Factorisation schemes for proton PDFs

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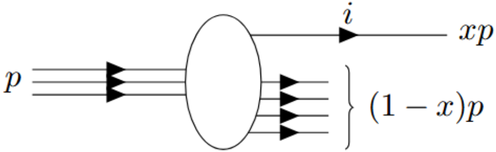
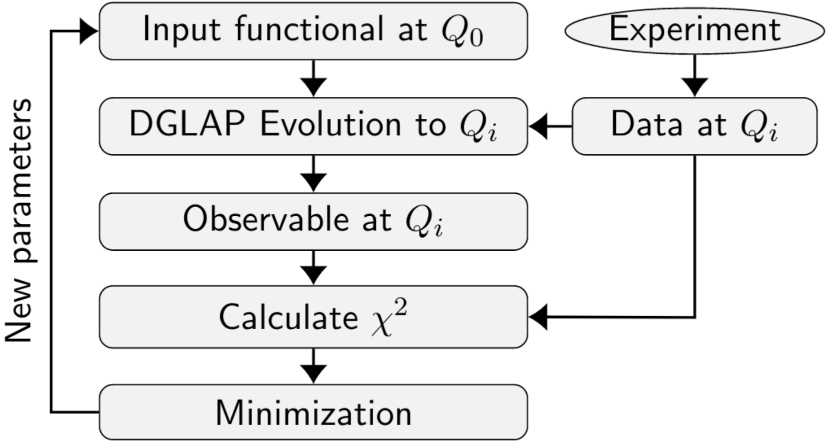
PDFs

- Probability densities to find a parton with a momentum fraction x inside of a hadron of momentum p



- Enter calculations involving hadrons in the initial state
- Non-perturbative, universal objects
- Fitted on experimental data

"Standard" fitting procedure



Factorisation schemes

- Collinear factorisation is assumed
- Relate a hadronic cross-section to the convolution of a coefficient function (perturbative) with a PDF (non-perturbative)
- DIS: $\sigma_{lh}(\mu_F, \mu_R) = \sigma_0 \sum_i f_i(\mu_F) \otimes C_i(\mu_F, \mu_R)$
- Drell-Yan like: $\sigma_{hh}(\mu_F, \mu_R) = \sigma_0 \sum_{i,j} f_i(\mu_F) \otimes f_j(\mu_F) \otimes C_{ij}(\mu_F, \mu_R)$
- Factorisation beyond leading-order not uniquely defined $\Rightarrow$ compensation between the coefficient functions and the PDF.
- Predictions from different schemes are all equally NLO-accurate but differ numerically $\Rightarrow$ Higher-order differences represent a genuine theory uncertainty
- While the DIS scheme was originally favoured, the $\overline{\mathrm{MS}}$ scheme is now the default choice in the majority of QCD calculations
- Since then several schemes were proposed with different motivations

Uncertainties

- Perturbative accuracy in order of α_s improved over recent years
- However several sources of uncertainties remaining:
  - Choice of PDF parametrisation: large freedom leading to uncertainties
  - What factorisation scale μ_F should be chosen? Remains an arbitrary choice
  - While $\overline{\mathrm{MS}}$ is the default scheme, it is arbitrary and the choice of factorisation scheme is another uncertainty source
- Increasing need to adress the remaining parametrisation, scale and scheme uncertainties
- Objective of this work: Compare factorisation schemes both analytically and numerically, assess their significance at several levels and consider the potential impact of the choice of scheme on phenomenology at the LHC

PDFs in different schemes

- 3 ways of obtaining PDFs in different schemes:
  - Direct fitting in a given scheme
  - Transforming the PDF at input scale, then evolving through DGLAP in new scheme (necessary to have splitting functions in the new scheme)
  - Transforming locally at each scale from a DGLAP evolved PDF in another scheme
- Focus on the last option
- Use modern PDF sets (`CT18NLO`, `MSHT20nlo`, `NNPDF40MC`) fitted and DGLAP-evolved in $\overline{\mathrm{MS}}$ scheme. Phys. Rev. D 103, 014013, Eur. Phys. J. C 81, 341 (2021), J. High Energ. Phys. 2024, 88 (2024)

