Confronting neutrino mixing-schemes with correlations of neutrino oscillation data

Szymon Zięba, University of Silesia

arXiv:2310.20681 PPNP (2024), arXiv:2403.13966 APPB (2024),

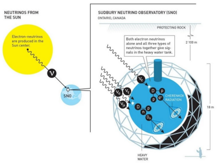
arXiv:25xx.xxxxx.
work in progress
with Janusz Gluza, Biswajit Karmakar

Matter To The Deepest 2025

Katowice, September 18, 2025

NATIONAL SCIENCE CENTRE POLAND

3ν oscillations and masses, $m_0 = m_{lightest}$



Sudbury Neutrino Observatory detects neutrinos from the Sun, where only electron-neutrinos are produced. The reactions between neutrinos and the heavy water in the tank yielded the possibility to measure both electron-neutrinos and all three types of neutrinos combined. It was discovered that the electron-neutrinos were fewer than expected, while the total number of all three types of neutrinos combined still corresponded to expectations. The conclusion was that some of the electron-neutrinos had changed into another identity.

To cite this section
MLA style: Popular information. NobelPrize.org. Nobel Prize Outreach AB 2024. Sun. 15 Sep 2024. <https://www.nobelprize.org/prizes/physics/2015/popular-information/>

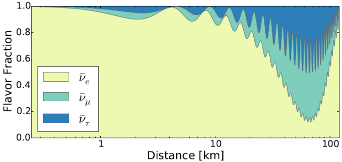
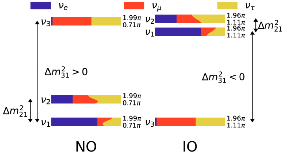


FIG. 1. **Illustration of neutrino oscillations.** The expected flavor composition of the reactor neutrino flux, for neutrinos of 4 MeV energy used as an example, is plotted as a function of distance to the reactor cores. arXiv:1503.01059



Taken from https://globalfit.astroparticles.es, updated arXiv:1806.11051, Figure 1

Normal mass ordering (NO)

$$
\begin{aligned}
m_1 &= m_0\,,\\
m_2 &= \sqrt{m_0^2 + \Delta m^2_{21}}\,,\\
m_3 &= \sqrt{m_0^2 + \Delta m^2_{31}}\,,
\end{aligned}
$$

Inverted mass ordering (IO)

$$
\begin{aligned}
m_1 &= \sqrt{m_0^2 - \Delta m^2_{21} - \Delta m^2_{32}}\,,\\
m_2 &= \sqrt{m_0^2 - \Delta m^2_{32}}\,,\\
m_3 &= m_0\,,
\end{aligned}
$$

$\Delta m^2_{21} > 0$ $\quad$ $|\Delta m^2_{31(32)}| > 0$ $\quad$ NO or IO ?

$\Sigma m_i < 0.12$ eV at 95% CL from Planck (CMB)

3ν mixing

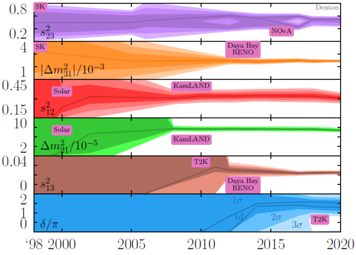
Neutrino flavor and mass eigenstates are related by

$$|\nu_\alpha\rangle = \mathrm{U}_{\alpha i}\,|\nu_i\rangle$$

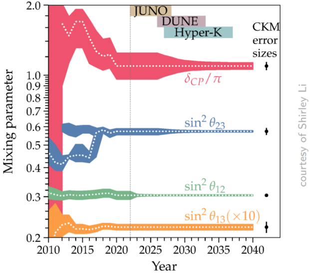
Pontecorvo–Maki–Nakagawa–Sakata parametrization of mixing matrix

$$\mathrm{U}_{\mathrm{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\delta \equiv \delta_{\mathrm{CP}}$.



Peter B. Denton talk, link here.



arXiv:2204.08668, Figure 2.1

Dirac or Majorana ? $(3+n)\nu$, n=0 ?

NO or IO ? $m_{lightest} \neq 0$?

flavor symmetry ? mixing scheme ?

θ_{23} octant ?

$\delta_{\mathrm{CP}} \neq 0$?

$\Sigma m_i < 0.12$ eV $\leftarrow$ this we know from Planck at 95% CL.

$\theta_{13} \neq 0$ $\leftarrow$ this we know from reactor experiments, e.g. Daya Bay.

Experimental data indicate that $\delta_{\mathrm{CP}} \neq 0$ and NO are favored.

