Next-to-next-to-leading order QCD$\otimes$QED corrections to semi-inclusive DIS

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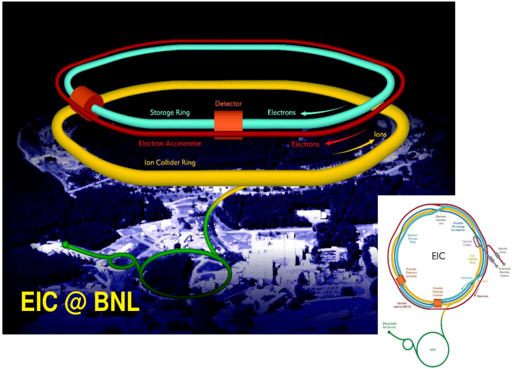
Electron-Ion Collider

A machine that will unlock the secrets of the strongest force in Nature

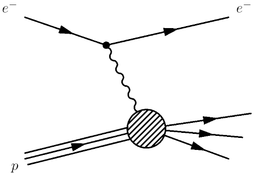


Introduction

DIS

$$l(k_l) + H(P) \to l(k_l') + X$$

- Only the scattered lepton is detected, while the remnants of the shattered nucleon are summed over.
- Depends on parton distribution function (PDF) of the incoming hadron.

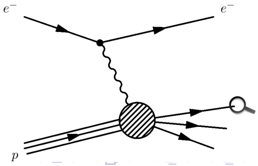


$$dPS\big|_{\text{SIDIS}} = dPS\big|_{\text{DIS}}\delta(z - \frac{P\cdot P_H}{P\cdot q})$$

Semi-inclusive DIS

$$l(k_l) + H(P) \to l(k_l') + H'(P_H) + X'$$

- In addition to the scattered lepton one of the final-state hadron is also detected, by applying an extra constraint on the phase space of tagged hadron.
- Depends on parton distribution dunction (PDF) of the incoming hadron and parton fragmentation function (FF) of the final state hadron.



State-of-the-art

Earlier works: NLO and beyond

- Processes involving fragmentation functions beyond the leading order in QCD
  Alteralli, Ellis, Martinelli and Pi, Volume 160, 1979
- Lepton-hadron processes beyond leading order in quantum chromodynamics
  Furmanski and Petronzio, Volume 11, 1982
- QCD Analysis of Unpolarized and Polarized Lambda - Baryon Production in Leading and Next-to-Leading Order
  de Florian, Stratmann and Vogelsang, arxiv:9711387
- Soft-Gluon Resummation for the Fragmentation of Light and Heavy Quarks at Large x
  Cacciari and Catani arxiv:0107138
- Next to leading order evolution of SIDIS processes in the forward region
  Daleo and Sassot, arxiv:0309073
- QCD resummation for semi-inclusive hadron production processes
  Anderle, Ringer and Vogelsang, arxiv:1212.2099
- Threshold resummation for polarized (semi-)inclusive deep inelastic scattering
  Anderle, Ringer and Vogelsang, arxiv:1304.1373
- Approximate NNLO QCD corrections to semi-inclusive DIS
  Abele, de Florian, and Vogelsang, arxiv:2109.00847
- Threshold resummation at N^3LL accuracy and approximate N^3LO corrections to semi-inclusive DIS
  Abele, de Florian, and Vogelsang, arxiv:2203.07928
- Charged hadron fragmentation functions at high energy colliders
  Borsa, Stratmann, de Florian, Sassot, arxiv:2311.17768
- Next-to-Next-to-Leading Order Global Analysis of Polarized Parton Distribution Functions
  Borsa, Stratmann, Vogelsang, de Florian, Sassot, arxiv:2407.11635

and more . . .

State-of-the-art

NNLO and beyond:

- Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering
  Goyal, Moch, Pathak, Rana, Ravindran, arxiv:2312.17711
- Next-to-Next-to-Leading Order QCD Corrections to Polarized Semi-Inclusive Deep-Inelastic Scattering
  Goyal, Lee, Moch, Pathak, Rana, Ravindran, arxiv:2404.09959
- NNLO phase-space integrals for semi-inclusive deep-inelastic scattering
  Ahmed, Goyal, Hasan, Lee, Moch, Pathak, Rana, Rapakoulias, Ravindran, arxiv:2412.16509
- NNLO QCD corrections to unpolarized and polarized SIDIS
  Goyal, Lee, Moch, Pathak, Rana, Ravindran, arxiv:2412.19309
- Soft and virtual corrections to semi-inclusive DIS up to four loops in QCD
  Goyal, Moch, Pathak, Rana, Ravindran, arxiv:2506.24078

Related Works:

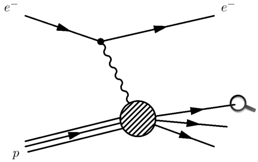
- Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD
  Bonino, Gehrmann, Stagnitto, arxiv:2401.16281
- Polarized Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD
  Bonino, Gehrmann, Löchner, Schönwald, Stagnitto, arxiv:2404.08597
- Antenna subtraction for processes with identified particles at hadron colliders
  Bonino, Gehrmann, Marcoli, Schürmann, Stagnitto, arxiv:2404.09925
- Identified Hadron Production in Deeply Inelastic Neutrino-Nucleon Scattering
  Bonino, Gehrmann, Löchner, Schönwald, Stagnitto, arxiv:2504.05376
- Neutral and Charged Current Semi-Inclusive Deep-Inelastic Scattering at NNLO QCD
  Bonino, Gehrmann, Löchner, Schönwald, Stagnitto, arxiv:2506.19926
- Single-valued representation of unpolarized and polarized semi-inclusive deep inelastic scattering at next-to-next-to-leading order
  Haug and Wunder, arxiv:2505.18109

Motivation

- Semi-inclusive Deep Inelastic Scattering (SIDIS) helps to study hadron structure and hadron fragmentation phenomena.
- With increase in experimental precision, one demands precise calculation from theoretical side as well.
- Fixed order results are sensitive to theoretical uncertainty which are coming from renormalisation scale dependence and factorisation scale dependence.
- We calculated NNLO corrections in QCD as well as QED for polarized and unpolarized scattering using the Feynman diagrammatic approach.
- Better constraints on FFs and (pol)PDFs with full flavour decomposition and precise mesurements at upcoming EIC.

