Decay rates of heavy neutrinos in the Grimus-Neufeld model

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outline

- the Grimus-Neufeld model (GNM)
  $=$ Standard Model (SM) $+$ one fermionic singlet $+$ two Higgs doublets
  not new: [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.
- the tiny seesaw scenario
- decays and rates

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Motivation for the Grimus-Neufeld model (GNM)

[G-N] p.15: *"As a concluding remark we want to point out the simplicity of the basic assumptions in our model and we stress once more that no exotic particles are needed in this scheme to generate naturally light neutrino masses. It is rather the interplay between two mechanisms, the seesaw mechanism and radiative mass production, which leads to this result."*

- the GNM is a minimal extension of the SM
  - that can accomodate the measured neutrino masses and mixings

    S. Draukšas, doi:10.15388/vu.thesis.614

    V. Dūdėnas and T. Gajdosik, Acta Phys. Polon. Supp. **15** (2022) no.2, 1

    S. Draukšas, *et al.*, Symmetry **11** (2019) no.11, 1418 doi:10.3390/sym11111418

    V. Dūdėnas, *et al.*, Acta Phys. Polon. B **48** (2017), 2235

  = general Two Higgs Doublet model + a single Majorana neutrino

- particle content:
  - gauge bosons and charged fermions $=$ the SM
  - 4 neutral Majorana fermions $\Leftrightarrow$ 3 SM neutrinos
  - 1 charged + 3 neutral Higgs bosons $\Leftrightarrow$ 1 SM Higgs boson

The GNM Lagrangian

- Gauge sector $\mathcal{L}_{\mathsf{G}}$ and Fermion-Gauge sector of the SM:
  - gauge group $U(1)_{\mathsf{Y}} \otimes SU(2)_{\mathsf{L}} \otimes SU(3)_{\mathsf{color}}$
  - gauge covariant derivative $D_\mu \psi$
  - and the Lagrangian $\quad \mathcal{L}_{\mathsf{G\text{-}F}} = \sum_\psi \bar{\psi}\, i \not{D}\, \psi \qquad (1)$

- Gauge-Higgs sector with the gauge covariant derivative $D_\mu \phi_a$
  and the Lagrangian $\quad \mathcal{L}_{\mathsf{G\text{-}H}} = (D^\mu \phi_a)^\dagger (D_\mu \phi_a) - V(\phi_a) \qquad (2)$

- Higgs sector: two Higgs doublets ϕ_a in the Higgs potential $V(\phi_a)$
  [H-ON] H. E. Haber and D. O'Neil, Phys. Rev. D **83** (2011) 055017 [arXiv:1011.6188 [hep-ph]]

- Fermion-Higgs sector with the Yukawa couplings (ignoring quarks)

$$\mathcal{L}_{\mathsf{F\text{-}H}} \quad = \quad -\bar{\ell}^0_{Lj}\, \phi_a\, Y^{\bar{a}}_{Ljk}\, e^0_{Rk} - \bar{\ell}^0_{Lj}\, \tilde{\phi}_{\bar{a}}\, \widetilde{Y}^a_{Lj} N^0 + h.c. \qquad (3)$$

with the adjoint Higgs doublet $\tilde{\phi}_{\bar{a}} = \epsilon \phi^*_a = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} (\phi^+_a)^* \\ (\phi^0_a)^* \end{pmatrix} =: \begin{pmatrix} \phi^{0*}_{\bar{a}} \\ -\phi^-_{\bar{a}} \end{pmatrix}$

- Majorana sector with the Majorana singlet N^0: $D_\mu N^0 = \partial_\mu N^0$

The bare GNM has parameters additionally to the "original" SM

- the (complex) singlet Majorana mass term M_R
- parameters in the Higgs sector – like a general 2HDM [H-ON]
  - using the Higgs basis fixes where the vev sits (ϕ_1)
    - * distinguishes the neutrino couplings between seesaw (ϕ_1) / loop (ϕ_2)
- the neutrino Yukawa coupling of the first Higgs doublet ϕ_1

  $(Y_N^{(1)})_j := \widetilde{Y}^1_{Lj} = \frac{\sqrt{2}}{v}(M_D)_j$ ... the "Dirac mass" term

  - is responsible for the seesaw : $y^2 = \sum_j |(Y_N^{(1)})_j|^2 = \frac{2 m_s m_4}{v^2}$ (4)
- the Yukawa couplings of the second Higgs doublet ϕ_2