Transformation from $\overline{\mathrm{MS}}$

- Transformed PDFs obtained from $\overline{\mathrm{MS}}$ ones through transformation kernels $\mathbb{K}_{ab}$
- $(xf)_a^{\mathrm{FS}}(x, \mu_F) = \sum_b \int_x^1 \mathrm{d}z \, \mathbb{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z, \mu_F) \; (xf_b)^{\overline{\mathrm{MS}}}\left(\frac{x}{z}, \mu_F\right)$
- $\mathbb{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z, \mu) = \delta_{ab}\,\delta(1-z) + \frac{\alpha_\mathrm{s}(\mu)}{2\pi}\, K_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z, \mu) + \mathcal{O}\left(\alpha_\mathrm{s}^2\right)$
- Transformed PDFs will have a $\overline{\mathrm{MS}}$ quark contribution and a $\overline{\mathrm{MS}}$ gluon contribution.
- To allow easier systematic comparison between schemes, kernels are decomposed as :
- $K(z) = \sum_{k=0}^1 a_k\, \mathcal{D}_k(z) + b(z)\log(1-z) + c(z)\log z + P(z) - \Delta\,\delta(1-z)$
- $\mathcal{D}_k(z) = \left[\frac{\log^k(1-z)}{1-z}\right]_+$

Various schemes

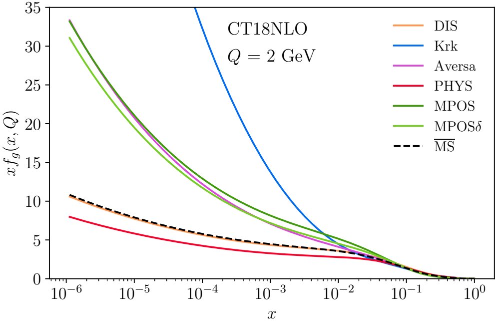
- Several schemes exist and are compared, among them the DIS, Aversa, Krk, PHYS schemes alongside the POS family of schemes.
- Different motivations for each of the schemes:
  - Krk: Scheme tailored for General Purpose Monte Carlo, aims to reduce the complexity of NLO matching to parton showers by absorbing Catani-Seymour dipole collinear contributions into PDFs. It is the basis for the KrkNLO method. Eur. Phys. J. C 76, 649 (2016)
  - POS family: Enforce positivity of PDFs, with MPOS ensuring momentum conservation via a soft-function in diagonal kernels and MPOSδ via a $\delta(1-z)$ termEur. Phys. J. C 84, 335 (2024)
  - PHYS: remove long distance interactions between massless QCD partons, which must be unphysical because of confinement J. High Energ. Phys. 2013, 156 (2013)

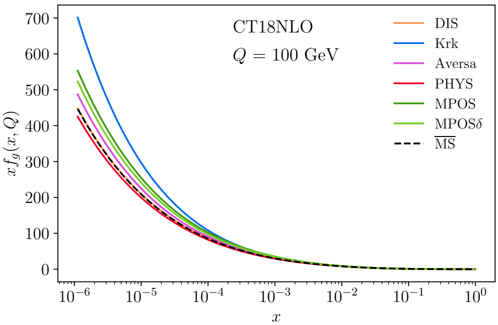
Example of kernels

$T_R^{-1} K_{qg}^{\overline{\mathrm{MS}}\to\mathrm{FS}}$	$\mathcal{D}_1$	$\mathcal{D}_0$	$\log(1-z)$	$\log(z)$	$P(z)$	$-\delta(1-z)$
AVERSA			$p_{qg}(z)$	$-p_{qg}(z)$		
DIS			$p_{qg}(z)$	$-p_{qg}(z)$	$-4p_{qg}(z)+3$	
KRK			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)+1$	
MPOS			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)$	
MPOSδ			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)$	
PHYS			$p_{qg}(z)$		$-p_{qg}(z)+1$	

- $p_{qg}(z) = z^2 + (1-z)^2$
- Encodes the $\overline{\mathrm{MS}}$ gluonic contribution to the transformed quark PDFs