Hadronic Cross section

Differential Hadronic cross section for
$e^-(k_l) + H(P) \rightarrow e^-(k_l') + H'(P_H) + X'$ is written as,



$$\frac{d^3\sigma_{e^-H}}{dxdydz} = \frac{2\pi y \alpha_e^2}{Q^4} L^{\mu\nu}(k_l, k_l', q) W_{\mu\nu}(q, P, P_H)$$

where, Leptonic Tensor is

$$L^{\mu\nu} = 2k_l^{\mu} k_l'^{\nu} + 2k_l'^{\mu} k_l^{\nu} - Q^2 g^{\mu\nu} - 2i\epsilon^{\mu\nu\sigma\lambda} q_\sigma s_{l,\lambda}$$

Hadronic tensor can be parametrized in terms of structure functions,

$$W^{\mu\nu} = F_1 \underbrace{\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right]}_{T_1^{\mu\nu}} + F_2 \underbrace{\left[\frac{1}{P.q} \left(P^\mu - \frac{P.q}{q^2} q^\mu \right) \left(P^\nu - \frac{P.q}{q^2} q^\nu \right) \right]}_{T_2^{\mu\nu}}$$

$$+ F_3 \left[\frac{i}{P.q} \varepsilon^{\mu\nu\sigma\lambda} q_\sigma P_\lambda \right] + g_1 \left[\frac{i}{P.q} \varepsilon^{\mu\nu\sigma\lambda} q_\sigma S_\lambda \right] + g_2 \left[\frac{i}{P.q} \varepsilon^{\mu\nu\sigma\lambda} q_\sigma \left(S_\lambda - \frac{S \cdot q}{P \cdot q} P_\lambda \right) \right]$$

Hadronic Cross section

Cross section is parametrized through structure functions,

- Unpolarized SIDIS: $d\sigma = \frac{1}{4}\sum_{s_l,S,s'_l,S_H} d\sigma^{s'_l,S_H}_{s_l,S}$,

$$\frac{d^3\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}\left[yF_1(x,z,Q^2) + \frac{(1-y)}{y}F_2(x,z,Q^2)\right] \quad (1)$$

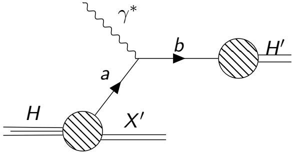
- Longitudinally polarized SIDIS:
$d\Delta\sigma = \frac{1}{2}\sum_{s'_l,S_H}\left(d\sigma^{s'_l,S_H}_{s_l=\frac{1}{2},S=\frac{1}{2}} - d\sigma^{s'_l,S_H}_{s_l=\frac{1}{2},S=-\frac{1}{2}}\right)$,

$$\frac{d^3\Delta\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}(2-y)g_1(x,z,Q^2) \quad (2)$$

Structure functions

$F_i(g_1)$ are Lorentz invariants which cannot be calculated in perturbation theory. We'll use Parton model to calculate them.
In parton model, we express F_i and g_1 as,

$$(g_1)F_i = x^{i-1}\sum_{a,b}\int_x^1 \frac{dx_1}{x_1}(\Delta)f_a(x_1,\mu_F^2)\int_z^1 \frac{dz_1}{z_1}D_b(z_1,\mu_F^2) \times (\Delta)\mathcal{C}_{i,ab}\left(\frac{x}{x_1},\frac{z}{z_1},Q^2,\mu_F^2\right). \quad (3)$$



- $(\Delta)f_a dx_1$: The probability of finding a parton of type 'a' which carries a momentum fraction x_1 of the parent hadron H.
- $D_b dz_1$: The probability that a parton of type 'b' will fragment into hadron H' which carries a momentum fraction z_1 of the parton.
- $(\Delta)\mathcal{C}_{i,ab}$ are the finite coefficient functions (CFs) that can be computed perturbatively, it is related to partonic cross section.

Coefficient functions, QCD$\otimes$QED

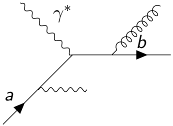
We are interested in series expansion of the CFs in strong coupling constant (a_s) and electromagnetic coupling (a_e),

$$\begin{aligned}(\Delta)\mathcal{C}_{i,ab} &= (\Delta)\mathcal{C}_{i,ab}^{(0,0)} \\ &+ a_s\ (\Delta)\mathcal{C}_{i,ab}^{(1,0)} + a_e\ (\Delta)\mathcal{C}_{i,ab}^{(0,1)} \\ &+ a_s^2\ (\Delta)\mathcal{C}_{i,ab}^{(2,0)} + a_e^2\ (\Delta)\mathcal{C}_{i,ab}^{(0,2)} + a_s a_e\ (\Delta)\mathcal{C}_{i,ab}^{(1,1)} \\ &+ \mathcal{O}\left(a_s^j a_e^k \Big|_{j+k\geq 3} \right). \end{aligned} \tag{4}$$

Coefficient functions

Computation of CFs starts from the parton level cross section denoted by $(\Delta)\hat{\sigma}_{i,ab}$, which is defined as,

$$\frac{d^3(\Delta)\hat{\sigma}_{i,ab}}{dxdydz} = \frac{(\Delta)\mathcal{P}_i^{\mu\nu}}{4\pi}\int \mathrm{dPS}_{X'+b}\,\Sigma\left|(\Delta)M_{ab}\right|^2_{\mu\nu}\delta\left(\frac{z}{z_1} - \frac{p_a\cdot p_b}{p_a\cdot q}\right)$$



here, $(\Delta)\mathcal{P}_i^{\mu\nu}$ are the projectors to project out CFs and $|(\Delta)M_{ab}|^2$ is the squared amplitude for the process $a(p_a) + \gamma^*(q) \to$ "b" $(p_b) + X'$.

