^{+0.07}_{-0.12}$	$\gamma_1 = -0.09 \pm 0.17$
CPL1 (SL; σ_{ap})	$w_0 = -0.15^{+1.27}_{-1.60}$	$w_1 = -6.95^{+7.25}_{-3.05}$	$\gamma_0 = 2.08 \pm 0.09$	$\gamma_1 = -0.09 \pm 0.17$
CPL1 (SL; σ_0)	$w_0 = -1.00^{+1.54}_{-1.05}$	$w_1 = -1.85^{+4.85}_{-6.75}$	$\gamma_0 = 2.14^{+0.07}_{-0.10}$	$\gamma_1 = -0.10 \pm 0.18$
CPL2 (SL; σ_{ap})	$w_0 = -0.16^{+1.21}_{-1.48}$	$w_1 = -6.25^{+6.25}_{-3.75}$	$\gamma_0 = 2.08$	$\gamma_1 = -0.09$
CPL2 (SL; σ_0)	$w_0 = -1.05^{+1.43}_{-1.77}$	$w_1 = -1.65^{+4.25}_{-6.35}$	$\gamma_0 = 2.14$	$\gamma_1 = -0.10$
CPL2 (SN)	$w_0 = -1.00 \pm 0.40$	$w_1 = -0.12^{+1.58}_{-2.78}$	☐	☐

[a]In our fits we separately considered observed velocity dispersions σ_{ap} and corrected velocity dispersions σ_0, $XCDM1$ corresponds to assumption of non-evolving power-law index γ, while $XCDM2$ assumes its evolution $\gamma(z) = \gamma_0 + \gamma_1 \, z_l$. Fixed prior of $\Omega_m = 0.315$ was assumed according to the Planck data. While fitting CPL parameters we assumed evolving lens mass density with γ_0 and γ_1 as free parameters (CPL1) and then fixed them at best-fit values (CPL2). For comparison fits of CPL parameters using Union2.1 supernovae data (SN) is shown.

S. Cao, M. Biesiada, R. Gavazzi, AP & Z.-H. Zhu (2015)

lenses as standard(izable) rulers – next move and future prospects

⇨ compilation of lenses **carefully chosen** from known catalogues:

The Lenses Structure and Dynamics (LSD)
Sloan Lens ACS Survey (SLACS)
and its extension – "SLACS for the Masses" (S4TM)
BOSS Emission-Line Lens Survey (BELLS)
BOSS for the GALaxy-Lyα EmitteR sYstems (BELLS GALLERY)
Strong Lensing Legacy Survey (SL2S)

a careful statistical analysis of the data in terms of observables to ensure the robustness of our sample

⇨ currently **the largest sample** with both high resolution imaging and stellar dynamical data

previously:
161 galaxy-scale strong lensing systems
Chen, et al. (2019)

To date, **several hundred galaxy-galaxy strong lenses have been discovered** in heterogeneous searches of photometric and spectroscopic survey data

Known lenses are rare because even the most massive galaxies are only capable of deflecting light by an arcsecond or two and only a small fraction of the sky has been observed to sufficient depth and with good enough image resoution to identify a typical Einstein ring.

⇨ The 4MOST Strong Lensing Spectroscopic Legacy Survey (4SLSLS)

- will provide pairs of redshifts for 10 000 strong-lensing galaxies (lenses) and background galaxies (sources)
- velocity dispersions will also be measured for 5000 lenses

strong lens candidates from:

The Euclid mission

esa
euclid

Legacy Survey of Space and Time (LSST)

VERA C. RUBIN OBSERVATORY

about 20 terabytes of data every night during the ten-year survey !

Collett, E. T. et al., The 4MOST Strong Lensing Spectroscopic Legacy Survey (4SLSLS)

gravitational lenses as a tool for quantum gravity

- **electromagnetic waves**

EM field equations

$$\partial_\mu F^{\mu\nu} = k j^\nu$$

EM field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

4-vector 'potential'

Lorentz gauge

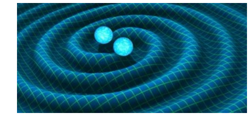
$$\partial_\mu A^\mu = 0$$

EM field equations
in terms of Lorentz gauge

$$\Box^2 A_\mu = \mu_0 j_\mu$$

VS

- **gravitational waves**



Einstein field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

κ

weak-field metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \text{where } |h_{\mu\nu}| \ll 1,$$

+ Lorentz gauge $\partial_\mu \bar{h}^{\mu\nu} = 0$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu} h.$$

$$\Box^2 \bar{h}^{\mu\nu} = 2\kappa T^{\mu\nu}$$

basic field equations of linearised GR
in terms of metric perturbation

$$\Box^2 \bar{h}^{\mu\nu} = 0$$

in vacuum

side effect of similarity
between EM and GW:

GW experience the same
geometric-optics effects
as EM waves!

cosmological redshift

gravitational redshift

gravitational lensing

in general relativity the graviton is predicted to be massless!

long range

The idea: In massive graviton scenario Einstein radius is slightly different when compared to the standard case

in SIS model +
standard physics:

$$\vartheta_E = 4\pi \frac{D_{ls}}{D_s} \frac{\sigma_v^2}{c^2}$$

in SIS model +
massive graviton:

$$\vartheta_{E,GW} = \vartheta_E \left(1 + \frac{m_{GW}^2 c^4}{2E^2}\right)$$

Lowenthal, PRD (1973)

small extra term

GR naturally lose it's applicability at curvature singularities i.e. the Planck length

$$l_{Pl} = \sqrt{\hbar G/c^3} \sim 10^{-33}\mathrm{cm}$$

or equivalently – Planck energy:

$$E_{PL} = \sqrt{\hbar c^3/G} \sim 10^{19}\mathrm{GeV}$$

sensitivity requirements for tests are very strict

- we need accuracy better than

$$\frac{E}{E_{Pl}} \sim 10^{-19}$$

no experimental indication which way is correct ?

effective phenomenology e.g.

existence of massive gravitons

https://www.particlezoo.net

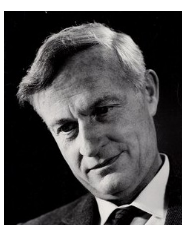
running nature of fundamental constants

violation of some basic principles

LIV

Fierz & Pauli (1930)

Wolfgang Pauli



Markus Eduard Fierz

first theory of a massive spin-2 field propagating on a flat spacetime

Fierz–Pauli mass term

$$\mathcal{L}_{\mathrm{FP}} = m^2 \left(h^{\mu\nu} h_{\mu\nu} - (\eta^{\mu\nu} h_{\mu\nu})^2 \right)$$

do not uniformly reduce to those of general relativity in the limit $m \to 0$

vDVZ discontinuity

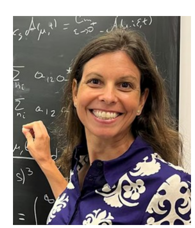
at small scales when one takes into account nonlinear effects!