$\theta_{13} \neq 0$, Daya Bay, RENO (2012)

BM, TB, GR, HG disfavored by non-zero θ_{13}.

$$\mathrm{U}_{\mathrm{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

$$\theta_{13} \neq 0^\circ \qquad \Downarrow \qquad \theta_{23} = 45^\circ$$

$$\mathrm{U}_0 = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Downarrow$$

$\theta_{12} = 45^\circ$, $s_{12} = 1/\sqrt{2}$ — Bimaximal Mixing

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta_{12} = 35,26^\circ$, $s_{12} = 1/\sqrt{3}$ — Tribimaximal Mixing

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta_{12} = 31,7^\circ$ — Golden Ratio Mixing

$$\begin{bmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta_{12} = 30^\circ$, $s_{12} = 1/2$ — Hexagonal Mixing

$$\begin{bmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Golden Ratio Mixing: $\mathrm{tg}\,\theta_{12} = 1/\varphi$, $\varphi = (1+\sqrt{5})/2$ being the golden ratio.

Non-zero θ_{13}: Successors of tribimaximal mixing

$$\mathrm{U_{TBM}} = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \qquad \mathrm{U_{PMNS}} \simeq \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0.15 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix},$$

```
In[688]:= Sin[8.6*Pi / 180]
          Sin[8.6*Pi / 180] ^2
Out[688]= 0.149535
Out[689]= 0.0223608
```

$$|\mathrm{U_{TM_1}}| = \begin{bmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{bmatrix}, \qquad |\mathrm{U_{TM_2}}| = \begin{bmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{bmatrix},$$

$$\mathrm{U_{TM_1}} = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{bmatrix},$$

$$\mathrm{U_{TM_2}} = \begin{bmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{bmatrix}.$$

Mixing-schemes structure: TM_1 and TM_2, partial μ-τ

Flavor and mass mixing:

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}.$$

TM_1 and TM_2 mixing-schemes structure

$$|\mathrm{U}_{\mathrm{TM}_1}| = \begin{bmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{bmatrix}, \quad |\mathrm{U}_{\mathrm{TM}_2}| = \begin{bmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{bmatrix}.$$

Partial μ-τ ($\mu 1$-$\tau 1$, $\mu 2$-$\tau 2$) reflection symmetry mixing structure:

$$|U_{\mu 1}| = |U_{\tau 1}|, \quad |U_{\mu 2}| = |U_{\tau 2}|.$$

TM_1 variant of more general $\mu 1$-$\tau 1$

TM_2 variant of more general $\mu 2$-$\tau 2$

Comparing the corresponding elements of the first column of $\mathrm{U}_{\mathrm{PMNS}}$ and $\mathrm{U}_{\mathrm{TM}_1}$.

$$|\mathrm{U}_{\mathrm{e}1}|^2 = c_{12}^2 c_{13}^2 = 2/3 \quad : \quad \theta_{12}^{\mathrm{TM}_1}(\theta_{13}) \quad : \quad s_{12}^2 = \frac{1-3s_{13}^2}{3-3s_{13}^2},$$

$$|\mathrm{U}_{\mu 1}|^2 = |\mathrm{U}_{\tau 1}|^2 = 1/6 \quad : \quad \delta_{\mathrm{CP}}^{\mathrm{TM}_1}(\theta_{13},\theta_{23}) \quad : \quad \cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}.$$

Comparing the corresponding elements of the second column of $\mathrm{U}_{\mathrm{PMNS}}$ and $\mathrm{U}_{\mathrm{TM}_2}$.

$$|\mathrm{U}_{\mathrm{e}2}|^2 = s_{12}^2 c_{13}^2 = 1/3 \quad : \quad \theta_{12}^{\mathrm{TM}_2}(\theta_{13}) \quad : \quad s_{12}^2 = \frac{1}{3-3s_{13}^2},$$

$$|\mathrm{U}_{\mu 2}|^2 = |\mathrm{U}_{\tau 2}|^2 = 1/3 \quad : \quad \delta_{\mathrm{CP}}^{\mathrm{TM}_2}(\theta_{13},\theta_{23}) \quad : \quad \cos\delta_{\mathrm{CP}} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}.$$

In partial μ-τ reflection symmetry, the mixing matrix symmetry is given by:

$$|U_{\mu 1}| = |U_{\tau 1}| \quad : \quad \delta_{\mathrm{CP}}^{\mu 1\text{-}\tau 1}(\theta_{13},\theta_{12},\theta_{23}) \quad : \quad \cos\delta_{\mathrm{CP}} = \frac{(c_{23}^2-s_{23}^2)(c_{12}^2 s_{13}^2 - s_{12}^2)}{4c_{12}s_{12}c_{23}s_{23}s_{13}},$$

$$|U_{\mu 2}| = |U_{\tau 2}| \quad : \quad \delta_{\mathrm{CP}}^{\mu 2\text{-}\tau 2}(\theta_{13},\theta_{12},\theta_{23}) \quad : \quad \cos\delta_{\mathrm{CP}} = \frac{(c_{23}^2-s_{23}^2)(c_{12}^2 - s_{12}^2 s_{13}^2)}{4c_{12}s_{12}c_{23}s_{23}s_{13}}.$$