$$\mathcal{P}_1^{\mu\nu} = \frac{1}{(D-2)}\left(T_1^{\mu\nu} + 2xT_2^{\mu\nu}\right)$$

$$\mathcal{P}_2^{\mu\nu} = \frac{2x}{(D-2)x_1}\left(T_1^{\mu\nu} + 2x(D-1)T_2^{\mu\nu}\right)$$

$$\mathcal{P}_3^{\mu\nu} = \Delta\mathcal{P}_1^{\mu\nu} = \frac{-i}{(D-2)(D-3)}\varepsilon^{\mu\nu\sigma\lambda}\frac{q_\sigma P_\lambda}{P.q} \tag{5}$$

Kinematics

Hadronic Kinematics :

P : Initial hadron momenta

k_l : Initial e^- momenta

q : Virtual photon momenta

q^2 : $-Q^2 < 0$

P_H : Momenta of hadron H'

y : $\frac{P.q}{P.k_l}$, Fractional energy loss by e^-

x : $\frac{Q^2}{2P.q}$, Bjorken-x, z : $\frac{P.P_H}{P.q}$

Partonic Kinematics :

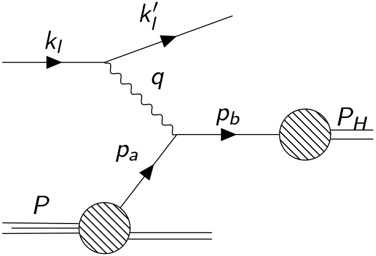
p_a : parton 'a' momenta

p_b : Tagged parton momenta 'b'

x_1 : $\frac{p_a}{P}$, z_1 : $\frac{P_H}{p_b}$

x' : $\frac{x}{x_1} = \frac{Q^2}{2p_a.q}$, z' : $\frac{z}{z_1} = \frac{p_a.p_b}{p_a.q}$

k_i : Momentum of real radiations

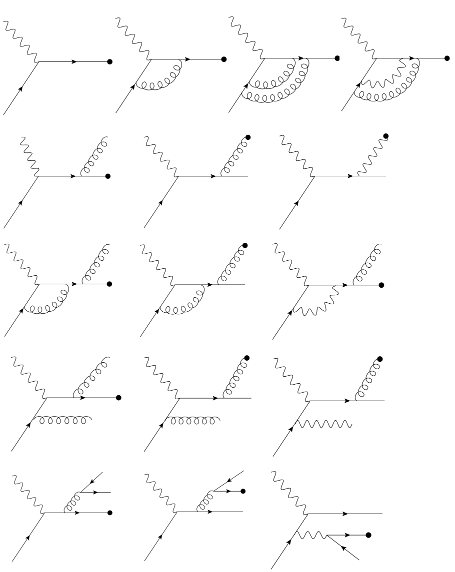


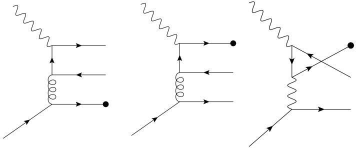
SIDIS subprocesses: QCD (QED)

LO		$\gamma^* q \to q$
NLO	1 Loop: (V)	$\gamma^* q \to q$
		$\gamma^* q \to q + g(\gamma)$
		$\gamma^* g(\gamma) \to q + \overline{q}$
NNLO	2 Loop: (VV)	$\gamma^* q \to q$
	1 Loop: (RV)	$\gamma^* q \to q + g(\gamma)$
		$\gamma^* q \to q + g(\gamma) + g(\gamma)$
		$\gamma^* q \to q + q_i + \overline{q}_i$
	1 Loop: (RV)	$\gamma^* g(\gamma) \to q + \overline{q}$
		$\gamma^* g(\gamma) \to q + \overline{q} + g(\gamma)$

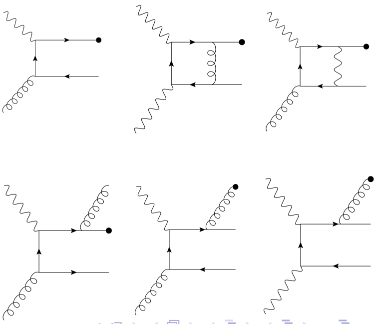
SIDIS subprocesses: Sample diagrams

Quark initiated:





Gluon (Photon) initiated:



Partonic Cross section

Beyond leading order, PCS gets contribution from loop diagrams as well as real emission diagrams which gives divergent integrals.

- In the high momentum region, the loop integral gives divergences, which are removed by renormalization procedure.
- Due to the presence of massless particles, further two types- soft and collinear divergences.
- Infrared divergences cancel among virtual and real emission processes[1], except for the collinear divergences related to the *a* and *b* partons in the initial state and the final fragmentation state. These divergences can be factored out into Altarelli-Parisi (AP) kernels (mass factorisation) at μ_F scale,
- We used Dimension regularization i.e. working in D $= 4 + \epsilon$ to regulate the divergences and used $\overline{\text{MS}}$ renormalization scheme.

[1] KLN theorem

Polarized case

In spin-dependent case, γ_5 and the Levi-Civita tensor appear, which are intrinsically four dimensinal objects, one need a presciption to define them in D$= 4 + \epsilon$. We chose Larin prescription [Larin '93]:

- Replace γ_5 by
$$\not{p}_a \gamma_5 = \frac{i}{3!} \, \varepsilon_{\mu\nu\sigma\lambda} p_a^\mu \gamma^\nu \gamma^\sigma \gamma^\lambda \, , \tag{6}$$
with $\varepsilon_{0123} = -\varepsilon^{0123} = 1$,
- Compute all matrix elements in D dimensions.
- Evaluate all Feynman loop integrals and phase-space integrals in D dimensions.
- Contract the Levi-Civita tensors in D dimensions using: $\varepsilon_{\mu_1\nu_1\rho_1\sigma_1}\varepsilon^{\mu_2\nu_2\rho_2\sigma_2} = 4!\delta^{\mu_2}_{[\mu_1} \ldots \delta^{\sigma_2}_{\sigma_1]}$
- To transform to $\overline{\text{MS}}$ scheme, we need to multiply an over all finite renormalisation constant [Matiounine, Smith, van Neerven '98].