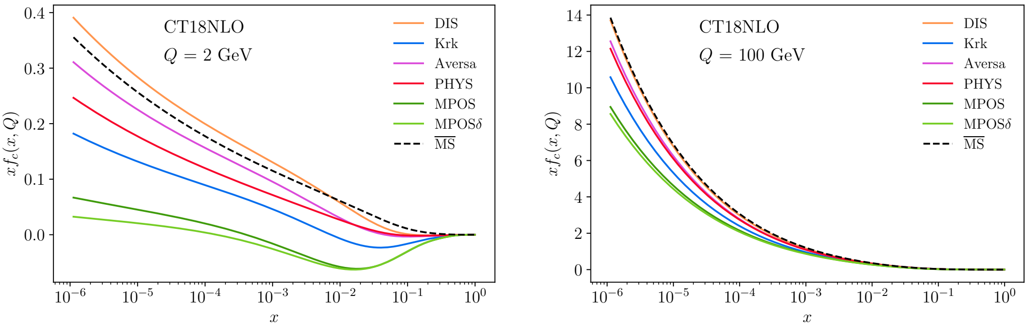
Schemes comparison: PDFs





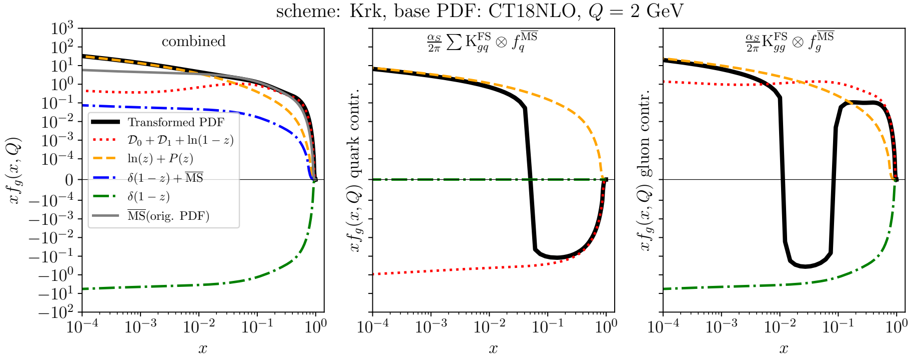
- Drastic differences at low-x and low scale, Krk up to one order of magnitude larger than $\overline{MS}$ PDF
- At higher scales, the magnitude of differences is reduced but the overall trend persists

Schemes comparison: PDFs



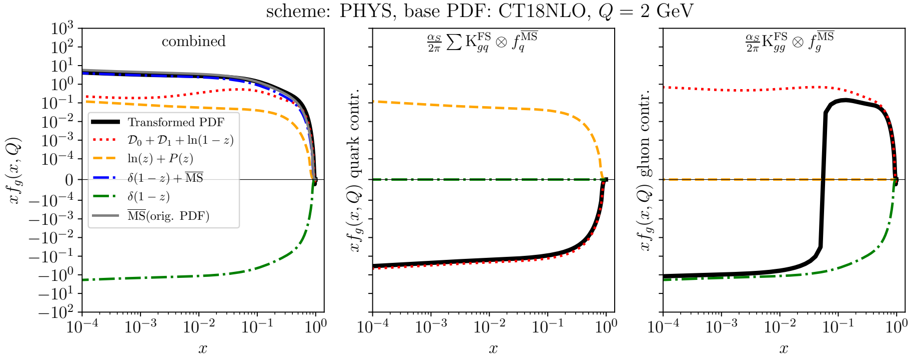
- Large spread between schemes at low scales, negativity for some schemes.
- Features washed out at higher scales due to the DGLAP evolution

Schemes comparison: Anatomy of PDFs



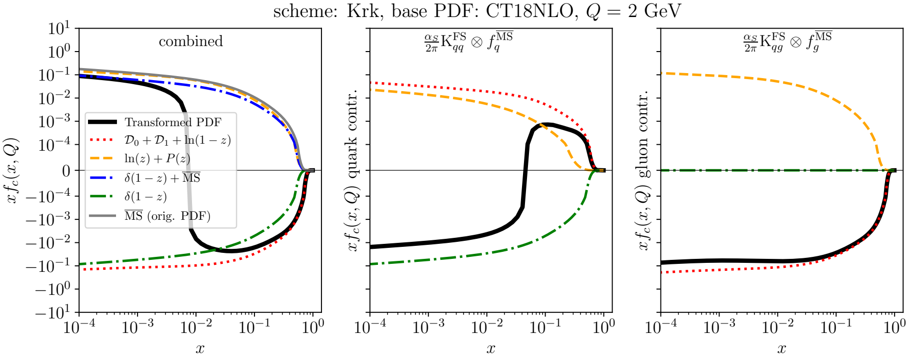
- NLO contribution comparable to LO contribution at low-x $\Rightarrow$ Breakdown of perturbation theory?
- $c(z) = -2p_{gg}(z)$ for $K^{Krk}_{gg} \Rightarrow ln(z)/z$ divergence at low x.

