Next-to-Soft-virtual resummation for SIDIS to NNLO+$\overline{\text{NNLL}}$

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Matter To The Deepest, University of Silesia, Katowice, Poland

Outline

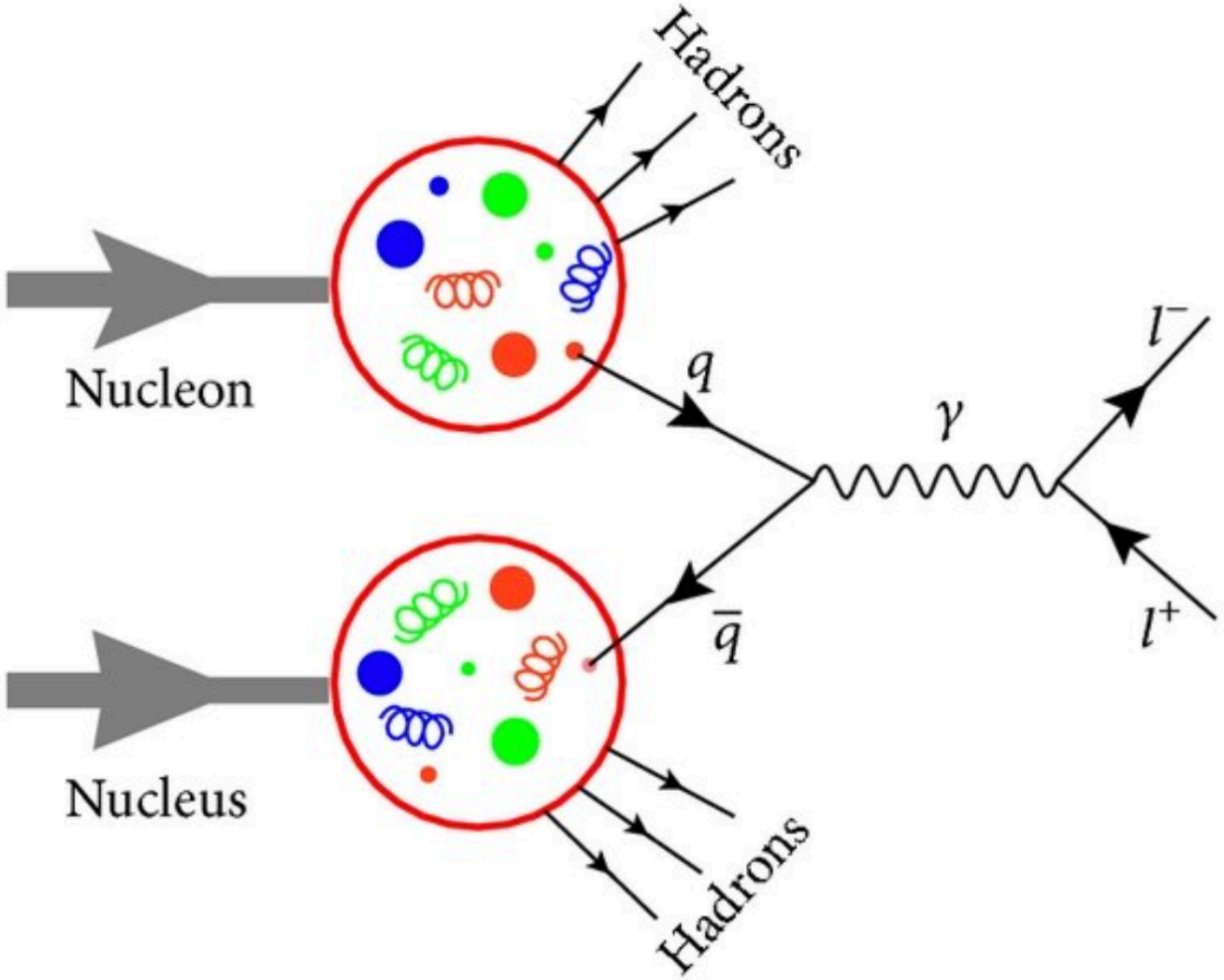
- Introduction
- Taking Forward…
- Threshold Expansion
- Formalism
- Predictions
- All Order Behaviour
- Numerical Result
- Summary and Future directions

Introduction

- Processes with identified final state hadrons play important roles in QCD. They provide crucial information on the Time-like splitting function and fragmentation function.
- Hadron production serves as a powerful probe of nucleon or nuclear structure.
- Hadron production data tests our key concepts in QCD at high energies such as factorization, universality of splitting functions, and perturbative calculations.
- Because electrons do not manifest any internal structure, they can be used as a precise probe of the more complicated nucleons and nuclei.

Drell-Yan Production at LHC

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2)\int \frac{dz}{z}\Phi_{ab}\left(\frac{\tau}{z},\mu_F^2\right)\Delta_{ab}(q^2,\mu_F^2,z)$$



Parton Distribution Function

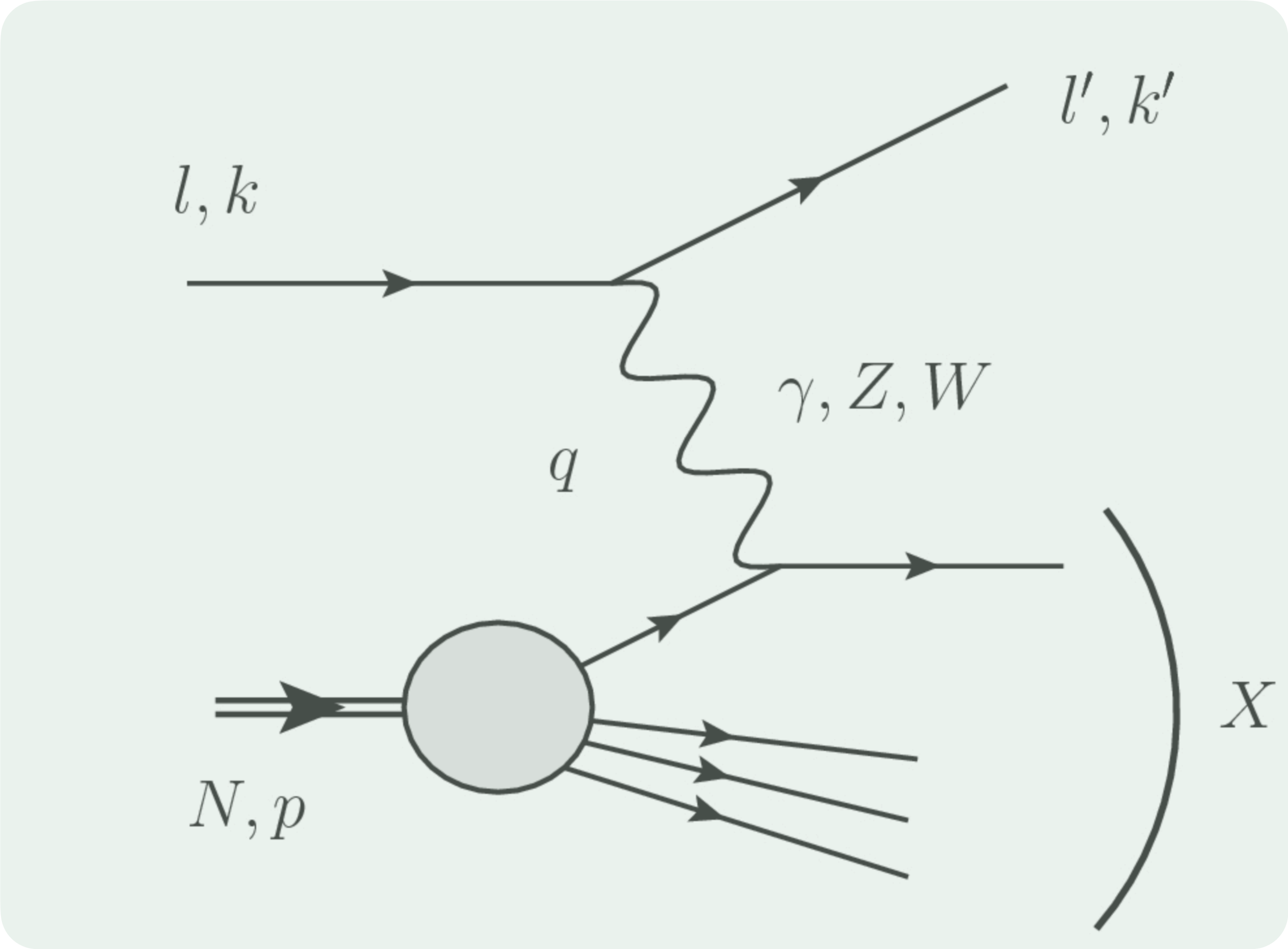
$$\Phi_{ab}(\mu_F^2,z) = \int \frac{dy}{y} f_a(y,\mu_F^2) f_b\left(\frac{z}{y},\mu_F^2\right)$$

Role of DIS

Inclusive DIS (Deep Inelastic Scattering),

lepton + hadron ⟶ lepton + X

One sums up all the particles in the final state, except the scattered lepton



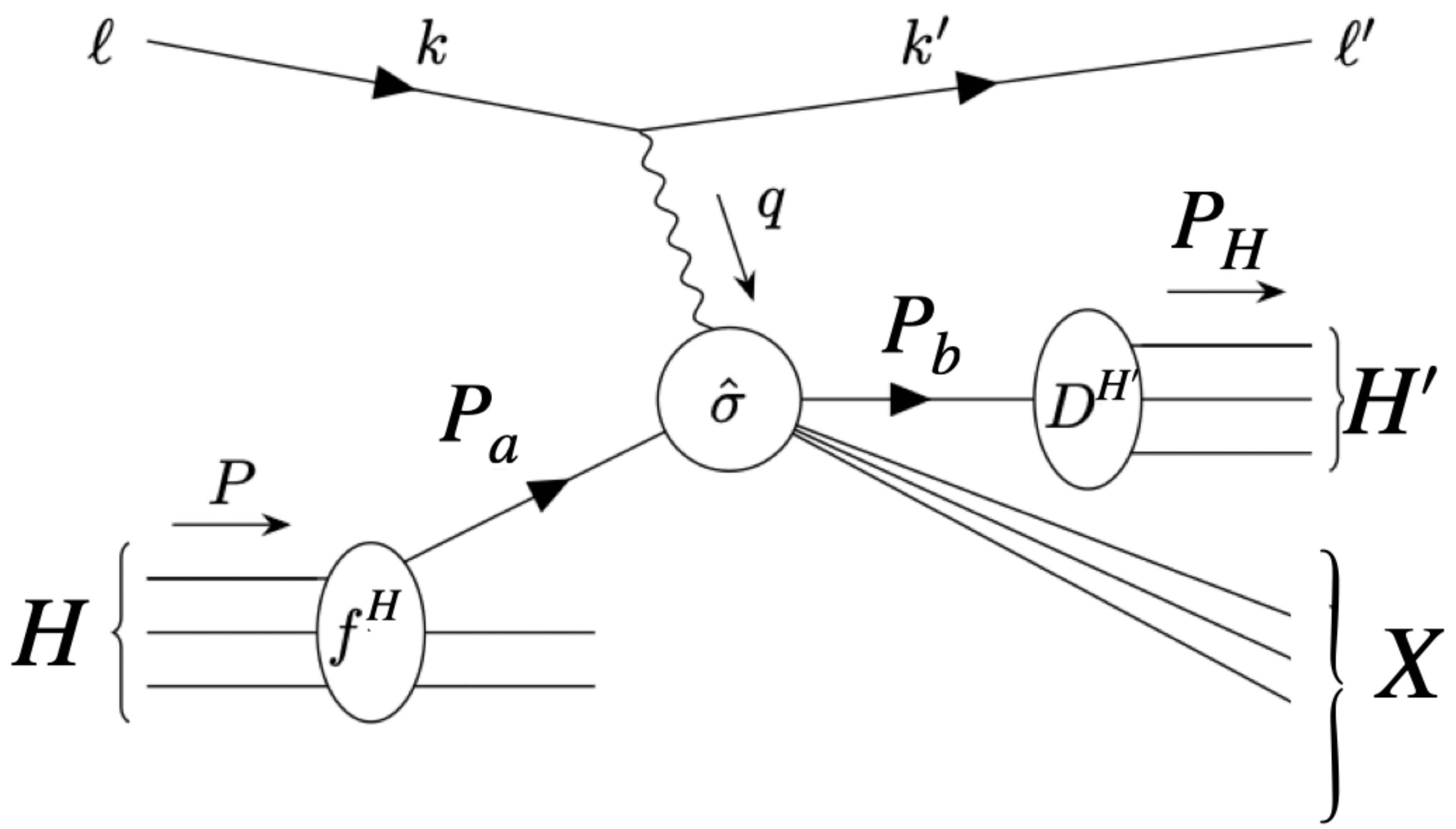
Depends on Parton Distribution Function (PDF) of Incoming hadron.