  $(Y_N^{(2)})_j := \widetilde{Y}^2_{Lj}$ to lepton doublets and neutral fermionic singlet N^0

  - is essential for the loop mass $\Rightarrow$ we have a general 2HDM

  $(Y_E^{(2)})_{jk} := Y^2_{Ljk}$ to lepton doublets and charged lepton singlets ℓ_{Rk}

  - is not restricted by neutrino data at one loop

- relates M_R and $(Y_N^{(1)})_k = i\,y\,u_{ks} \quad (Y_N^{(2)})_k := d\,u_{kt} + i\,d'u_{ks}$ (5)

  to PMNS and $\widehat{m}$: $U_{(\nu)}{}^\dagger M_\nu^F = \mathrm{diag}(m_o, m_t, m_s, m_4) U_{(\nu)}{}^\top =: \widehat{m} U_{(\nu)}{}^\top$ (6)

- the massless state $\nu''_o = \nu'_o$ with mass $m_o = 0$ is untouched
  - since it does not couple to any Higgs
- the tree level states $\nu'_{t,s}$ mix into one-loop mass eigenstates $\nu''_{2,3}$
  - m_2 and m_3 can be determined from measured mass differences . . . see (7), next slide
- the transformation chain: [DGKKS2022] V. Dūdėnas *et al.*, JHEP **09** (2022), 174

flavor eigenstate mass state

$$\begin{pmatrix} 0^{0\ell}_{3\times3} & \frac{v}{\sqrt{2}} Y_N^{(1)} \\ \frac{v}{\sqrt{2}} Y_N^{(1)\top} & M_R \end{pmatrix} \xrightarrow{\tilde{V}} \begin{pmatrix} 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & 0^{1\ell} \\ 0^{1\ell} & 0^{0\ell} & 0^{0\ell} & 0^{0\ell} \\ 0^{1\ell} & 0^{0\ell} & 0^{0\ell} & i\frac{vy}{\sqrt{2}} \\ 0^{1\ell} & 0^{0\ell} & i\frac{vy}{\sqrt{2}} & M_R \end{pmatrix} \xrightarrow{\tilde{S}} \begin{pmatrix} 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & 0^{1\ell} \\ 0^{1\ell} & \multicolumn{2}{c}{\hat{\Sigma}} & 0^{1\ell} \\ 0^{1\ell} & & & 0^{1\ell} \\ 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & m_4 + 0^{1\ell} \end{pmatrix} \xrightarrow{\tilde{R}} \hat{m}$$

$\Vert$			$\Vert$
M_ν^F			$\tilde{U}^* M_\nu^F \tilde{U}^\dagger$
$\nu_\alpha := \{\nu_i^0, N^0\}$	ν'_α	$\approx \nu'_\alpha$	ν''_α
$Y^{(i)}$	$Y^{(i\prime)}$	$\approx Y^{(i\prime)}$	$Y^{(i\prime\prime)}$

  - allows a convenient parametrization of the GNM

the tiny seesaw

- the physical light masses (of neutral fermions) are determined:

$$m_o = m_1 = 0 \;, \quad m_2 = \sqrt{\Delta m^2_{\mathrm{sol}}} \;, \quad m_3 = \sqrt{\Delta m^2_{\mathrm{atm}}} \tag{7}$$

  - but $m_4 \sim |M_R|$ is a free parameter
  - implementing this model in FlexibleSUSY exhibits an instability:
    * for stable loop level Higgs masses we are limited to $m_4 < 10^6\mathrm{GeV}$

- using the seesaw relation $$y^2 = \sum_j |(Y_N^{(1)})_j|^2 = \frac{2 m_s m_4}{v^2} \tag{4}$$
  - we see, that y becomes a small parameter !

$\Rightarrow$ motivates the definition of the tiny seesaw scenario $$y \le 10^{-7} \tag{8}$$

sidestep: what happens when $y \to 0$ (i.e. $(Y_N^{(1)})_j \to 0$) ?