Schemes comparison: Anatomy of PDFs



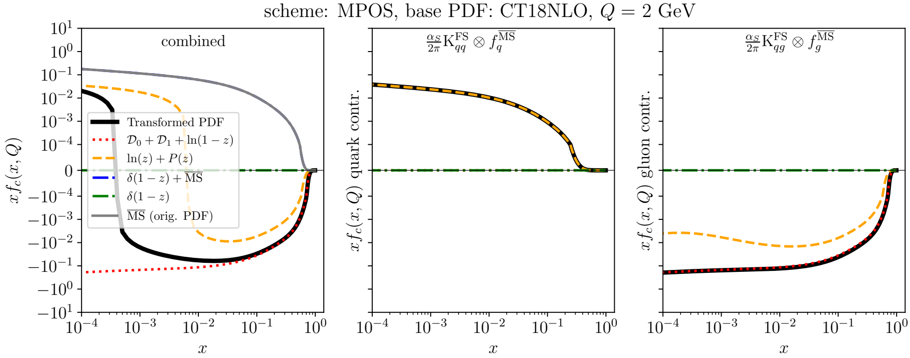
- Smaller NLO contribution compared to LO
- No divergences in the kernels

Schemes comparison: Anatomy of PDFs



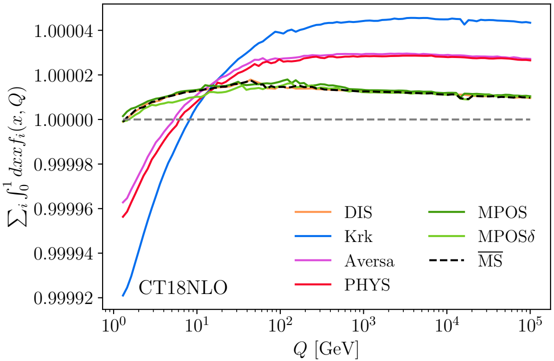
- Scale close to threshold $\Rightarrow$ Small charm quark contribution compared to gluon one
- Negativity driven by $b(z)$ coefficient in K^{Krk}_{qg} (twice that of other schemes)
- Small mitigation from +1 part of $P(z)$

Schemes comparison: Anatomy of PDFs



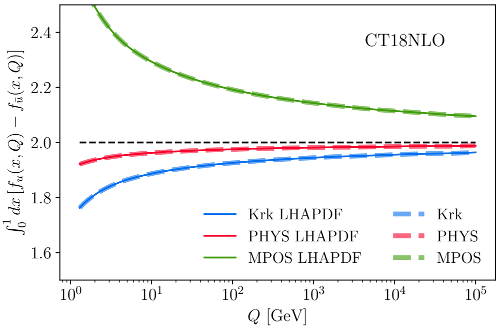
- Scale close to threshold $\Rightarrow$ Small charm quark contribution compared to gluon one
- No mitigation in the gluon contribution (even opposite effect!)
  $\Rightarrow$ Bigger negativity region

Schemes comparison: Momentum sum rule



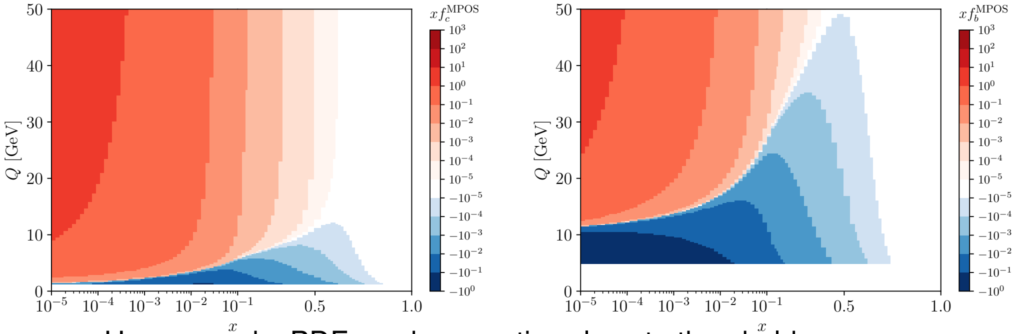
- $\sum_i \int_0^1 dx\ xf_i(x, Q)$
- Deviation of the order of 10^{-5}
- $\overline{MS}$ PDF already deviating from 1
- Very similar results between MPOS and MPOSδ
- Different schemes $\Rightarrow$ Different valence/sea quarks/gluon momentum fractions