Computation Details

Computation Steps:

- Generation of set of Feynman diagrams using '`QGRAF`', which gives partonic level diagrams in symbolic form.
- Convert the output of `QGRAF` in '`FORM`' form to get amplitude for individual diagrams using in-house codes.
- Used `FORM` extensively to do symbolic calculation like Lorentz contractions, Dirac algebra, handling Gell-Mann matrices.
- Used `FORM` for calculation of the $(\Delta)\mathcal{C}_{i,ab}^{(j,k)}$.

To get the cross section we have to perform loop integrations and phase space integrations (done in D=$4+\epsilon$, dimensional regularisation).

[1] `Qgraf`: Nogueira '91 [2] `FORM`: Vermaseren '00; Kuipers, Ueda, Vermaseren, Vollinga '12

Loop Integrals

Using the fact that the integral of a total derivative vanishes within DR and the property of scaleless integral, one gets linear Integration-by-parts (IBP) identities to write Loop integrals in terms of the basis of integrals called Master Integrals (MIs).

$$\int d^D l \frac{\partial}{\partial l^\mu}\left[\frac{\{l^\mu, p_i^\mu\}}{D_1^{\nu_1} D_2^{\nu_2} \ldots D_n^{\nu_n}}\right] = 0\,. \tag{7}$$

Following steps are performed,

- Choose integral families,
- Map the loop integrals (large set) onto these integral families by momentum shifts (`Reduze`).
- Reduce these integrals into the MIs, used `LiteRed`, which generates IBP reduction rules.
- Solve MIs [Matsuura, Marck, Neerven '89], [Gehrmann, Huber, Maitre '05] .

[1] `Reduze`: Manteuffel, Studerus '12 [2] `LiteRed`: Lee '13

Phase Space Integrals

3-Body Phase Space, $p_a + q \to$ "p_b" $+ k_1 + k_2$,

$$\int_{z'} [dPS]_3 =$$

$$\frac{1}{(2\pi)^{2D-3}} \int d^D k_1 \int d^D k_2 \int d^D p_b \delta(k_1^2)\delta(k_2^2)\delta(p_b^2)\delta^D(p_a + q - p_b - k_1 - k_2)\delta(z' - \frac{p_a.p_b}{p_a.q})$$

To solve the phase space integrals, we'll use Reverse Unitarity method[1], for converting phase space integral into loop integral and performing reduction to get set of MIs.

$$(2\pi i)\ \delta\left(p^2\right) \to \frac{1}{p^2 + i\varepsilon} - \frac{1}{p^2 - i\varepsilon}\,. \tag{8}$$

We get total '20' MIs in phase space calculation. Results of these MI's are not there in literature, so we solved them using different methods.

[1] Anastasiou, Melnikov, Petriello '03

Solving RR MIs

We used two approches to solve these MIs:

1) Using brute force method: choosing an appropriate frame (we chose the $k_1 - k_2$ COM frame)[1].
2) Using Differential equation method: setting up a system of DEs by taking derivative with respect to external parameters, here $\{x', z', Q^2\}$, ("Feynman Trick"). Given a boundry condition, this system can be solved.

[1] See talk by A. Rapakoulias

Differential equation demonstration

Example: 1. We want to solve,

$$F(t) = \int_0^1 \frac{x^t - 1}{\log(x)} dx \tag{13}$$

Taking derivative wrt to t, $\frac{dF}{dt} = \int_0^1 x^t dx = \frac{1}{1+t}$. Using, $F(0) = 0$, we get $F(t) = \log(1+t)$.

Example: 2. Family of Integrals:

$$I_n(\alpha) = \int_0^\infty e^{-\alpha x^2} x^n dx \tag{14}$$

Note we can now take derivative wrt α, we get, $-\frac{\partial}{\partial \alpha} I_n = I_{n+2}$ we get two basic integrals (MIs), $n = \{0, 1\}$.

Differential equation method

The differential eq. system of 20 MIs is obtain using `LiteRed`,

$$\frac{\partial \vec{f}}{\partial x'} = \hat{A}_1(x', z', \epsilon)\vec{f}$$
$$\frac{\partial \vec{f}}{\partial z'} = \hat{A}_2(x', z', \epsilon)\vec{f} \tag{15}$$

Since the system is integrable this puts constraint on $\hat{A}_i$,
Integrability condition:

$$\frac{\partial \hat{A}_1}{\partial z'} - \frac{\partial \hat{A}_2}{\partial x'} + [\hat{A}_1, \hat{A}_2] = 0 \tag{16}$$

Note, by construction $\hat{A}_1$ and $\hat{A}_2$ are lower triangle matrices.