(shorter tan the Compton wavelength of the graviton)

de Rham-Gabadadze-Tolley (dRTG) gravity (2010)

Ghost-free massive gravity

Andrew Tolley



Claudia de Rham

Gregory Gabadadze

Searching for QG – lesson from GW detection history

do GWs really exist and, if yes, can we detect them directly ?

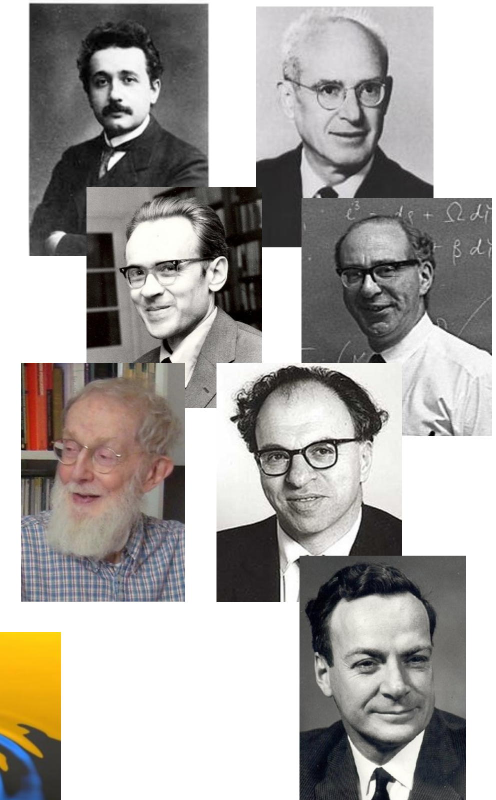
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \Rightarrow \quad T_{\mu\nu} = \frac{c^4}{8\pi G} G_{\mu\nu}$$

stress tensor ($T_{\mu\nu}$) — strain tensor ($G_{\mu\nu}$)

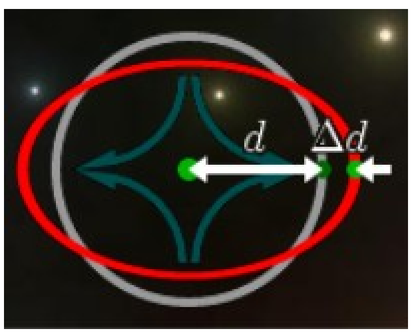
(analogy to the Hooke's Law)

$$\frac{c^4}{8\pi G} \approx 10^{43}\, N$$

elastisity modulus



physical effect
of gravitational wave: $h = \Delta L / L$



wave intensity:
relative amplitude
of deformation!

$$h = \frac{2\Delta d}{d}$$

For $1M_\odot \Rightarrow R_s = 2GM_\odot/c^2 = 3$ km

If $v \approx c$, then at $r = 15$ Mpc:

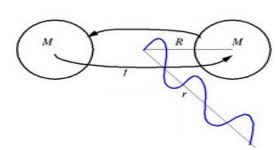
$$h \sim 3 \times 10^{-21}$$

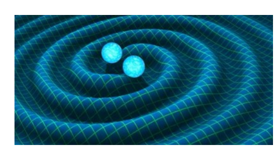
!

$\Delta L = h\, L \lesssim 4{\times}10^{-16}$ cm

h: $\lesssim 10^{-21}$ — L: 4 km

the proton radius is ~ 10^{-13} cm ...





GWs generation mechanisms effective in systems with

- sizes of the order of Schwarzchild radius
- velocities of the order of speed of light in vacuum

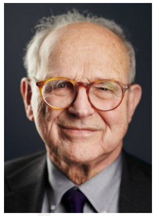
⇓

compact object binaries:
BH-BH, BH-NS, NS-NS

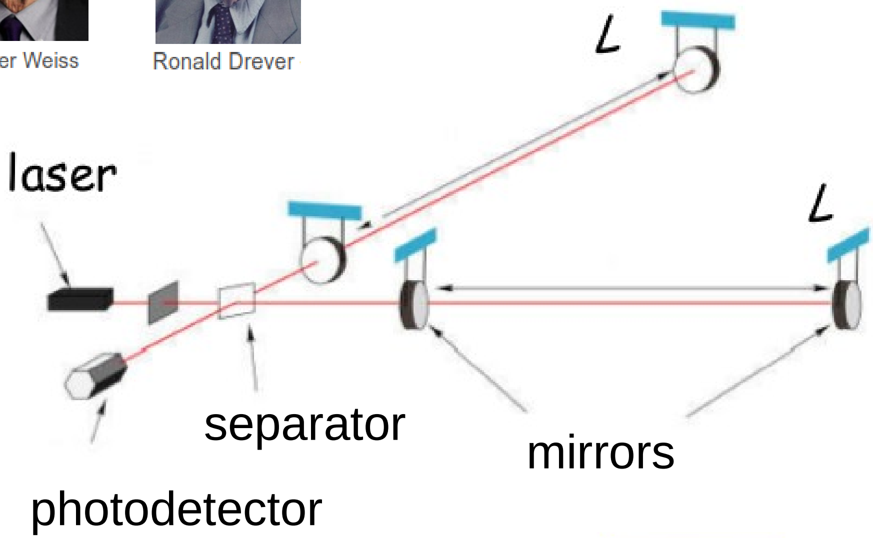
Interferometric detectors:

