Leptogenesis Assisted by Bubbles

Matter to the Deepest - Sept. 19

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+ Work in Progress

Phase Transition Basics

Scalar Potentials depend on temperature/density

- Field coupled to a hot bath -> real thermal excitations in the plasma contribute to the free energy.

- Generate **temperature dependent** corrections to quantities like the scalar potential.

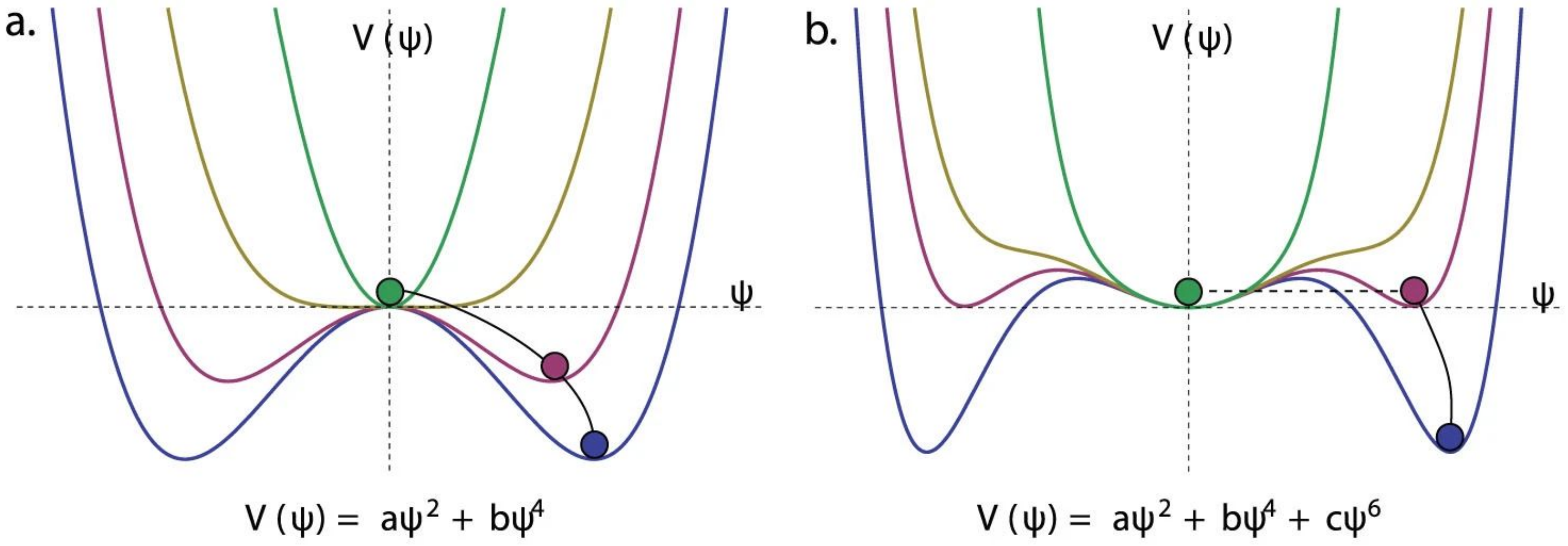
$$V(\phi, T) = V_0(\phi) + \sum_i V_{CW}(m_i^2(\phi) + \Pi_i) + \sum_i V_T(m_i^2(\phi) + \Pi_i)$$

- At very high temperatures this is dominated by a piece quadratic in temperature -> *generally* leads to symmetry **restoration** at high T.

$$\mathcal{O}(T^2\phi^2),\ \mathcal{O}(T\phi^3),\ \mathcal{O}(\phi^4\log(\phi^2/T^2),\ (\phi/T)^{2n}$$

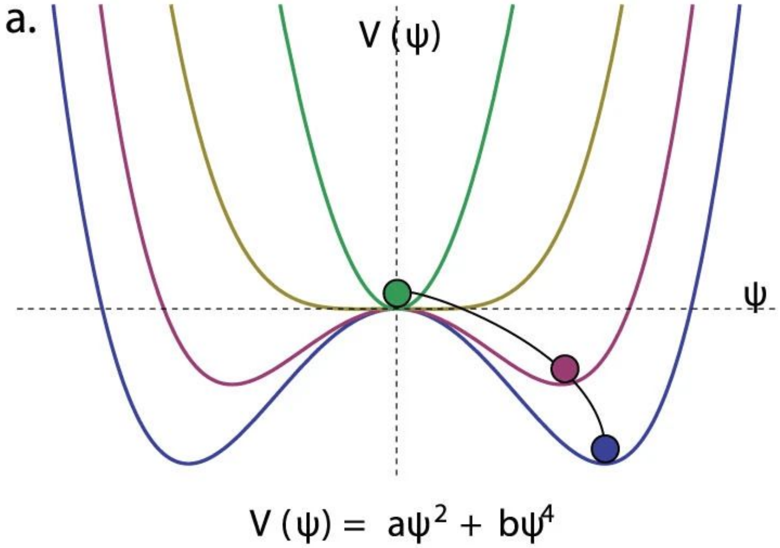
Potential vs Temperature

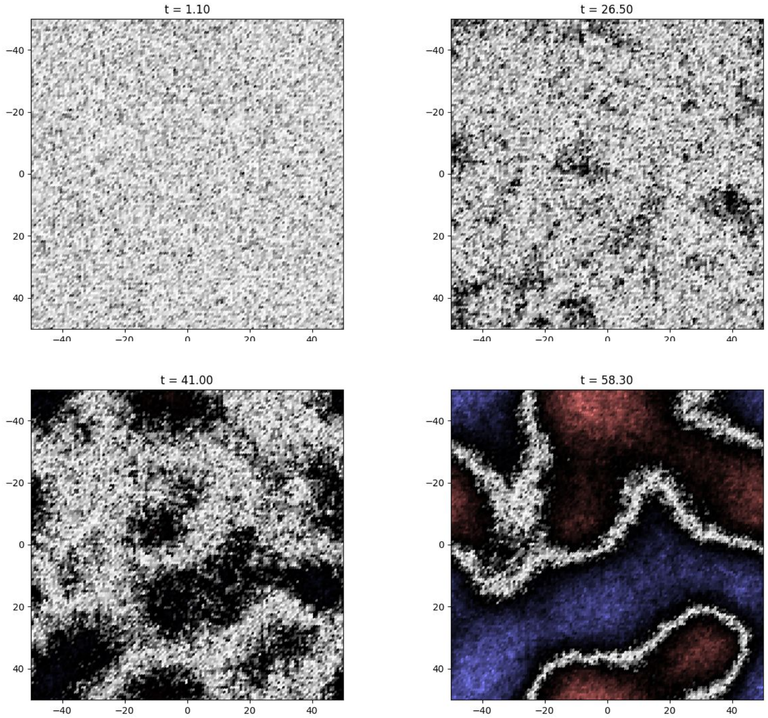
- What can happen to the shape as the temperature is decreasing?



2nd Order Transitions

- What can happen to the shape as the temperature is decreasing?



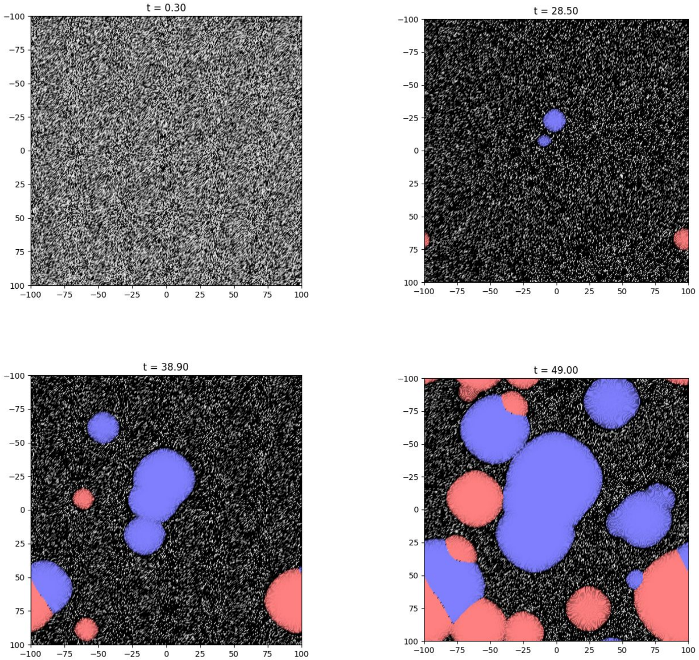


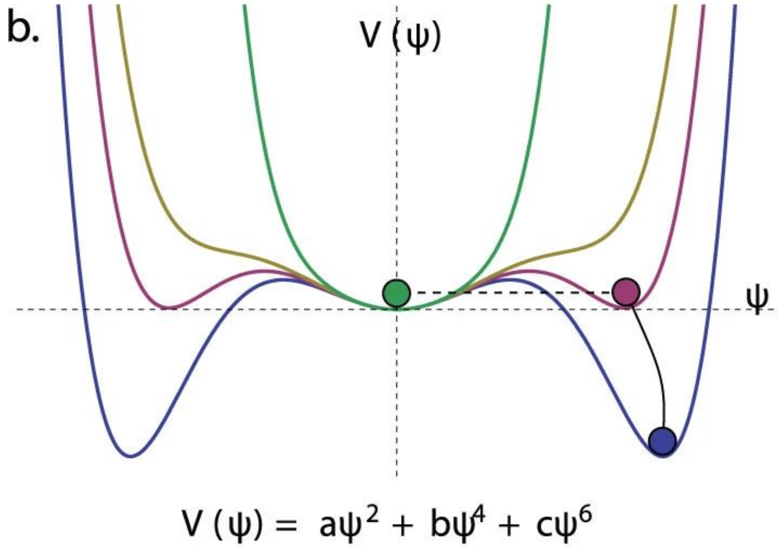
Minimum itself moves, scalar stays at minimum.
(Near) homogeneous transition.

Animations available on YT at : @tdutka_phys

1st Order Transitions

- What can happen to the shape as the temperature is decreasing?





There is a barrier, scalar is trapped. Can escape via fluctuations - inhomogeneous transition.

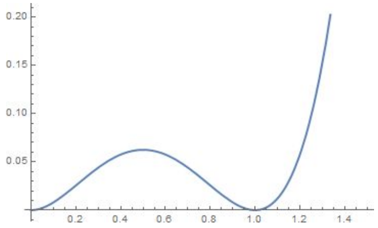
Animations available on YT at : @tdutka_phys

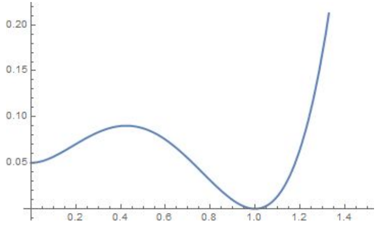
General features of FOPTs

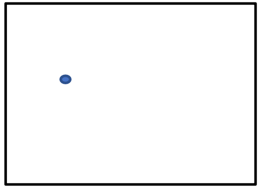
Supercooled: $T_{\mathrm{nucl}} \ll T_{\mathrm{crit}}$

T

T_{crit}



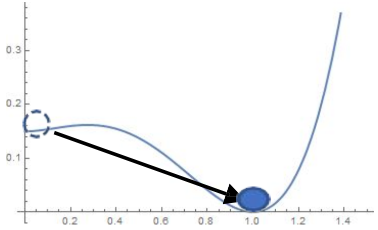


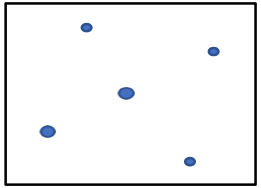


$\Gamma_n \neq 0, \Gamma_n \ll H^4$

Bubble nucleation is possible, but bubbles are diluted away by the spacetime expansion outside bubbles

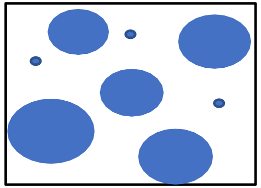
T_{nucl}

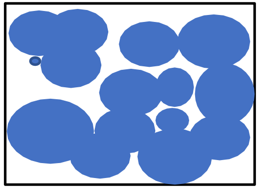




$\boldsymbol{\Gamma_n \simeq H^4}$

One bubble nucleation per one Hubble patch within one Hubble time





nucleation → **expansion** **collision**

Figures courtesy of Tae Hyun Jung

Supercooled First Order Phase Transitions

(Recent) popularity in studying (supercooled) phase transitions in cosmological contexts.

- Can produce a secondary, smaller period of 'thermal inflation' which can help dilute the abundances of some long-lived BSM particles which could spoil the predictions of BBN, e.g. gravitino, moduli… hep-ph/9510204, hep-ph/9602263, 0801.4197, 1412.7814….

- Can produce sizeable gravitational wave signals, through collision and motion of true-vacuum bubbles within the thermal plasma. 1809.08242, 1811.11169, 2007.15586, 2208.11697, 2303.02450….