- $\mathcal{L}_{\mathrm{GNM}}$ gains an additional Z_2 symmetry : $$\phi_2 \leftrightarrow -\phi_2 \,, \quad N_R \leftrightarrow -N_R \tag{9}$$

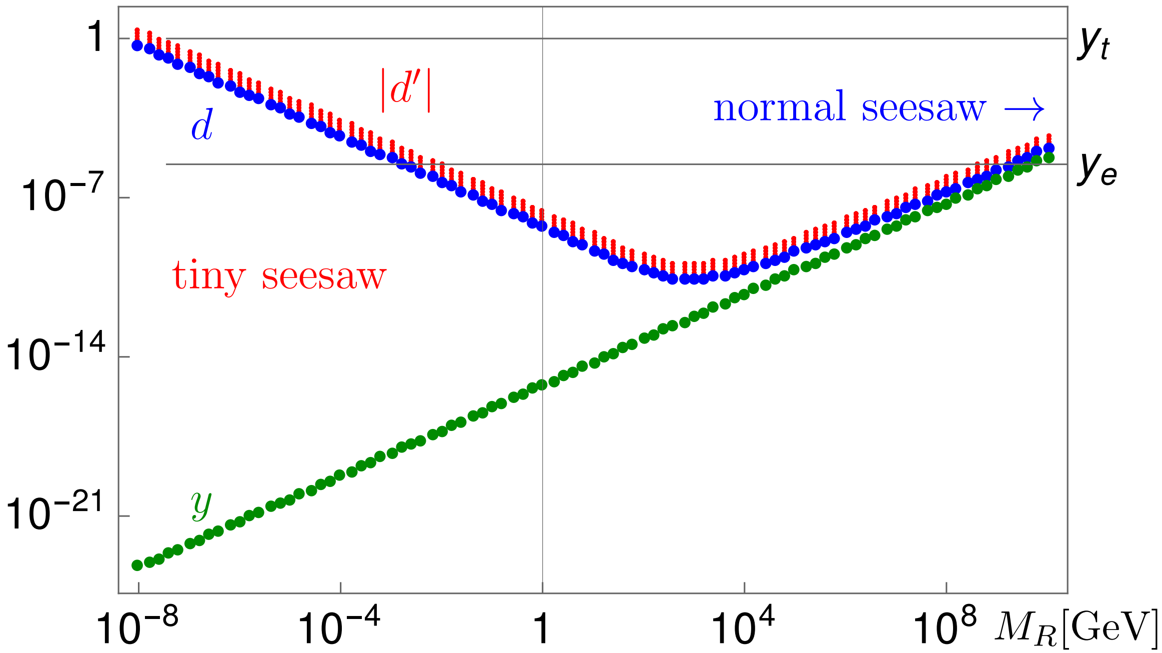
$\Rightarrow$ the tiny seesaw scenario has an approximate Z_2 symmetry
  - "technical" naturalness

features of the tiny seesaw scenario

- the seesaw scale becomes smaller than the EW scale : $m_4 < v$ (10)
- the loop inducing coupling $(Y_N^{(2)})_k = d\, u_{kt} + i\, d' u_{ks}$ becomes large (11)
  - determined by $m_2 m_3 = m_t m_s = \det[\hat{\Sigma}] = d^2 m_3^{\text{tree}} \Lambda$ (12)

    with the loop function of the neutral Higgses

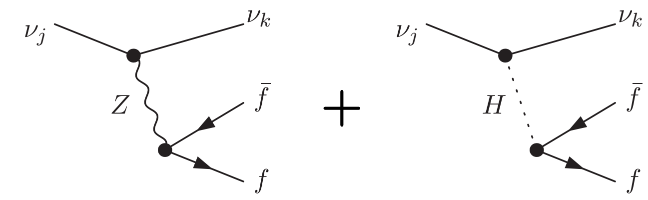
$$\Lambda = \frac{m_4}{32\pi^2}[B_0(0, m_4^2, m_{A^0}^2) - B_0(0, m_4^2, m_{H^0}^2)] \propto \frac{m_4}{32\pi^2}\lambda_5 \qquad (13)$$



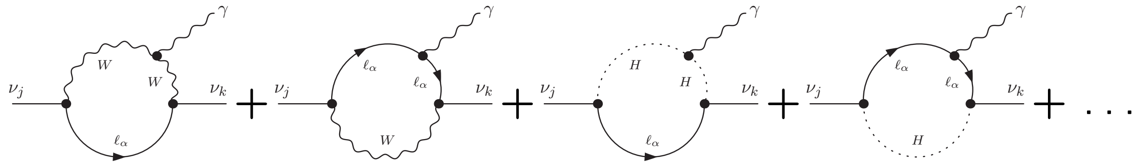
The Yukawa coupling parameters y, d, and $|d'|$ in dependence on the seesaw scale M_R for the benchmark point B1 of B. Hespel, *et al.*, JHEP **09** (2014), 124, i.e. for Higgs masses $m_{H^0} = 300\,\text{GeV}$, $m_{A^0} = 441\,\text{GeV}$, and $m_{H^\pm} = 442\,\text{GeV}$. The range for $|d'|$ comes from varying its phase $\phi' = \arg[d']$.