Schemes comparison: Number sum rule



- $N_q = \int_0^1 dx\, (f_q(x,Q) - f_{\bar{q}}(x,Q))$
- $N_q^{\mathrm{FS}} = N_q^{\overline{\mathrm{MS}}} \left\{ 1 + \frac{\alpha_s(\mu)}{2\pi} \int_0^1 \mathrm{d}z\, K_{qq}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z) \right\}$
- Possible modification of the number sum rule
  $\Rightarrow$ Less natural to impose during fits
- Modification scale-dependent but can be 0 (if $K_{qq}^{\overline{\mathrm{MS}}\to\mathrm{FS}}$ is a plus-distribution)
- Perfect agreement between analytical value and value obtained from transformed LHAPDF sets

Schemes comparison: Positivity

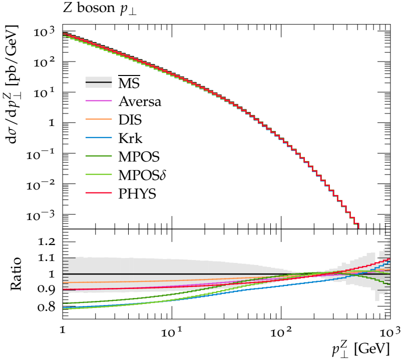


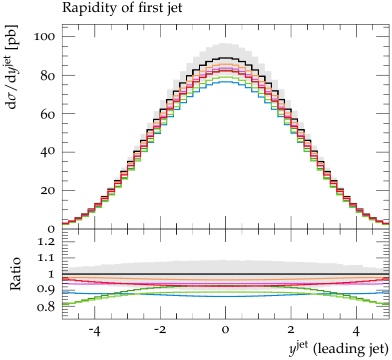
- Heavy quarks PDF can be negative close to threshold
- Strong negative gluon contribution and small input HQ distribution
- Adding intrinsic charm reduces negativity at low Q and high-x (for charm PDF)
- Need for quark-mass effects in kernels for heavy flavors?

Impact on phenomenology

- Choice of factorisation scheme for a NLO process compensated by a modification of the partonic cross-section at the same order.
- Some observables only non-zero for real emission kinematics $\Rightarrow$ effectively-LO calculation, uncompensated factorisation scheme dependence
- Same effects on these observables in LO $pp \to X + j$ as in NLO $pp \to X$
- Chosen processes: $pp \to Z + j$ and $pp \to H + j$
- Using Herwig 7 alongside transformed `NNPDF40MC` PDFs and $\alpha_S(M_Z) = 0.118$
- Set-up appropriate for LHC Run II data: center of mass energy of 13 TeV and fiducial cuts similar to ATLAS and CMS.

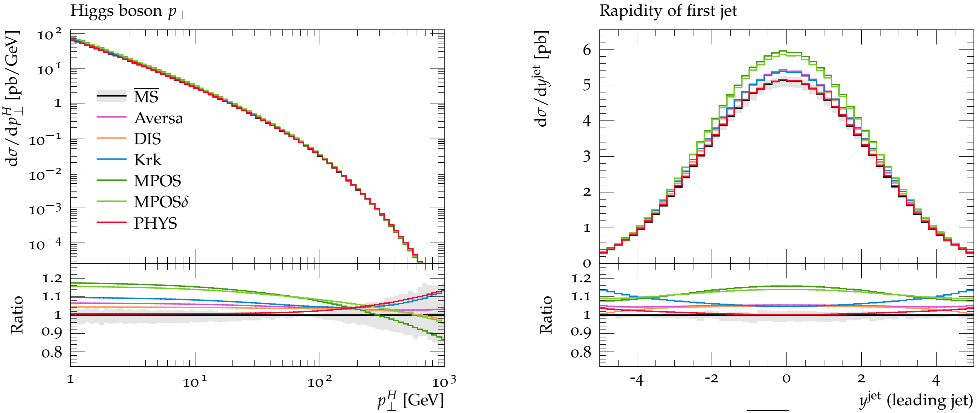
Impact on phenomenology: Z boson + jet





- Smaller distributions in other schemes compared to $\overline{\text{MS}}$
- Factorisation scheme uncertainty can go as far as 20%, exceeding $\overline{\text{MS}}$ scale variation uncertainty