- Supercooled phase transitions can be used as explanations for other problems in particle cosmology, e.g. PBH production, dark matter filtering, baryogenesis catalysts.. 1912.04238, 2110.04271, 2206.04691, 2206.09923, 2304.00908, 2305.10759….

- Constructing supercooled models is not difficult, just require relatively flat potentials, most commonly classically scale invariant or SUSY models.

Supercooled First Order Phase Transitions

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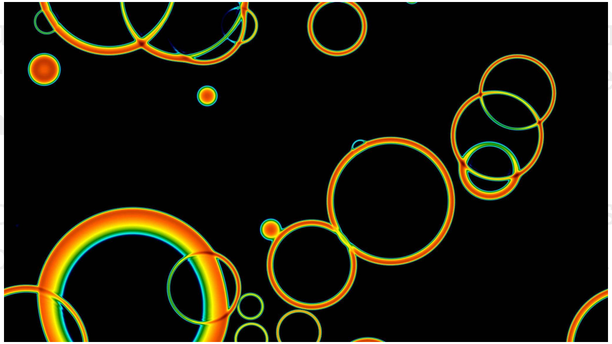
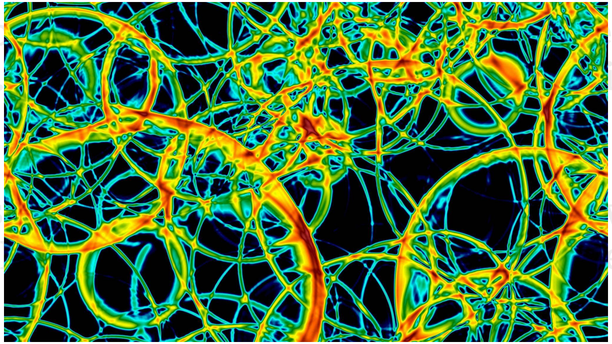
- Su … n b … r pr … e.g. … ltering, ba…

- Co … s is … vely flat po … ca … els.



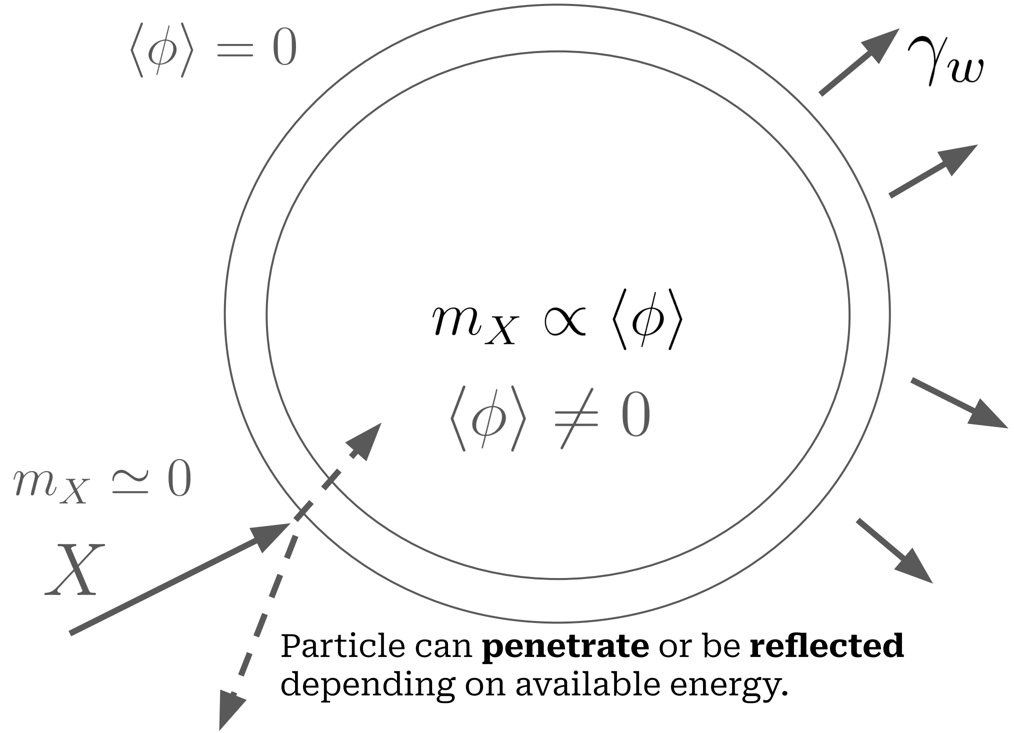


Figures from 1705.01783

Particle Cosmology during Supercooling

Particle Physics and Supercooling

Define a number of relevant parameters involved in cases where FOPT bubbles interact with plasma particles:



Particle can **penetrate** or be **reflected** depending on available energy.

$$T_{\text{crit}}$$

$$\downarrow$$

$$T_{\text{perc}}/T_{\text{nuc}}$$

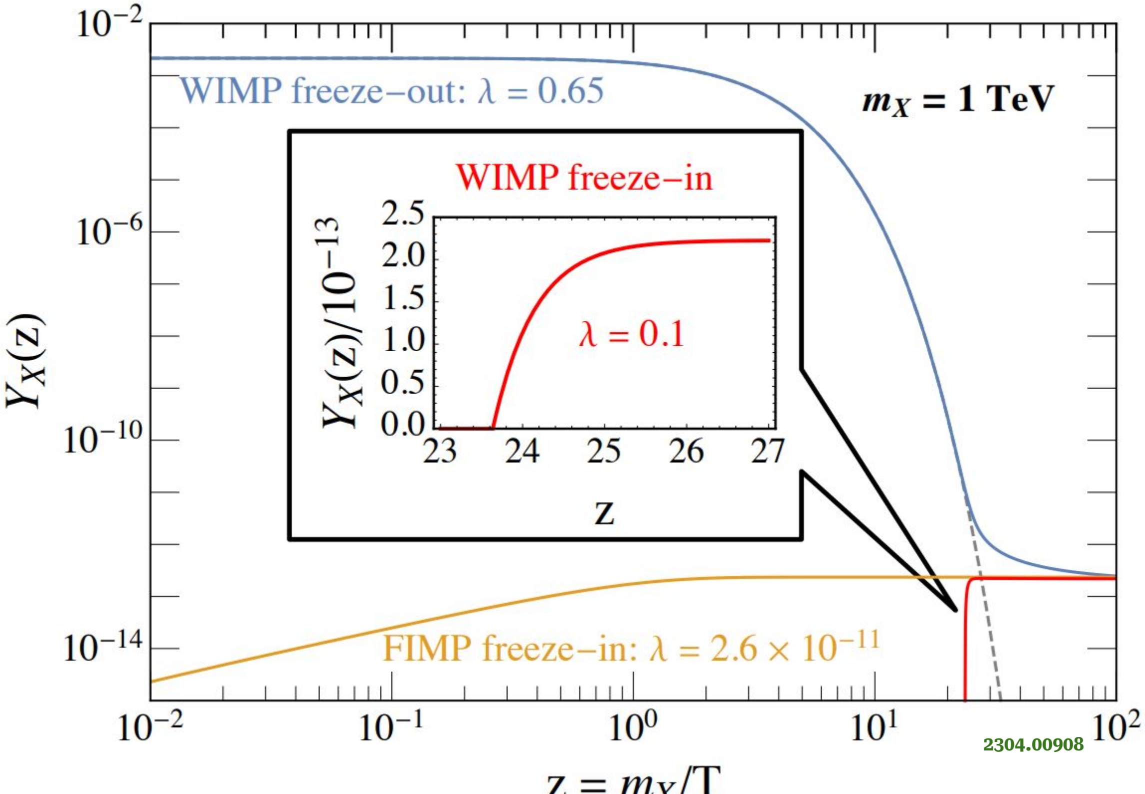
$$\downarrow$$

$$T_{\text{reh}}$$

$$\sim \left(\Delta V + \rho_{\text{plasma}}(T_{\text{nuc}})\right)^{1/4}$$

Particle Physics and Supercooling

Dilution scenario (beyond the usual e-folding situation)



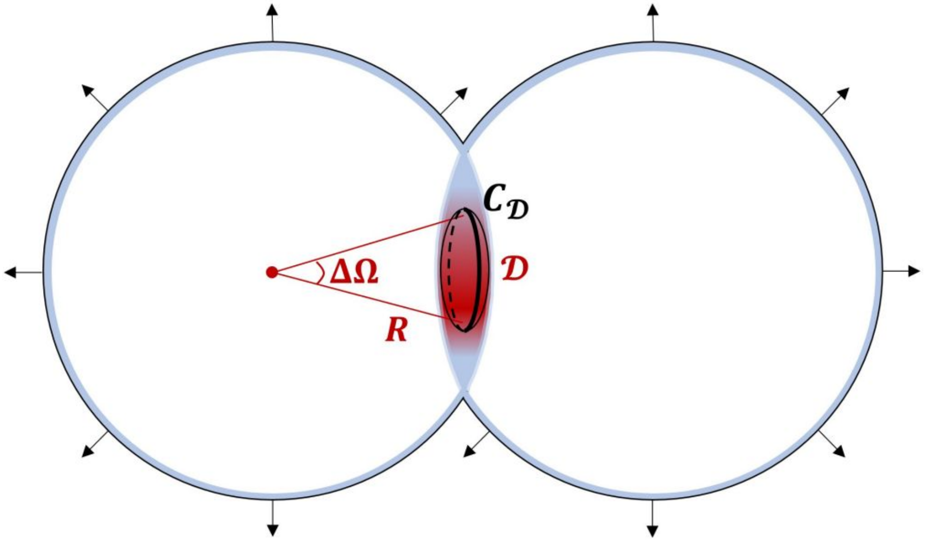
Freeze-in DM model with strong couplings:

Strong FOPT removes any pre-existing thermal DM density

reheating: $Y_X \propto \left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3$

Particle Physics and Supercooling

PBH formation: a number of proposals



2110.04271

Collapse of overdense regions related to inhomogeneous vacuum decay

DM candidate with a motivated production mechanism

2106.05637, 2210.14094, 2212.14037

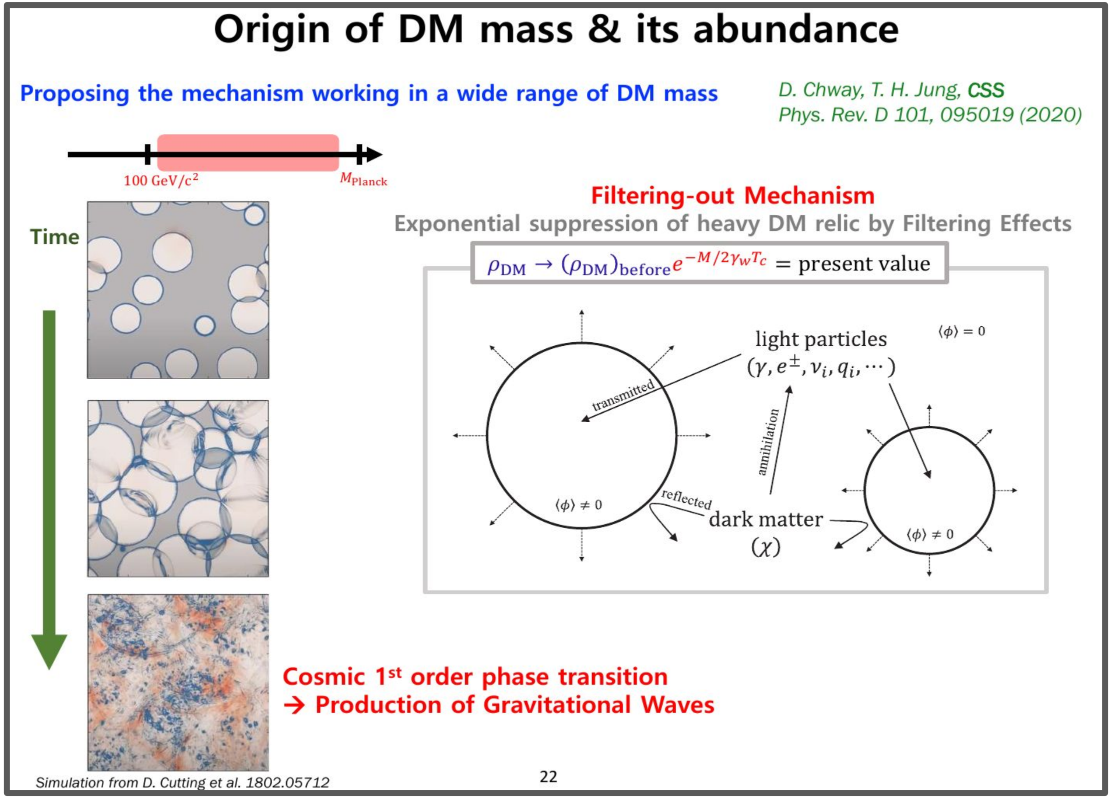
Particle Physics and Supercooling

"**Squeezing**" scenario

$$T_{\rm nuc} < \gamma_w T_{\rm nuc} \ll m_X$$

Only a few DM particles can penetrate the bubble, from the Boltzmann tail.