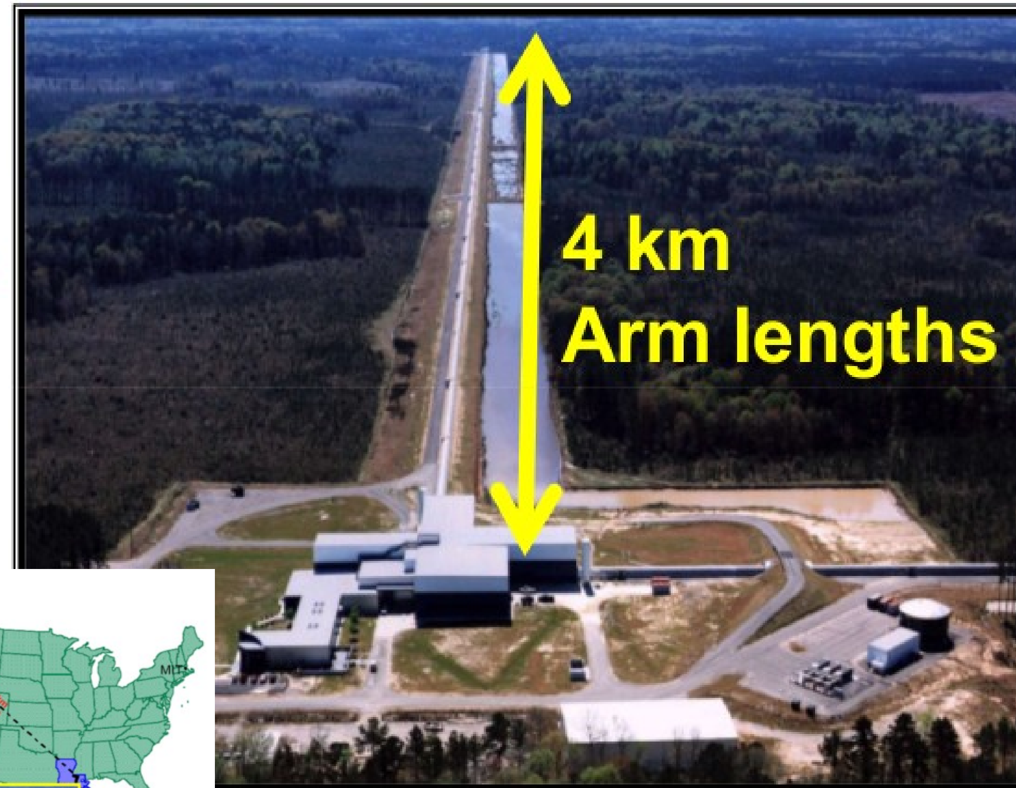
+ KAGRA observation run started on 25 February 2020

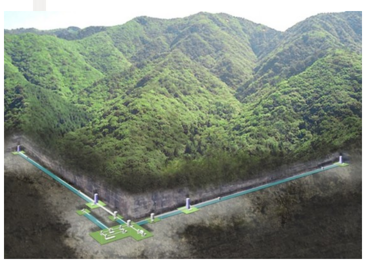
GEO600, TAMA300, LIGO, VIRGO


Rainer Weiss


Ronald Drever






KAGRA detector
in Kamioka mine

LIGO – two detectors!

PSR B1913+16

GW signal doscovered 100 years after formulation of GR

apart from Hulse and Tylor indirect proof (Nobel Prize 1993)

PRL 116, 061102 (2016)

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS

week ending
12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

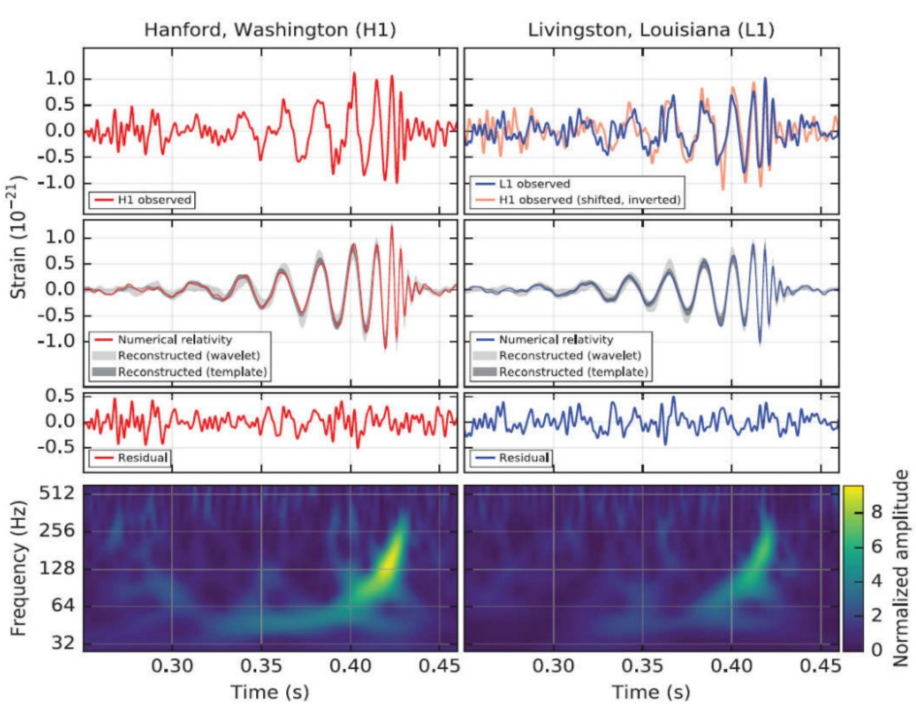
Kip
Thorne

Barry C.
Barish

Reiner
Weiss

2017 Nobel Prize in Physics

„For decisive contributions to the
LIGO detector and the observation of gravitational waves."



GW150914

- first evidence for the existence of BBH
- first evidence for relativistic evolution up to the merger stage of BBH systems
- validation of gravitational radiation formulas

Estimated source parameters 90% credible interval.

Quantity	Value	Upper/Lower error estimate	Unit
Primary black hole mass	**36.2**	+5.2 -3.8	M sun
Secondary black hole mass	**29.1**	+3.7 -4.4	M sun
Final black hole mass	**62.3**	+3.7 -3.1	M sun
Final black hole spin	**0.68**	+0.05 -0.06	
Luminosity distance	**420**	+150 -180	Mpc
Source redshift, *z*	**0.09**	+0.03 -0.04	
Energy radiated	**3.0**	+0.5 -0.5	M sun

PHYSICAL REVIEW LETTERS

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Speed of Gravitational Waves from Strongly Lensed Gravitational Waves and Electromagnetic Signals

Xi-Long Fan, Kai Liao, Marek Biesiada, Aleksandra Piórkowska-Kurpas, and Zong-Hong Zhu

