



JAGIELLONIAN UNIVERSITY
IN KRAKÓW

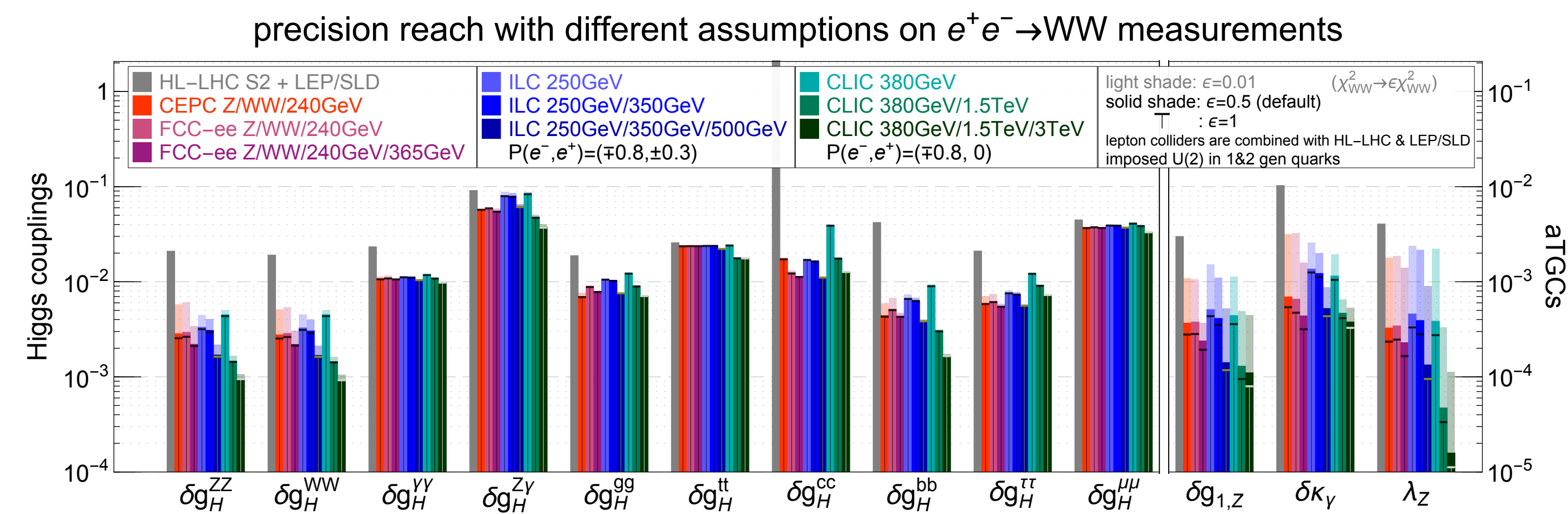
YFS for Future and Present e^+e^- Colliders

Alan Price
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Physics Landscape at Higgs Factories

- ❖ Higgs couplings measured to a few %
- ❖ Self coupling with 50% precision
- ❖ Top-quark pole mass uncertainty of 500 MeV
- ❖ Flavour physics observables improved by about one order of magnitude compared to today
- ❖ Improvement on direct Dark matter limits
- ❖ Possible surprises?



[J. De Blas et al JHEP 12 \(2019\) 117](#)