The remaining one are reflected (squeezed) into false vacuum pockets and eventually disappear.



Courtesy of Chang Sub Shin

Particle Physics and Supercooling

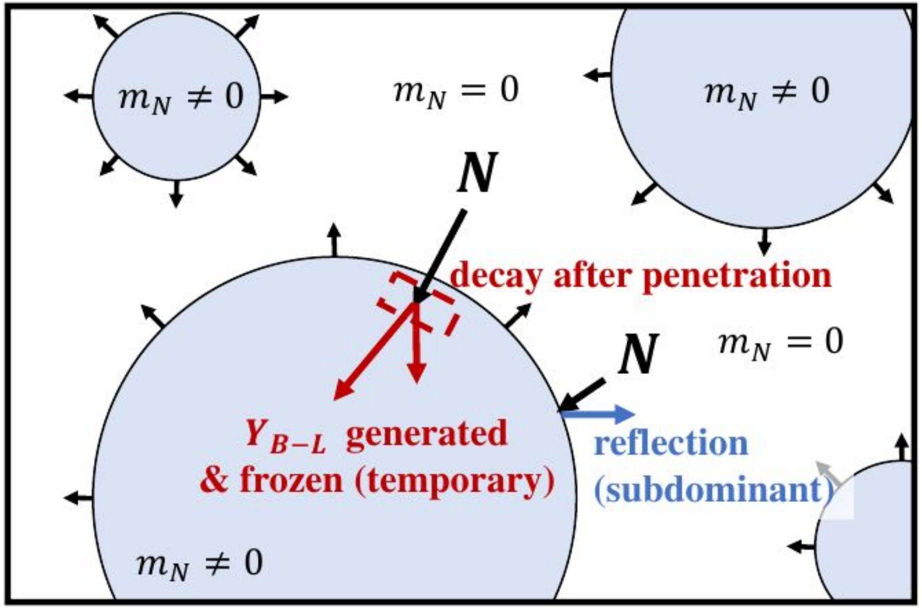
"**Mass-gain**" scenario

$$T_{\rm perc} < m_X < \gamma_w T_{\rm perc}$$

Naively mass-gaining particle cannot penetrate the wall.

Large velocity of bubble wall leads to large boost factor and particles are 'swept' away by expanding bubble.

$T = T_{\rm nuc}$



Chun, TPD, Jung, Nagels, Vanvlasselaer

‘Typical’ Leptogenesis

(Thermal) Vanilla Leptogenesis

$$\mathcal{L} \supset \frac{1}{2}(M_N)_{ij}\overline{N_i^c}N_j + (Y_D)_{\alpha i}\,\bar{\ell}_\alpha H N_i + h.c.$$

Fukugita, Yanagida, 1986

Sakharov Conditions:

- **B violation**
- **C & CP violation**
- **Departure from thermal equilibrium**

$$Y_D \sim \mathcal{O}(1) \implies M_N \sim 10^{14}\ \mathrm{GeV}$$

$$Y_D \sim \mathcal{O}(10^{-5}) \implies M_N \sim 10^{4}\ \mathrm{GeV}$$

$$m_\nu \simeq \frac{Y_D^2 v_{\mathrm{EW}}^2}{M_N}$$

(Thermal) Vanilla Leptogenesis

$$\mathcal{L} \supset \frac{1}{2}(M_N)_{ij}\overline{N_i^c}N_j + (Y_D)_{\alpha i}\,\overline{\ell}_\alpha H N_i + h.c.$$

Fukugita, Yanagida, 1986

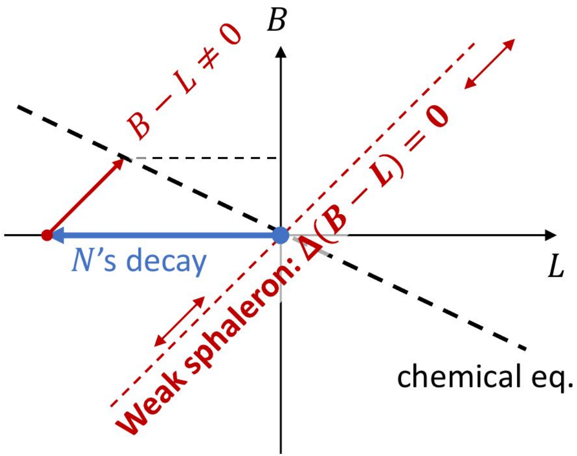
Sakharov Conditions:

- **B violation**

  $L(N,\ell) = 1$
  $L(H) = 0$

- **C & CP violation**

- **Departure from thermal equilibrium**



$$T_{\rm lepto} \gtrsim 140\ \mathrm{GeV}$$
$$\implies M_N \gtrsim 140\ \mathrm{GeV}$$

(Thermal) Vanilla Leptogenesis

$$\mathcal{L} \supset \frac{1}{2}(M_N)_{ij}\overline{N_i^c}N_j + (Y_D)_{\alpha i}\,\overline{\ell}_\alpha H N_i + h.c.$$

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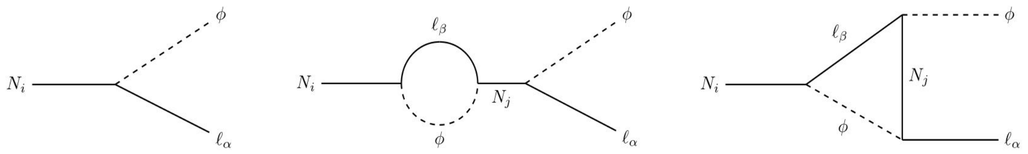


Can be easily included, need at least 2 RHNs (also for m_ν)

- **Departure from thermal equilibrium**

(Thermal) Vanilla Leptogenesis

$$\mathcal{L} \supset \frac{1}{2}(M_N)_{ij}\overline{N_i^c}N_j + (Y_D)_{\alpha i}\,\overline{\ell}_\alpha H N_i + h.c.$$

Fukugita, Yanagida, 1986

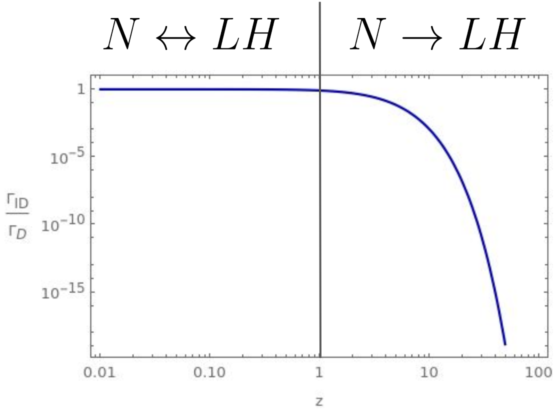
Sakharov Conditions:

- **B violation**

- **C & CP violation**

- **Departure from thermal equilibrium**

Induced by universe expansion **but** ‘smooth’ departure from equil.



$T \to 0 \implies \frac{\Gamma_{\rm ID}(T)}{\Gamma_{\rm D}(T)} \to 0$ $\quad \frac{\Gamma_{\rm ID}(z)}{\Gamma_{\rm D}(z)} = \frac{1}{2}z^2 K_2(z)$

Parameterising Thermal Leptogenesis

(1) (2) (3) (4)

$$Y_B \simeq Y_{N_1}\, \epsilon^1_{\mathrm{CP}}\, \kappa_{\mathrm{sph}}\, \kappa_{\mathrm{wash}}$$

$$Y_B \simeq 8.75 \times 10^{-11} \implies M_1 \simeq \frac{10^9\ \mathrm{GeV}}{\kappa_{\mathrm{wash}}}$$

hep-ph/0202239

hep-ph/0401240

$\kappa_{\mathrm{wash}} \simeq 5 \times 10^{-3} \implies$ Scale is typically much higher, ‘strong-washout leptogenesis’, need assumptions to saturate bound. (Unless $m_{\nu_1} \lesssim 10^{-3}$ eV and quite specific couplings.)

$$\implies M_1 \gtrsim (\mathrm{a\ few}) \times 10^{10} - 10^{11}\ \mathrm{GeV}$$

No precise, model-independent, lower bound as serious treatment of flavour effects varies this bound.

Bubble-assisted Leptogenesis

How can bubbles modify the dynamics of the vanilla leptogenesis scenario?

Shuve, Tamarit: 1704.01979

Baldes, et al: 2106.15602

Huang, Xie: 2206.04691
Dasgupta, et al: 2206.07032

Chun, **TPD**, Jung, Nagels, Vanvlasselaer

Cataldi, Shakya: 2407.16747

Zhao, Zhang, Wu: 2403.18630

Dichtl, Nava, Pascoli, Sala: 2312.09282

Huang, Xu: 2312.06380

Arakawa, Lu, Takhistov: 2409.12228

Setup

$$\mathcal{L} \supset \frac{1}{2}(Y_N)_{ij}\overline{N_i^c}\Phi N_j + (Y_D)_{\alpha i}\,\bar{\ell}_\alpha H N_i + h.c.$$

$$\downarrow$$

$$M_N = Y_N\langle\Phi\rangle$$

New scalar is assumed to undergo a **first-order phase transition**

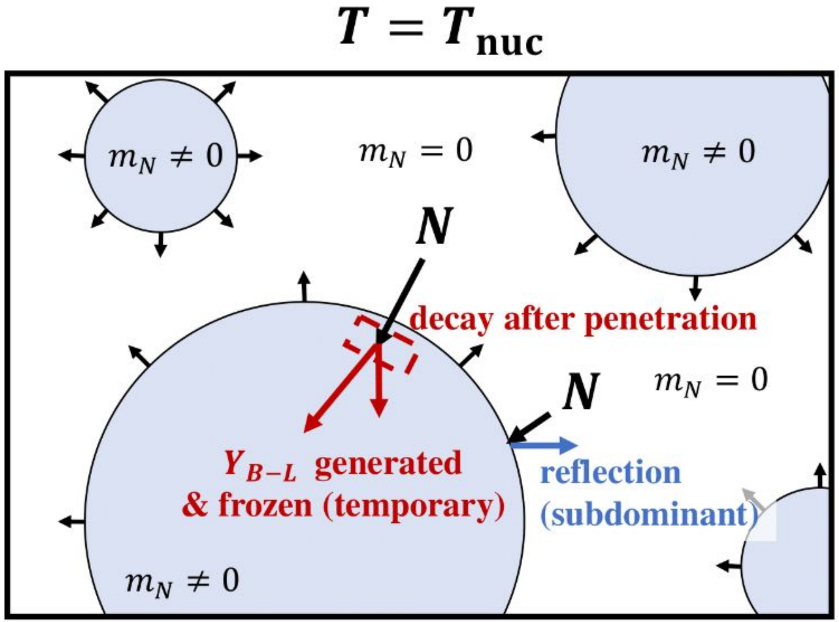
Much stronger departure from thermal equilibrium compared to usual thermal leptogenesis

$\langle\Phi\rangle \neq 0$	$\langle\Phi\rangle = 0$
$M_N \neq 0$	$M_N = 0$
$n_N^{\mathrm{eq}} \propto e^{-M_N/T}$	$n_N^{\mathrm{eq}} \propto T^3$

Larger population of RHNs available to decay *if they can penetrate the bubbles* - $\gamma_w > 1$

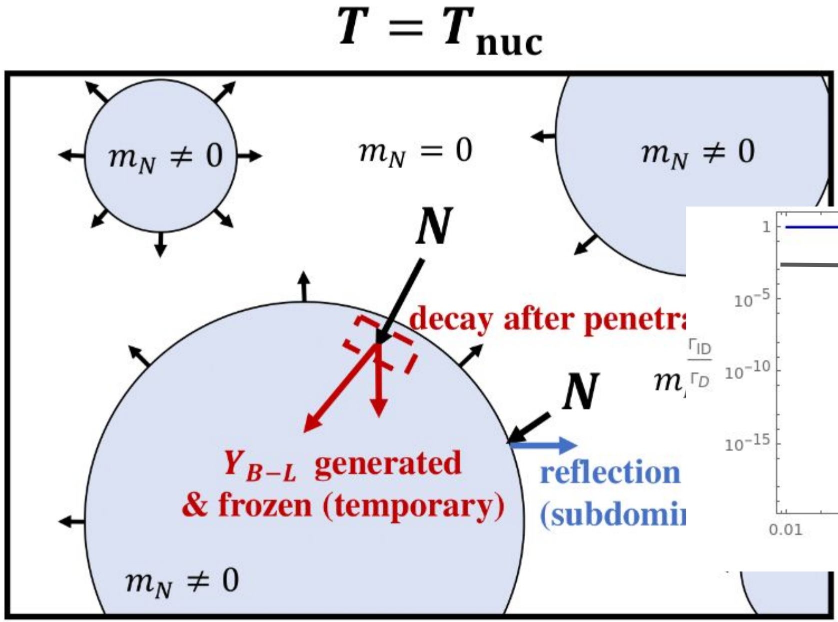
Bubbles begin to expand around the nucleation temperature

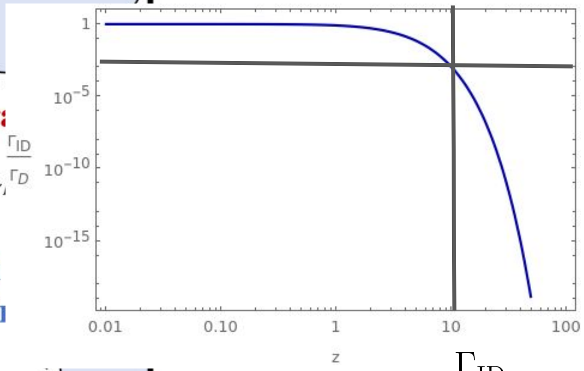
As bubbles expand



If $M_N \gg T_{\mathrm{nuc}}$ is satisfied (and RHNs penetrate) inverse decays could freeze out immediately after penetration $\kappa_{\mathrm{wash}} = 1$?