Canonical Bases

We change the bases from $\vec{f} \to \vec{J} = \hat{T}^{-1}\vec{f}$, such that the DEq system becomes [Henn '13],

$$\left\{ \begin{array}{ll} \frac{\partial \vec{f}}{\partial x'} & = \hat{A}_1(x', z', \epsilon)\vec{f} \\ \frac{\partial \vec{f}}{\partial z'} & = \hat{A}_2(x', z', \epsilon)\vec{f} \end{array} \right\} \to \left\{ \begin{array}{ll} \frac{\partial \vec{J}}{\partial x'} & = \epsilon\, \hat{\mathcal{A}}_1(x', z')\vec{J} \\ \frac{\partial \vec{J}}{\partial z'} & = \epsilon\, \hat{\mathcal{A}}_2(x', z')\vec{J} \end{array} \right\}$$

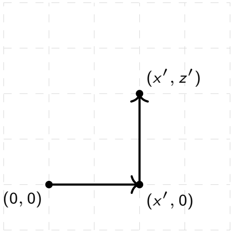
Here, $\hat{\mathcal{A}}_1 = \hat{T}^{-1}(\hat{A}_1\hat{T} - \frac{d\hat{T}}{dx'})$, Similar of $\hat{\mathcal{A}}_2$. $\hat{T}$ is found using Libra [Lee '20]. The solution of the system is as follows:

$$\vec{J}(x', z', \epsilon) = \mathcal{P}exp\left[\epsilon \int_{x_0', z_0'}^{x', z'} (\hat{\mathcal{A}}_1 dx_1' + \hat{\mathcal{A}}_2 dz_1')\right]\vec{J}(x_0', z_0', \epsilon) \tag{18}$$

Now, this path ordered exponential can be solved order-by-order in ϵ.

Path In-Dependence

- Note, since the left hand side $\vec{J}(x', z', \epsilon)$ is path independent, we can exploit this fact, eg. if we know boundry of the integral, let say, at (x'_1, z'_1) other than (x'_0, z'_0), we can fix the constant in $\vec{J}(x'_0, z'_0, \epsilon)$.
- We choose (x'_0, z'_0) to be $(0, 0)$ and calculated the boundary of MIs in the $k_1 - k_2$ frame at $x' \to 1$ and $z' \to 1$, keeping terms of the form $(1 - x')^{(a\epsilon + b)}$ and $(1 - z')^{(c\epsilon + d)}$ explicitly.



Alphabet

While solving the integrals we encounter following denominators:

$$
\begin{aligned}
&x', z', (1-x'), (1+x'), (1-z'), (z'-x'), (z'+x'),\\
&(1+x'z'), (1-z')^2+4x'z', (1+x')-4x'z',\\
&\sqrt{x'}, \sqrt{z'}, \sqrt{(1-z')^2+4x'z'}, \sqrt{(1+x')^2-4x'z'}.
\end{aligned}
$$

We defined[1] the iterated integrals using these letters, example:

$$
G(r_2, r_1, z') = \int_0^{z'} \frac{dz_2}{z_2\sqrt{(1-z_2)^2+4x'z_2}} \int_0^{z_2} \frac{dz_1}{\sqrt{(1-z_1)^2+4x'z_1}} \tag{19}
$$

After solving the MIs, the results are expressed in iterated integrals, we cross-checked our results numerically against different methods.

[1] Bonciani, Degrassi, Vicini '10

To obtain the finite partonic cross-section

Cross section Calculation

- Expanding the results of master integrals up to sufficient order in ϵ.
- Then substituted them into partonic cross section calculation to obtain the divergent cross section (in $\epsilon \to 0$).
- To separate the singularities at $x' \to 1$ or $z' \to 1$, we use Plus distribution. For example,

$$
\begin{aligned}
\frac{(1-x)^{\epsilon}}{1-x} &= \frac{1}{\epsilon}\delta(1-x) + \sum_{k=0}^{\infty} \frac{\epsilon^k}{k!}\left[\frac{\ln^k(1-x)}{1-x}\right]_+ \\
&= \frac{1}{\epsilon}\delta(1-x) + \sum_{k=0}^{\infty} \frac{\epsilon^k}{k!} D_k(x)
\end{aligned}
$$

$$\int_0^1 dx\ f(x)\big[g(x)\big]_+ = \int_0^1 dx \Big(f(x) - f(1)\Big) g(x)$$

here, $\big[g(x)\big]_+ = g(x)$ for $x \neq 1$

Double distribution:

$$\int_0^1 dx \int_0^1 dz \frac{f(x,z)}{[1-x]_+[1-z]_+} =$$

$$\int_0^1 dx \int_0^1 dz \frac{f(x,z) - f(1,z) - f(x,1) + f(1,1)}{(1-x)(1-z)}$$

Cross section Calculation

- Pole structure of the cross section is $\frac{1}{\epsilon^{2\mathrm{l}}}$ at l loop.
- Collect all the results of subprocesses and do the coupling constant renormalization at μ_R^2 scale to get UV finite result.
- And after adding all the renormalized subprocesses, we observed the cancellation of $\frac{1}{\epsilon^4}$, $\frac{1}{\epsilon^3}$ at NNLO and $\frac{1}{\epsilon^2}$ at NLO.
- The left-over pole terms due to initial state and final state collinear singularities are factorised after mass factorisation procedure at μ_F^2 scale and absorbed in bare PDFs and FFs.

$$\frac{d(\Delta)\hat{\sigma}_{i,ab}(x',z',\epsilon)}{x'^{i-1}} = (\Delta)\Gamma_{c\leftarrow a}(x',\mu_F^2,\epsilon) \otimes (\Delta)\mathcal{C}_{i,cd}(x',z',\mu_F^2,\epsilon)\tilde{\otimes}\tilde{\Gamma}_{b\leftarrow d}(z',\mu_F^2,\epsilon)\,.$$

here, $(\Delta)\Gamma_{c\leftarrow a}$ and $\tilde{\Gamma}_{b\leftarrow d}$ are AP kernels (divergent) corresponding to initial and final leg respectively.

$$A(x')\otimes C(x',z')\tilde{\otimes}B(z') = \int_{x'}^{1}\frac{dx_1}{x_1}\int_{z'}^{1}\frac{dz_1}{z_1}A(x_1)C\left(\frac{x'}{x_1},\frac{z'}{z_1}\right)B(z_1)$$

Cross section Calculation: Polarized case

- For the polarized case, since we used Larin's scheme to compute the partonic cross sections, to cancel the initial-state collinear divergences, spin-dependent AP kernels in the same scheme were used.
- After getting the finite PCS, transformed the result to $\overline{\mathrm{MS}}$ scheme by overall renormalization constant.