SIDIS ?

SIDIS (Semi-Inclusive Deep Inelastic Scattering),

lepton + hadron $\longrightarrow$ lepton + hadron + X

In SIDIS, in addition to the scattered lepton, we tag on one of the outgoing hadron



Depends on Parton Distribution Function (PDF) of Incoming hadron and Fragmentation Function (FF) of Outgoing hadron.

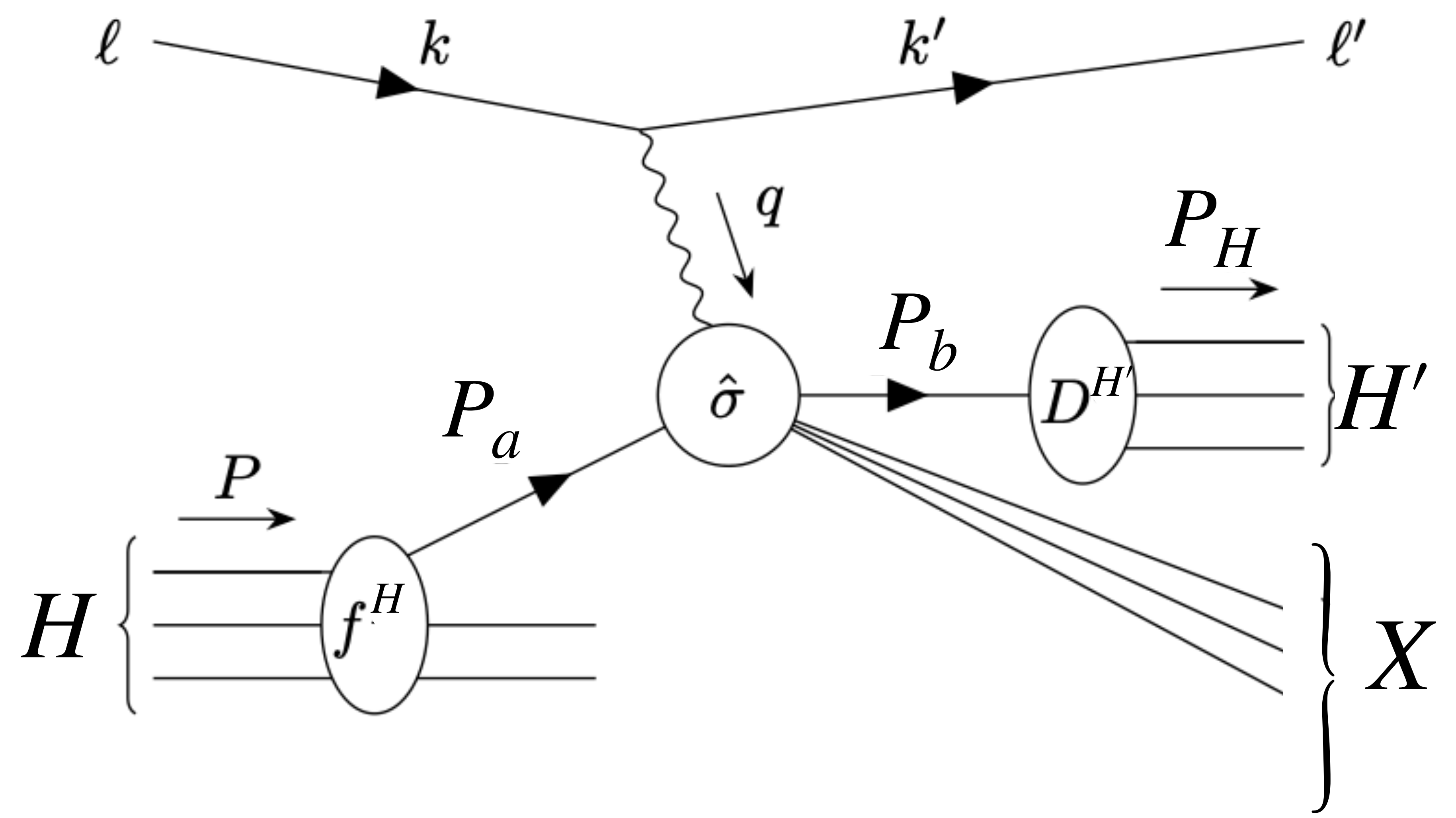
Theoretical Framework

- Semi-Inclusive Deep Inelastic process,

$$l(k_l) + H(P) \to l(k'_l) + H'(P_H) + X$$

- At Partonic level cross-section,

$$q/g(P_a) + \gamma^*(q) \to q/g(P_b) + \mathcal{X}$$



where $P_a^2 = P_b^2 = 0$ and $q^2 = -Q^2$. Our kinematic variables are

$$x = \frac{Q^2}{2P.q} \Rightarrow x' = \frac{Q^2}{2P_a.q}$$ and $$z = \frac{P.P_H}{P.q} \Rightarrow z' = \frac{P_a.P_b}{P_a.q}$$

Taking Forward …

- Processes involving fragmentation functions beyond leading order in QCD [Altarelli, Ellis, Martinelli, So-Young, Pi ’79]
- Approximate NNLO QCD corrections to semi-inclusive DIS [Abele, De Florian, Vogelsang ’21]
- Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering [Goyal, Moch, VP, Rana, Ravindran ‘24]
- Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD [Bonino, Gehrmann, Stagnitto ‘24]
- Next-to-Next-to-Leading Order QCD Corrections to Polarized Semi-Inclusive Deep-Inelastic Scattering [Bonino, Gehrmann , Löchner , Schönwald, Stagnitto '24]

Taking Forward …

- Next-to-Next-to-Leading Order QCD Corrections to Polarized Semi-Inclusive Deep-Inelastic Scattering [Goyal, Lee, Moch, VP, Rana, Ravindran ‘24]

- NNLO phase-space integrals for semi-inclusive deep-inelastic scattering [Ahmed, Goyal, Hasan, Lee, Moch, VP, Rana, Rapakoulias, Ravindran ‘25] [1]

- NNLO QCD corrections to unpolarized and polarized SIDIS [Goyal, Lee, Moch, VP, Rana, Ravindran ‘25] [2]

- Single-valued representation of unpolarized and polarized semi-inclusive deep inelastic scattering at next-to-next-to-leading order [Haug, Wunder ‘25]

1* See talk by A. Rapakoulias, 2* See talk by S. Goyal

SIDIS with Photon Exchange

- The Unpolarised cross section can be written as,

$$\frac{d^3\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}\left[y\ F_1(x,z,Q^2) + \frac{(1-y)}{y}\ F_2(x,z,Q^2)\right]$$

F_J are called structure functions and E is the energy of the incoming lepton and y is the inelasticity.

- Similarly, the spin-dependent cross-section is found to be

$$\frac{d^3\Delta\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}(2-y)\ g_1(x,z,Q^2)$$

- Note that g_2 does not contribute since we restrict ourselves to longitudinally polarized hadron in the initial state.

Threshold Expansion

In pQCD, The Structure Function can be factorised as :

$$(g_1)F_i(x,z,Q^2) = \sum_{a,b=q,\bar{q},g} \int_x^1 \frac{dx_1}{x_1} \underbrace{(\Delta)f_{a/\mathrm{P}}\left(x_1,\mu_F^2\right)}_{\text{Parton distribution function}} \int_z^1 \frac{dz_1}{z_1} \underbrace{D_{\mathrm{H'/b}}\left(z_1,\mu_F^2\right)}_{\text{Fragmentation function}} \underbrace{(\Delta)\mathscr{C}_{i,ab}}_{\text{Coefficient function}}\left(\frac{x}{x_1},\frac{z}{z_1},Q^2,\mu_F^2\right)$$

Perturbative Structure of coefficient function at threshold : x' →1, z'→1

$$\underbrace{\mathscr{C}_i}_{\text{Finite}} \rightarrow \mathscr{C}_i^{virt}\,\delta(1-x')\delta(1-z') + \sum_{k,l} D_k(x')D_l(z')\mathscr{C}_{i,kl}(x',z') \;\; + \underbrace{\mathscr{O}(1-x')(1-z')}_{\text{Beyond NSV}}$$

- Our goal is to calculate the perturbatively calculable quantity

$$(\Delta)\mathscr{C}_{i,ab}\left(\frac{x}{x_1},\frac{z}{z_1},Q^2,\mu_F^2\right)$$

Why Resummation?

- The Fixed Order CFs fail in certain regions because of presence of Large Logarithms.

- In the threshold region, fixed order CFs at each order in perturbation become comparable.

- These Logarithms originate from the partonic configuration where partons are Soft and/or Collinear to each other.

- To make sensible predictions, we need to resum these large Logarithms to all orders in perturbation theory.

Threshold Expansion

Perturbative Structure of coefficient function at threshold : x' →1, z'→1

$$\mathscr{C}_i \to \mathscr{C}_i^{virt}\, \delta(1-x')\delta(1-z') + \sum_{k,l} D_k(x')D_l(z')\mathscr{C}_{i,kl}(x',z') \;+ \mathscr{O}(1-x')(1-z')$$

Finite *Beyond NSV*

$$\delta(1-x')\left(\frac{\ln^i(1-z')}{1-z'}\right)_+$$

$$\left(\frac{\ln^i(1-x')}{1-x'}\right)_+\left(\frac{\ln^j(1-z')}{1-z'}\right)_+$$

Soft-Virtual (SV)