As bubbles expand





$$\frac{\Gamma_{\mathrm{ID}}}{\Gamma_{\mathrm{D}}} \simeq 10^{-3}$$

If $M_N \gg T_{\mathrm{nuc}}$ is satisfied (and RHNs penetrate) inverse decays could freeze out immediately after penetration $\kappa_{\mathrm{wash}} = 1?$ - fast bubbles req.

$$(\gamma_w T_{\mathrm{nuc}} > M_N > T_{\mathrm{nuc}})$$

Steps to evaluate final asymmetry

- **(i) For given Lagrangian parameters evaluate phase transition dynamics**

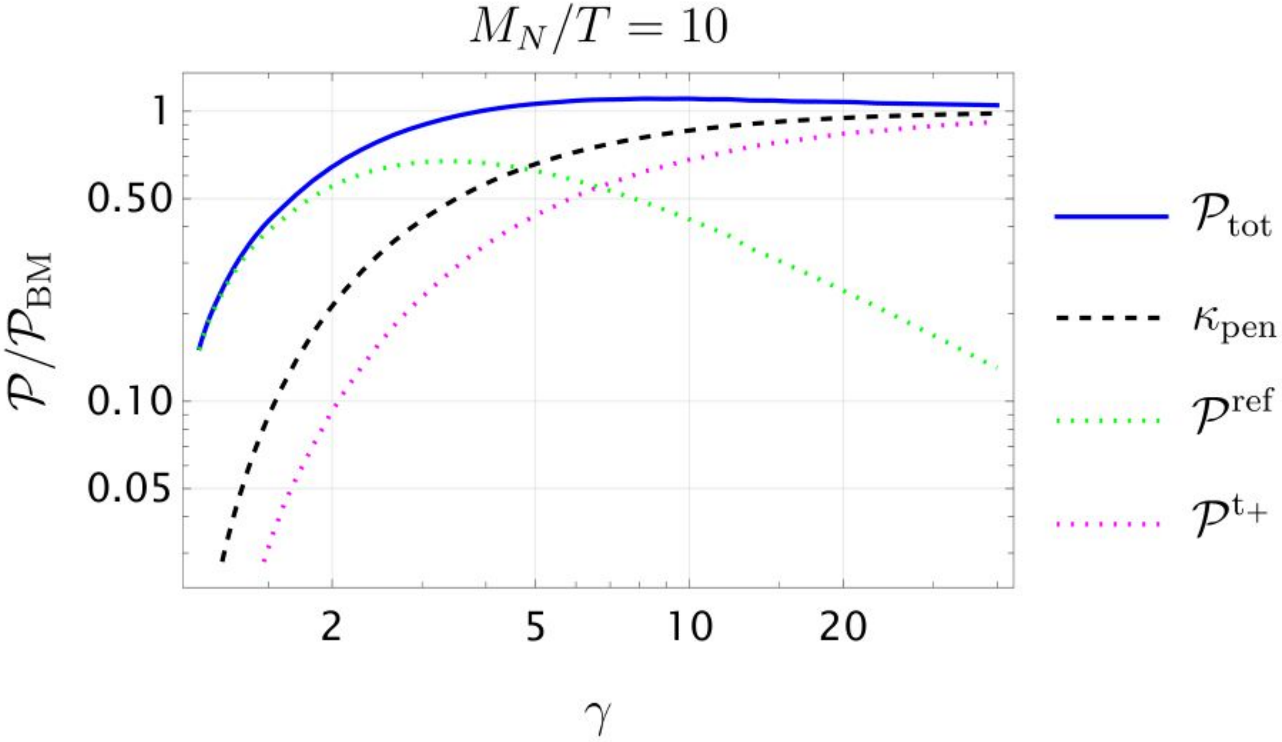
  Bubble-wall velocity, penetration rate of RHNs, FOPT properties

  $\gamma_w \gg 1$ (ideally) $\kappa_{\rm pen} \simeq 1$ (ideally) $T_{\rm nuc},\, T_{\rm reh},\, \alpha_n,\, \beta_{\rm PT}$

  $$\gamma_w T_{\rm nuc} > M_N > T_{\rm nuc}$$

  We **don't** want reflections along the bubble wall ideally.

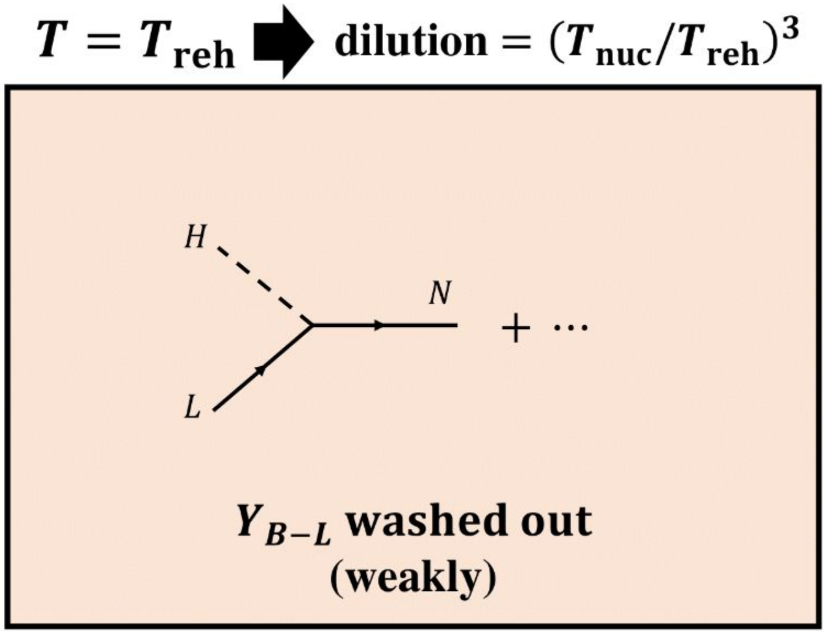
Step (i) - RHN Penetration



Penetration potential of RHNs much more positive compared to initial estimates

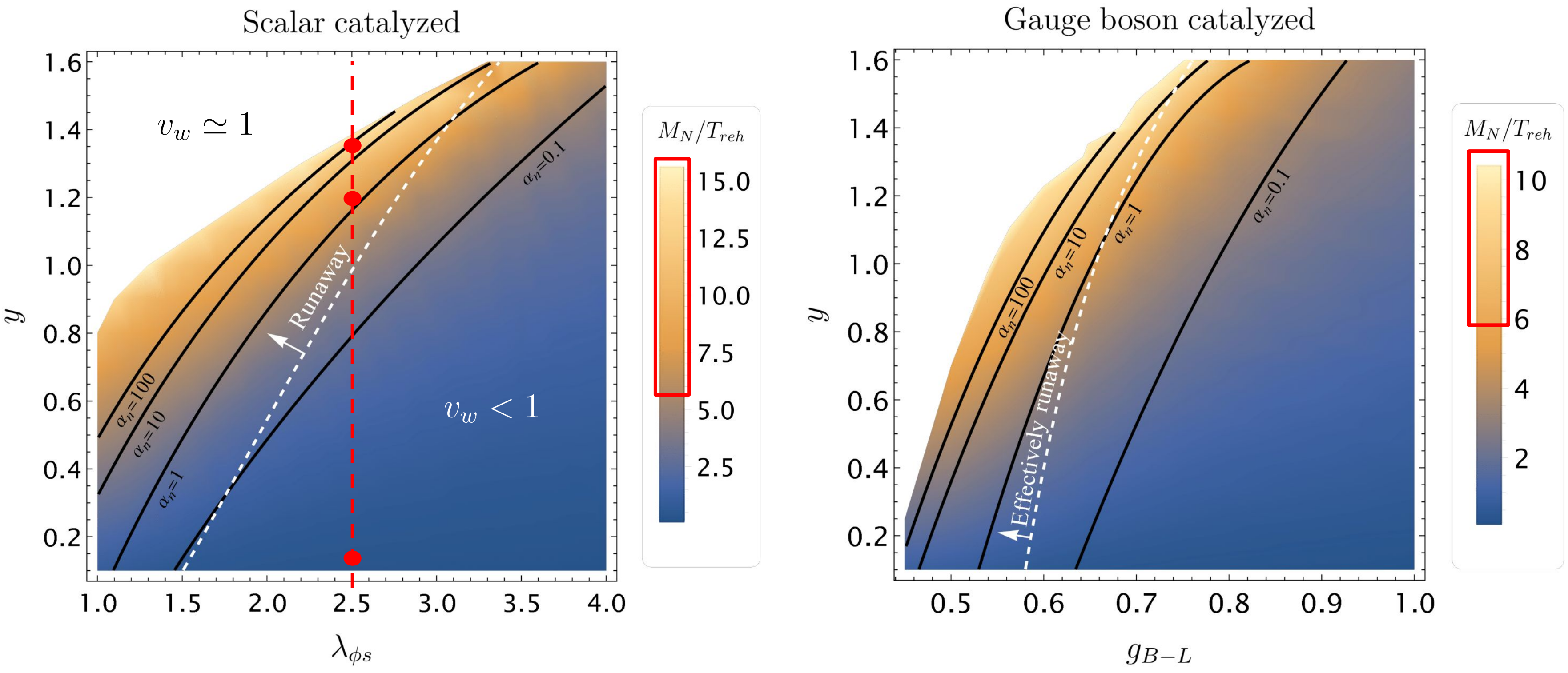
Shuve, Tamarit: 1704.01979

After bubbles collide



Universe reheats and the final asymmetry is diluted.
$M_N \gg T_{\text{reh}}$ also required to avoid washout

Step (i) - Mass/Temp. Hierarchies



Numerically find $M_N/T_{\rm reh} \gtrsim \mathcal{O}(5)$ required to avoid *conventional* washout

Steps to evaluate final asymmetry

- **(i) For given Lagrangian parameters evaluate phase transition dynamics**

  Bubble-wall velocity, penetration rate of RHNs, FOPT properties

  $\gamma_w \gg 1$ $\kappa_{\rm pen} \simeq 1$ $T_{\rm nuc},\, T_{\rm reh},\, \alpha_n,\, \beta_{\rm PT}$

- **(ii) Within the bubbles, RHNs decay and generate an asymmetry**

  Solve the Boltzmann equations *including any unavoidable washout channels*

$$\mathcal{L} \supset Y_N NN\phi + g_{B-L}\left(NN + ff\right) A_\mu$$

$$NN \to \phi\phi, ff$$

$$\left(\frac{T_{\rm reh}}{M_N} \propto \frac{\Delta V}{v_\phi^4} < 1\right) \implies m_\phi \ll M_N?$$

Interactions which remove RHNs without generating asymmetry: depletions

Steps to evaluate final asymmetry

- **(i) For given Lagrangian parameters evaluate phase transition dynamics**

  Bubble-wall velocity, penetration rate of RHNs, FOPT properties

  $\gamma_w \gg 1$ $\kappa_{\rm pen} \simeq 1$ $T_{\rm nuc},\, T_{\rm reh},\, \alpha_n,\, \beta_{\rm PT}$

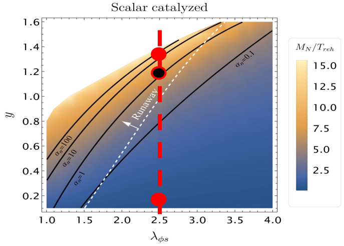
- **(ii) Within the bubbles, RHNs decay and generate an asymmetry**