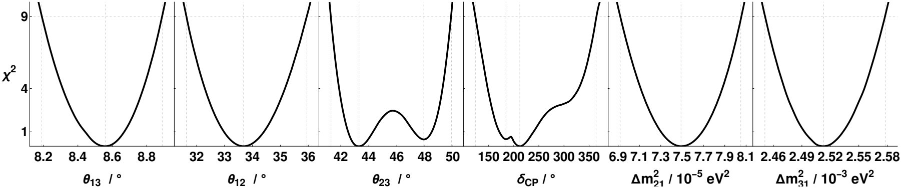
Summary of relations arising from mixing-schemes structure

TM_1	TM_2
$s_{12}^2 = \frac{1-3s_{13}^2}{3-3s_{13}^2}$	$s_{12}^2 = \frac{1}{3-3s_{13}^2}$
$\cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}$	$\cos\delta_{\mathrm{CP}} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}$
$\mu 1 - \tau 1$	$\mu 2 - \tau 2$
$\cos\delta_{\mathrm{CP}} = \frac{(c_{23}^2-s_{23}^2)(c_{12}^2 s_{13}^2 - s_{12}^2)}{4c_{12}s_{12}c_{23}s_{23}s_{13}}$	$\cos\delta_{\mathrm{CP}} = \frac{(c_{23}^2-s_{23}^2)(c_{12}^2 - s_{12}^2 s_{13}^2)}{4c_{12}s_{12}c_{23}s_{23}s_{13}}.$

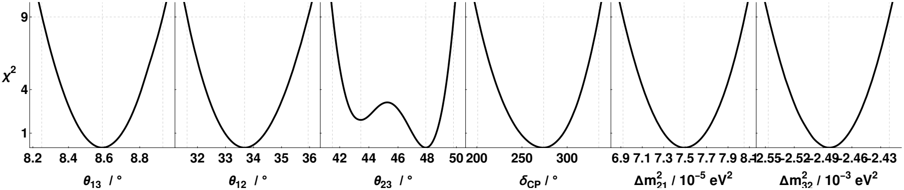
How to confront the mixing schemes with experimental data ?

Oscillation parameters, 3σ ranges, b-fs, NuFIT v6.0 (2025), "model independent"

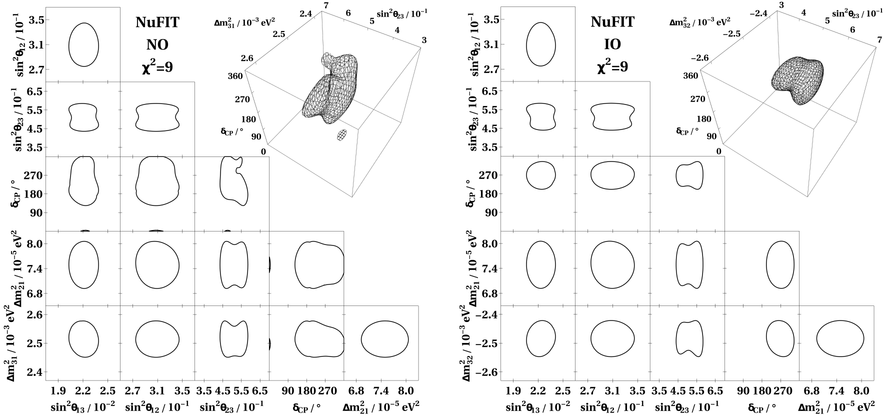
NO



IO



Oscillation parameters, correlations, NuFIT v6.0 (2025), "model independent"



Currently, up to 3D datasets are available.

Mathematica output based on (interpolated) NuFIT v6.0 datasets (correlations).

```
In[686]:= {{"", "sin²θ23=0.57", "Δm²31=0.00255 eV²", "δCP=290°"},
  {"χ²", deltachisq["NO"][{"T23"}][5.7], deltachisq["NO"][{"DMA"}][2.55], deltachisq["NO"][{"DCP"}][290]}} // TableForm
Out[686]//TableForm=
      sin²θ23=0.57   Δm²31=0.00255 eV²   δCP=290°
  χ²  2.71635        3.07873             2.82341
```

$\longleftarrow$ **treated independently, each $\chi^2 \sim 3$**

```
In[687]:= {{"", "sin²θ23=0.57 ∧ Δm²31=0.00255 eV²"}, {"χ²", deltachisq["NO"][{"T23", "DMA"}][5.7, 2.55]}} // TableForm
Out[687]//TableForm=
      sin²θ23=0.57 ∧ Δm²31=0.00255 eV²
  χ²  4.68546
```

$\longleftarrow$

```
In[688]:= {{"", "Δm²31=0.00255 eV² ∧ δCP=290°"}, {"χ²", deltachisq["NO"][{"DMA", "DCP"}][2.55, 290]}} // TableForm
Out[688]//TableForm=
      Δm²31=0.00255 eV² ∧ δCP=290°
  χ²  7.56033
```

$\longleftarrow$ **correlations of two, $\chi^2 \sim 5 \text{ to } 13$**

```
In[689]:= {{"", "sin²θ23=0.57 ∧ δCP=290°"}, {"χ²", deltachisq["NO"][{"T23", "DCP"}][5.7, 290]}} // TableForm
Out[689]//TableForm=
      sin²θ23=0.57 ∧ δCP=290°
  χ²  13.454
```