- Only diagonal Channels

$$\delta(1-x')\ln^i(1-z')$$

$$\left(\frac{\ln^i(1-x')}{1-x'}\right)_+ \ln(1-z')$$

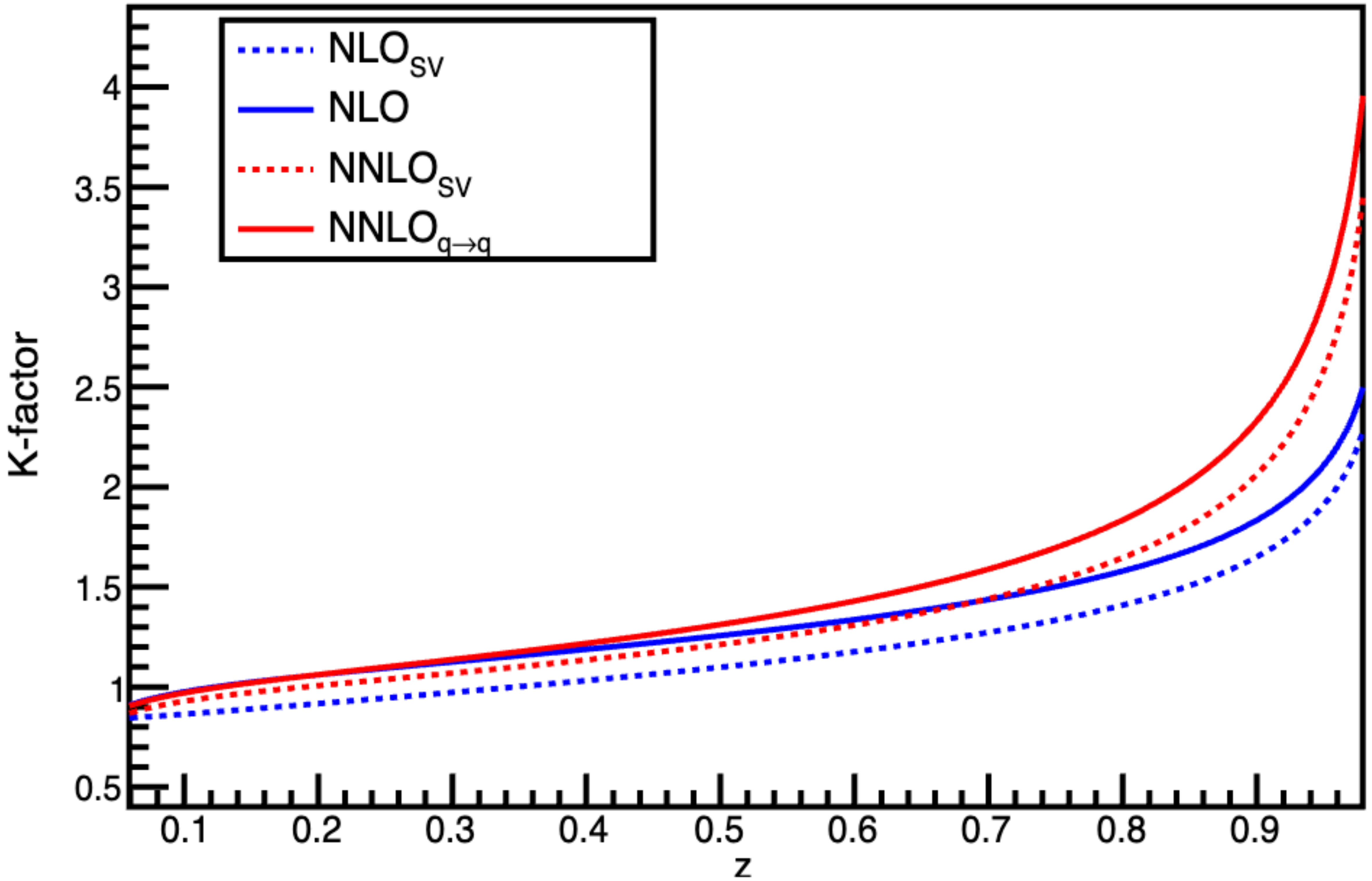
Next to Soft-Virtual (NSV)

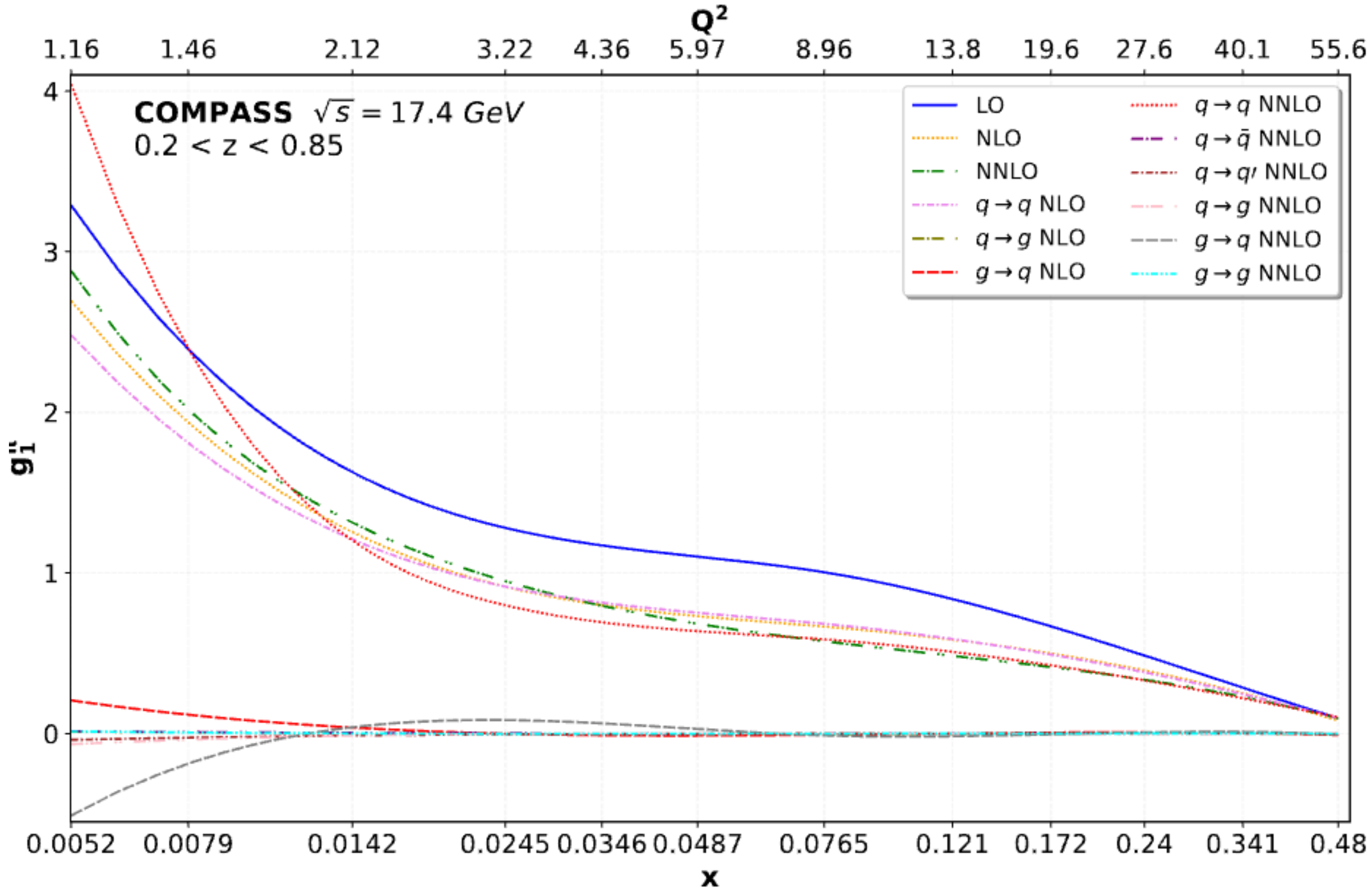
Suppressed to SV

- Diagonal and off-Diagonal Channels

These distributions give large contribution in threshold region,Hence we need to resum these distributions upto NSV.

Threshold Expansion

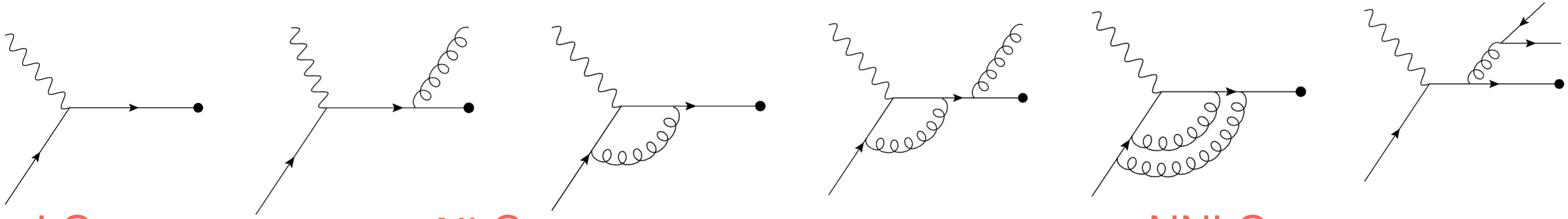




- From Plots, One can easily tell that contribution of $q \longrightarrow q$ channel is dominant. That's why we are resumming this channel only.
- SV part is contributing comparable to $q \longrightarrow q$ channel. Hence, to make more precise we want to include Next to SV part also.

Taking Forward …

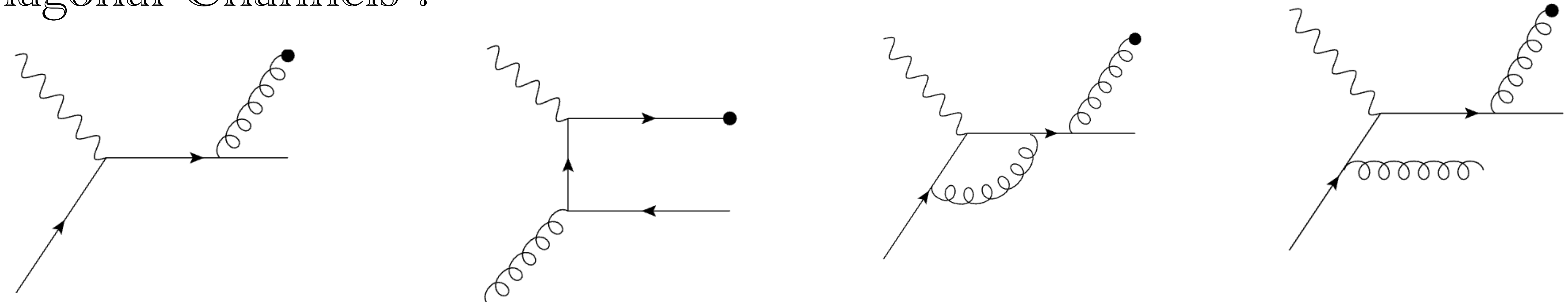
Diagonal Channels :



LO NLO NNLO

- Diagonal Channels gives SV + NSV + higher Order terms.

Off-Diagonal Channels :



- Off-Diagonal Channels gives NSV + higher Order terms.

Formalism

The Decomposition formula for Diagonal CFs can be written as :

$$(\Delta)\mathscr{C}^{\mathrm{SV+NSV}}_{J,qq} = \left|\hat{F}_q(Q^2,\varepsilon)\right|^2 \otimes \left((\Delta)\Gamma\right)^{-1}_{q\leftarrow q}(x',\mu_F^2,\varepsilon) \otimes (\Delta)\hat{\mathcal{S}}_{J,qq}(Q^2,x',z',\varepsilon) \otimes \left(\tilde{\Gamma}\right)^{-1}_{q\leftarrow q}(z',\mu_F^2,\varepsilon)$$

- $\left|\hat{F}_q(Q^2,\varepsilon)\right|^2$: FormFactor, coming from virtual correction.
- $\left((\Delta)\Gamma\right)^{-1}_{q\leftarrow q}(x',\mu_F^2,\varepsilon)$: Space-like AP kernels, coming initial-state collinear singularity.
- $(\Delta)\hat{\mathcal{S}}_{J,qq}(Q^2,x',z',\varepsilon)$: Soft function, coming from threshold limit of real emission diagrams.
- $\left(\tilde{\Gamma}\right)^{-1}_{q\leftarrow q}(z',\mu_F^2,\varepsilon)$: Time-like AP kernels, coming from final-state collinear singularity.

Each building block obeys 1st Order Evolution differential equations wrt (μ_F^2 or Q^2).