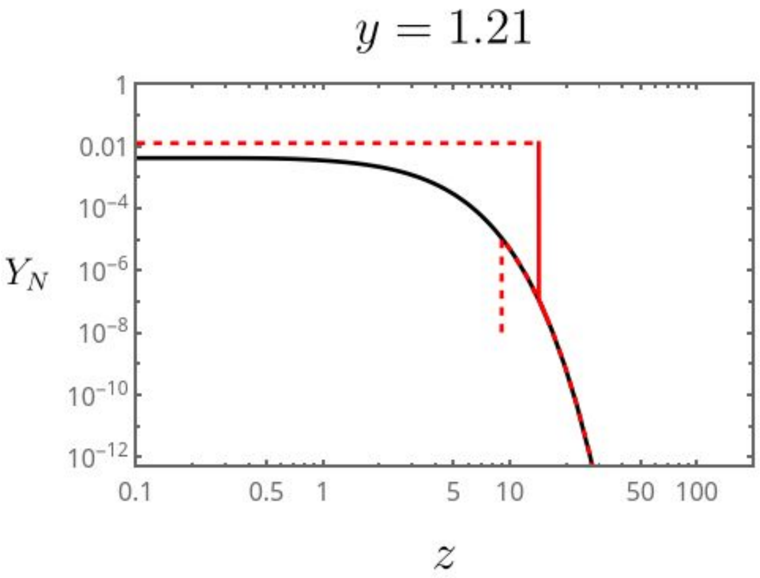
  Solve the Boltzmann equations *including any unavoidable washout channels*

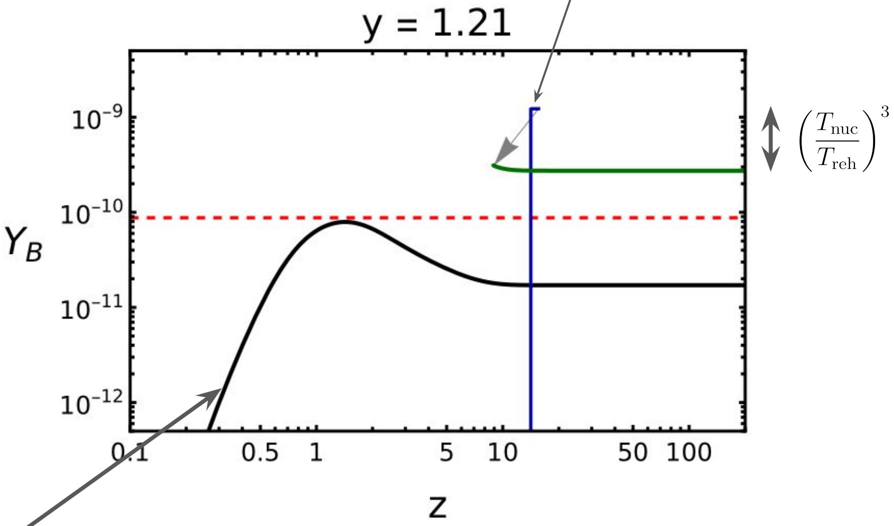
- **(iii) After bubbles collide the universe reheats, dilution + washout may occur**

$$T_{\rm reh} \simeq T_{\rm nuc}\,(1+\alpha)^{1/4}$$

Step (ii + iii)



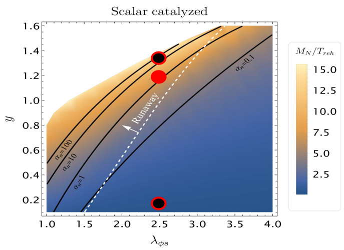




Bubbles nucleate, leptogenesis begins - little washout

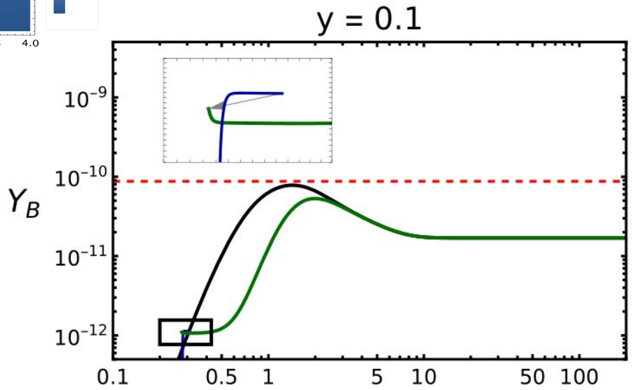
Comparison to conventional leptogenesis

reheating after collisions



Step (ii + iii)

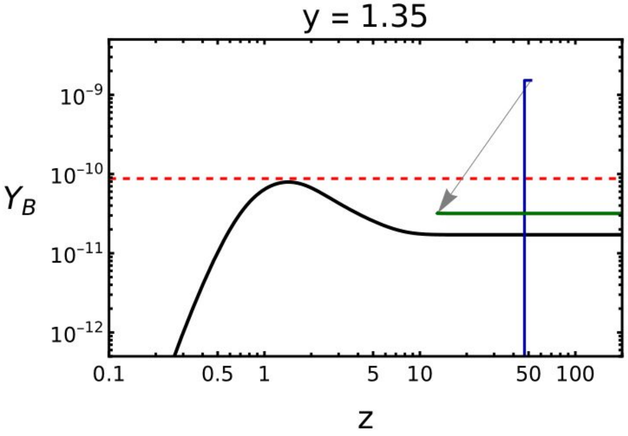
$\alpha < 1$



Phase transition too weak, no dilution from reheating but RHNs remain in equilibrium:

$$M_N < T_{\rm nuc}, T_{\rm reh}$$

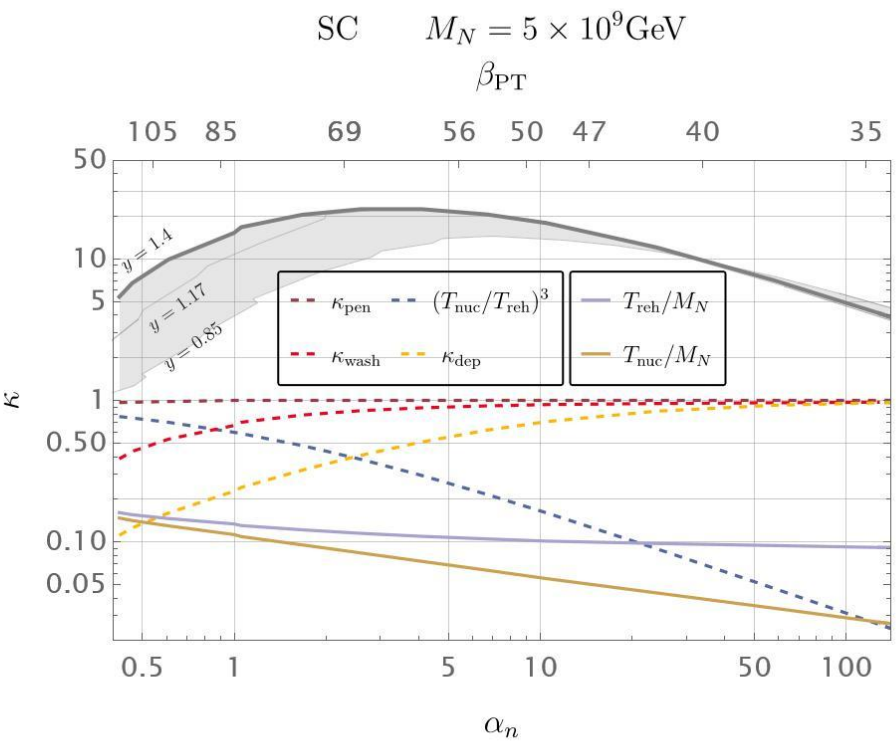
$\alpha \gg 1$

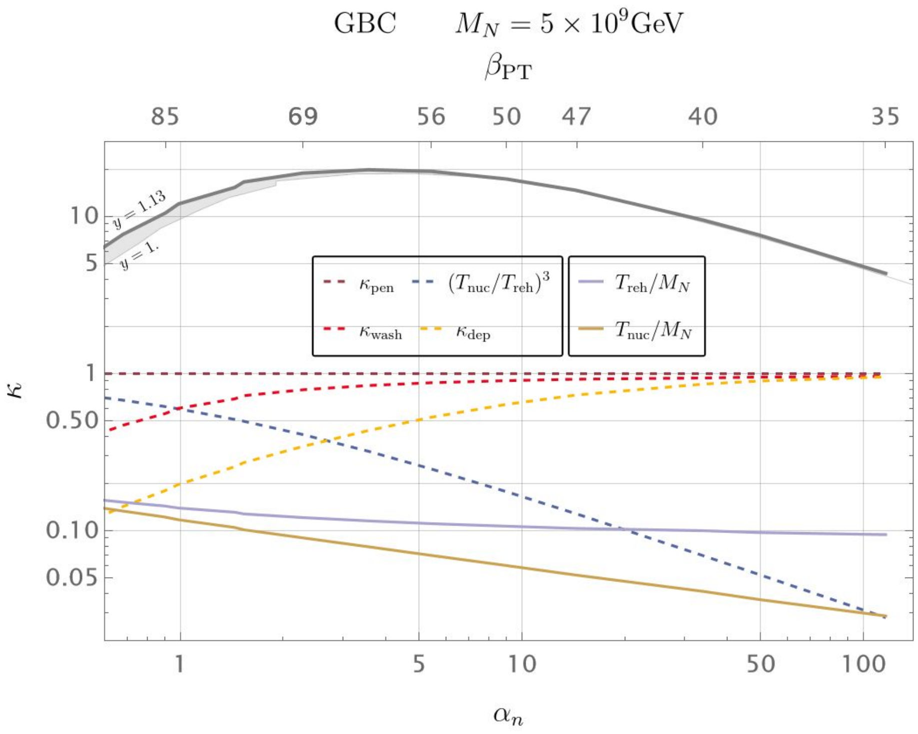


Phase transition too strong, extremely large reheating - final asymmetry is strongly diluted from bubble collisions

$$\left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3$$

Results

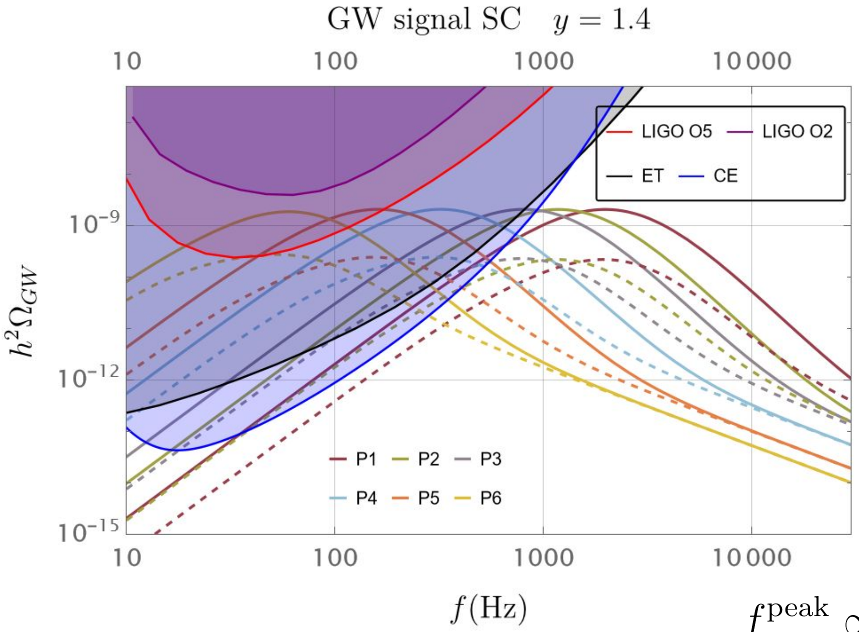


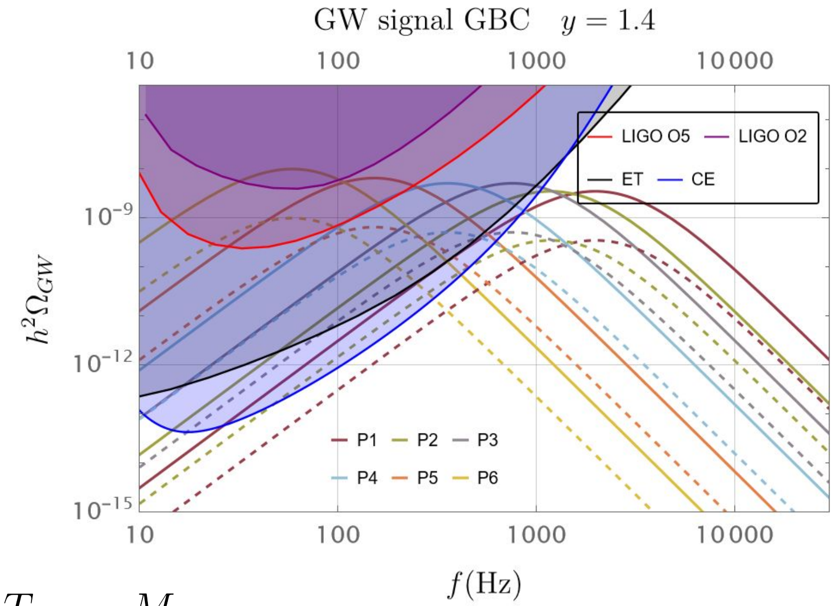


Significant enhancement in generated asymmetry compared to conventional scenario.