$$
\begin{aligned}
g_1 =& \Delta f_c^L \otimes \Delta \mathcal{C}^L_{1,cb} \tilde{\otimes} D_b^{\overline{\mathrm{MS}}} \\
=& \Delta f_c^L \otimes Z_{ca} \otimes Z_{ad}^{-1} \otimes \Delta \mathcal{C}^L_{1,db} \tilde{\otimes} D_b^{\overline{\mathrm{MS}}} \\
=& \Delta f_a^{\overline{\mathrm{MS}}} \otimes \Delta \mathcal{C}^{\overline{\mathrm{MS}}}_{1,ab} \tilde{\otimes} D_b^{\overline{\mathrm{MS}}}
\end{aligned} \tag{22}
$$

- Z_{ac} is known upto NNLO in QCD[1,2], we also extracted for QED case.

[1] Matiounine, Smith, Neerven '98 [2] Moch, Vermaseren, Vogt '14

Results: NNLO

$$(\Delta)\mathcal{C}_i^{(i,j)} = \underbrace{\sum_r C_i^r h_r(x',z')}_{SV} + \sum_\beta \Big((\Delta) C^\beta_{i,x'}(x') Z_\beta(z') + (\Delta) C^\beta_{i,z'}(z') X_\beta(x') \Big) + (\Delta) R_i(x',z')$$

	Distributions	Constants/Functions
$h_r(x',z')$ $(q\gamma^* \to q)$	$\delta(1-z')\delta(1-x')$ $\delta(1-x')D_i(z'),\ i=0,\cdots,3$ $\delta(1-z')D_i(x')$ $D_j(x')D_j(z'),\ j=0,\cdots,2$	$\pi,\ \zeta_2,\zeta_3,\zeta_4$
$Z_\beta(z')$	$\delta(1-z')$ $D_j(z')$	$\pi,\zeta_2,\zeta_3,\log(f_{x'}),$ $Li_2(f_{x'}), Li_3(f_{x'})$
$X_\beta(x')$	$\delta(1-x')$ $D_j(x')$	$\pi,\zeta_2,\zeta_3,\log(f_{z'}),$ $Li_2(f_{z'}), Li_3(f_{z'})$
$R_I(x',z')$		$\pi,\log(f_{x',z'}),\zeta_2,\zeta_3,\zeta_4,$ $Li_2(f_{x',z'}),$ $Li_3(f_{x'}), Li_3(f_{z'})$

Results and Checks

- We calculated NNLO QCD$\otimes$QED corrections to SIDIS process, which requires evaluation of new master integrals.
- We agreed SV Limit of CFs i.e. terms containing double distributions against known results[1].
- Integrating z' for specific subprocess, we got inclusive results and are crosschecked against known in the literature[2].
- Independent calculation for CFs by another group, found numerical aggreement for (un)polarized QCD corrections.
- Verified abelianization, QCD$\rightarrow$ QCD$\otimes$QED[3,4].
- New QED Time-like splitting functions using SIDIS and using those, extracted polarized space-like splitting functions and cross check with [de Florian, Conte '25].

[1] Abele, de Florian, Vogelsang '21 [2] Neerven, Zijlstra '91

[3] de Florian, Der, Fabre '18

[4] Hameed, Banerjee, Chakraborty, Dhani, Mukherjee, Rana, Ravindran '19

QCD $\otimes$ QED Splitting functions

Time-Like Splitting functions (Preliminary):

$$\begin{aligned} \text{LO} &: \tilde{\mathrm{P}}^{(0,1)}_{qq}, \tilde{\mathrm{P}}^{(0,1)}_{q\gamma}, \tilde{\mathrm{P}}^{(0,1)}_{\gamma q}, \tilde{\mathrm{P}}^{(0,1)}_{\gamma\gamma}, \\ \text{NLO} &: \tilde{\mathrm{P}}^{(0,2)}_{\gamma q}, \tilde{\mathrm{P}}^{(0,2),\mathrm{NS}}_{qq}, \tilde{\mathrm{P}}^{(0,2),\mathrm{S}}_{qq}, \tilde{\mathrm{P}}^{(0,2)}_{q\bar{q}}, \tilde{\mathrm{P}}^{(1,1)}_{gq}, \tilde{\mathrm{P}}^{(1,1)}_{\gamma q} \end{aligned} \tag{23}$$

Space-like polarized Splitting functions[1]:

$$\begin{aligned} \text{LO} &: \Delta\mathrm{P}^{(0,1)}_{qq}, \Delta\mathrm{P}^{(0,1)}_{q\gamma}, \Delta\mathrm{P}^{(0,1)}_{\gamma q}, \Delta\mathrm{P}^{(0,1)}_{\gamma\gamma} \\ \text{NLO} &: \Delta\mathrm{P}^{(0,2)}_{q\gamma}, \Delta\mathrm{P}^{(0,2),\mathrm{NS}}_{qq}, \Delta\mathrm{P}^{(0,2),\mathrm{S}}_{qq}, \Delta\mathrm{P}^{(0,2)}_{q\bar{q}}, \Delta\mathrm{P}^{(1,1)}_{qg}, \Delta\mathrm{P}^{(1,1)}_{q\gamma} \end{aligned} \tag{24}$$

[1] de Florian, Conte '25 [2] de Florian, Sborlini, Rodrigo '15

[3] de Florian, Sborlini, Rodrigo '16

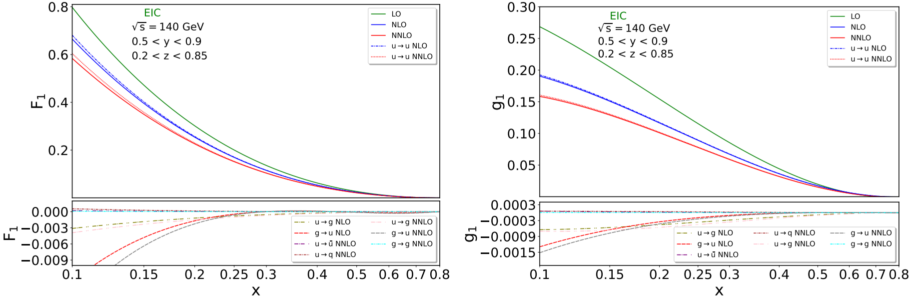


Figure: Contributions from all partonic channels to the SFs F_1 (left panel) and g_1 (right panel) as a function of x for the EIC at $\sqrt{s} = 140$ GeV.