Formalism

Form Factor, $\hat{F}_c$:

Satisfies: $$Q^2 \frac{d}{dQ^2} \ln \hat{F}_c(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \frac{1}{2}\Bigg(\underbrace{K\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)}_{\text{Pole}} + \underbrace{G\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)}_{\text{Finite}} \Bigg)$$

Functional Form: $$\ln \hat{F}_c(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=-\infty}^{i+1} \mathscr{L}_c^{(i,j)} \frac{1}{\epsilon^j}$$

Expressed in terms of : $$\mathscr{L}_c^{(i,j)} = \{A^c, B^c, f^c, g^c\}$$

Process Independent : $\{A^c, B^c, f^c\}$ Process dependent : g^c

Formalism

DGLAP Kernel, Γ_{cc} and $\tilde{\Gamma}_{cc}$:

★ Collinear Singular Terms

★ Evolution Equation: $\mu_F^2 \dfrac{d}{d\mu_F^2}\Gamma_{cc}(\mu_F^2,\xi) = \dfrac{1}{2}\mathrm{P}_{cc}(\mu_F^2,\xi)\otimes\Gamma_{cc}(\mu_F^2,\xi)$

★ Functional Form:

$$\mathrm{P}_{cc}(\mu_F^2,\xi) = 2\left(\underbrace{\frac{A^c(\mu_F^2)}{(1-\xi)_+} + B^c(\mu_F^2)\delta(1-\xi)}_{SV} + \underbrace{\mathrm{C}^c(\mu_F^2)\ln(1-\xi) + \mathrm{D}^c(\mu_F^2)}_{NSV}\right)$$

★ A and B are same for Space-like and Time-like splitting function but C and D are different for Space-like and Time-like splitting function.

Formalism

Soft-Collinear Function, $\hat{\mathcal{S}}_c$:

Functional Form of ansatz:

$$\frac{1}{2}\ln\mathcal{S}(Q^2,x',z',\varepsilon)=\sum_{i=1}^{\infty}\hat{a}_s^i\left(\frac{Q^2(1-x')(1-z')}{\mu^2}\right)^{i\frac{\varepsilon}{2}}S_\varepsilon^i\left[\frac{(i\varepsilon)^2}{4(1-x')(1-z')}\hat{\varphi}_{d,q}^{SV,(i)}(\varepsilon)+\frac{i\varepsilon}{4(1-x')}\hat{\varphi}_{d,z',q}^{NSV,(i)}(z',\varepsilon)+\frac{i\varepsilon}{4(1-z')}\hat{\varphi}_{d,x',q}^{NSV,(i)}(x',\varepsilon)\right],$$

With the help of energy evolution equation of $\mathcal{S}(Q^2,x',z,\varepsilon)$, we derive its functional form till 4-loop.

Similar to Form-Factor it also satisfies:

$$Q^2\frac{d}{dQ^2}\ln\mathcal{S}(Q^2,x',z',\varepsilon)=\frac{1}{2}\Bigg(\underbrace{\overline{K}\Big(\hat{a}_s,\frac{\mu_R^2}{\mu^2},\varepsilon\Big)}_{\text{Pole}}+\underbrace{\overline{G}\Big(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\varepsilon\Big)}_{\text{Finite}}\Bigg)$$

All order behaviour

In order to study all-order behaviour, we formulated an integral representation for

$$(\Delta)\mathscr{C}^{\rm sv+nsv}_{J,qq} = \mathscr{C}\exp\left(\Psi^q_d(Q^2,\mu_F^2,x',z',\varepsilon)\right)\Bigg|_{\varepsilon=0} ,$$

$$\Psi^q_d = \frac{\delta(\bar{x}')}{2}\left(\left\{\int_{\mu_F^2}^{Q^2\bar{z}'}\frac{d\lambda^2}{\lambda^2}\mathscr{P}^q\left(a_s(\lambda^2),\bar{z}'\right) + \mathscr{Q}^q\left(a_s(Q_2^2),\bar{z}'\right)\right)_+ + \frac{1}{4}\left(\frac{1}{\bar{x}'}\left\{\mathscr{P}^q\left(a_s(Q_{12}^2),\bar{z}'\right) + 2\tilde{L}^q\left(a_s(Q_{12}^2),\bar{z}'\right)\right.\right.$$

$$\left.\left. +Q^2\frac{d}{dQ^2}\left(\mathscr{Q}^q\left(a_s(Q_2^2),\bar{z}'\right) + 2\varphi^q_{d,z',f}\left(a_s(Q_2^2),\bar{z}'\right)\right)\right\}\right)_+ + \frac{1}{2}\delta(\bar{x}')\,\delta(\bar{z}')\ \ln\left(g^q_{d,0}\left(a_s(\mu_F^2)\right)\right) + \left(\bar{x}'\leftrightarrow\bar{z}'\right)$$

- $g^q_{d,0}$ is process dependent and get contribution from FF and $\mathscr{S}^q$
- $\mathscr{P}^q$: From Splitting Kernel, after pole cancellation from Γ_{qq} and $\mathscr{S}^q$
- $\mathscr{Q}^q$: Finite part of SV and NSV from $\mathscr{S}^q$

All order behaviour

In order to study all-order behaviour, we formulated an integral representation for

$(\Delta)\mathscr{C}_{i,qq}^{\mathrm{SV+NSV}}$

$$\ln\mathscr{C}^{(SV+NSV)} = \frac{\delta(\bar{x}')}{2}\left(\left\{\int_{\mu_F^2}^{Q^2\bar{z}'}\frac{d\lambda^2}{\lambda^2}\mathscr{P}^q\left(a_s(\lambda^2),\bar{z}'\right)+\mathscr{Q}^q\left(a_s(Q_2^2),\bar{z}'\right)\right)_+ + \frac{1}{4}\left(\frac{1}{\bar{x}'}\left\{\mathscr{P}^q\left(a_s(Q_{12}^2),\bar{z}'\right)+2\tilde{L}^q\left(a_s(Q_{12}^2),\bar{z}'\right)\right.\right.$$

$$\left.\left.+Q^2\frac{d}{dQ^2}\left(\mathscr{Q}^q\left(a_s(Q_2^2),\bar{z}'\right)+2\varphi^q_{d,z',f}\left(a_s(Q_2^2),\bar{z}'\right)\right)\right\}\right)_+ + \frac{1}{2}\delta(\bar{x}')\,\delta(\bar{z}')\ln\left(g^q_{d,0}\left(a_s(\mu_F^2)\right)\right)+\left(\bar{x}'\leftrightarrow\bar{z}'\right)$$

$$\mathscr{P}^q(a_s,z') = \boxed{2A^c(a_s)\mathscr{D}_0} + \boxed{2\tilde{L}(a_s(q_{12}^2,\overline{z'}))}$$

$$\mathscr{Q}^q_d(a_s,\bar{z}') = \boxed{\frac{2}{\bar{z}'}G^q_d(a_s)} + \boxed{2\varphi^q_{d,z',f}(a_s,\bar{z}')}$$

SV NSV

$$\tilde{L}^q = C(a_s)\ln(\overline{z'}) + D(a_s)$$

$\overline{x'} = 1-x', \overline{z'} = 1-z'$

$\mathrm{Q}_1^2 = \mathrm{Q}^2\overline{x'}, \mathrm{Q}_2^2 = \mathrm{Q}^2\overline{z'}$

SV Predictions

- Due to the differential equations that $F, \mathcal{S}, \Gamma$ satisfies, the CFs $(\Delta)\mathscr{C}_i^{(SV+NSV)}$ exhibit an exponential structure, which helps to predict certain higher order terms.

GIVEN				PREDICTIONS FOR SV		
$\Psi_d^{q,(1)}$	$\Psi_d^{q,(2)}$	$\Psi_d^{q,(3)}$	$\Psi_d^{q,(n)}$	$(\Delta)\mathcal{C}_{J,cc}^{(2)}$	$(\Delta)\mathcal{C}_{J,cc}^{(3)}$	$(\Delta)\mathcal{C}_{J,cc}^{(i)}$
$\delta_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=0$ $j'=1,0$				$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2, \{1\}$ $j'=3, \{2\}$	$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=4, \{3\}$ $j'=5, \{4\}$	$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2i-2, \{2i-3\}$ $j'=2i-1, \{2i-2\}$
	$\delta_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2,1,0$ $j'=3,\ldots,0$				$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=4,3,\{2,1\}$ $j'=5,4,\{3,2\}$	$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2i-2,\ldots,\{2i-4, 2i-5\}$ $j'=2i-1,\ldots,\{2i-3, 2i-4\}$
		$\delta_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=4,\ldots,0$ $j'=5,\ldots,0$				$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2i-2,\ldots,\{2i-5,\ldots,2i-7\}$ $j'=2i-1,\ldots,\{2i-4,\ldots,2i-6\}$
			$\delta_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}$, $\mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2n-2,\ldots,0$ $j'=2n-1,\ldots,0$			$\mathcal{D}^j_{\overline{x}'}\mathcal{D}^k_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{x}'}\delta_{\overline{z}'}, \mathcal{D}^{j'}_{\overline{z}'}\delta_{\overline{x}'}$ $j+k=2i-2,\ldots,\{2i-n-2,\ldots,2i-2n-1\}$ $j'=2i-1,\ldots,\{2i-n-1,\ldots,2i-2n\}$

NSV Predictions

GIVEN, SV +				PREDICTIONS FOR NSV		
$\Psi_d^{q,(1)}$	$\Psi_d^{q,(2)}$	$\Psi_d^{q,(3)}$	$\Psi_d^{q,(n)}$	$(\Delta)\mathcal{C}_{J,cc}^{(2)}$	$(\Delta)\mathcal{C}_{J,cc}^{(3)}$	$(\Delta)\mathcal{C}_{J,cc}^{(i)}$
$\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k},$ $\mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $j+k=0$ $j'=1,0$				$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=2$ $j'=3$	$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=4$ $j'=5$	$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}, \mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=2i-2$ $j'=2i-1$
	$\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k},$ $\mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $j+k=2,1,0$ $j'=3,\ldots,0$				$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=4,3,\{2\}$ $j'=5,4$	$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}, \mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=2i-2, 2i-3, \{2i-4\}$ $j'=2i-1, 2i-2$
		$\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k},$ $\mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $j+k=4,\ldots,0$ $j'=5,\ldots,0$				$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}, \mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=2i-2,\ldots,\{2i-5, 2i-6\}$ $j'=2i-1,\ldots,2i-3$
			$\mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}$ $j+k=2n-2,\ldots,0$ $j'=2n-1,\ldots,0$			$\mathcal{D}_{\overline{x}'}^{j}\mathcal{L}_{\overline{z}'}^{k}, \mathcal{D}_{\overline{z}'}^{j}\mathcal{L}_{\overline{x}'}^{k}, \mathcal{L}_{\overline{x}'}^{j'}\delta_{\overline{z}'}, \mathcal{L}_{\overline{z}'}^{j'}\delta_{\overline{x}'}$ $j+k=2i-2,\ldots,\{2i-n-2,\ldots,2i-2n\}$ $j'=2i-1,\ldots,2i-n$

where, $\delta_{\overline{\xi}} = \delta(1-\xi)$, $\mathscr{D}_{\overline{\xi}}^{j} = \left[\dfrac{\ln^j(1-\xi)}{1-\xi}\right]_+$ and $\mathscr{L}_{\overline{\xi}}^{j} = \ln^j(1-\xi)$ with $\xi = x', z'$.