Strong-washout leptogenesis close to the Davidson-Ibarra bound

Gravitational waves



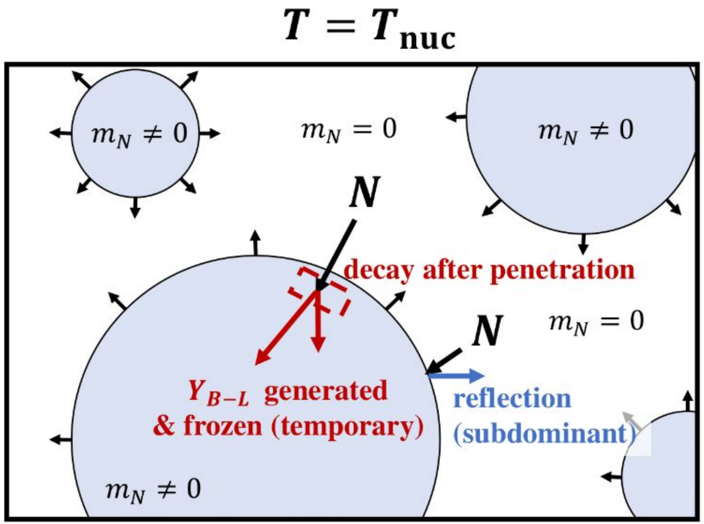


$$f^{\mathrm{peak}} \propto T_{\mathrm{reh}} \sim M_N$$

Considered production of GWs *during* the FOPT, e.g. bubble wall collision and sound waves for future GW detectors.

Peak frequency shifts with RHN mass: $\mathrm{P1}\!: M_N = 6 \times 10^9\ \mathrm{GeV} \ \rightarrow\ \mathrm{P6}\!: M_N = 10^8\ \mathrm{GeV}$

Future Explorations - Slow Transitions

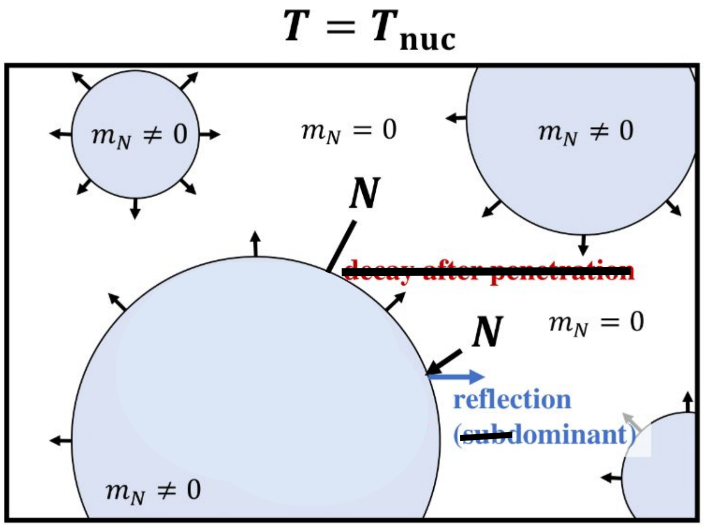


$\gamma_{\text{bubble}} \gg 1$

$\kappa_{\text{pen}} \simeq 1$

$m_N\Big|_{t=t_{\text{lepto}}} \neq 0$

Leptogenesis through sudden mass-gain decay.



$\gamma_{\text{bubble}} \simeq 1$

$\kappa_{\text{pen}} \to 0$

$m_N\Big|_{t=t_{\text{lepto}}} = 0$

Squeezed leptogenesis via CPV scatterings?

Hyun

Conclusions

- Bubble-assisted leptogenesis can allow for a strong departure from thermal equilibrium
  - Conventional washout can be fully suppressed
  - New channels ($NN \to \phi\phi$) become relevant
  - Dilution from reheating
    - Can enhance the final asymmetry sizeably (~20) for masses close to DI bound, but not maximally

- Enhancement cannot be arbitrary applied to other (low-scale) leptogenesis models
  - Strong annihilation for smaller masses
  - Enhancement disappears below $M_N \simeq 10^7\ \mathrm{GeV}$

- Are there different potentials beyond this toy model which can sizeably change these results?

- Works currently in progress: Is the kinematically inverse regime where RHNs are unable to penetrate barrier -> get squeezed into false-vacuum pockets a viable source of CP asymmetry?

Fin

Backup

Setup details

$$\mathcal{L} \supset \frac{1}{2}(Y_N)_{ij}\overline{N_i^c}\Phi N_j + (Y_D)_{\alpha i}\,\overline{\ell}_\alpha H N_i + h.c.$$

Assume a classically scale-invariant potential

$$V^{\text{tree}}(\Phi, \dots) \supset \lambda\,|\Phi|^4$$

which develops a flat direction at some scale $\lambda(\mu_*) = 0 \quad (\langle\Phi\rangle \sim \mu_*)$

Scale-invariance radiatively broken, naturally can generate a strong phase transition. Importantly, this predicts a light scalar.

Setup details

RHNs (fermions) destabilise the CW potential and are necessary for leptogenesis

Required to introduce new bosons for stability:

Consider two cases:

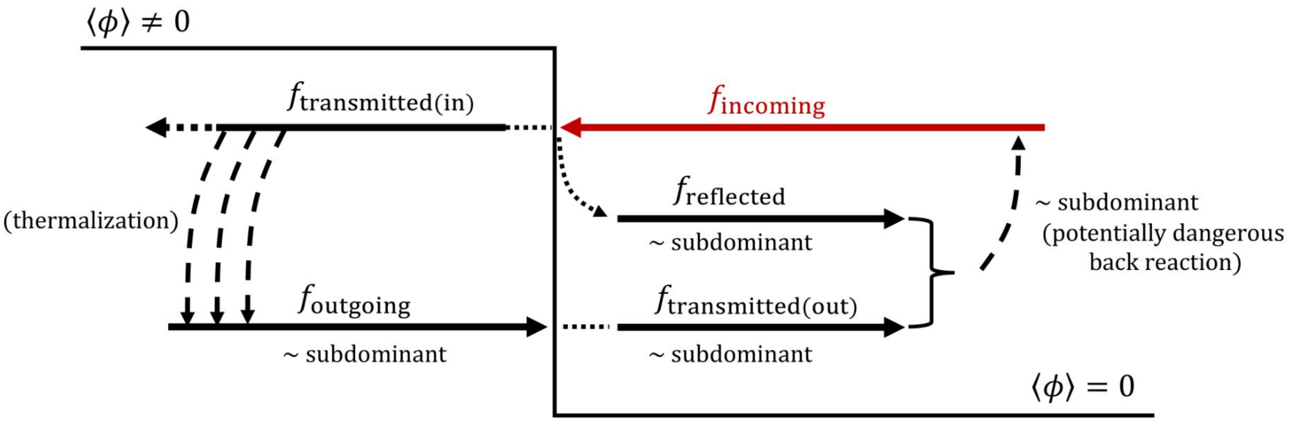
- **Gauged** $U(1)_{B-L}$ **(GC, Coleman-Weinberg):**

$$m_A(\phi) = 2g_{B-L}\phi$$

- **Introduce additional singlet scalar (SC, Gildener-Weinberg):**

$$\mathcal{L} \supset \frac{1}{2}\lambda_{s\phi}s^2 \left|\Phi\right|^2 \qquad m_s^2(\phi) = \lambda_{s\phi}\phi^2$$

RHN Penetration



$$\kappa_{\text{pen}} = \frac{\int_{p_z<-M_N} d^3p\, f_{\text{incoming}}}{\int_{p_z<0} d^3p\, f_{\text{incoming}}}$$

$$f_{\text{incoming,bubbleframe}} \simeq \frac{1}{e^{\gamma(E+vp_z)/T_{\text{nuc}}} \pm 1}$$

RHN Penetration

$$\mathcal{P}^i = \int \frac{d^3p}{(2\pi)^3}\,(\Delta p) f = \int \frac{dp_z\,dp_\perp\,2\pi p_\perp}{(2\pi)^3}\,(\Delta p) f$$

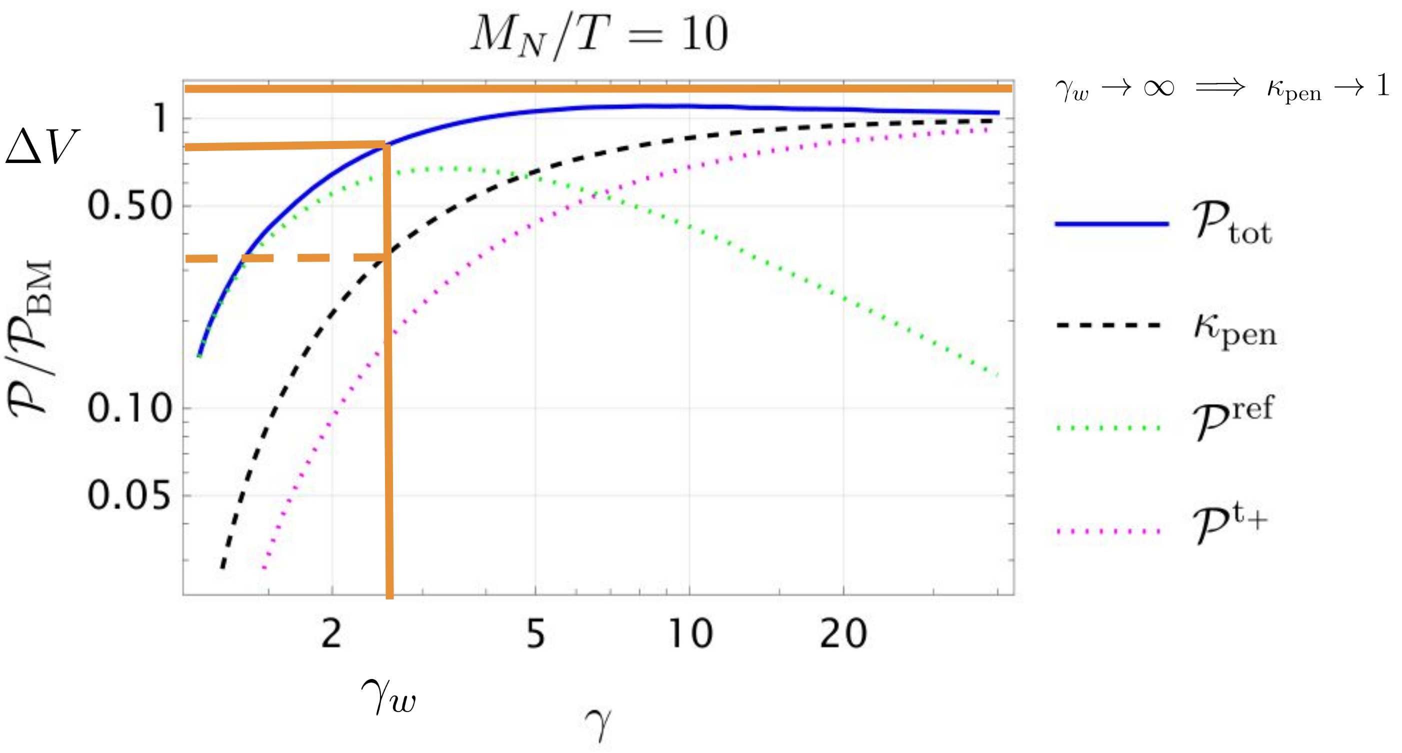
$$(\Delta p)_{\text{reflection}} = 2p_z$$

$$(\Delta p)_{\text{trans,incoming}} = p_z + \sqrt{p_z^2 - M_X^2}$$

$$(\Delta p)_{\text{trans,outgoing}} = p_z - \sqrt{p_z^2 - M_X^2}$$

$$\Delta V + \mathcal{P}(v_w) \geq 0$$

RHN Penetration



Washout

$$Y_D = i\frac{\sqrt{2}}{v_{\rm EW}}U^* m_n^{1/2} R^T M^{1/2}$$

$$RR^T = 1$$

$$\kappa_{\rm wash}\left(K \equiv \frac{\Gamma_{N_1}}{H(M_1)}\right)$$

$$\Gamma_{N_I} = \frac{1}{8\pi}(Y_D^\dagger Y_D)_{II} M_{N_I} = \frac{1}{4\pi v_{\rm EW}^2} M_{N_I}^2 \left(m_{\nu_1}\left|R_{I1}\right|^2 + m_{\nu_2}\left|R_{I2}\right|^2 + m_{\nu_3}\left|R_{I3}\right|^2\right)$$