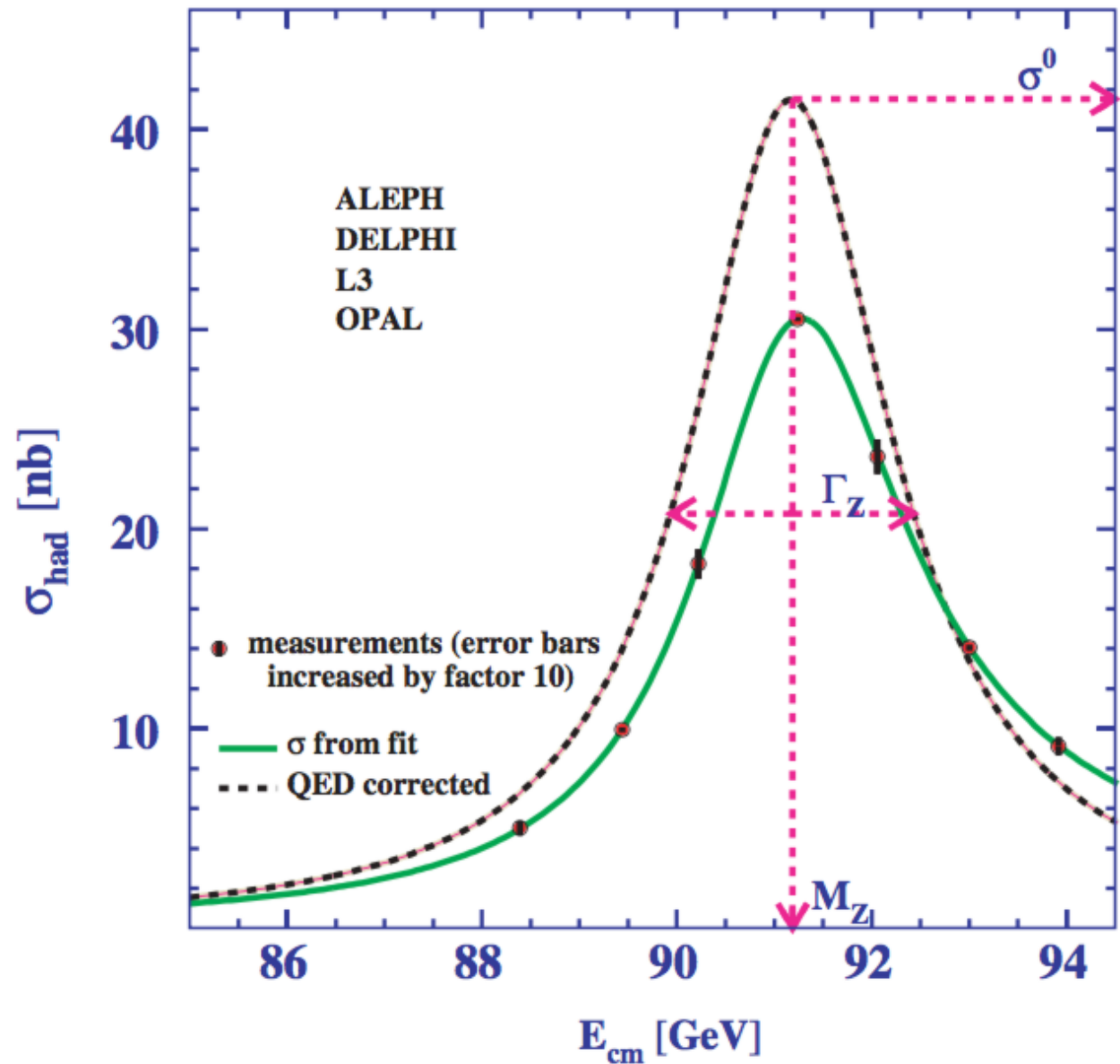
An electron-positron Higgs factory is the highest-priority next collider. -EUROPEAN STRATEGY FOR PARTICLE PHYSICS