decays of the heavier neutrinos: diagrams

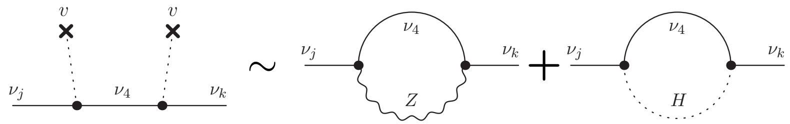
- the GNM has no symmetry that prevents decays $\Rightarrow$ they happen
  - some decays happen already in the SM
  - rates depend on couplings and phasespace
    - $\nu_j \to \nu_k + \bar{f} + f$ allowed iff $m_j > m_k + m_{\bar{f}} + m_f$, requires mixing of νs



    - $\nu_j \to \nu_k + \gamma$ allowed iff $m_j > m_k$, requires loop



    - the GNM is built on: "size(seesaw) $\sim$ size(loop)"



- in the tiny seesaw we assume $m_4 \ll m_Z < m_h$ . . . and ignore $f = e^-$
  - treelevel boson propagators become constants
  - the treelevel decays have only neutrinos in the final state
    - simplification for kinematics: $m_{\nu_{\text{final}}} \to 0$

decays of the heavier neutrinos: rates

- tree- and loop-level decays have different final state
  - for the overview we analyse the tree-level amplitudes Γ_Z, Γ_{H_1}, and Γ_{H_2} separately:

$$\Gamma_Z = \frac{G_F^2}{96\pi^3} s_\theta^2 c_\theta^2 m_4^5 \sim \frac{g^4}{m_Z^4} \frac{m_3 m_4}{(m_4 + m_3)^2} m_4^5 \sim m_4^4 \tag{14}$$

$$\Gamma_{H_1} \simeq \frac{y^4 (c_\theta^2 - s_\theta^2)}{24576\pi^3 m_{H_1}^4} s_\theta^2 c_\theta^2 m_4^5 \sim \frac{m_3^2 m_4^2}{m_W^4 m_h^4} \frac{(m_4 - m_3) m_3 m_4}{(m_4 + m_3)^4} m_4^5 \sim m_4^5 \tag{15}$$

$$\Gamma_{H_2} \simeq \frac{30 d^4 + 15 d^2 |d'|^2 + 8 |d'|^4}{12288\pi^3 m_{H_2}^4} s_\theta^2 m_4^5 \sim \frac{m_2^2/m_4^2}{m_{H^0}^4} \frac{m_3}{m_4 + m_3} m_4^5 \sim m_4^2 \tag{16}$$

  - rates scale differently with the seesaw scale m_4

- the SM-like loop contribution $\quad \Gamma_W = 27 \frac{\alpha_{\text{em}}}{4\pi} \frac{G_F^2}{96\pi^3} s_\theta^2 c_\theta^2 m_4^5 \ll \Gamma_Z \quad (17)$