Washout

$$Y_D = i\frac{\sqrt{2}}{v_{\mathrm{EW}}}U^* m_n^{1/2} R^T M^{1/2}$$

$$RR^T = 1$$

$$\kappa_{\mathrm{wash}}\left(K \equiv \frac{\Gamma_{N_1}}{H(M_1)}\right)$$

$$\Gamma_{N_I} = \frac{1}{8\pi}(Y_D^\dagger Y_D)_{II} M_{N_I} = \frac{1}{4\pi v_{\mathrm{EW}}^2} M_{N_I}^2 \left(m_{\nu_1}|R_{I1}|^2 + m_{\nu_2}|R_{I2}|^2 + m_{\nu_3}|R_{I3}|^2\right)$$

$$\Gamma_{N_1} \sim \frac{2M_1 m_\nu}{8\pi v_{\mathrm{EW}}^2} \simeq 65\left(\frac{M_1}{10^9\ \mathrm{GeV}}\right)^2\ \mathrm{GeV}$$

$$(m_\nu = m_{\mathrm{atm}})$$

Strong/Weak Washout

$$Y_D = i\frac{\sqrt{2}}{v_{\mathrm{EW}}}U^* m_n^{1/2} R^T M^{1/2} \qquad RR^T = 1$$

$$\left((Y_D)^\dagger Y_D\right)_{IJ} = \frac{2}{v_{\mathrm{EW}}^2}\sqrt{M_{N_I}}\sqrt{M_{N_J}}\sum_k m_{\nu_k} R_{jk} R_{ik}^*$$

$$\Gamma_{N_I} = \frac{1}{8\pi}(Y_D^\dagger Y_D)_{II} M_{N_I} = \frac{1}{4\pi v_{\mathrm{EW}}^2} M_{N_I}^2 \left(m_{\nu_1}\left|R_{I1}\right|^2 + m_{\nu_2}\left|R_{I2}\right|^2 + m_{\nu_3}\left|R_{I3}\right|^2\right)$$

To achieve weak washout in Type-I:

$$R_{I2}, R_{I3} \ll R_{I1} \qquad m_{\nu_1} \lesssim 10^{-3}\ \mathrm{eV}$$

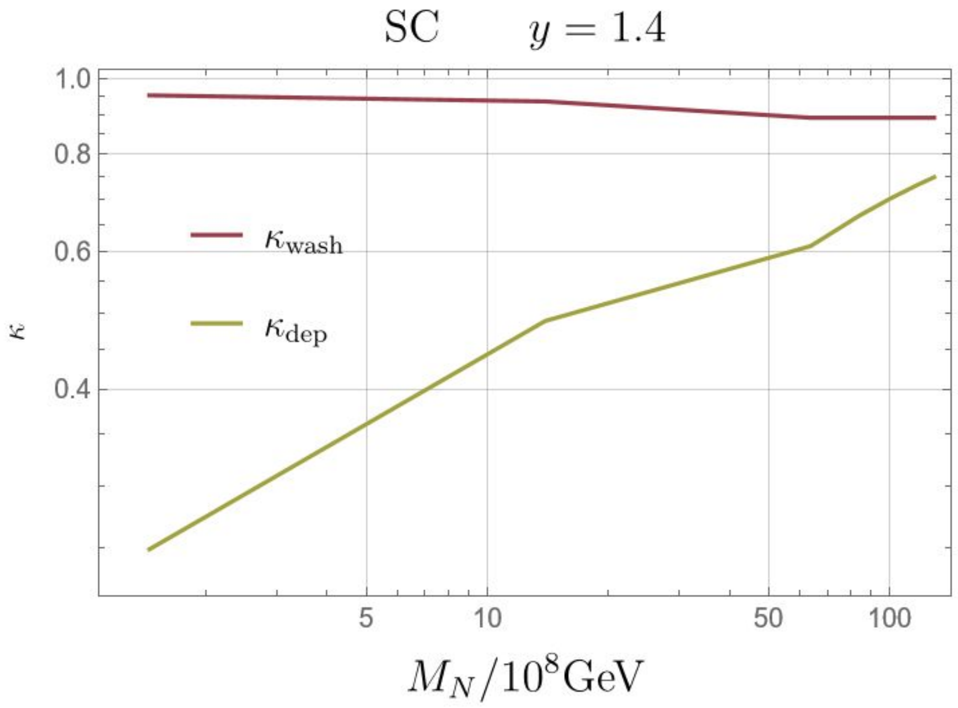
GW Benchmarks

	SC			GBC		
	$M_N/10^8\text{GeV}$	$\frac{n_B^{FOPT}}{n_B^{\text{thermal}}}$	α_n	$M_N/10^8\text{GeV}$	$\frac{n_B^{FOPT}}{n_B^{\text{thermal}}}$	α_n
P1	62	26	4.4	60	22	4.8
P2	34	21	6	37	17	5
P3	26	18	6.	25	15	9
P4	10	12	10	10	10	9.6
P5	4.2	8	25	4	5.6	15
P6	1.4	3.5	25	1.4	2.9	33

$$\Omega^0_{\mathrm{GW}} \sim 0.1 - 0.01$$

simulation of energy transmission during sound waves

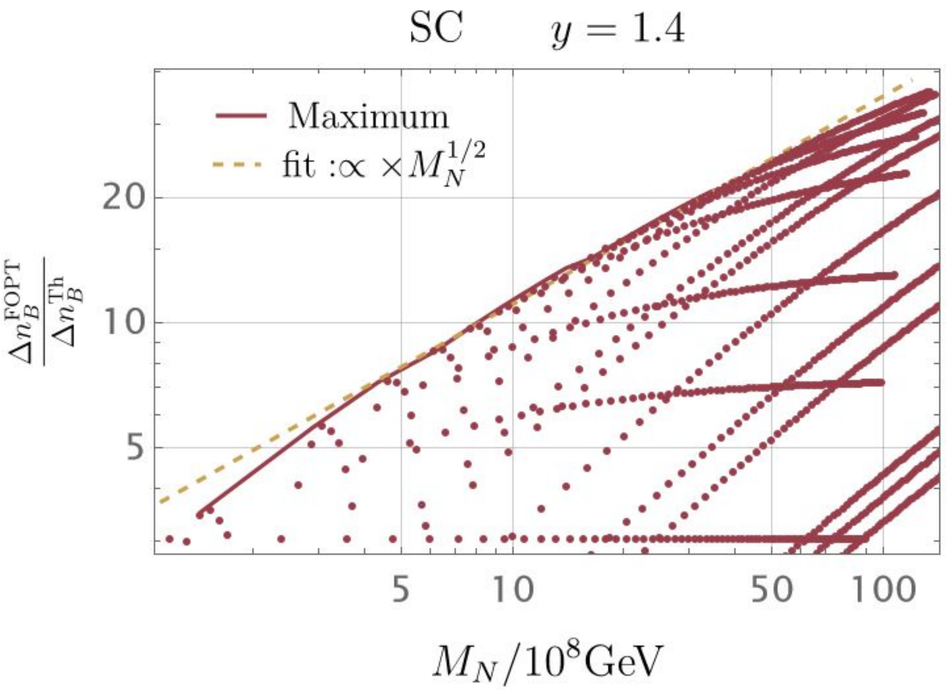
Lower-bound for bubble enhancement

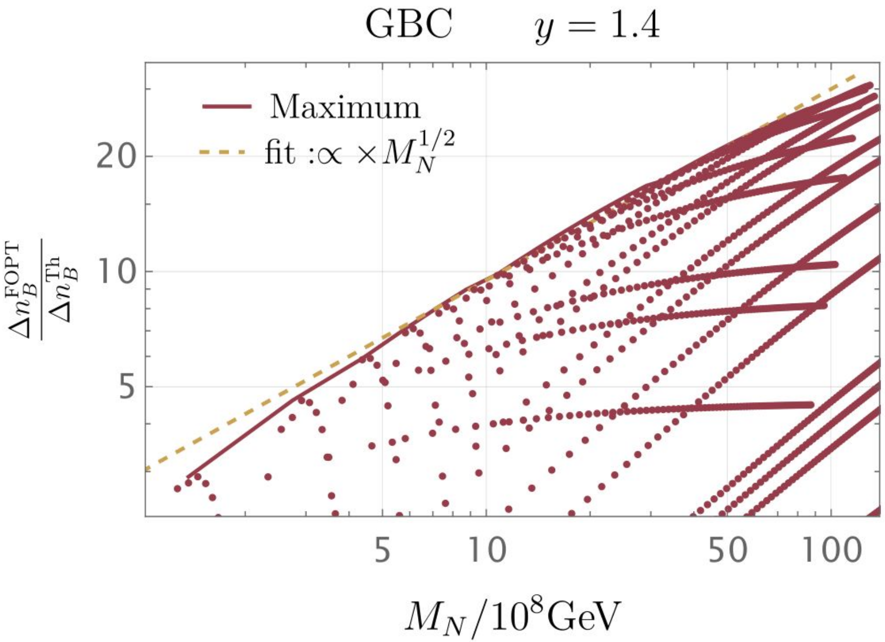


$M_N/T_{\rm nuc} = 5$

$$\Gamma_D \sim y_D^2 M_N \propto M_N^2 \qquad \Gamma_{NN\to\phi\phi} = \langle \sigma_{NN\to\phi\phi} v\rangle n_N \sim y^4 \left(\frac{T}{M_N}\right)^4 M_N \propto M_N$$

Bounded from below?





$$\frac{n_B^{\rm FOPT,max}}{n_B^{\rm thermal}} \sim \left(\frac{M_N}{10^7\mathrm{GeV}}\right)^{1/2}$$

Lower bound below which bubble-assisted dynamics generate a *suppression* of the final asymmetry

$$NN \to \phi\phi$$

BE details

$$zHs\,Y_N'(z) = -\bar{\gamma}_D \left(\frac{Y_N}{Y_N^{(\mathrm{eq})}} - 1 \right) - 2\gamma_{NN\to\phi\phi} \left(Y_N^2 - \left(Y_N^{(\mathrm{eq})} \right)^2 \right) + [NN \to ff]$$

$$zHs\,Y_{B-L}'(z) = -\epsilon_{\mathrm{CP}} \bar{\gamma}_D \left(\frac{Y_N}{Y_N^{(\mathrm{eq})}} - 1 \right) - \frac{1}{2}(c_L + c_H)\,\gamma_D \frac{Y_{B-L}}{Y^{(\mathrm{eq})}}$$

$$\bar{\gamma}_D \equiv \sum_I \gamma_D(N_I) = \sum_I n_{N_I}^{(\mathrm{eq})} \frac{K_I(z)}{K_2(z)} \Gamma_D(N_I)$$

$$\epsilon_{\mathrm{CP}}\,\bar{\gamma}_D \equiv \sum_I \epsilon_I \gamma_D(N_I)$$

$$\gamma_{NN\to\phi\phi} \equiv \frac{1}{9} s^2 \sum \langle \sigma v \rangle_{N_I N_I \to \phi\phi}$$

Temperature (GeV)	c_l	c_H	$c_H + c_L$
10^{11-12}	$\frac{6}{35}$	$\frac{95}{460}$	~ 0.38
10^{8-11}	$\frac{5}{53}$	$\frac{47}{358}$	~ 0.22
$\ll 10^8$	$\frac{7}{79}$	$\frac{8}{79}$	~ 0.19

BE details

$$n_{N_I}^{(0)} = \kappa_{\rm pen}\,\frac{2\cdot\frac{3}{4}\cdot\zeta(3)}{\pi^2}T_{\rm nuc}^3 \qquad z_{\rm col}\sim e^{H\Delta t_{\rm PT}}z_{\rm nuc}\sim 1.1 z_{\rm nuc}$$

Step (ii)

$$Y_N(z_{\rm nuc}) = \frac{3n_{N_I}^{(0)}}{s(T_{\rm nuc})},\quad Y_{B-L}(z_{\rm nuc}) = 0,\quad z_{\rm nuc} = \frac{M_N}{T_{\rm nuc}}$$

$$\tilde{Y}_N \equiv Y_N(z_{\rm col}),\quad \tilde{Y}_{B-L}\equiv Y_{B-L}(z_{\rm col})$$

Step (iii)

$$Y_N\left(z_{\rm reh}\right) = \tilde{Y}_N\left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3,\quad Y_{B-L}\left(z_{\rm reh}\right) = \tilde{Y}_{B-L}\left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3,\quad z_{\rm reh} = \frac{M_N}{T_{\rm reh}}$$