Theory Requirements

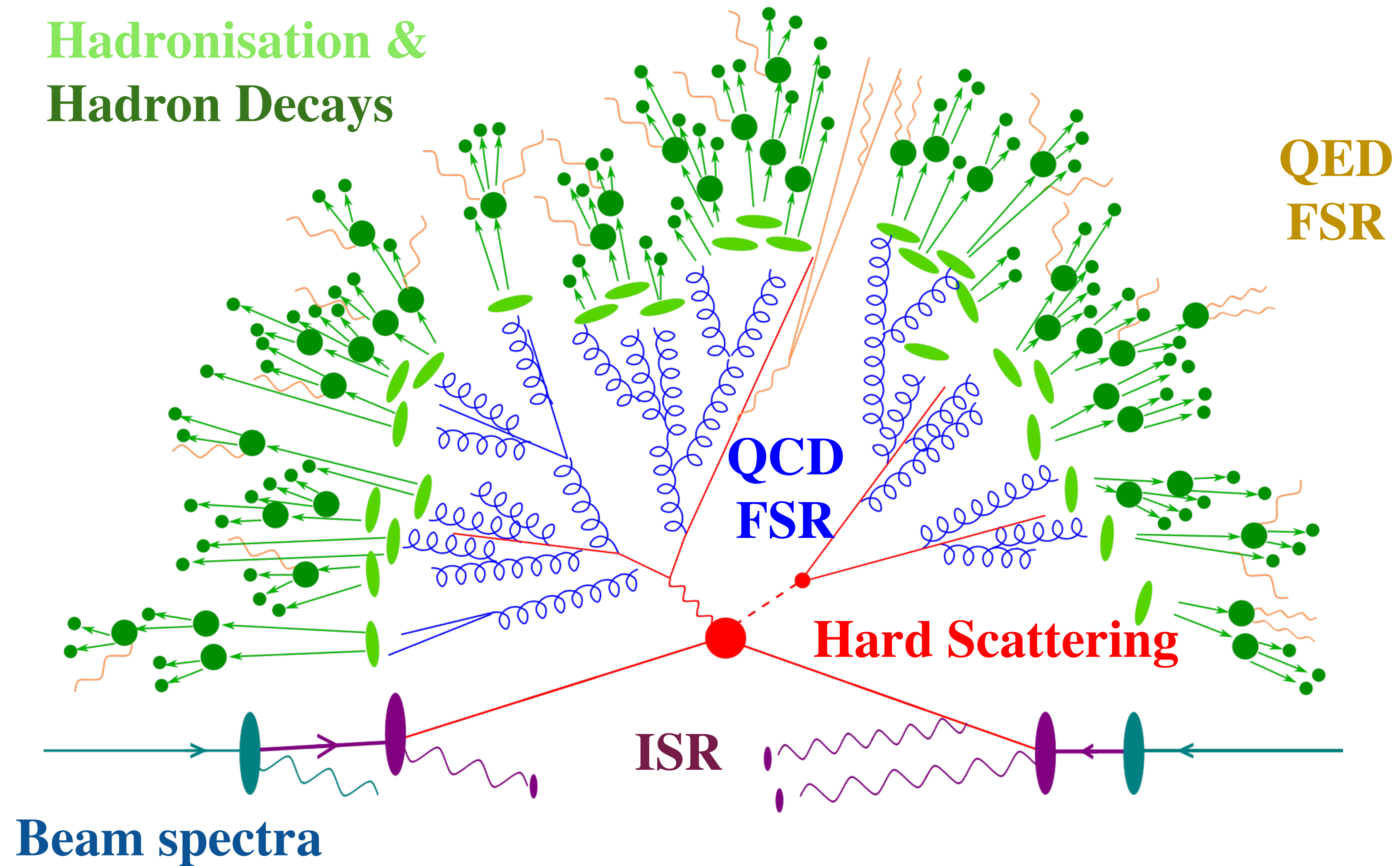
- ❖ Factor 5-200 reduction of experimental error
- ❖ QED effects of 0.1% could be included in LEP error budget
- ❖ Future colliders will deliver full LEP Statistics in minutes
- ❖ Limited room for theory uncertainties =>
State of the art MC simulations

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [29]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [29]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [34]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
σ_{had}^0 [nb]	σ_{had}^0 [29]	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [29]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$ [35]	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2 \theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [34]	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2 \theta_W^{eff} \times 10^5$	$\langle \mathcal{P}_\tau \rangle, A_{FB}^{pol,\tau}$ [29]	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO [36]	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [29]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