NSV in Mellin Space

- To convert the convoluted exponent into normal exponent we need to go into Mellin-Space.
- Solving the integral repesentation in Mellin Space, we get :

$$(\Delta)C^{\vec{N}}_{J,qq} = \int_0^1 dx' x'^{N_1-1} \int_0^1 dz' z'^{N_2-1} (\Delta)\mathscr{C}^{\mathrm{sv+nsv}}(x', z')$$

- Threshold limit $\{x', z'\} \longrightarrow \{1,1\}$ in Mellin space corresponds to $\{N_1, N_2\} \longrightarrow \{\infty, \infty\}$
- Taking till 1/N corrections from SV and NSV terms :

SV **NSV**

$$\left(\frac{\ln(1-z')}{1-z'}\right)_+ \rightarrow \boxed{\frac{\ln^2 N_2}{2}} - \boxed{\frac{\ln N_2}{2N_2} + \frac{1}{2N_2}} + \mathcal{O}\left(\frac{1}{N_2^2}\right)$$

$$\ln^k(1-z') \rightarrow \boxed{\frac{\ln^k N_2}{2N_2}} + \mathcal{O}\left(\frac{1}{N_2^2}\right)$$

NSV in Mellin Space

SV NSV

$$(\Delta)C^{\vec{N}}_{J,qq} = 1 + a_s\left[c_1^2\ \ln^2 N_1 N_2 + \ldots + c_1^0 + d_1^1\ \frac{\ln^2 N_1 N_2}{N_1} + \ldots + d_1^0 \frac{1}{N_1} + \mathcal{O}(\frac{1}{N_1^2})\right]$$

$$+a_s^2\left[c_2^4\ \ln^4 N_1 N_2 + \ldots + c_2^0 + d_2^3\ \frac{\ln^3 N_1 N_2}{N_1} + \ldots + d_2^0 \frac{1}{N_1} + \mathcal{O}(\frac{1}{N_1^2})\right]$$

$$+\ldots+$$

$$+a_s^n\left[c_n^{2n}\ \ln^{2n} N_1 N_2 + \ldots + c_n^0 + d_2^{2n-1}\ \frac{\ln^{2n-1} N_1 N_2}{N_1} + \ldots + d_n^0 \frac{1}{N_1} + \mathcal{O}(\frac{1}{N_1^2})\right]$$

$$+\ (N_1 \leftrightarrow N_2)$$

NSV in Mellin Space

- Solving the integral repesentation in Mellin Space, we get :

$$(\Delta)C^{\vec{N}}_{J,qq} = \tilde{g}^q_{d,0}(Q^2,\mu_F^2)\exp\left(\overline{G}^{\vec{N}}_{d,q}(Q^2,\mu_F^2)\right)$$

- Here the resumed expression take the following form :

$$\overline{G}^{\vec{N}}_{d,q} = \underbrace{g^q_{d,1}(\omega)\ln N_1 + \sum_{i=0}^{\infty} a_s^i\left(\frac{1}{2}g^q_{d,i+2}(\omega)\right.}_{SV} + \underbrace{\left.\frac{1}{N_1}\overline{g}^q_{d,i}(\omega)\right) + \frac{1}{N_1}\left(h^q_{d,0}(\omega,N_1) + \sum_{i=1}^{\infty} a_s^i h^q_{d,i}(\omega,\omega_1,N_1)\right.}_{NSV}\left.\vphantom{\sum_{i=1}^{\infty}}\right) + (N_1 \leftrightarrow N_2, \omega_1 \leftrightarrow \omega_2)$$

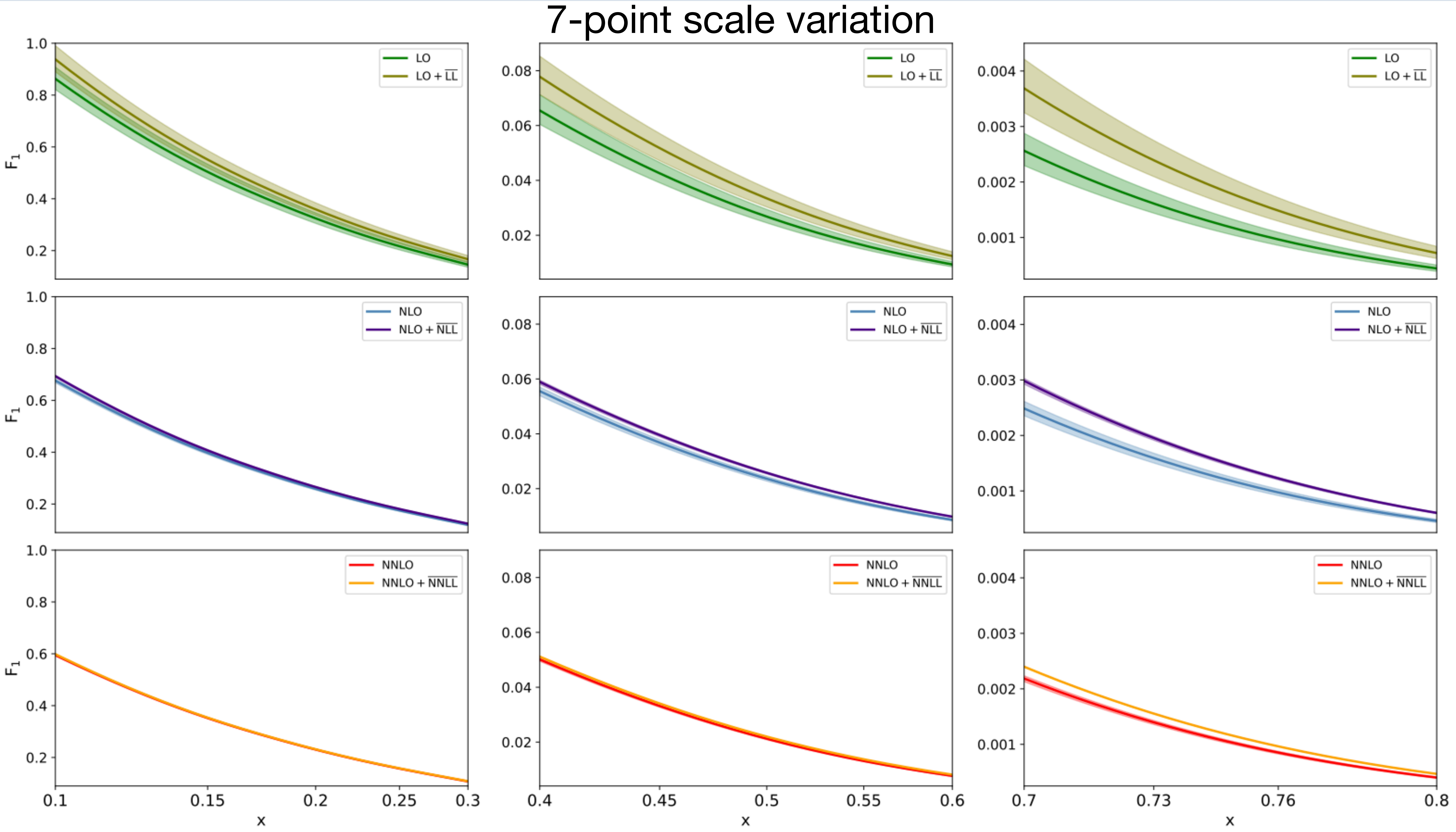
where $\omega = a_s\beta_0 \ln N_1 N_2$ and $\omega_l = a_s\beta_0 \ln N_l$

All order prediction

GIVEN	PREDICTIONS - SV and NSV Logarithms					Logarithmic Accuracy
Resummed Exponents Upto	$\mathcal{C}^{(2)}_{d,qq,\vec{N}}$	$\mathcal{C}^{(3)}_{d,qq,\vec{N}}$	$\mathcal{C}^{(4)}_{d,qq,\vec{N}}$	$\cdots$	$\mathcal{C}^{(n)}_{d,qq,\vec{N}}$	
$\tilde{g}^q_{d,0,0}, g^q_{d,1},$ $\bar{g}^q_{d,0}, h^q_{d,0}$	$L_1^i L_2^j\|_{i+j=4}$ $L_{N_1}^i L_2^j\|_{i+j=3}$ $L_1^i L_{N_2}^j\|_{i+j=3}$	$L_1^i L_2^j\|_{i+j=6}$ $L_{N_1}^i L_2^j\|_{i+j=5}$ $L_1^i L_{N_2}^j\|_{i+j=5}$	$L_1^i L_2^j\|_{i+j=8}$ $L_{N_1}^i L_2^j\|_{i+j=7}$ $L_1^i L_{N_2}^j\|_{i+j=7}$	$\cdots$	$L_1^i L_2^j\|_{i+j=2n}$ $L_{N_1}^i L_2^j\|_{i+j=2n-1}$ $L_1^i L_{N_2}^j\|_{i+j=2n-1}$	$\overline{\text{LL}}$
$\tilde{g}^q_{d,0,1}, g^q_{d,2}$ $\bar{g}^q_{d,1}, h^q_{d,1}$		$L_1^i L_2^j\|_{i+j=6,5,4}$ $L_{N_1}^i L_2^j\|_{i+j=5,4}$ $L_1^i L_{N_2}^j\|_{i+j=5,4}$	$L_1^i L_2^j\|_{i+j=8,7,6}$ $L_{N_1}^i L_2^j\|_{i+j=7,6}$ $L_1^i L_{N_2}^j\|_{i+j=7,6}$	$\cdots$	$L_1^i L_2^j\|_{i+j=2n,\ldots,2n-2}$ $L_{N_1}^i L_2^j\|_{i+j=2n-1,2n-2}$ $L_1^i L_{N_2}^j\|_{i+j=2n-1,2n-2}$	$\overline{\text{NLL}}$
$\tilde{g}^q_{d,0,2}, g^q_{d,3}$ $\bar{g}^q_{d,2}, h^q_{d,2}$			$L_1^i L_2^j\|_{i+j=8,\ldots,4}$ $L_{N_1}^i L_2^j\|_{i+j=7,\ldots,5}$ $L_1^i L_{N_2}^j\|_{i+j=7,\ldots,5}$	$\cdots$	$L_1^i L_2^j\|_{i+j=2n,\ldots,2n-4}$ $L_{N_1}^i L_2^j\|_{i+j=2n-1,\ldots,2n-3}$ $L_1^i L_{N_2}^j\|_{i+j=2n-1,\ldots,2n-3}$	$\overline{\text{NNLL}}$

$$L_1^i = \ln^i(N_1), L_{N_1}^i = \frac{\ln^i(N_1)}{N_1}, L_2^j = \ln^j(N_2), L_{N_2}^j = \frac{\ln^j(N_2)}{N_2}$$