Thermal Potential

$$V(\phi, T) = V_0(\phi) + \sum_i V_{CW}(m_i^2(\phi) + \Pi_i) + \sum_i V_T(m_i^2(\phi) + \Pi_i)$$

$$V_{CW}(m_i^2(\phi)) = (-1)^{2s_i} g_i \frac{m_i^4(\phi)}{64\pi^2}\left[\log\left(\frac{m_i^2(\phi)}{\mu^2}\right) - c_i\right]$$

$$V_T(m_i^2(\phi)) = \pm \frac{g_i}{2\pi^2} T^4 J_{\mathrm{B,F}}\left(\frac{m_i^2(\phi)}{T^2}\right), \qquad J_{\mathrm{B,F}}(y^2) = \int_0^\infty dx\ x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right],$$

$$(\Delta t)^{-1}_{\mathrm{PT}} \sim -\frac{d(S_3/T)}{dt}\bigg|_{T=T^{\mathrm{nuc}}} \equiv H_{\mathrm{reh}}\beta_{\mathrm{PT}}$$

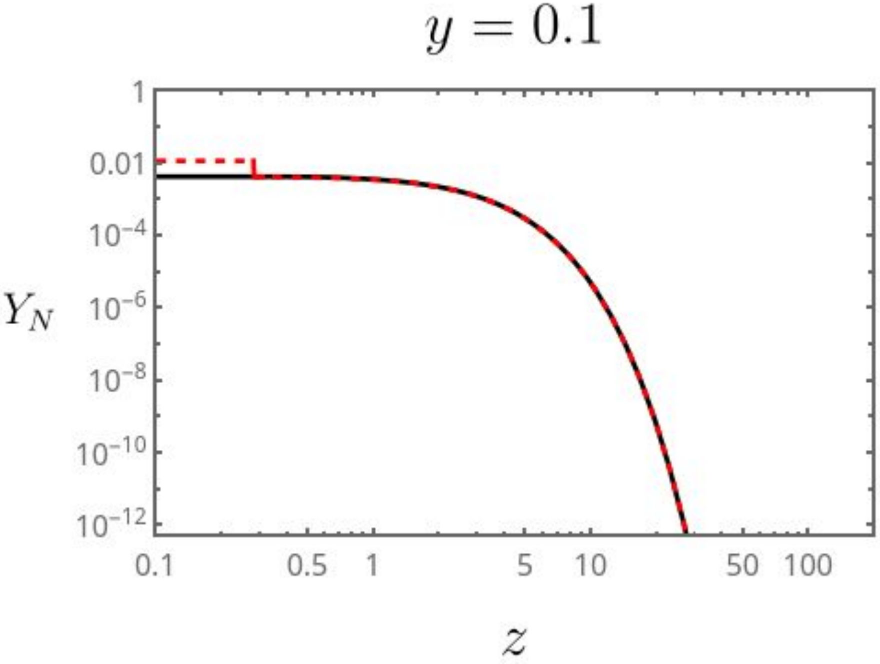
$$\rho(T_{\mathrm{reh}}) \simeq \rho(T_{\mathrm{nuc}}) + \Delta V$$

$$\Gamma_{\mathrm{nuc.-rate}}(T = T_{\mathrm{nuc}}) = H(T_{\mathrm{nuc}})^4$$

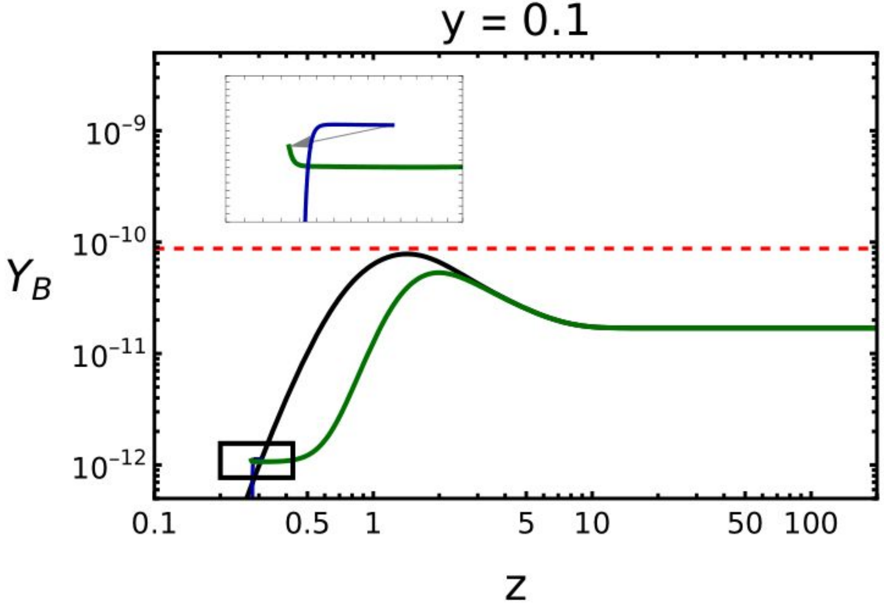
$$\alpha_n \equiv \frac{\Delta V}{\rho(T_{\mathrm{nuc}})}$$

$$\left(\frac{T_{\mathrm{nuc}}}{T_{\mathrm{reh}}}\right)^3 \simeq (1+\alpha_n)^{-3/4}$$

BEs

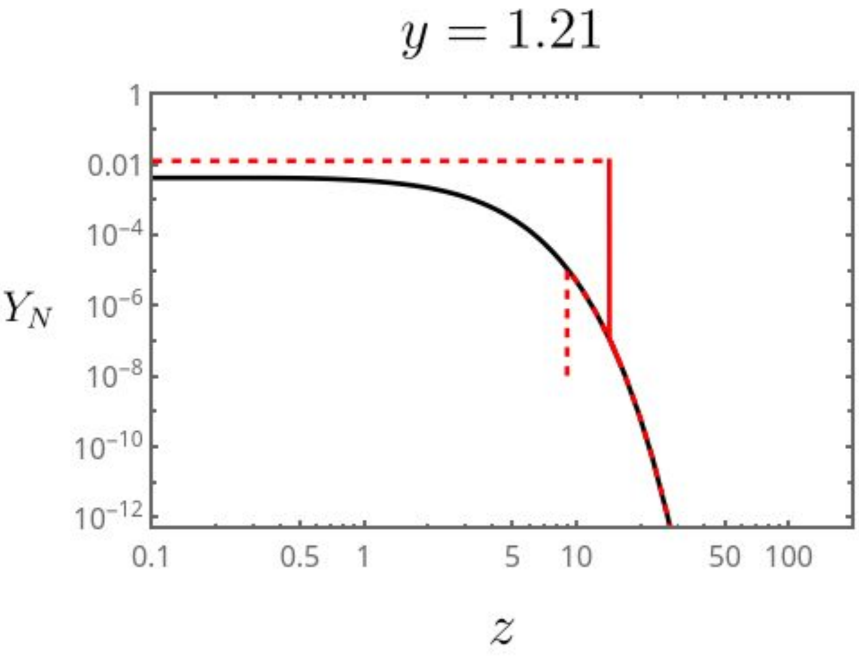


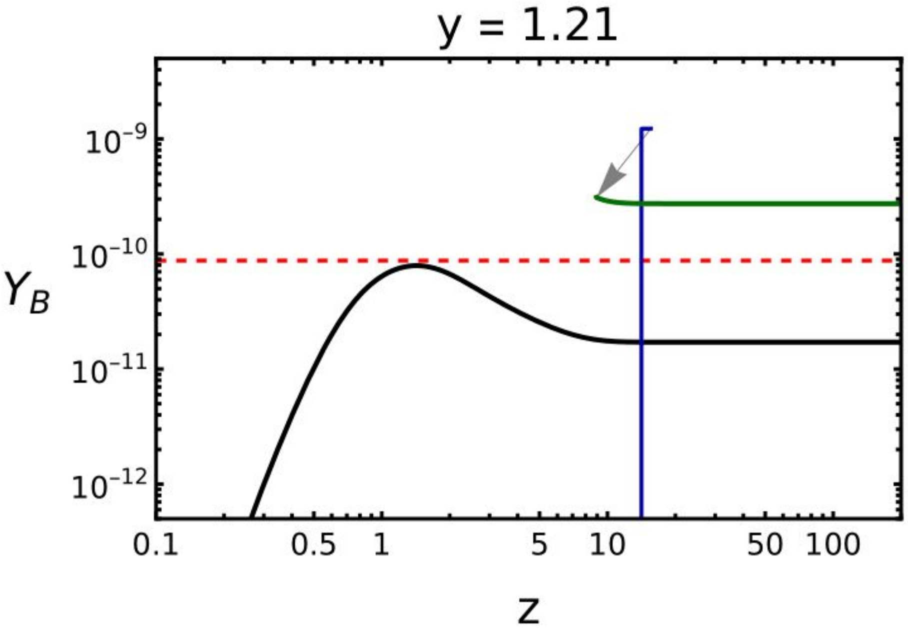
$y = 0.1$
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Y_N
0.1
0.5
1
5
10
50
100
z



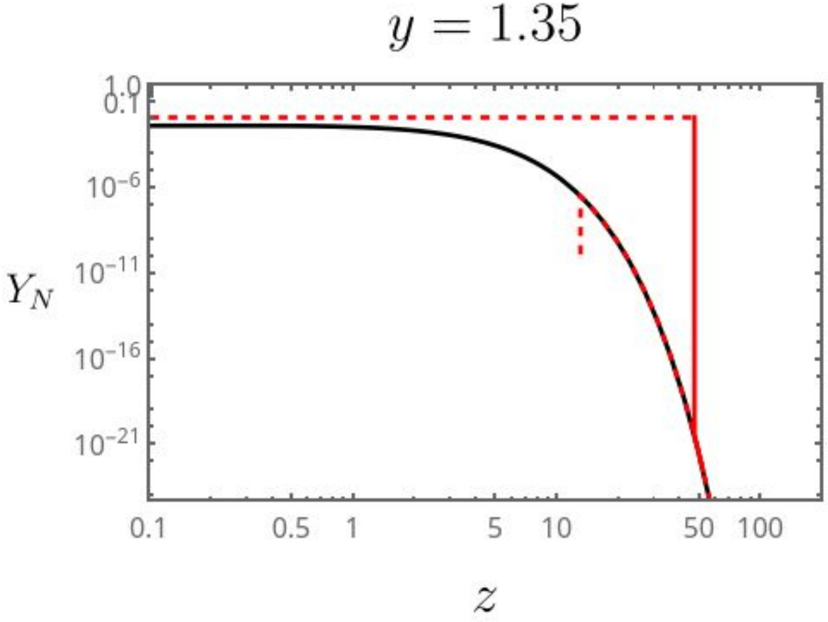
y = 0.1
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Y_B
0.1
0.5
1
5
10
50
100
z

BEs

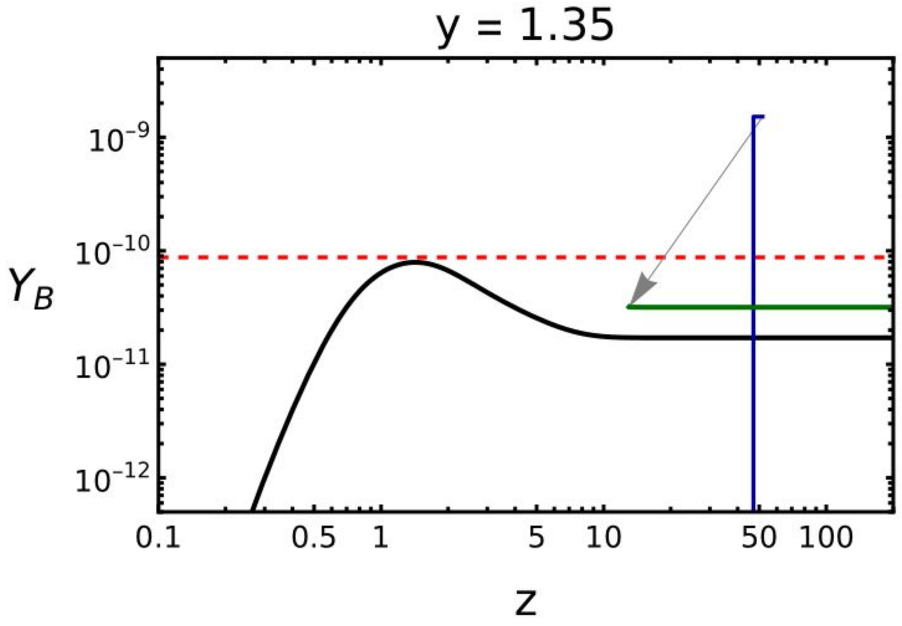

$y = 1.21$
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Y_N
0.1
0.5
1
5
10
50
100
z


y = 1.21
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Y_B
0.1
0.5
1
5
10
50
100
z

BEs



$y = 1.35$
1.0
0.1
10^{-6}
10^{-11}
10^{-16}
10^{-21}
Y_N
0.1
0.5
1
5
10
50
100
z



y = 1.35
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Y_B
0.1
0.5
1
5
10
50
100
z