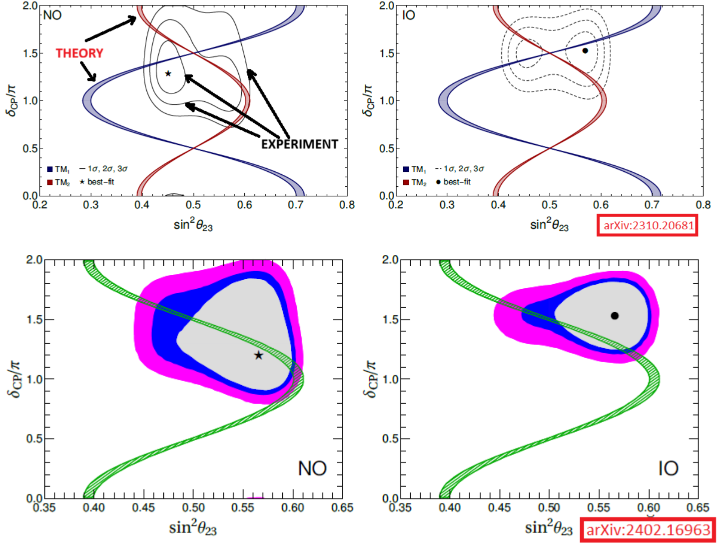
$\longleftarrow$

```
In[690]:= {{"", "sin²θ23=0.57 ∧ Δm²31=0.00255 eV² ∧ δCP=290°"}, {"χ²", deltachisq["NO"][{"T23", "DMA", "DCP"}][5.7, 2.55, 290]}} // TableForm
Out[690]//TableForm=
      sin²θ23=0.57 ∧ Δm²31=0.00255 eV² ∧ δCP=290°
  χ²  17.4936
```

$\longleftarrow$ **correlation of three, $\chi^2 \sim 17$**

Correlations improve the probability output through the χ^2 value.

Confronting mixing schemes with oscillation data (examples)



⇒ **Next, we will combine theory with experiment which are flashed independently on the plots.**

THEORY

⇩

MIXING SCHEME RELATIONS

⇩

COMBINE

EXPERIMENT

⇩

OSCILLATION PARAMTERS CORRELATIONS

⇩

EXTEND TO FULL RANGE ⇦ **COMBINE**

Extend to full range..

We assume, that for random variables (X_i) with corresponding best-fits (bf_i) and standard deviations (σ_i), the relation holds:

$$\chi^2(X_1, \ldots, X_i) = \sum_i \left(\frac{X_i - bf_i}{\sigma_i} \right)^2.$$

Extending the number of random variables, we have:

$$\chi^2(X_1, \ldots, X_i) \geqslant \max \left\{ \chi^2(X_1, \ldots, X_{k-1}, X_{k+1}, \ldots, X_i), \quad k \in \{1 \ldots i\} \right\},$$

no risk of excluding allowed region because of "$\geqslant$"

since for any $k \in \{1 \ldots i\}$

$$\chi^2(X_1, \ldots, X_i) = \sum_i \left(\frac{X_i - bf_i}{\sigma_i} \right)^2 \geqslant \sum_{i \neq k} \left(\frac{X_i - bf_i}{\sigma_i} \right)^2 = \chi^2(X_1, \ldots, X_{k-1}, X_{k+1}, \ldots, X_i),$$

and in the context of neutrino parameters e.g.

$$\chi^2(\theta_{23}, \Delta m^2_{31}, \delta_{\mathrm{CP}}) \geqslant \max \left\{ \chi^2(\Delta m^2_{31}, \delta_{\mathrm{CP}}), \chi^2(\theta_{23}, \delta_{\mathrm{CP}}), \chi^2(\theta_{23}, \Delta m^2_{31}) \right\}$$

INDEED: $17.5 \geqslant \max\{7.6, 13.5, 4.7\}$ ← **numbers taken from slide 12**

..and reduce (when needed).

To reduce the number of random variables, we have:

$$\chi^2(X_1, ..., X_{k-1}, X_{k+1}, ..., X_i) = \sum_{i \neq k} \left(\frac{X_i - bf_i}{\sigma_i} \right)^2 = \sum_{i \neq k} \left(\frac{X_i - bf_i}{\sigma_i} \right)^2 + \left(\frac{bf_k - bf_k}{\sigma_k} \right)^2$$
$$= \chi^2(X_1, ..., X_k = bf_k, ..., X_i),$$

and in the context of neutrino parameters e.g.

$$\chi^2(\theta_{23}, \delta_{\mathrm{CP}}) = \chi^2(\theta_{23}, \theta_{13}^{bf}, \delta_{\mathrm{CP}}).$$

A full (6D) correlations directly from experiment would further constrain the results.

The extension formula may still be useful due to numerical complexity and memory requirements of 6D data sets.