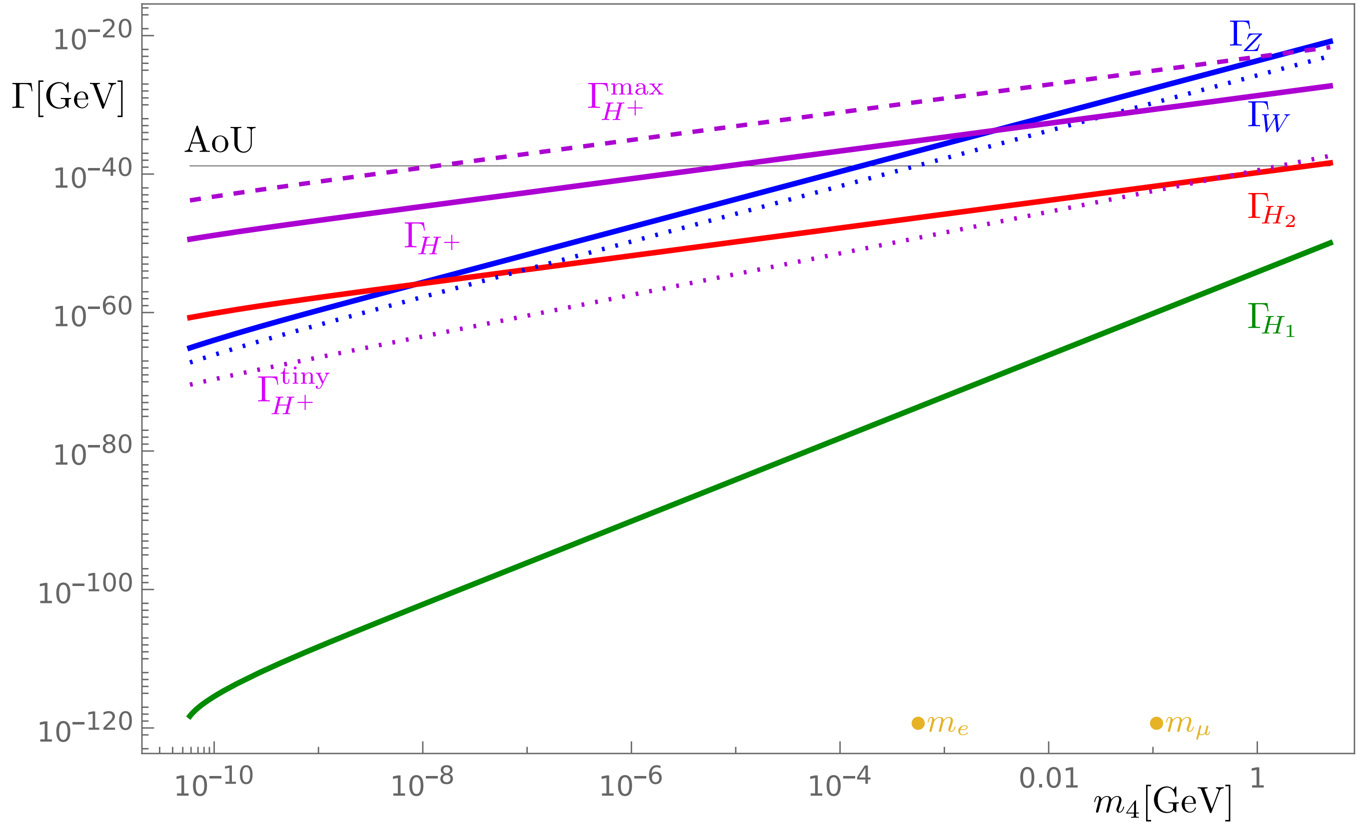
- the H^+ loop gives $\quad \Gamma_{H+} \simeq \frac{\alpha_{\text{em}}}{4\pi} \frac{d^2 + |d'|^2}{256\pi^3 m_{H+}^4} c_\theta^2 |Y_E^{(2)}|^2 m_{\ell-}^2 m_4^3 \sim m_4^2 \quad (18)$
  - no seesaw suppression $s_\theta^2 = m_3/(m_4 + m_3)$
  - and "unrestricted" Yukawa coupling $Y_E^{(2)} < 2\pi$

decays of the heavier neutrinos: rates

Rates using the benchmark point B1, i.e. Higgs masses $m_{H^0} = 300\,\mathrm{GeV}$, $m_{A^0} = 441\,\mathrm{GeV}$, and $m_{H^\pm} = 442\,\mathrm{GeV}$. AoU describes the rate where the neutrino has a lifetime as long as the age of the universe.

For describing the Yukawa coupling $Y_N^{(2)}$ were taken $\omega_{22} = \pi/3$ and $r = \pi/6$ as representative model parameters. For definitions see V. Dūdėnas *et al.*, JHEP **09** (2022), 174 .

For the charged lepton Yukawa coupling $Y_E^{(2)}$ we took as maximal value 2π ($\Gamma_{H^+}^{\max}$), as "normal" value 10^{-2} (Γ_{H^+}), and as tiny value y ($\Gamma_{H^+}^{\text{tiny}}$).