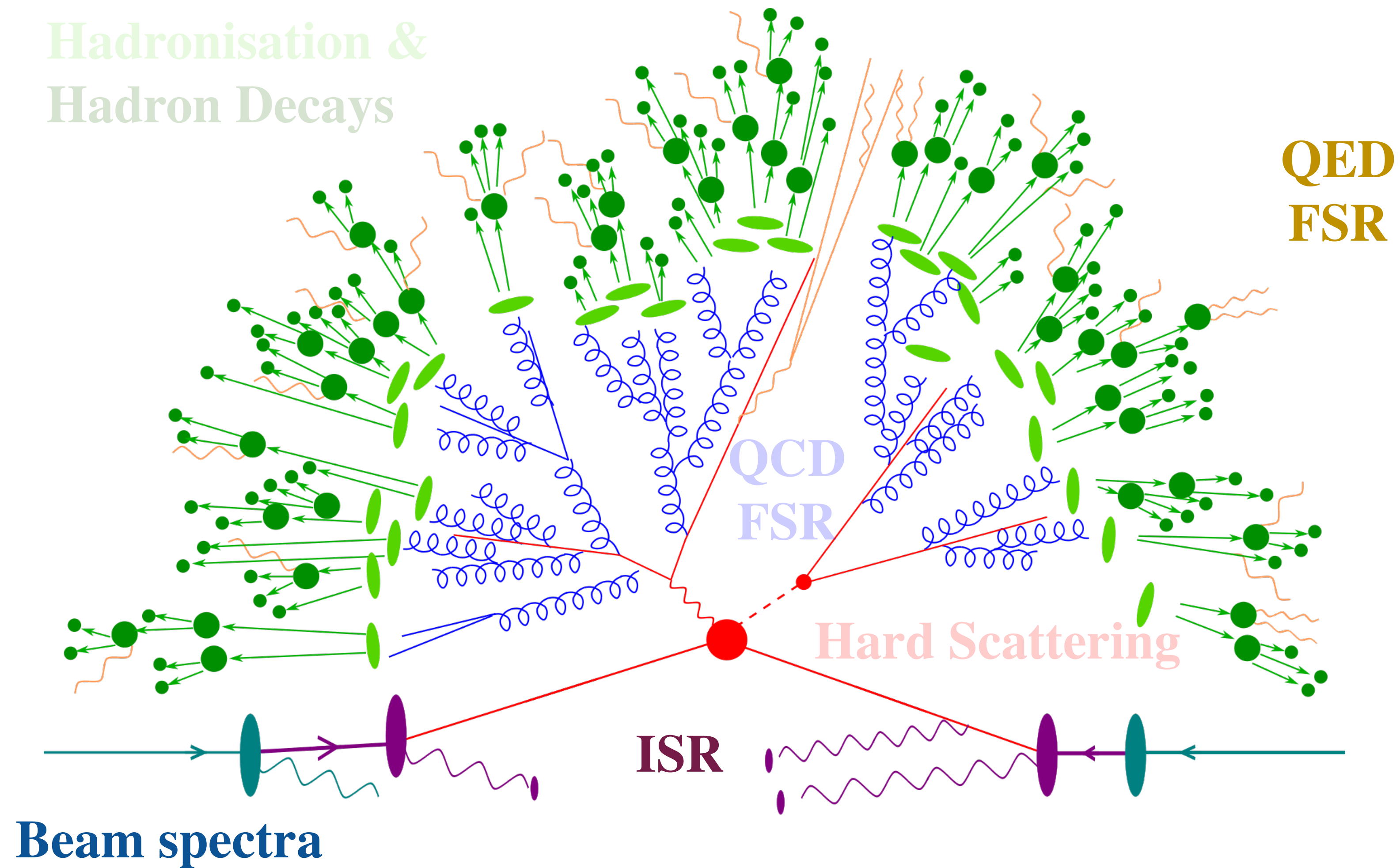
S.Jadach and M.Skrzypek, Eur. Phys. J.C 79, no.9, 756 (2019)



Anatomy of e^+e^- Collision



Anatomy of e^+e^- Collision



How to treat QED Corrections?