Impact on phenomenology: Higgs + jet



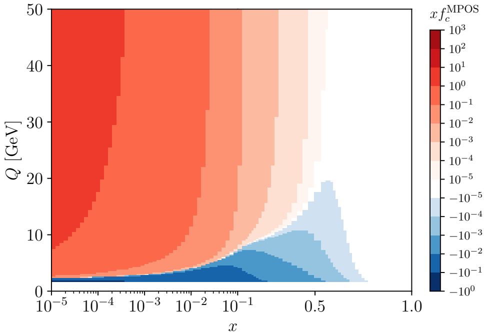
- Bigger distributions in other schemes compared to $\overline{\text{MS}}$
- Factorisation scheme uncertainty exceeding $\overline{\text{MS}}$ scale variation uncertainty

Conclusions

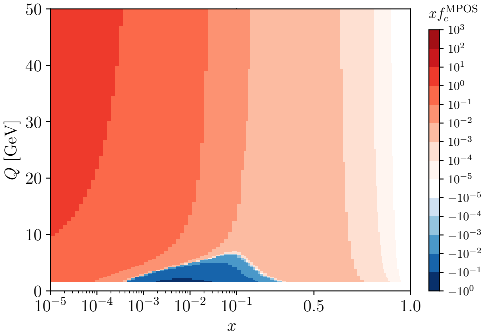
- PDFs are essential objects when dealing with hadronic processes
- Several factorisation schemes exist, with their own motivations and aims, creating a source of uncertainty
- We performed the first-ever systematic comparison of schemes
- At the PDF level, different schemes may lead to peculiar behaviours: negativity for heavy flavours, NLO contribution for gluon PDFs of the same magnitude as the LO contribution, deviation from the number sum rule...
- The choice of factorisation scheme has an impact on LHC phenomenology: scheme uncertainty up to 20%, sometimes exceeding factorisation scale uncertainty
- Next step: Revisit the transformation procedure and derive splitting functions in other schemes, in order to perform the transformation to a different scheme at the input scale, before performing the DGLAP evolution in the chosen scheme directly

Back-up

Effect of intrinsic charm on positivity

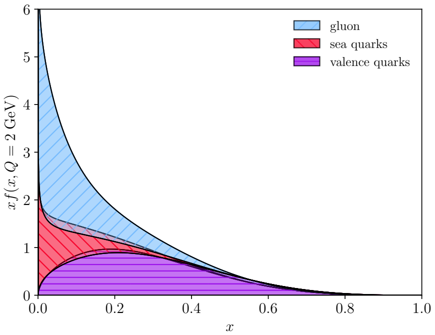


`NNPDF40MC` (perturbative charm)

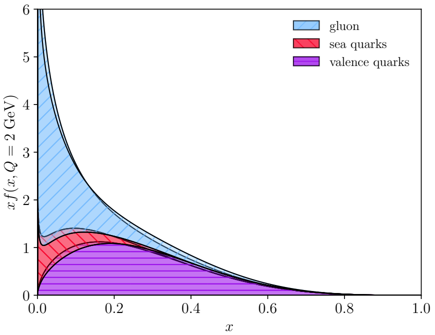


`NNPDF40` (intrinsic charm)

Modification of the momentum distributions



$\overline{\text{MS}}$ (background) vs DIS (foreground)



MPOS (background) vs MPOSδ (foreground)

Set-up for $pp \to Z + j$

- Center of mass energy of 13 *TeV*
- $p_T^{\ell_{1,2}} > 25$ *GeV* , $\left|\eta^{\ell_{1,2}}\right| < 3.5, M_{\ell\ell} \in [66, 116]$ *GeV*
- Jet identified by anti-k_T algorithm with jet-radius $R = 0.4$ and minimum transverse momentum of 10 *GeV*.
- $\mu_R = \mu_F = M_{\tau\tau}$
- In case of $\overline{\text{MS}}$, μ_F is varied by $\mu_F \in \{\frac{1}{2}, 1, 2\} M_{\tau\tau}$

Set-up for $pp \to H + j$

- Center of mass energy of 13 *TeV*, Higgs decays to a tau pair
- $p_T^{\tau_{1,2}} > 25$ *GeV* , $|\eta^{\tau_{1,2}}| < 3.5, M_{\tau\tau} \in [115, 135]$ *GeV*
- Jet identified by anti-k_T algorithm with jet-radius $R = 0.4$ and minimum transverse momentum of 10 *GeV*.
- $\mu_R = \mu_F = M_{\ell\ell}$
- In case of $\overline{\mathrm{MS}}$, μ_F is varied by $\mu_F \in \{\frac{1}{2}, 1, 2\} M_{\ell\ell}$