- the loop decay with H^+ dominates in the tiny seesaw regime
  - as long as $Y_E^{(2)}$ does not vanish
- Γ_Z still gives a good lifetime estimate for larger masses m_4
  - when the lifetime becomes shorter than the age of the universe (AoU)

Summary of the GNM

- the GNM extends the SM with a Higgs doublet and a Majorana singlet
  - the neutrinos become Majorana particles
    - the lightest neutrino stays mass-less at one loop
  - neutrino oscillations determine the neutrino Yukawa couplings $Y_N^{(i)}$
    - allows predictions of Lepton Flavor violating processes

- An approximate Z_2 symmetry defines the tiny seesaw scenario
  - motivates suppression of the charged lepton Yukawa coupling $Y_E^{(2)}$
  - stabilizes the numerical renormalization in FlexibleSUSY
  - y and λ_5 interpolate between seesaw and radiative neutrino masses

  $\Rightarrow$ the GNM can be seen as generalization of Dark matter models
    - in terms of predicting Lepton Flavor violating processes

- loop decays of neutrinos are important in the tiny seesaw scenario
  - but still leave the heavy neutrino "stable" (compared to AoU)

Thank you

for discussion

and comments

and of course for the conference! ☺

Backup slides

the normal seesaw can be understood from the diagonalization of

$$A = \begin{pmatrix} 0 & iy \\ iy & M \end{pmatrix} = S \begin{pmatrix} m_3 & 0 \\ 0 & m_4 \end{pmatrix} S^\top = S\widehat{m}S^\top \tag{19}$$

- with $$\det[A] = y^2 = m_3 m_4 \quad \text{and} \quad \mathrm{Tr}[A] = M = m_3 + m_4 \tag{20}$$
- and $$S = \left(\begin{smallmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{smallmatrix} \right) \quad , \text{ where } s_\theta^2 = \frac{m_3}{m_3 + m_4} \ll 1 \tag{21}$$

the matrix chain, slide 5, $V \to S \to R$ gives the steps

- flavor $\to$ treelevel $\to$ seesaw $\to$ one loop mass eigenstates

$\Rightarrow$ allows definition of mass eigenstate Yukawa couplings

$$Y_N^{(1)} = \frac{iy|z|^{-1/2}}{e^{i\phi_R}}(0, -e^{i\omega_{32}} \sin r, e^{i\omega_{22}} \cos r) \tag{22}$$

$$Y_N^{(2)} = \mathrm{sign}(\Lambda)\sqrt{\frac{m_2}{|z\,\Lambda|}}(0, e^{i\omega_{22}} \cos r, \tfrac{m_3}{m_2} e^{i\omega_{32}} \sin r) \tag{23}$$

where $$z = e^{2i\omega_{22}} \cos^2 r + \tfrac{m_3}{m_2} e^{2i\omega_{32}} \sin^2 r = \frac{m_3}{m_3^{\mathrm{tree}}} \tag{24}$$

defines ω_{32} in terms of the other parameters.

- Λ is given by the Higgs loop, eq. (13).

$$\begin{array}{ccccccc}
\text{flavor eigenstate} & & & & & & \text{mass state} \\
\begin{pmatrix} 0^{0\ell}_{3\times3} & \frac{v}{\sqrt{2}} Y_N^{(1)} \\ \frac{v}{\sqrt{2}} Y_N^{(1)\top} & M_R \end{pmatrix} & \xrightarrow{\tilde{V}} & \begin{pmatrix} 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & 0^{1\ell} \\ 0^{1\ell} & 0^{0\ell} & 0^{0\ell} & 0^{0\ell} \\ 0^{1\ell} & 0^{0\ell} & 0^{0\ell} & i\frac{vy}{\sqrt{2}} \\ 0^{1\ell} & 0^{0\ell} & i\frac{vy}{\sqrt{2}} & M_R \end{pmatrix} & \xrightarrow{\tilde{S}} & \begin{pmatrix} 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & 0^{1\ell} \\ 0^{1\ell} & & & 0^{1\ell} \\ 0^{1\ell} & & \hat{\Sigma} & 0^{1\ell} \\ 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & m_4 + 0^{1\ell} \end{pmatrix} & \xrightarrow{\tilde{R}} & \hat{m} \\
\| & & & & & & \| \\
M_\nu^F & & & & & & \tilde{U}^* M_\nu^F \tilde{U}^\dagger \\
\nu_\alpha := \{\nu_i^0, N^0\} & & \nu'_\alpha & & \approx \nu'_\alpha & & \nu''_\alpha \\
Y^{(i)} & & Y^{(i\prime)} & & \approx Y^{(i\prime)} & & Y^{(i\prime\prime)}
\end{array}$$