TM$_1$ mixing relations + extended to full range correlations, example:

$$\sin^2\theta_{12} = \frac{1-3\sin^2\theta_{13}}{3-3\sin^2\theta_{13}}$$

$$\downarrow$$

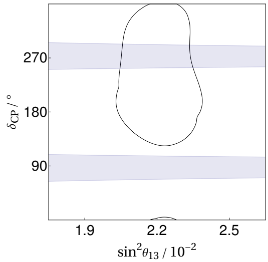
$$\chi^2(\theta_{13}, \theta_{12}, \theta_{23}, \delta_{\mathrm{CP}}, \Delta m^2_{21}, \Delta m^2_{31})$$

$$\uparrow$$

$$\cos\delta_{\mathrm{CP}} = \frac{(1-5\sin^2\theta_{13})(2\sin^2_{23}-1)}{4\sin_{13}\sin\theta_{23}\sqrt{2(1-3\sin^2\theta_{13})(1-\sin^2\theta_{23})}}$$

For any choice of parameters, the equations for these parameters must be solved first.

NO and IO need to be treated separately.

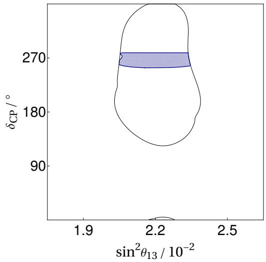


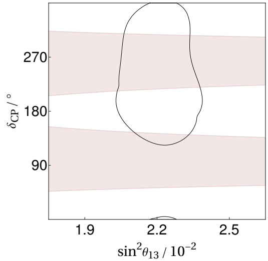
THEORY AND EXPERIMENT

FLASHED INDEPEDENTLY

TM1

COMBINED



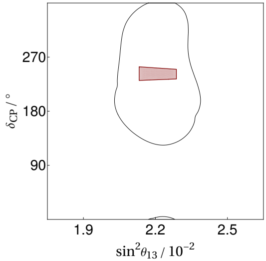


THEORY AND EXPERIMENT

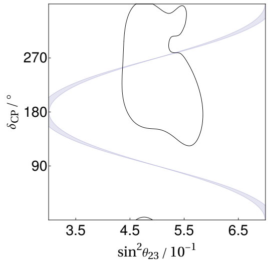
FLASHED INDEPEDENTLY

TM2

COMBINED



NO, $\chi^2 < 9$

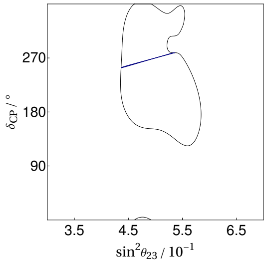


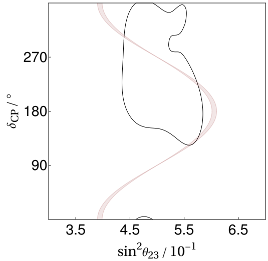
THEORY AND EXPERIMENT

FLASHED INDEPEDENTLY

TM1

COMBINED



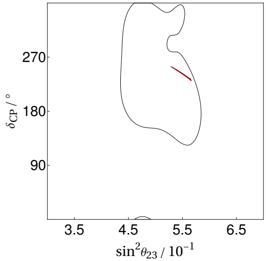


THEORY AND EXPERIMENT

FLASHED INDEPEDENTLY

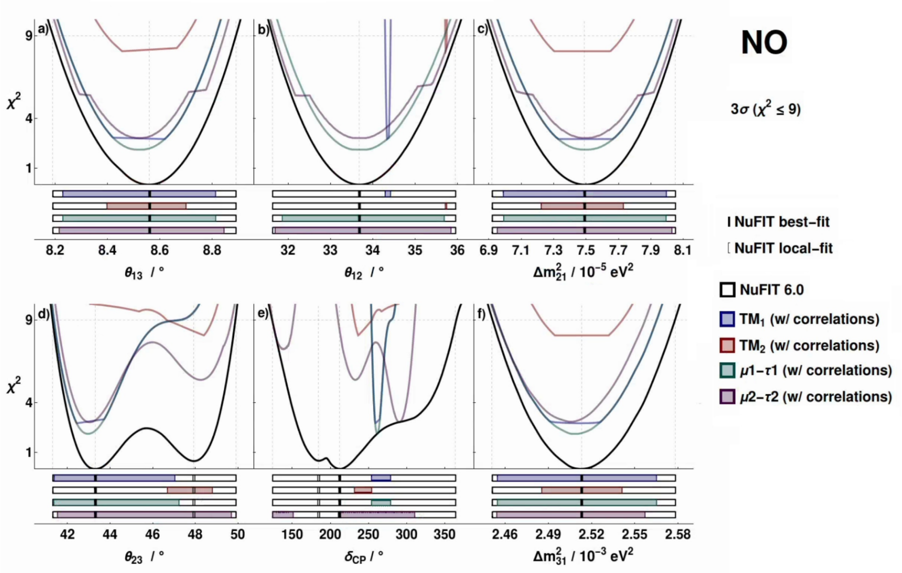
TM2

COMBINED



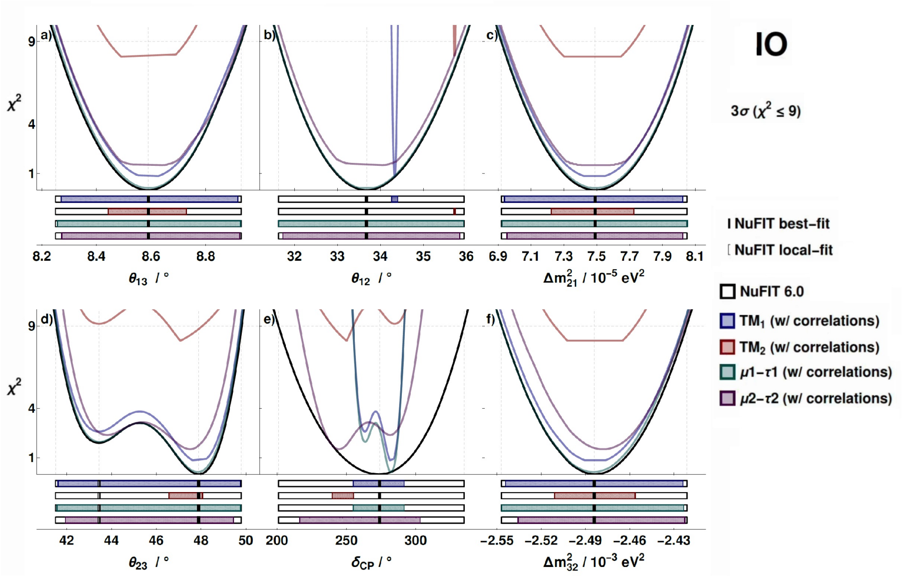
NO, $\chi^2 < 9$

Extracted ranges of model's parameters from complete correlations (NuFIT v6.0)

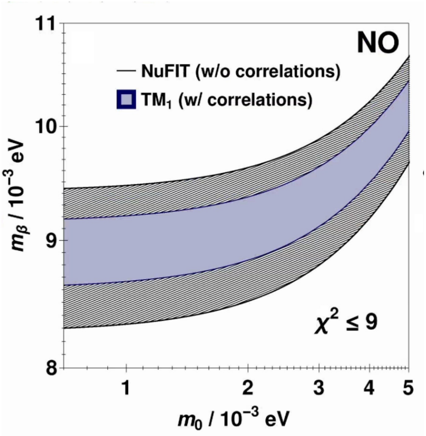