Phys. Rev. Lett. **118**, 091102 – Published 2 March 2017

difference between time delays measured **independently** in **GW and EM windows**

$$\Delta t_\gamma - \Delta t_{GW}$$

$$\vartheta_E \neq \vartheta_{E,GW}$$

$$\Downarrow$$

$$\Delta t_{GW} \neq \Delta t_\gamma$$

$$\Delta t_{SIS} = \frac{32\pi^2}{H_0}\left(\frac{\sigma}{c}\right)^4 y \frac{\tilde{r}(z_l)\tilde{r}(z_l,z_s)}{\tilde{r}(z_s)}$$

source-lens misalignment $y = \beta/\vartheta_E$

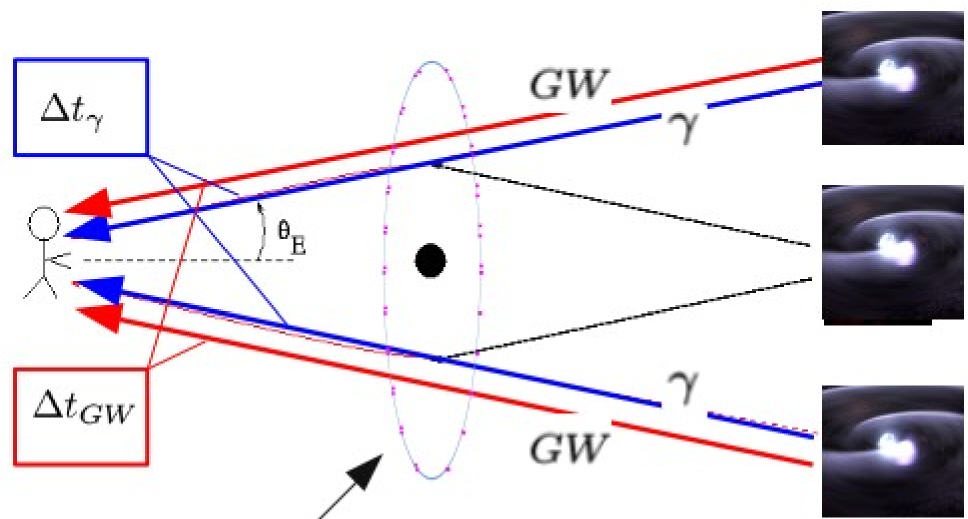
- method based on modified dispersion relation and thus **independent of a particular non-standard model of gravity**
- method is differential in nature and thus - **free from any assumptions regarding intrinsic timelag between EM and GW signal emission**

general form for bound on v_{GW} valid for a broad set lens models

timing accuracy

$$1 - \left(\frac{v_{GW}}{c}\right)^2 \leq \frac{\delta T}{\Delta t_\gamma F_{\rm lens}(z_l, z_s)}$$

factor related to lens model and cosmology $F_{\rm lens}(z_l, z_s) \sim O(1)$



time delay is produced at lens location – **results doesn't depend strongly on cosmology**

we can constrain **directly** speed of GWs with lensing **!**

See also:
T. E. Collett and D. Bacon, Phys. Rev. Lett.118, 091101 (2017)

Fan, Liao, Biesiada, AP & Zhu (2017)

lenses as a tool for quantum gravity – future prospects

LSC LIGO Scientific Collaboration

VIRGO Listening to the cosmic whisper

+ KAGRA

started on 25 February 2020

A2 A bird's eye view image of KAGRA. The 3-km arms and the center part are illustrated.

O1 from 12 September 2015 to 19 January 2016,

O2 from 30 November 2016 to 25 August 2017,

O3a from 1 April to 30 September 2019

O3b from 1 November 2019 suspended in March 2020

O4 currently planned to end on 7 October 2025 at 15:00 UTC

Hundreds of GW signals registered so far !

GW170817 the first NS-NS merger

GW190814 probably the first mixed BH-NS merger

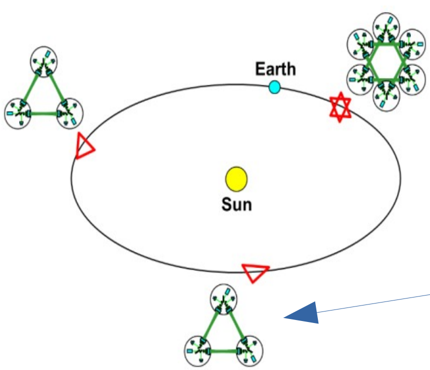
B.P.Abbott et al. [LSC, Virgo Collab.], Phys. Rev. X 13(4):041039 (2023)
R.Abbott et al. [LSC, Virgo Collab.], arXiv:2010.14527 [gr-qc] (2020)
R.Abbott et al. [LSC, Virgo Collab.], arXiv:2010.14533 [astro-ph] (2020)
B.P.Abbott et al. [LSC, Virgo Collab.], Phys. Rev. X 9, 031040 (2019)

Next generation GW detectors:

DECIGO Space Gravitational Wave Antenna

https://decigo.jp/index_E.html

Fig. 2. Conceptual design of DECIGO. One cluster of DECIGO consists of three drag-free spacecraft. FP cavities are used to measure a change in the arm length.



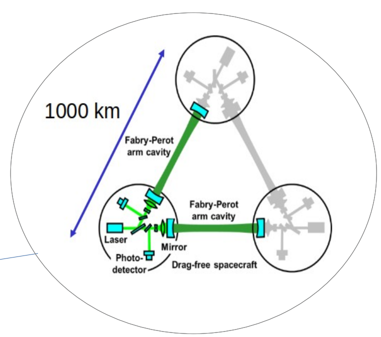


Fig. 1. Orbit of DECIGO. Four clusters of DECIGO are put in the heliocentric orbit: two at the same position and the other two at different positions.

Nakamura T., et al. (2016)
Kawamura, S., et al. (2019)

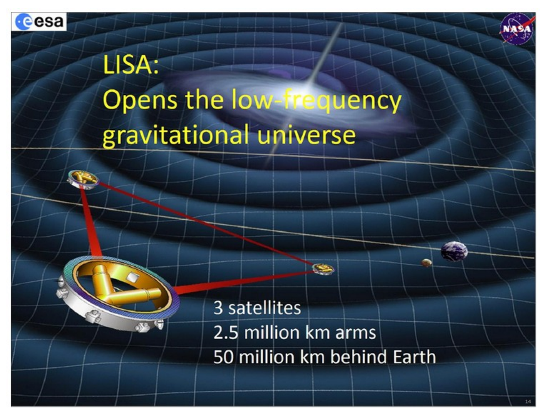
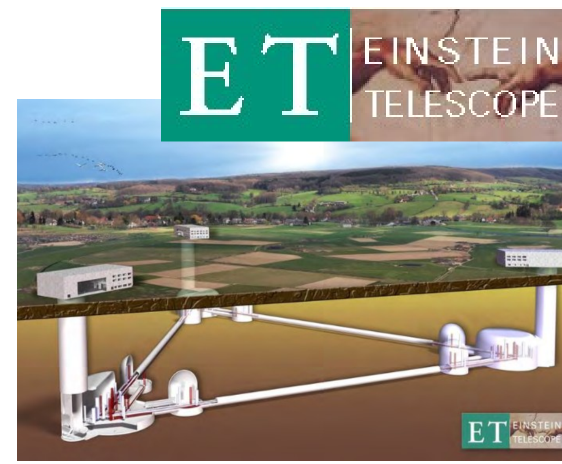


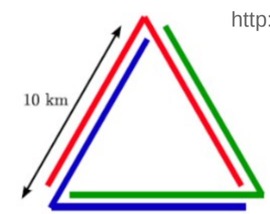
Figure 2: Artist's rendition of the LISA mission (image credit: ESA)

Goldstein, A., et al., *ApJL* **2017**, *848*, *L14*.