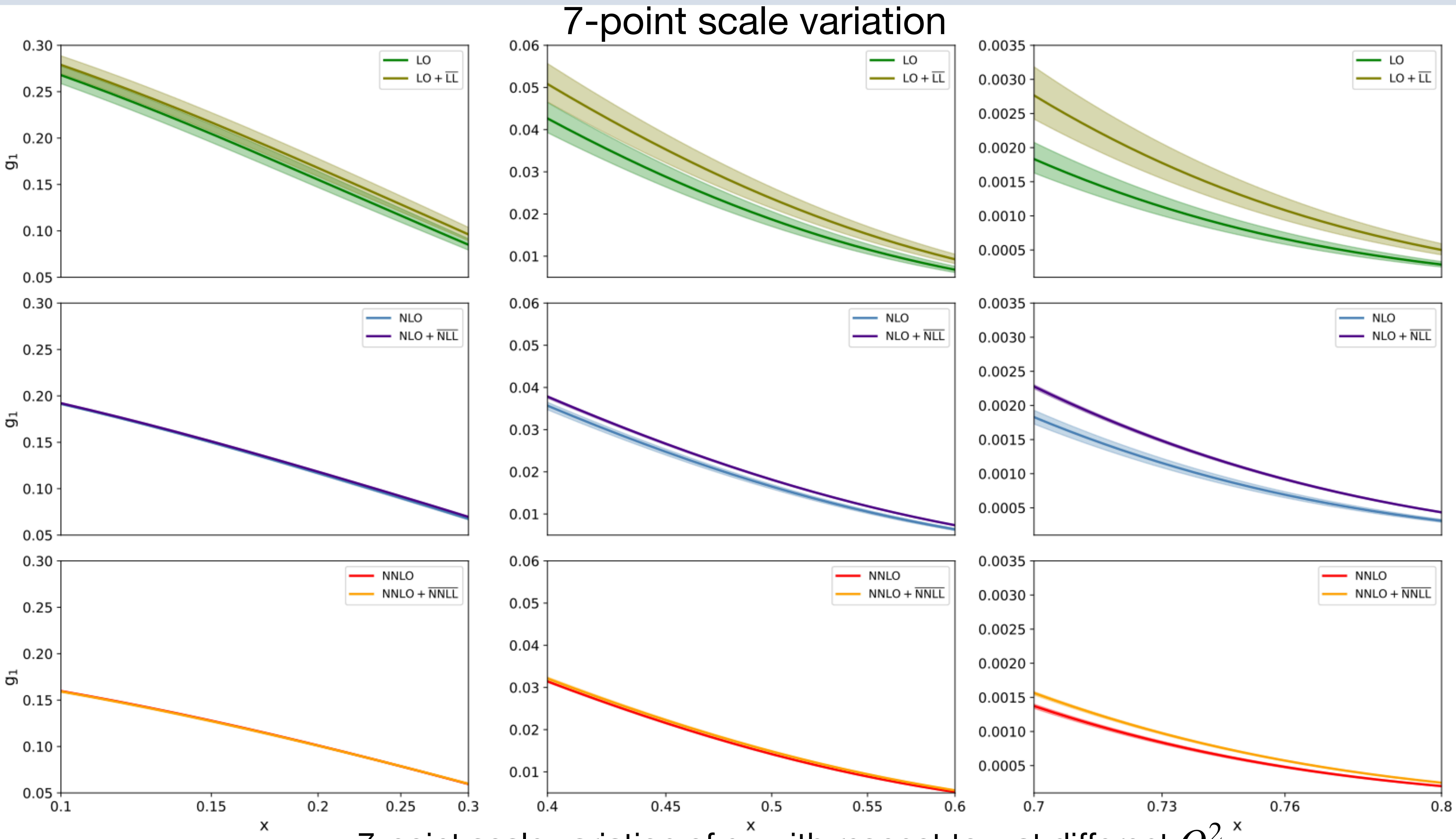
Numerical Result



7-point scale variation of F_1 with respect to x at different Q^2

- ABMP16 PDF sets and NNFF10PIp FF sets at respective orders.
- Integration ranges are $y \in [0.5, 0.9]$ and $z \in [0.2, 0.85]$.

Numerical Result

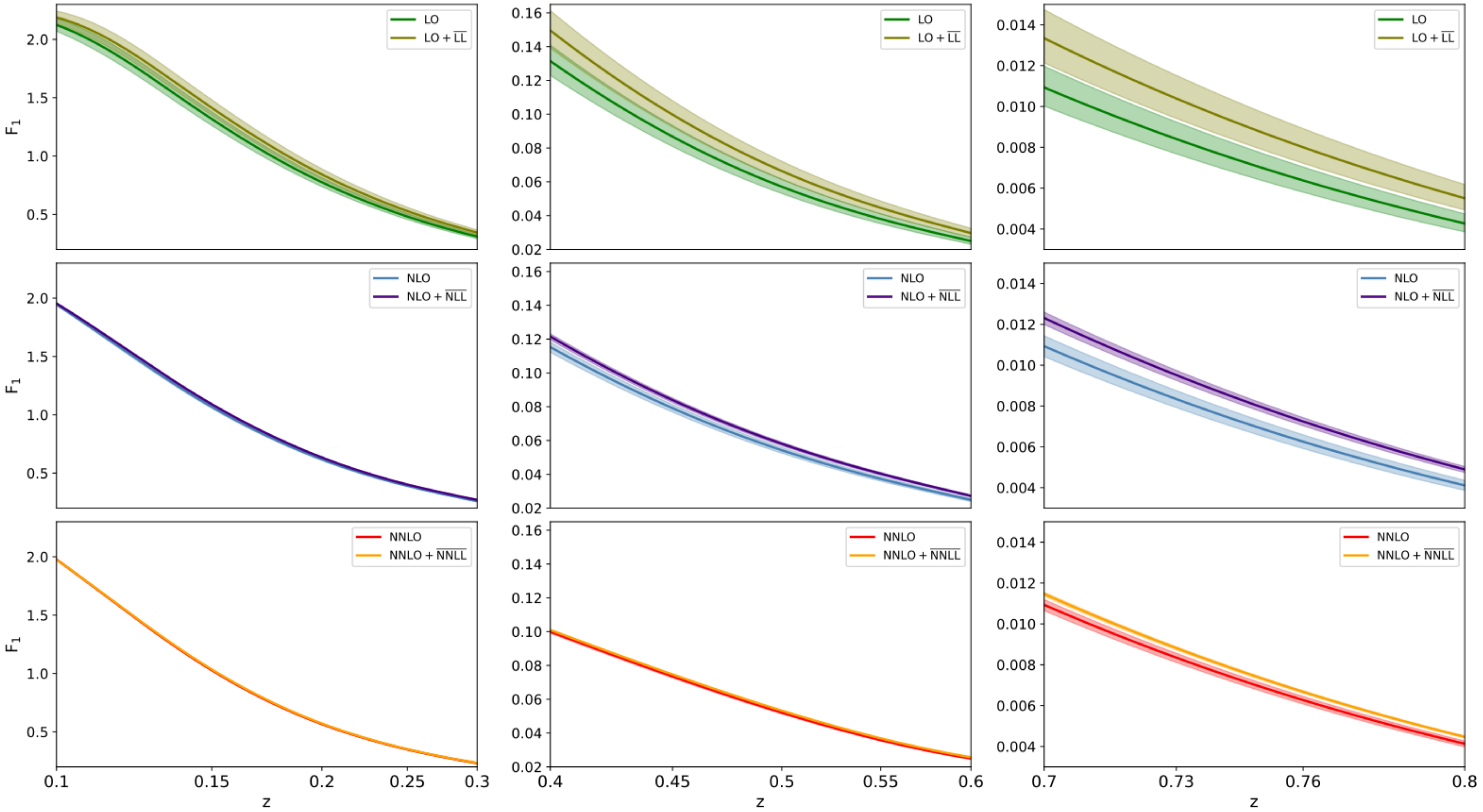


7-point scale variation of g_1 with respect to x at different Q^2

- BDSSV24 PDF sets and NNFF10PIp FF sets at respective orders.
- Integration ranges are $y \in [0.5, 0.9]$ and $z \in [0.2, 0.85]$.

Numerical Result

7-point scale variation

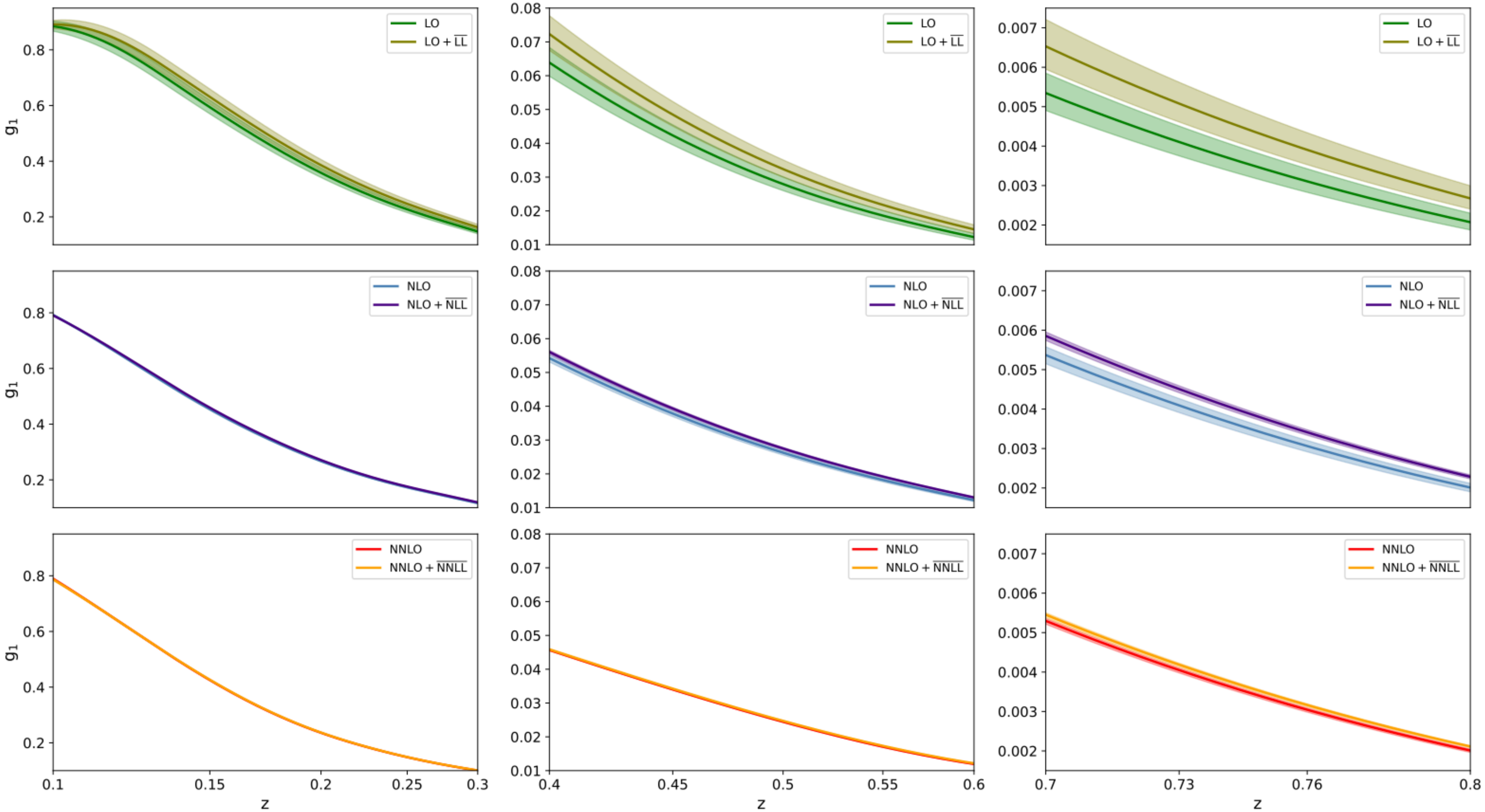


7-point scale variation of F_1 with respect to z

- ABMP16 PDF sets and NNFF10PIp FF sets at respective orders.
- Integration ranges are $x \in [0.1, 0.8]$ and $y \in [0.5, 0.9]$.

Numerical Result

7-point scale variation



7-point scale variation of g_1 with respect to z

- BDSSV24 PDF sets and NNFF10PIp FF sets at respective orders.
- Integration ranges are $x \in [0.1, 0.8]$ and $y \in [0.5, 0.9]$.

Numerical Result

- These plots clearly demonstrate how, in the large x and/or large z regions, the contribution of resummed terms is significant. Furthermore, they illustrate, how resummed predictions substantially reduce the theoretical uncertainties arising from the choice of μ_R and μ_F .

- We found that at each logarithmic order the resummed contributions are larger than the corresponding fixed order ones.

- Including these logarithms to all orders through resummation reduces the dependence on the renormalisation and factorisation scales and hence improves the reliability of our predictions.

Summary

- The CFs exhibit an exponential behaviour, which allows us to predict all order prediction for certain SV+NSV logarithms.
- By formulating a integral representation, we propose an SV+NSV resummation framework in double Mellin Space which is first of the kind.
- We have extended the resummation of NSV logarithms till N2LL accuracy.
- We find that NSV contributions are significant, hence for better theoretical prediction we need to resum them.