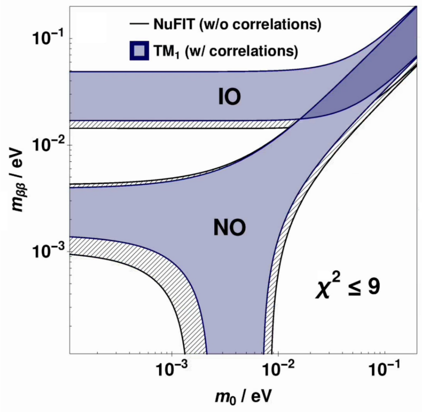
χ^2_{min}: $\mathrm{TM}_1 = 2.8$, $\mathrm{TM}_2 = 8.1$, $\mu 1 - \tau 1 = 2.1$, $\mu 2 - \tau 2 = 2.8$

Extracted ranges of model's parameters from complete correlations (NuFIT v6.0)



χ^2_{min}: $\mathrm{TM}_1 = 0.9$, $\mathrm{TM}_2 = 8.1$, $\mu1-\tau1 = 0.1$, $\mu2-\tau2 = 1.5$


NO
— NuFIT (w/o correlations)
TM1 (w/ correlations)
$\chi^2 \leq 9$
11
10
9
8
1
2
3
4
5
m_β / 10^{-3} eV
m_0 / 10^{-3} eV


— NuFIT (w/o correlations)
TM1 (w/ correlations)
IO
NO
$\chi^2 \leq 9$
10^{-1}
10^{-2}
10^{-3}
$m_{\beta\beta}$ / eV
m_0 / eV

Summary

Multidimensional correlation data on neutrino oscillation parameters analysis allows for better theoretical model predictions

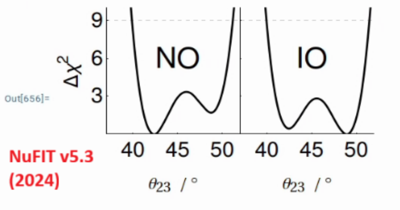
With the presented approach, we

- test TM_1, TM_2 and partial μ-τ mixing-schemes over the full range of parameters at a given χ^2 level
- some parameters are exceptionally constrained
  - θ_{12} for TM_1 and TM_2,
  - θ_{23} for TM_2,
  - δ_{CP} for all considered mixing-schemes,
  - Δm^2_{21} and $\Delta m^2_{31(32)}$ for TM_2
- find tighter, model-dependent constraints on the effective neutrino masses

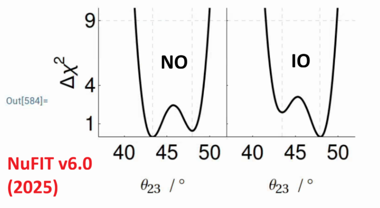
The method is general and can be applied to any model providing mixing-scheme relations in the form analytical expressions involving the neutrino oscillation parameters.