Yagi, K. & Seto, N. 2011, PRD, 83, 044011



http://www.et-gw.eu/



single triangular detector unit is equivalent for two standard L-shaped interferometers rotated by 45°

f: 0.1 mHz – 100 mHz

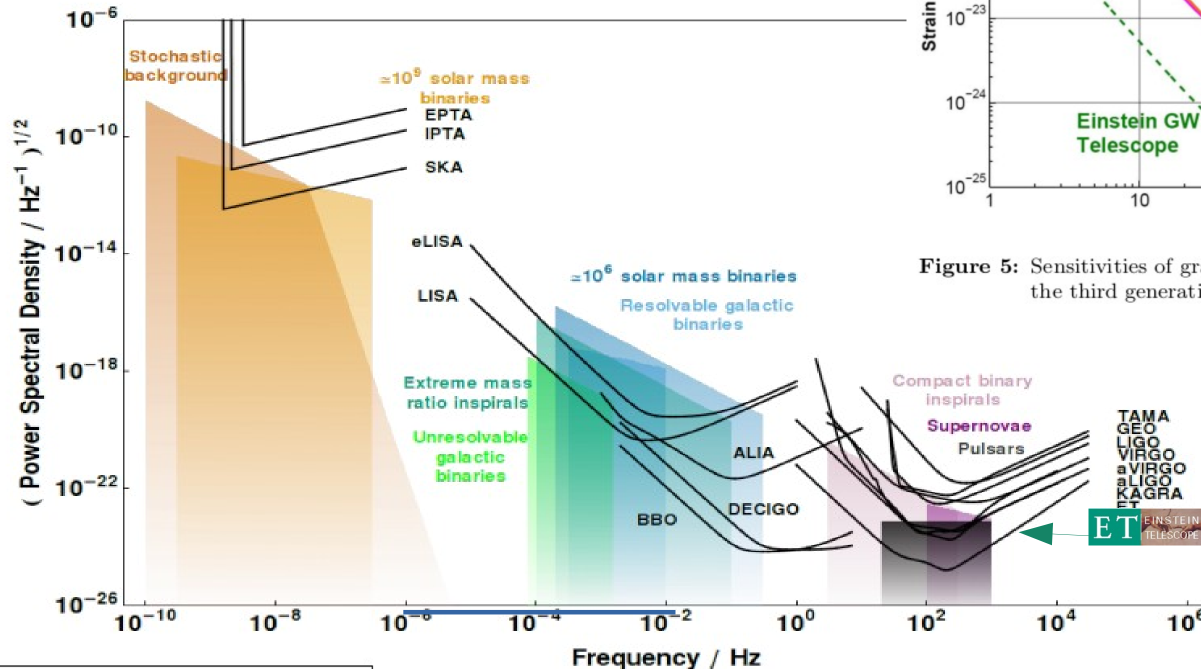
f: 1 mHz – 100 Hz

multifrequency

GW astrophysics !

broadening the GW spectrum to lower frequencies (lower than 1 Hz)

inaccessible from the ground
due to irremovable **seismic noise**



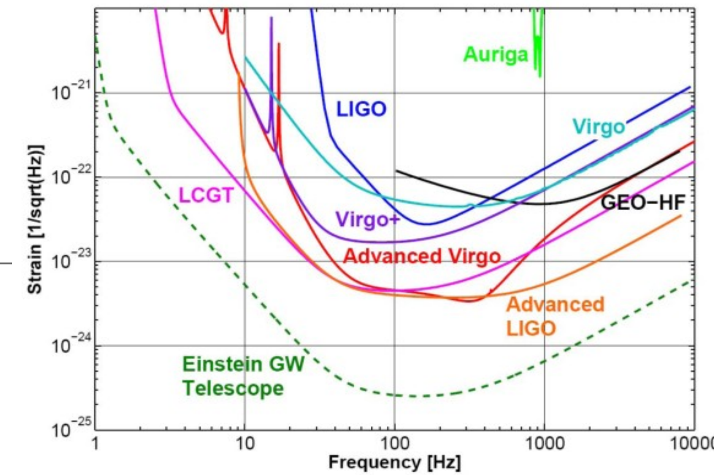
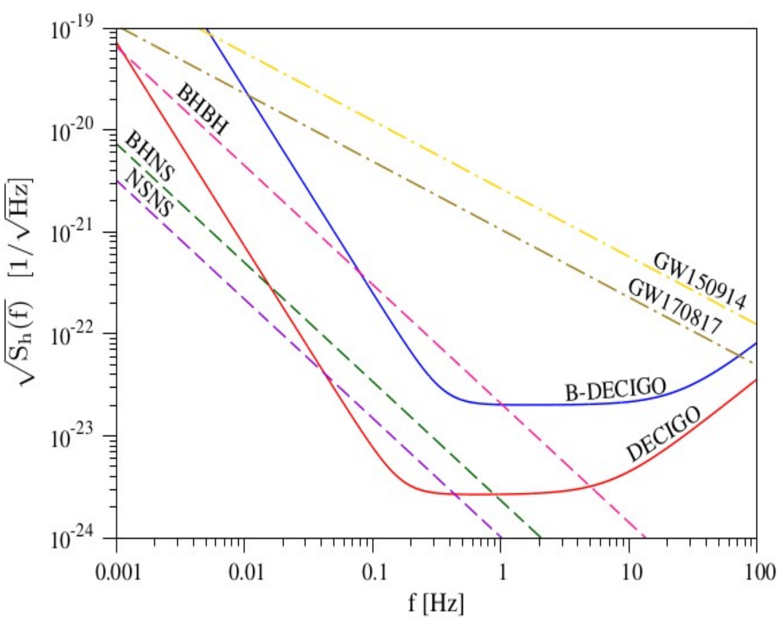


Figure 5: Sensitivities of gravitational wave detectors from the first to the third generation.