- PDF sets: `NNPDF31`(F_1) and `BDSSV24NLO`, `BDSSV24NNLO`(g_1)
- FF set: `NNFF10PIp`
- $u \to u$ channel dominates, corrections are negative.

Plots

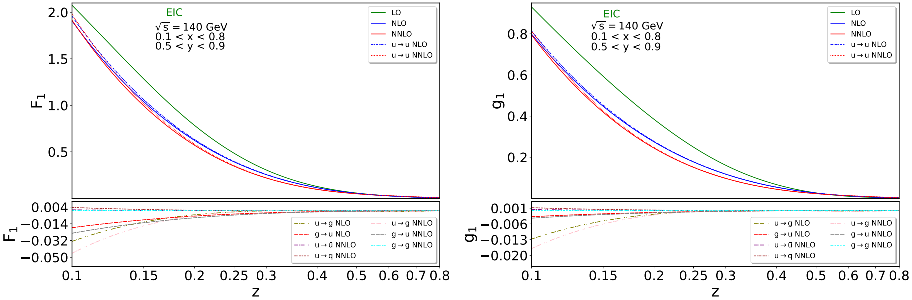


Figure: Now as a function of z

- PDF sets: NNPDF31(F_1) and BDSSV24NLO, BDSSV24NNLO(g_1)
- FF set: NNFF10PIp
- $u \to u$ channel dominates.

Plots

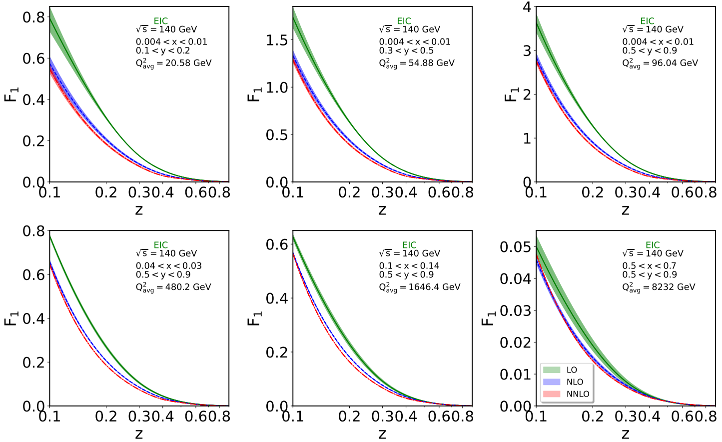
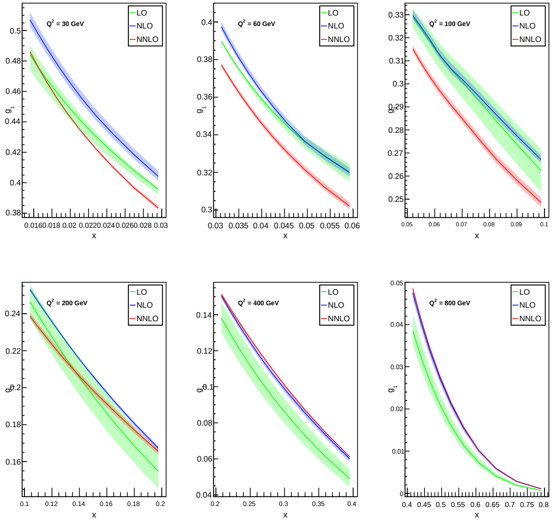


Figure: Scale variation of F_1 as a function z for six different values of Q^2 at EIC.

- Scale variation, $\mu_R^2, \mu_F^2 \in \left[Q^2_{\text{avg}}/2, 2Q^2_{\text{avg}}\right]$ with $1/2 \leq \mu_R^2/\mu_F^2 \leq 2$,
- PDF sets: `NNPDF31`, FF set: `NNFF10PIp`,

Plots



PDF set: `MAPPDF10NLO`, `MAPPDF10NNLO` and FF set: `MAPFF10NLO`, `MAPFF10NNLO`

Shows the 7-scale variation, the inclusion of NNLO corrections reduces the scale dependence when compared to the previous orders.

Comparison Plot

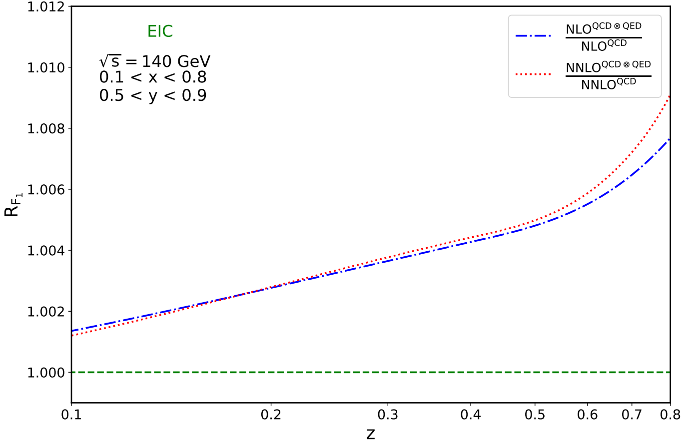


Figure: Ratio of F_1 as a function z at EIC.