Thank you
for your attention.

3ν mixing




```
In[658]:= {deltachisq[#][{"T13"}][0] & /@ {"NO", "IO"},
  deltachisq[#][{"DCP"}][0] & /@ {"NO", "IO"}}
Out[658]= {{1674.27, 1664.84}, {11.0731, 14.4343}}
```




```
In[585]:= {deltachisq[#][{"T13"}][0] & /@ {"NO", "IO"},
  deltachisq[#][{"DCP"}][0] & /@ {"NO", "IO"}}
Out[585]= {{1774.37, 1768.02}, {8.03189, 19.3677}}
```

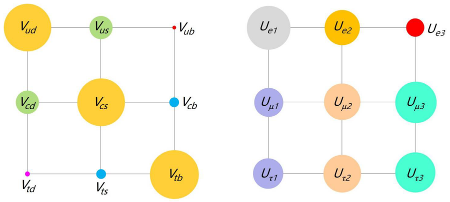
Left: Mathematica output based on NuFIT v5.3 data (2024). **Right:** Mathematica output based on NuFIT v6.0 data (2025).

NO

$$\begin{pmatrix} 0.800 \to 0.843 & 0.519 \to 0.580 & 0.143 \to 0.155 \\ 0.354 \to 0.461 & 0.533 \to 0.620 & 0.635 \to 0.769 \\ 0.277 \to 0.485 & 0.589 \to 0.616 & 0.620 \to 0.759 \end{pmatrix}$$

IO

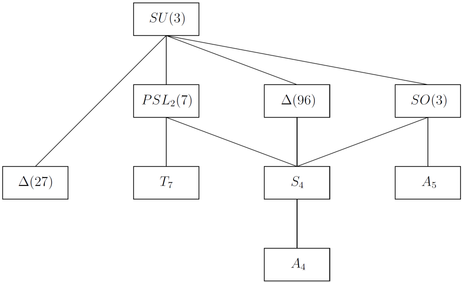
$$\begin{pmatrix} 0.800 \to 0.843 & 0.519 \to 0.580 & 0.143 \to 0.156 \\ 0.288 \to 0.479 & 0.567 \to 0.606 & 0.635 \to 0.772 \\ 0.246 \to 0.527 & 0.585 \to 0.603 & 0.616 \to 0.759 \end{pmatrix}$$



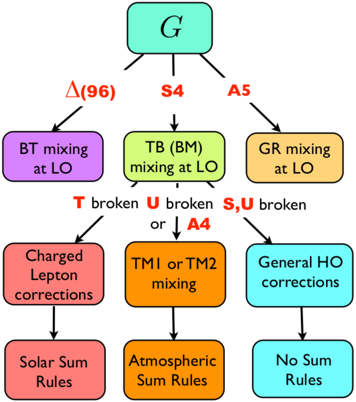
(a) CKM quark flavor mixing (b) PMNS lepton flavor mixing

Left: Mixing matrix ($|\mathrm{U_{PMNS}}|$) ranges in 3σ, NuFIT v5.3. **Right:** Taken from arXiv:2210.11922, Figure 2.