Remark:



DECIGO sensitivity significantly affected by unresolved BH-BH systems; B-DECIGO affected much less

$$t_c = 1.03 \times 10^6\ \mathrm{s}\ (\mathcal{M}_z/30.1\ M_\odot)^{-5/3}(f/0.1\ \mathrm{Hz})^{-8/3}$$

Isoyama, S. et al., Prog. Theor. Exp. Phys. 073E01 (2018)

GW150914 and **GW170817** could have been visible in (B-)DECIGO band for ~ 10 days and ~ 7 yrs prior to coalescence with large numbers of GW cycles

!

A. Piórkowska-Kurpas et al, ApJ 908 196 (2021)

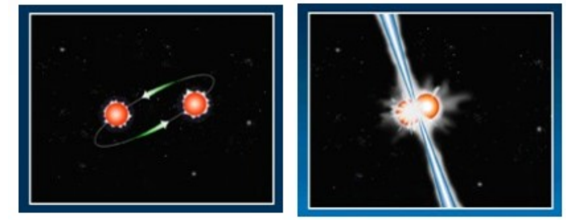
lenses as a tool for quantum gravity – perspectives:

for galaxy-galaxy strong lensing with $z_l = 1$ and $z_s = 2$

$$1-\left(\frac{v_{GW}}{c}\right)^2 \leq 4.26 \times 10^{-10} \left(\frac{\delta T}{1\ ms}\right)\left(\frac{\sigma}{250\ km/s}\right)^{-4}\left(\frac{y}{0.1}\right)^{-1}$$

$$\Delta t_{\gamma,GW} = \frac{1}{2H_0}(1+z_s)^2 I_2(0,z_s) \Rightarrow 1-\left(\frac{v_{GW}}{c}\right)^2 \leq 9.92 \times 10^{-22}$$

lensed NS-NS mergers !



jet collimation

~10% of NS-NS systems will be aligned as to give observable SGRBs

EM counterpart of NS-NS or NS-BH mergers visible as:

- kilonovae duration of order of days — P.S.Cowperthwaite and E.Berger, ApJ 814, 25 (2015)
- short GRBs duration of order of 0.1 - 1s — D.B. Fox et al., Nature 437, 845 (2005)
- FRB duration of order of ms — D. J. Champion et al., MNRAS 10.1093 (2016); D.Thornton et al., Science 341, 53 (2013)

Let's do some statistics...

Detection rates for aLIGO and ET

Source	BNS	NS-BH	BBH
Rate (Mpc^{-1} Myr^{-1})	0.1–6	0.01–0.3	2×10^{-3}–0.04
Event Rate (yr^{-1}) in aLIGO	0.4–400	0.2–300	2–4000
Event Rate (yr^{-1}) in ET	$\mathcal{O}(10^3–10^7)$	$\mathcal{O}(10^3–10^7)$	$\mathcal{O}(10^4–10^8)$

M. Biesiada et al. JCAP10(2014)080

Yearly detection rate for DECIGO

Evolutionary scenario	standard	optimistic CE	delayed SN	high BH kicks
NS-NS				
low-end metallicity	233.1	119.	335.5	3054.4
high-end metallicity	439.6	203.9	707.3	8807.7
BH-NS				
low-end metallicity	2688.9	1239.5	1838.6	1877.6
high-end metallicity	2000.	1314.6	1614.5	1613.7
BH-BH				
low-end metallicity	207755.2	384698.	178991.7	20125.8
high-end metallicity	166436.	360001.5	145583.5	15379.5
TOTAL				
low-end metallicity	210677.2	386056.5	181165.8	25057.8
high-end metallicity	168875.6	361520	147905.3	25800.9

A. Piórkowska-Kurpas et al, ApJ 908 196 (2021)

ET EINSTEIN TELESCOPE — DECIGO Space Gravitational Wave Antenna — NASA esa LISA Laser Interferometer Space Antenna

Big catalogs of inspiral events up to cosmological distances

Some of them would be gravitationally lensed

First Multimessenger Transient

GW170814 GWs LIGO/VIRGO

⇩ ~1.7 s

GRB 170817A γ-rays Fermi/GBM

⇩ 11 hours after the merger

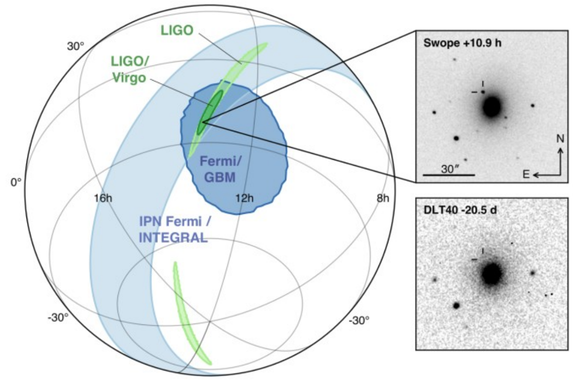
SSS17a / AT 2017gfo bright optical transient in NGC 4993

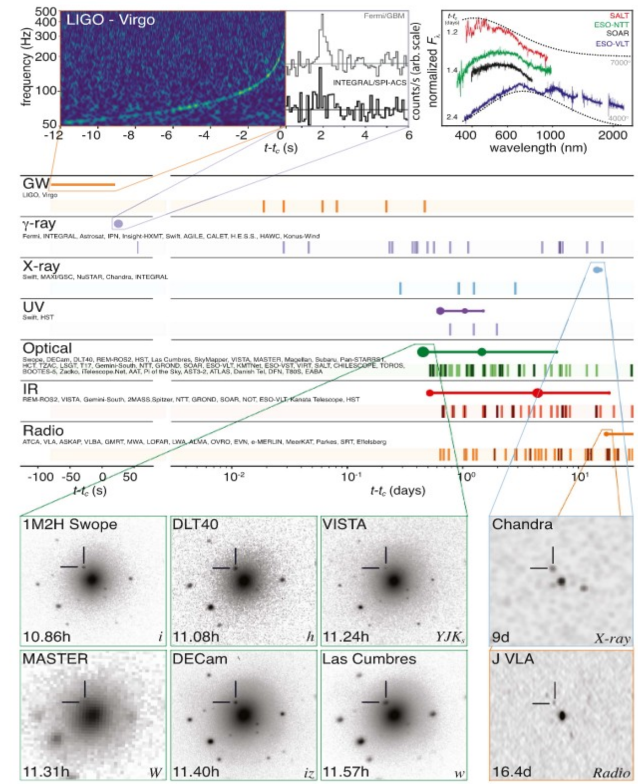
⇩ multi-wavelength evolution within the first 12–24 hr

Follow-up observations:

- UV-blue transient ~15h after merger
- X-ray emission ~9 days after merger
- radio emission ~16 days after merger