Collinear Resummation

- ❖ Collinear logs are resummed with universal PDF ($P_T = 0$)
- ❖ Recently matched to NLO
- ❖ Combined with Parton Shower to generate photon emissions
- ❖ Beyond NLO becomes tricky

[S.Frixione et.al JHEP 03 \(2020\)](#)

[Jadach et.al, Z.Phys.C 49 \(1991\)](#)

[577-584, Europhys. Lett. 17 \(1992\)](#)

[123-128](#)

$$d\sigma(L, \hat{L}) = \alpha^k \sum_n \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log \left(\frac{Q^2}{E_\gamma^2} \right) \quad L = \log \left(\frac{Q^2}{m_e^2} \right)$$

Soft Resummation

- ❖ Soft logs resummed to infinite order using the YFS theorem
- ❖ Provides a robust scheme for the inclusion of real and virtual corrections at any order.

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[See talk by Francesco Ucci](#)

[S.Frixione et.al JHEP 03 \(2020\)](#)

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Soft Resummation

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- ❖ Provides a robust scheme for the inclusion of real and virtual corrections at any order.

Yennie, Frautschi, and Suura showed how to reorder the entire perturbative series such that all IR divergences are resummed

It also provides an analytical treatment of the multi-photon phasespace

[See talk by B.Ward](#)

ANNALS OF PHYSICS: **13**: 379–452 (1961)

The Infrared Divergence Phenomena and High-Energy Processes*

D. R. YENNIE†

School of Physics, University of Minnesota, Minneapolis, Minnesota

S. C. FRAUTSCHI‡

Department of Physics, University of California, Berkeley, California

AND

H. SUURA

Department of Physics, Nihon University, Tokyo, Japan

A general treatment of the infrared divergence problem in quantum electrodynamics is given. The main feature of this treatment is the separation of the infrared divergences as multiplicative factors, which are treated to all orders of perturbation theory, and the conversion of the residual perturbation expansion into one which has no infrared divergence, and hence no need for an infrared cutoff. In the infrared factors, which are exponential in form, the infrared divergences arising from the real and virtual photons cancel out in the usual way. These factors can then be expressed solely in terms of the momenta of the initial and final charged particles and an integral over the region of phase space available to the undetected photons; they do not depend upon the specific details of the interaction. Electron scattering from a static potential is treated in considerable detail, and several other examples are discussed briefly. As an important byproduct of the general treatment, it is found that when the infrared contributions are separated in a particular way, they dominate the radiative corrections at high energies and together with certain “magnetic terms” and vacuum polarization corrections seem to give all the contributions proportional to $\ln(E/m)$. All of these corrections can be easily estimated (in most cases) simply from a knowledge of the external momenta of the charged particles; this then provides a very powerful and accurate way of estimating radiative corrections to high-energy processes.

* Supported in part by the U. S. Atomic Energy Commission, Contract AT(11-1)-50.

† Various refinements were added to the manuscript (particularly in Appendix C) during the academic year 1960–1961 while this author was a National Science Foundation Senior Fellow visiting the University of Paris. He is grateful to Professor M. M. Lévy for the hospitality afforded by the Laboratoire de Physique Théorique et Hautes Énergies at Orsay.

‡ Supported by National Science Foundation Grant.