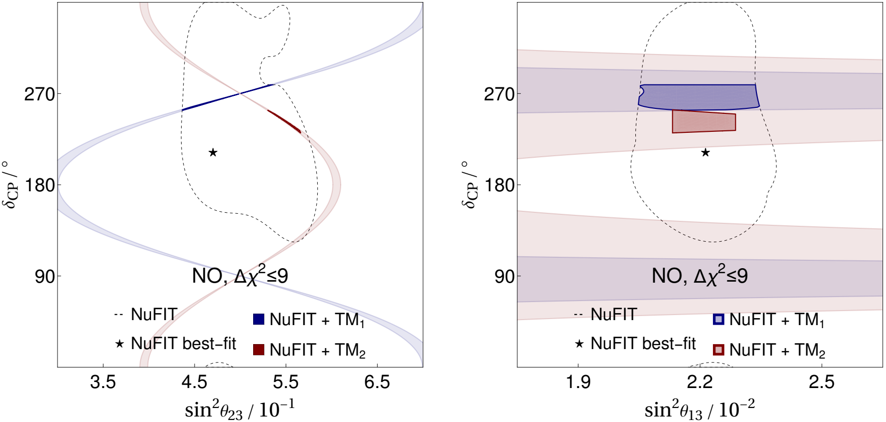
Flavor symmetries and mixing-schemes



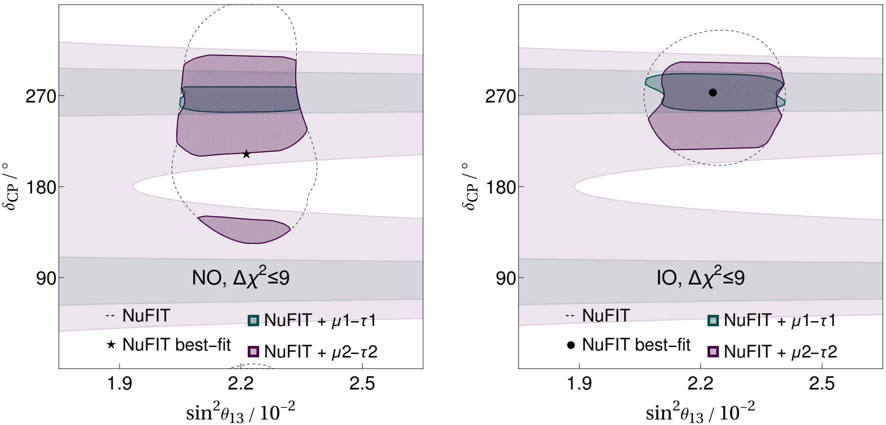
arXiv:1301.1340 S.F. King, C. Luhn



Experiment / Theory / Combined at $\chi^2 < 9$ (NuFIT v6.0)



Experiment / Theory / Combined at $\chi^2 < 9$ (NuFIT v6.0)



Effective neutrino mass

Effective electron neutrino mass:

$$m_\beta^2 = c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2 = \begin{cases} \text{NO:} & m_0^2 + \Delta m_{21}^2 c_{13}^2 s_{12}^2 + \Delta m_{3\ell}^2 s_{13}^2 \,, \\ \text{IO:} & m_0^2 - \Delta m_{21}^2 c_{13}^2 c_{12}^2 - \Delta m_{3\ell}^2 c_{13}^2 . \end{cases}$$

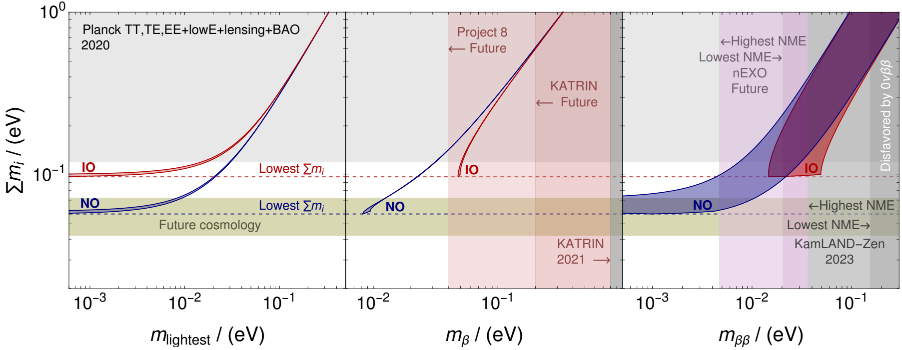
Effective Majorana mass:

$$m_{\beta\beta} = \left| \sum_i m_i U_{ei}^2 \right| = \left| m_1 c_{13}^2 c_{12}^2 e^{i2\alpha_1} + m_2 c_{13}^2 s_{12}^2 e^{i2\alpha_2} + m_3 s_{13}^2 e^{-i2\delta_{\rm CP}} \right|$$

$$= \begin{cases} \text{NO:} & m_0 \left| c_{13}^2 c_{12}^2 e^{i2(\alpha_1 - \delta_{\rm CP})} + \sqrt{1 + \frac{\Delta m_{21}^2}{m_0^2}}\, c_{13}^2 s_{12}^2 e^{i2(\alpha_2 - \delta_{\rm CP})} + \sqrt{1 + \frac{\Delta m_{3\ell}^2}{m_0^2}}\, s_{13}^2 \right| \\ \text{IO:} & m_0 \left| \sqrt{1 - \frac{\Delta m_{3\ell}^2 + \Delta m_{21}^2}{m_0^2}}\, c_{13}^2 c_{12}^2 e^{i2(\alpha_1 - \delta_{\rm CP})} + \sqrt{1 - \frac{\Delta m_{3\ell}^2}{m_0^2}}\, c_{13}^2 s_{12}^2 e^{i2(\alpha_2 - \delta_{\rm CP})} + s_{13}^2 \right| . \end{cases}$$

Here $m_0 = m_1$ (m_3) and $\ell = 1$ (2) in $\Delta m_{3\ell}^2$ for NO (IO).

Neutrino mass



Planck: $\sum_i m_i < 0.12$ eV, 95% CL, arXiv:1807.06209

KATRIN: $m_\beta < 0.8$ eV, 90% CL, arXiv:2105.08533

KamLAND-Zen: $m_{\beta\beta} <$ (0.036 to) 0.156 eV, 90% CL, arXiv:2203.02139