[Multi-messenger Observations of a Binary Neutron Star Merger; ApJL, 848:L12, 2017]





⇨ **lensed** NS-NS mergers

GW lensing in ET discussed in papers:

A. Piórkowska et al. **JCAP10(2013)022** (NS-NS only)

M. Biesiada et al. **JCAP10(2014)080** (full DCO: NS-NS, BH-NS, BH-BH)

X. Ding et al. **JCAP12(2015)006** (relaxing intrinsic SNR=8 demand; magnification bias)

robust prediction: **50-100 lensed DCO events per year**

BH-BH systems contribute 91 – 95%;
NS-NS systems 1 – 4%

~a few lensed NS-NS /yr

ET EINSTEIN TELESCOPE

Einstein Telescope

- *Increased sensitivity great expectations*
- *Big catalogs of inspiral events up to cosmological distances*
- *Multi-messenger astrophysics*
- *Some of them would be gravitationally lensed*

results corrected for Earth's rotation effect:
L. Yang et al. **ApJ 874, 139 (2019)**

THE ASTROPHYSICAL JOURNAL

Inspiraling Double Compact Object Detection and Lensing Rate: Forecast for DECIGO and B-DECIGO

Aleksandra Piórkowska-Kurpas[1,2], Shaoqi Hou[3], Marek Biesiada[1,4], Xuheng Ding[3,5], Shuo Cao[1], Xilong Fan[3], Seiji Kawamura[6], and Zong-Hong Zhu[1,3]

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DECIGO
Space Gravitational Wave Antenna

SNR above threshold of 8

Yagi & Seto 2011
Isoyama et al. 2018

optical depth corrected for finite duty cycle of detector

$$\dot{N}_{lensed}(z_s) = \int_0^{z_s} \tau_{\Delta t}(z_s, y_{max}, T_{surv}) \frac{d\dot{N}(>\rho_0)}{dz} dz$$

merger rates according to **Dominik et al. 2013**
https://www.syntheticuniverse.org
StarTrack code

50 lensed events per year
few lensed events per year

→ confusion noise of unresolved systems influence our ability to detect inspiraling DCO systems

! only BH-BH systems

4 binary evolution scenarios:

2 galaxy metallicity evolution models
„high-end" „low-end"

1. Standard
2. Optimistic Common Envelope (OCE)
3. Delayed SN explosion
4. High BH kick

expected numbers of lensed GW events from inspiraling DCOs

$T_{surv} = 4\ yrs.$

DECIGO

Evolutionary scenario	standard	optimistic CE	delayed SN	high BH kicks
NS-NS				
low-end metallicity	0.	0.	0.	0.07
high-end metallicity	0.	0.	0.	0.29
BH-NS				
low-end metallicity	0.2	0.02	0.15	0.38
high-end metallicity	0.21	0.03	0.2	0.39
BH-BH				
low-end metallicity	66.91	58.12	62.86	10.04
high-end metallicity	65.07	71.28	61.41	8.46

B-DECIGO

Evolutionary scenario	standard	optimistic CE	delayed SN	high BH kicks
NS-NS				
low-end metallicity	0.	0.	0.	0.
high-end metallicity	0.	0.	0.	0.
BH-NS				
low-end metallicity	0.2	0.02	0.15	0.38
high-end metallicity	0.21	0.03	0.2	0.39
BH-BH				
low-end metallicity	9.25	5.42	9.2	2.73
high-end metallicity	13.66	10.94	14.78	2.25

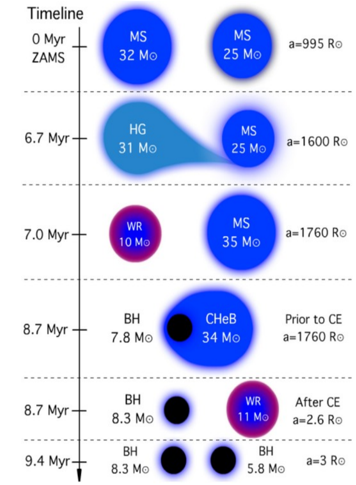
DECIGO

standard 4yrs	optimistic CE 4yrs	delayed SN 4yrs	high BH kicks 4yrs
6.1	53.4	6.4	6.1
6.0	56.7	6.5	6.2
6.2	9.4	2.9	0.7
5.7	9.8	2.7	0.7
146.4	324.0	125.2	11.4
125.6	312.0	106.4	9.4

B-DECIGO

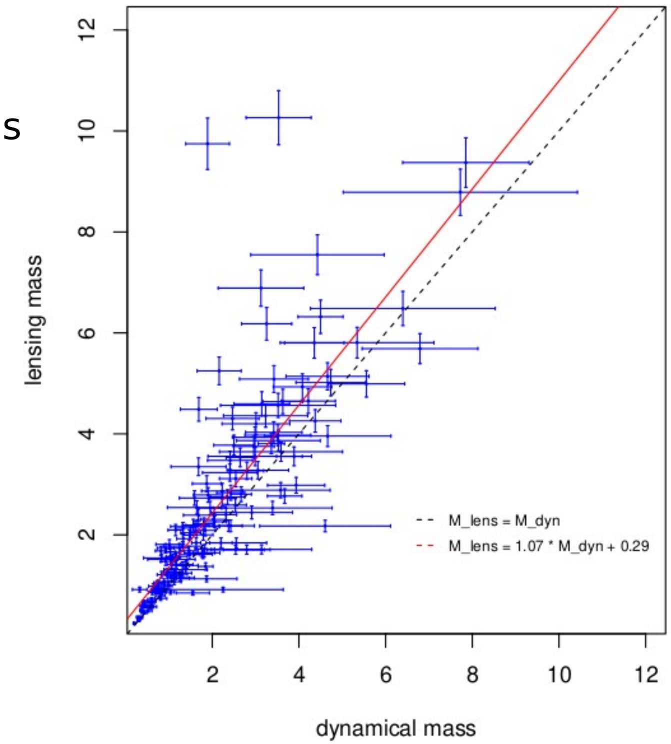
standard 4yrs	optimistic CE 4yrs	delayed SN 4yrs	high BH kicks 4yrs
0.0001	0.0004	0.0001	0.0001
0.0002	0.0004	0.0002	0.0001
0.07	0.2	0.04	0.006
0.03	0.1	0.02	0.005
48.8	134.1	40.3	3.1
38.3	121.2	31.3	2.4

lensing rates calculated if all accessible sources were resolvable ...