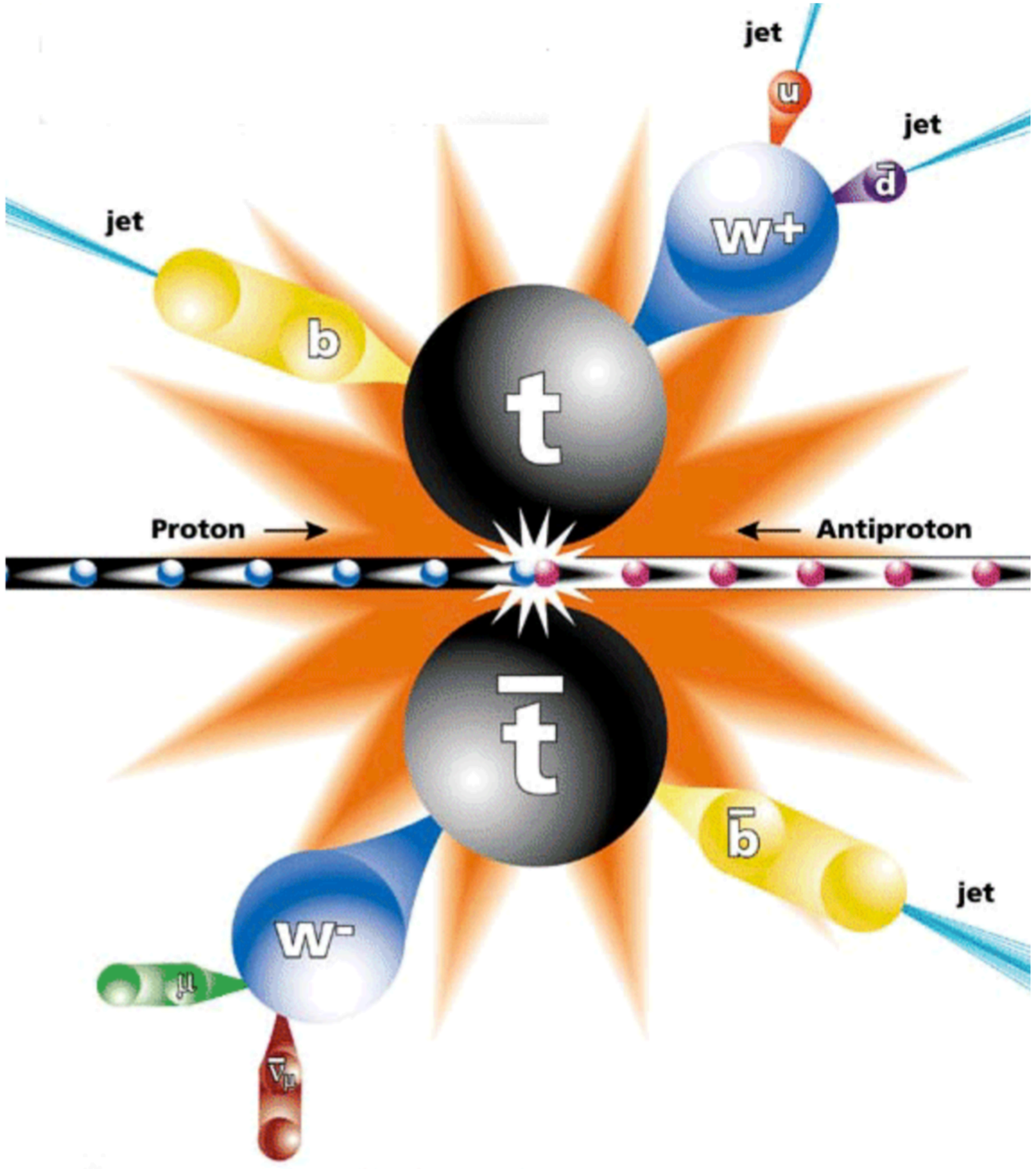
Future directions

- Modify the our formalism to accommodate off-diagonal channels.
- We are currently investigating the numerical impact of NNLO QED corrections on the existing NNLO+NNLL QCD results. We expect significant improvement in result.
- We have also extracted Time-like Splitting and Space-like Polarised pure QED and mixed QCD $\otimes$ QED Splitting function, which is not known in literature.[1]
- We are also currently investigating parallely the numerical impact of Neutral and Charged current intermediate processes. We expect improvement in result.
- We also plan to calculate QCD $\otimes$ EW correction to SIDIS.

1* See talk by S. Goyal

Thank You !