- PDF sets: `LUXqed`, FF set: `NNFF10PIp`

Comparison Plot

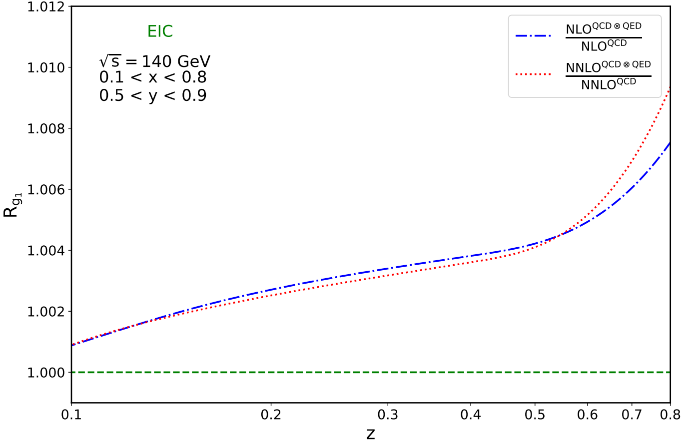


Figure: Ratio of g_1 as a function z at EIC.

- PDF sets: `BDSSV24`, FF set: `NNFF10PIp`

Conclusion and Future Plans

- EIC will unravel the mysteries of strong force. Theoretical precision studies are extremely necessary to fully exploit the EIC data.
- We calculated NNLO QCD$\otimes$QED corrections to SIDIS process.
- We calculated 20 new master integrals using different approaches.
- Extracted Time-like splitting functions.
- The NNLO predictions will be important to put more constraints on (pol)PDFs and FFs very precisely.
- Noticed significant reduction in the dependence of unphysical scales.
- We have resummed SV and NSV logarithms for diagonal channels in $\vec{N}$-space till $\overline{\text{NNLL}}$ accuracy[1].
- Work in progress: Numerical implementation of neutral and charged current interactions in SIDIS.

[1] see talk by V. Pathak

Thank you for listening :)

Reference slides:

Mass Factorization

After coupling constant renormalization at scale $\mu_R = \mu_F$, we can write mass factorisation, for eg. at NLO for $a, b = q$ process:

$$\left(\frac{1}{\mu_F^2}\right)^{\frac{\epsilon}{2}} d\hat{\sigma}^{(1)}_{1,qq} = \delta(1-x') \otimes \mathcal{C}^{(1)}_{1,qq} \tilde{\otimes} \delta(1-z') + \delta(1-x') \otimes \mathcal{C}^{(0)}_{1,qq} \tilde{\otimes} \tilde{\Gamma}^{(1)}_{qq} + \Gamma^{(1)}_{qq} \otimes \mathcal{C}^{(0)}_{1,qq} \tilde{\otimes} \delta(1-z'),$$

at NNLO for $a, b = q$ process:

$$\left(\frac{1}{\mu_F^2}\right)^{\epsilon} d\hat{\sigma}^{(2)}_{1,qq} + \frac{2\beta_0}{\epsilon}\left(\frac{1}{\mu_F^2}\right)^{\frac{\epsilon}{2}} d\hat{\sigma}^{(1)}_{1,qq} = \delta(1-x') \otimes \mathcal{C}^{(2)}_{1,qq} \tilde{\otimes} \delta(1-z') + \delta(1-x') \otimes \mathcal{C}^{(1)}_{1,qb'} \tilde{\otimes} \tilde{\Gamma}^{(1)}_{qb'} + \delta(1-x') \otimes \mathcal{C}^{(0)}_{1,qq} \tilde{\otimes} \tilde{\Gamma}^{(2)}_{qq} + \Gamma^{(1)}_{a'q} \otimes \mathcal{C}^{(0)}_{1,a'b'} \tilde{\otimes} \tilde{\Gamma}^{(1)}_{qb'} + \Gamma^{(1)}_{a'q} \otimes \mathcal{C}^{(1)}_{a'q} \tilde{\otimes} \delta(1-z') + \Gamma^{(2)}_{qq} \otimes \mathcal{C}^{(0)}_{1,qq} \tilde{\otimes} \delta(1-z').$$

Transformation Matrix $\hat{T}$

$$
\left(\begin{array}{cccccccccccccccccccc}
-\frac{(-1+xp)(-1+zp)}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{(1+ep)^2}{4\,ep^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1+ep}{2\,ep} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp}{4\,ep^2\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+zp)\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2}{2\,ep^2} & -\frac{(1+ep)^2}{ep^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2\,(1+ep)^2\,(-1+zp)}{zp} & \frac{(1+ep)^2\,(xp-zp)}{8\,zp} & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\,(1+ep)^2\,(1+xp) & \frac{(1+ep)^2\,(1+xp)\,(1+zp)}{4\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,\sqrt{xp}}{4\,ep^2\,\sqrt{zp}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,xp^2}{4\,ep^2\,(1+xp)\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp}{4\,ep^2\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{2\,ep^2\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(1+xp)\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,xp}{2\,ep^2\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp\,(-1+zp)}{4\,ep^2\,zp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & \frac{(1+ep)^2\,xp}{4\,ep^2\,zp} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+zp)\,zp} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,xp^2\,(1+zp)}{4\,ep^2\,(-1+xp)\,zp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp}
\end{array}\right)
$$

IBP-Tadpole

Lets Consider One Loop massive Integral (Tadpole diagram),

$$I_\nu = \int \frac{d^D k}{(k^2 - m^2)^\nu}$$

Using IBP identity,i.e.

$$\int d^D k \frac{\partial}{\partial k^\mu}\left[\frac{k^\mu}{(k^2 - m^2)^\nu}\right] = 0$$

we get,

$$(D - 2\nu) I_\nu - 2\nu m^2 I_{\nu+1} = 0$$
$$I_\nu = \frac{(D - 2\nu + 2)}{2(\nu - 1) m^2} I_{\nu-1}$$

So any integral with $\nu \geq 1$ can be expressed recursively in terms of one integral I_1 (MI)