lenses as a tool for quantum gravity – next step:

for SIS model



analysis performed on the sample of
167 strong gravitational lensing systems

Observation: discrepancy between M_{lens} and M_{dyn}

⇒ Within the Einstein radius, we suppose that $M_{lens} = M_{dyn}$

mass inside the Einstein radius: $$M_{lens} = \frac{c^2}{4G}\frac{D_s D_l}{D_{ls}}\theta_E^2$$

dynamical mass inside the aperture projected to lens plane
(from solving the Jeans equation):

$$M_{dyn} = \frac{\pi}{G}\sigma_{ap}^2 R_E \left(\frac{R_E}{R_{ap}}\right)^{2-\gamma} f(\gamma) = \frac{\pi}{G}\sigma_{ap}^2 D_l \theta_E \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

$$f(\gamma) = -\frac{1}{\sqrt{\pi}}\frac{(5-2\gamma)(1-\gamma)}{3-\gamma}\frac{\Gamma(\gamma-1)}{\Gamma(\gamma-3/2)} \times \left[\frac{\Gamma(\gamma/2-1/2)}{\Gamma(\gamma/2)}\right]^2$$

⇒ For the SIS mass model, $\gamma = 2$ and $f(\gamma) = 1$,

$$M_{\rm dyn}(<\theta_E) = \frac{\pi}{G}\sigma_{\rm ap}^2 D_l \theta_E = \frac{\pi}{G}\sigma_{\rm ap}^2 R_E.$$

Idea:

$$\theta_E^{obs} = \theta_E + \Delta\theta_E$$

'pure' Einstein radius
i.e. Einstein radius as is expected within standard theory

some correction which depends on the details of a particular QG model (i.e. on its parameters)

⇓

$$\theta_E^{obs} = (1+\alpha)\theta_E$$

Li et al. ApJ (2018)

e.g. within massive photon scenario:

$$\theta_E^{\gamma} = (1+\alpha_\gamma)\,\theta_E, \text{ where } \alpha_\gamma = \frac{m_\gamma^2 c^4}{2E^2}$$

Lowenthal, PRD (1973)

the stringest upper limit for the photon mass is:
(from pulsar timing and fast radio bursts FRBs data)

$$m_\gamma \leq 9.52 \times 10^{-46} kg$$

(which is equivalent to $m_\gamma \leq 5.34 \times 10^{-10} eVc^{-2}$)

Wang et al. (2024)

$$\alpha_\gamma \geq 9.3 \times 10^{-10}$$

Li et al. ApJ (2018)

$$M_{lens} = M_{dyn}$$

$$\frac{c^2}{4G}\frac{D_l D_s}{D_{ls}}\theta_E^2 = \frac{\pi}{G}\sigma_{ap}^2 D_l \theta_E \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

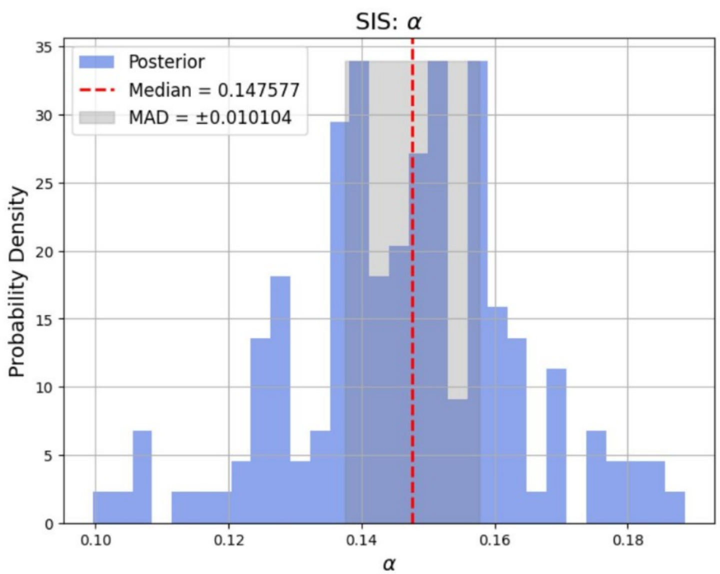
$$(\theta_E)^{th} = (1+\alpha)\left[4\pi\left(\frac{\sigma_{ap}}{c}\right)^2 \frac{D_{ls}}{D_s}(\theta_{ap})^{\gamma-2} f(\gamma)\right]^{\frac{1}{\gamma-1}}$$

$$\chi^2 = \sum_i \frac{\left[(\theta_E)_i^{\rm obs} - (\theta_E)_i^{\rm th}\right]^2}{\left(\Delta(\theta_E)_i^{\rm obs}\right)^2 + \left(\Delta(\theta_E)_i^{\rm th}\right)^2}$$

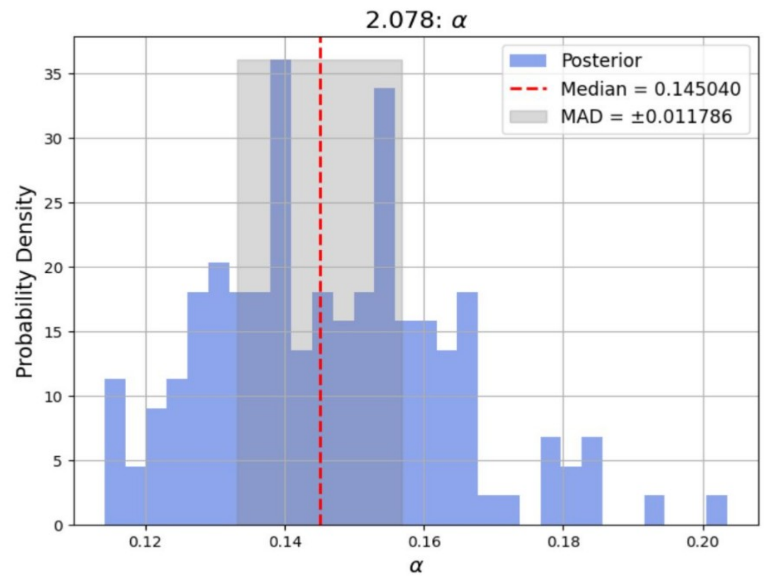
Preliminary results

H0=73.6
Om=0.334

SIS model (γ=2.0)



power-law model (γ=2.078)



Mass discrepancy between strong lensing and galaxy dynamics observations

A. Piórkowska-Kurpas,[1★] S. Geng,[2] M. Biesiada[2]
[1]*Institute of Physics, University of Silesia, 75 Pułku Piechoty 1, 41-500 Chorzów, Poland*
[2]*National Centre for Nuclear Research , Pasteura 7, 02-093 Warsaw, Poland*


Thank you for your attention .