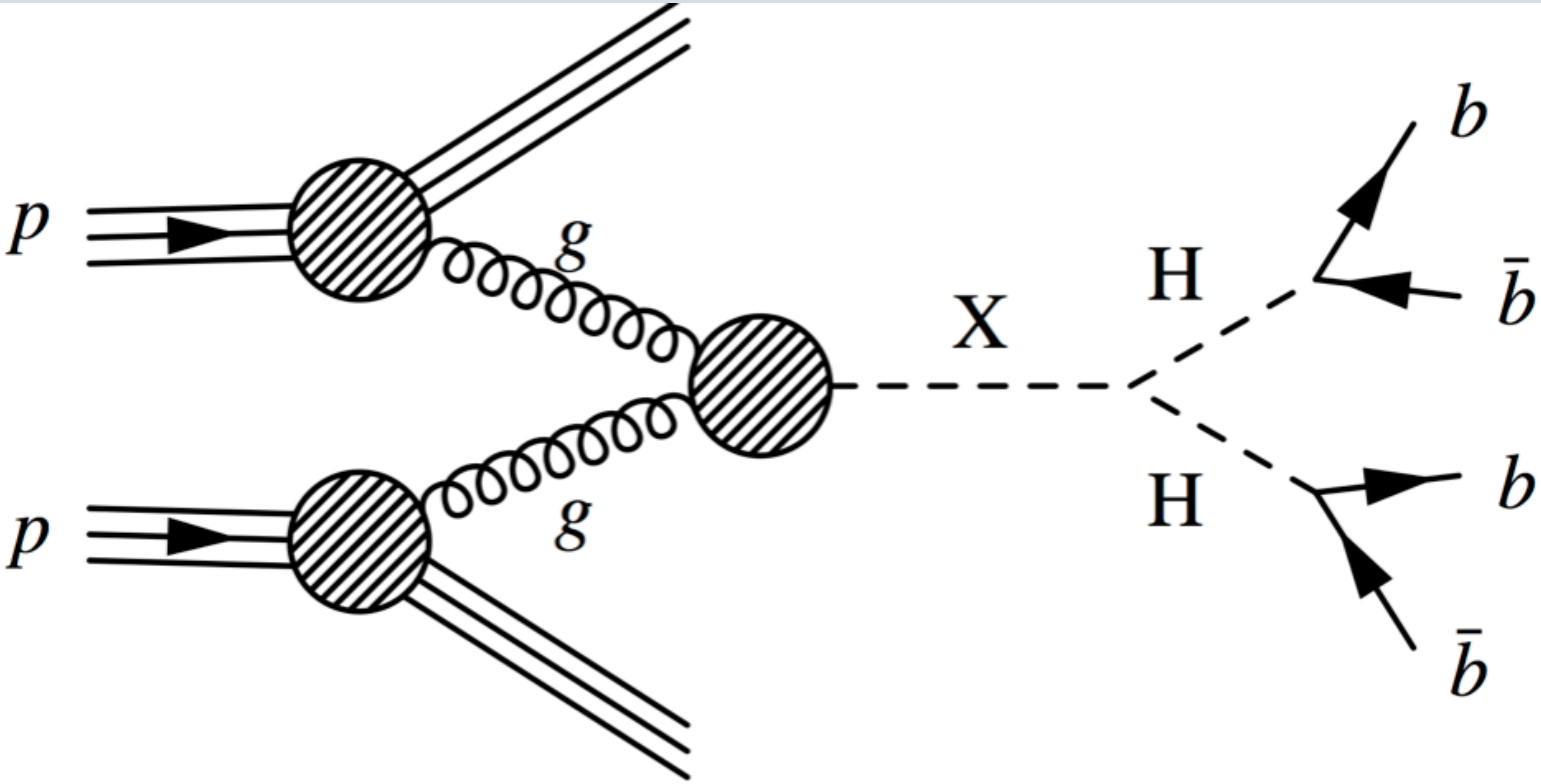
Top pair Production at the LHC



Parton Distribution Function

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

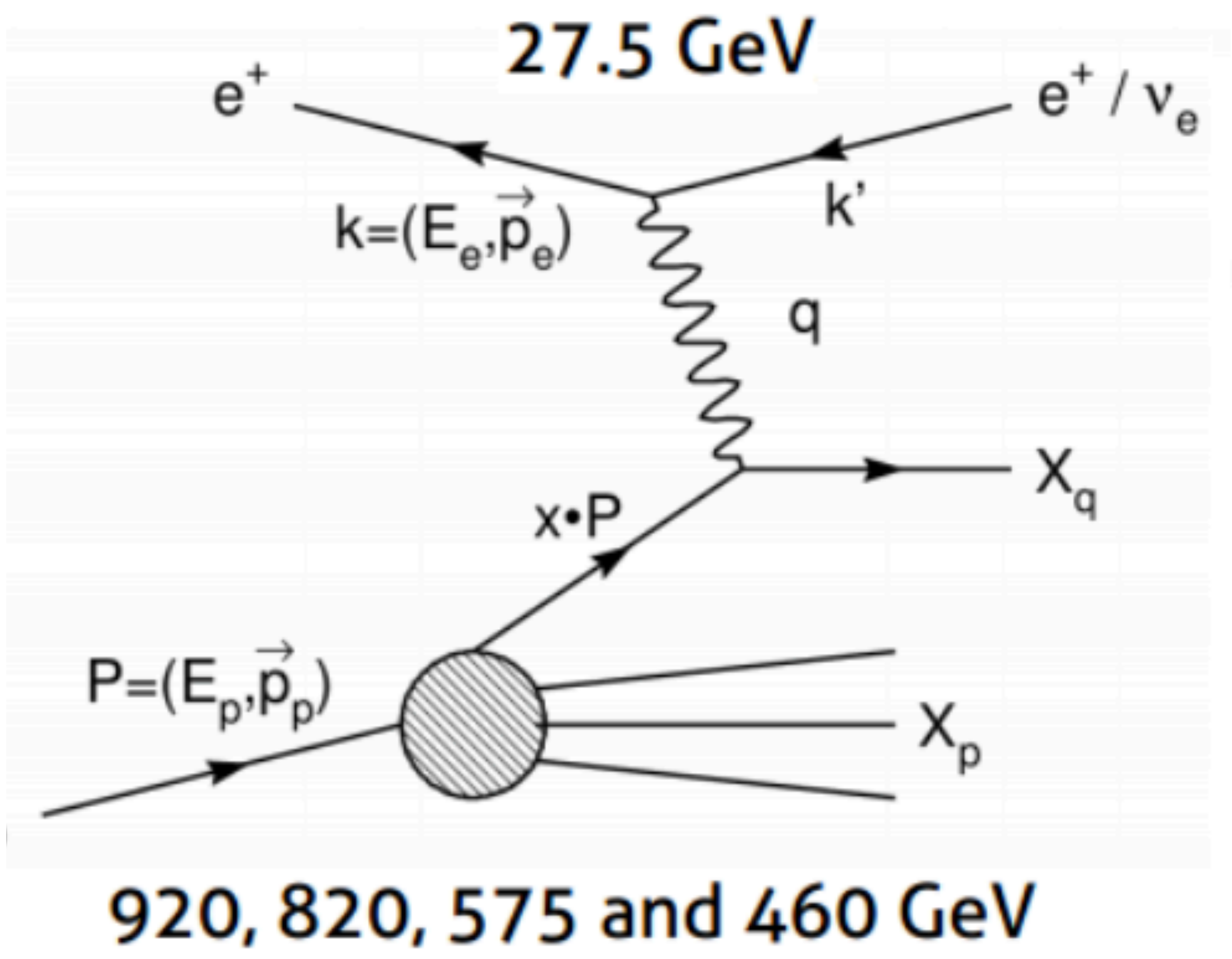
Higgs Production at the LHC



Parton Distribution Function

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

HERA — world only $e^{\pm}p$ collider



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ABM, ABKM

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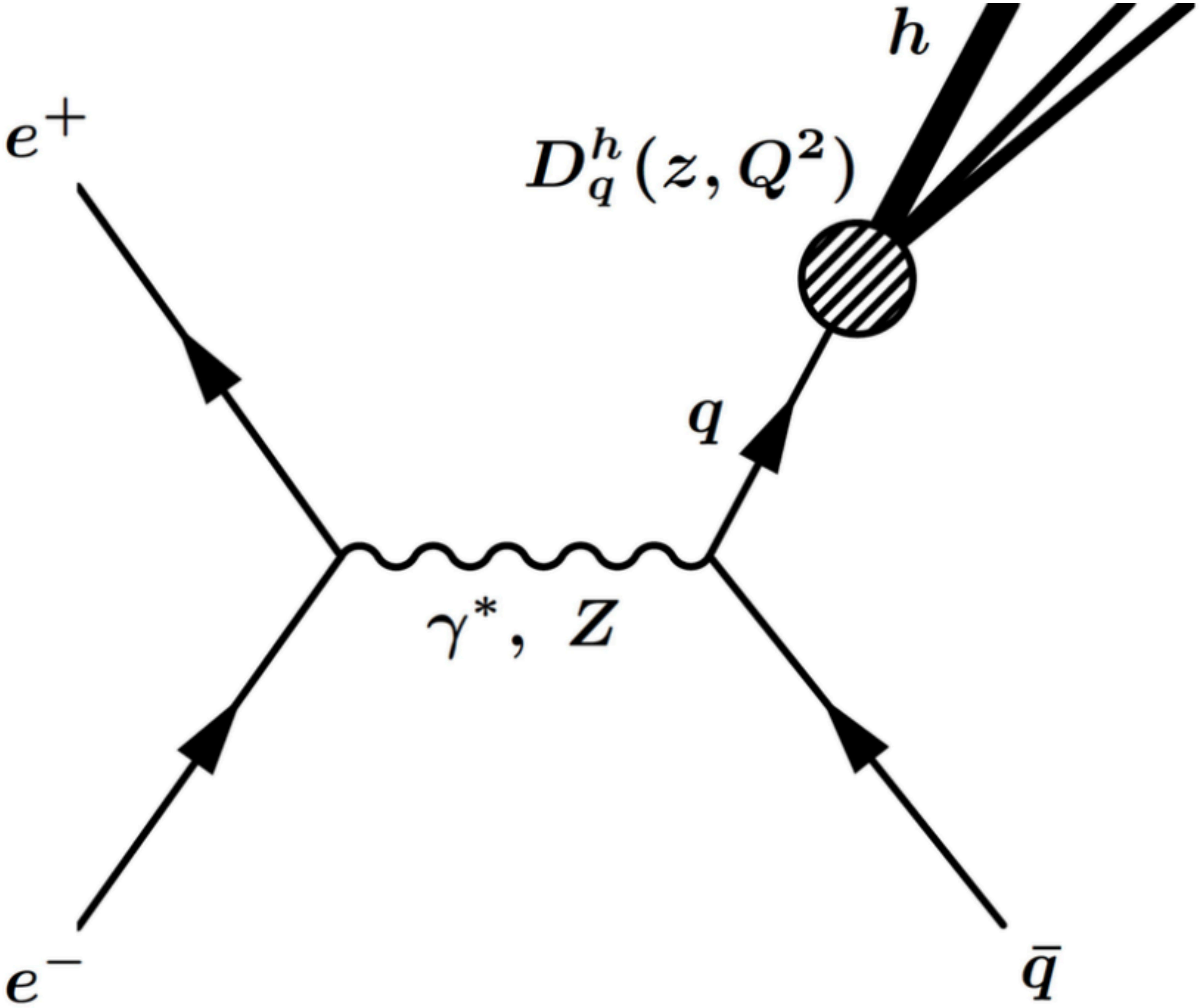
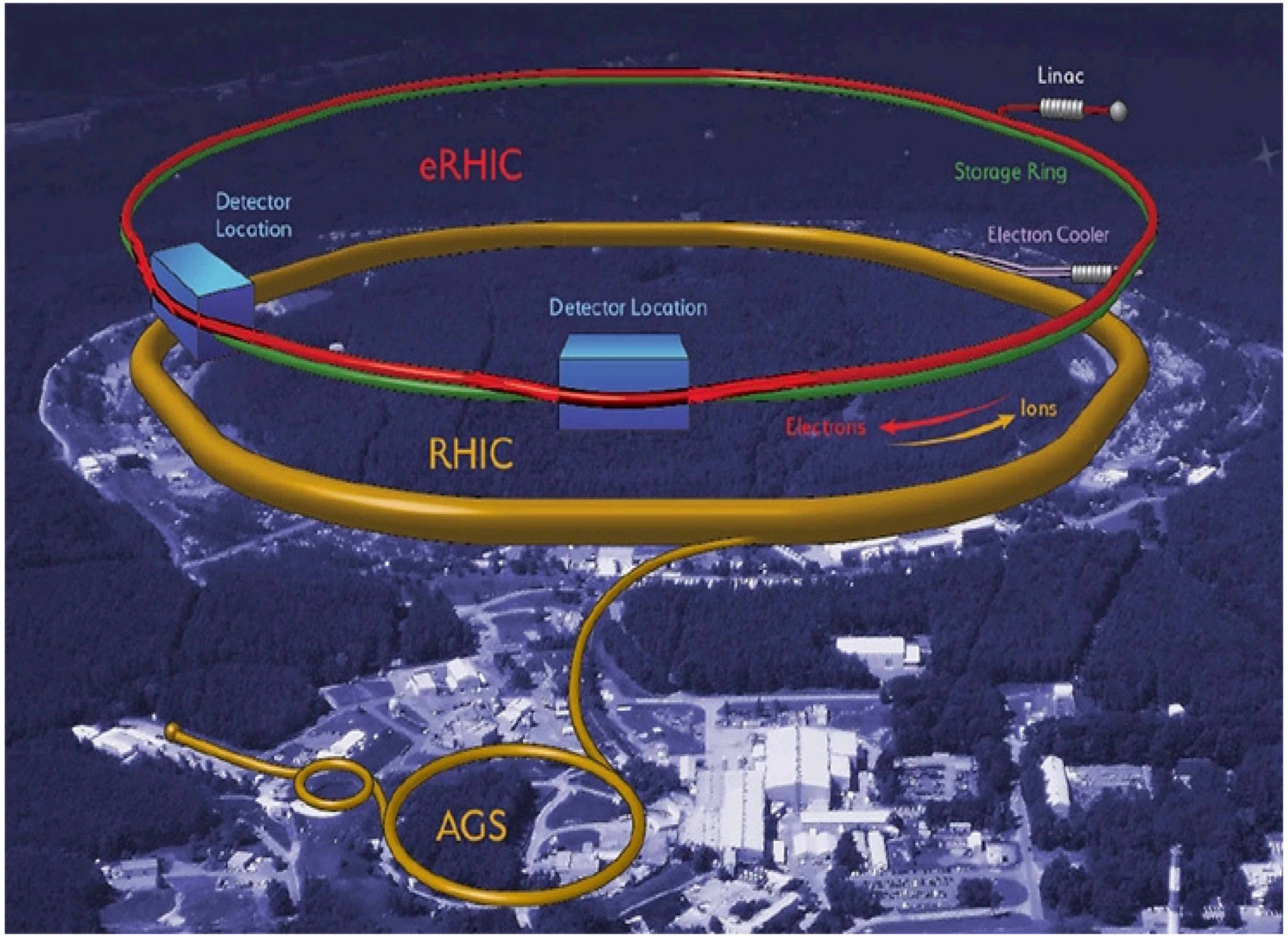
Fragmentation Function

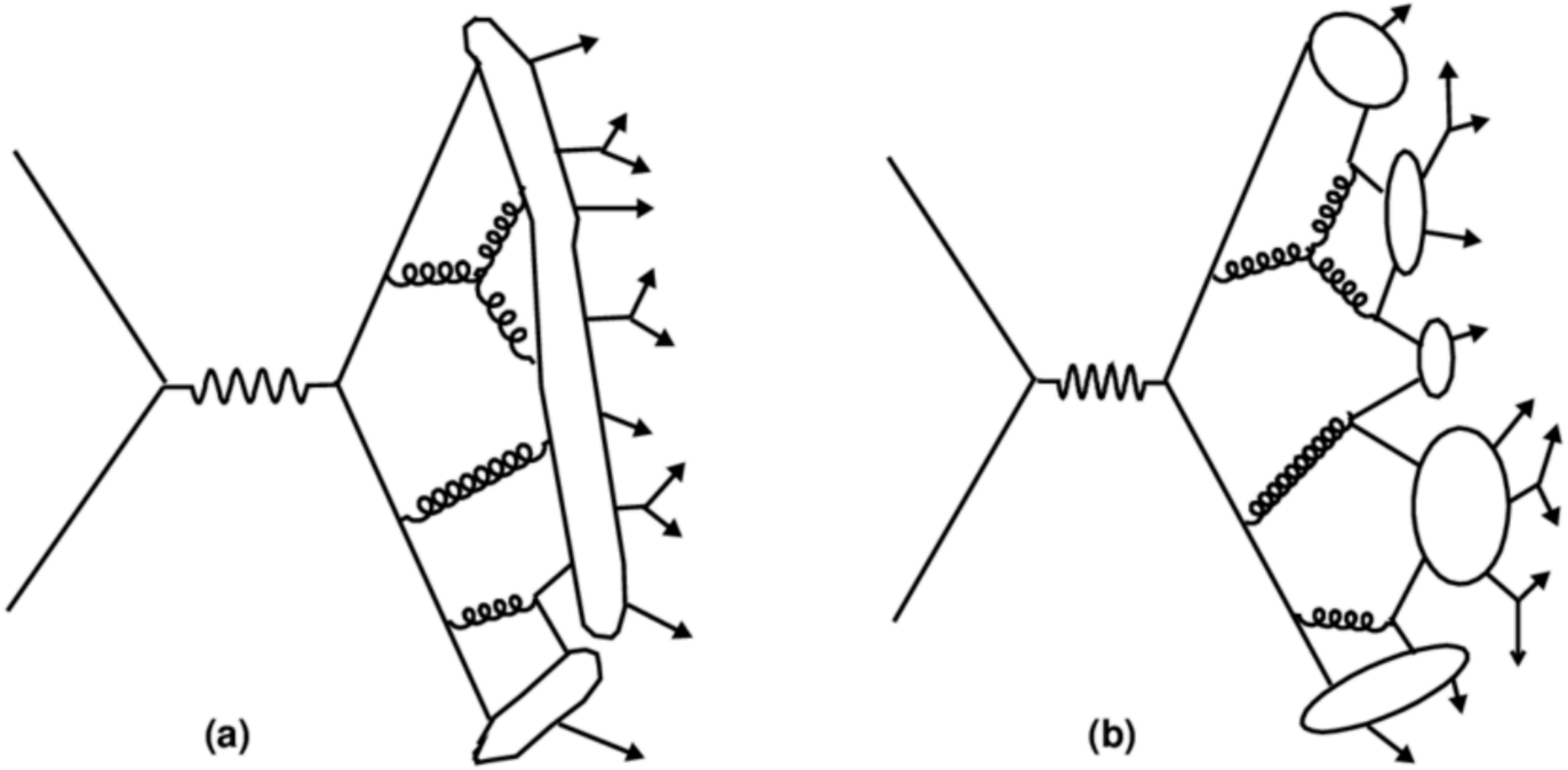


EIC Goals

- Precision 3D imaging of hadrons.
- Solving the proton spin puzzle.
- Gluon saturation and Color glass condensate.
- Quark and gluon confinement.
- Mass problem of nucleons.



Hadronization



Fragmentation Function:

Probability of a Parton converting to a Hadron