YFS Master Equation

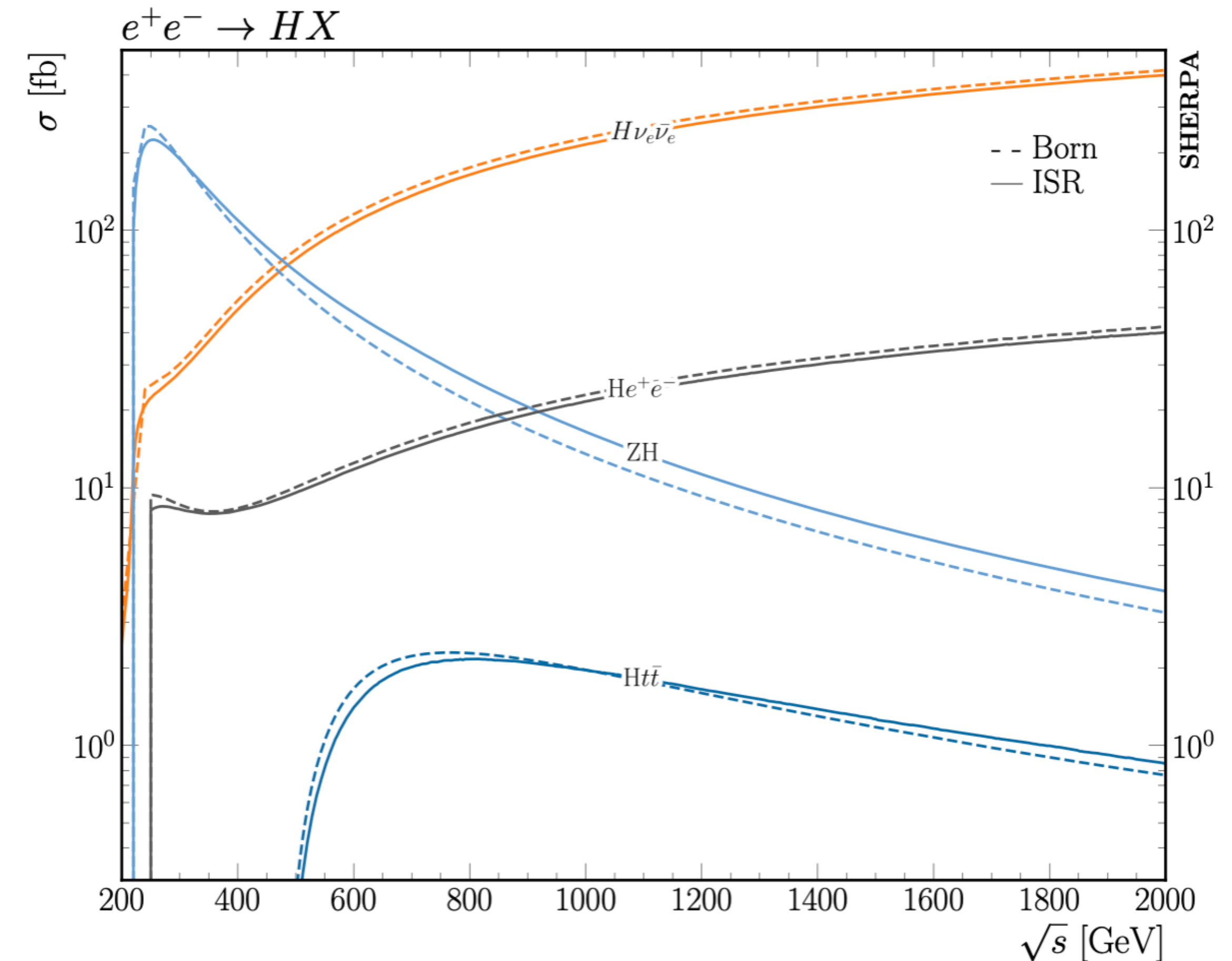
$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i) \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{s}(k_j)} + \sum_{\substack{j,k=1 \\ j < k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{s}(k_j)\tilde{s}(k_k)} + \dots \right)$$

This expression contains **no approximations**. It does require any further matching. The accuracy is limited by how far you can calculate the betas

$$Y(\Omega) = \sum_{i < j} \mathcal{R}e B_{ij}(\Phi_n) + \tilde{B}_{ij}(\Phi_{n+1})$$

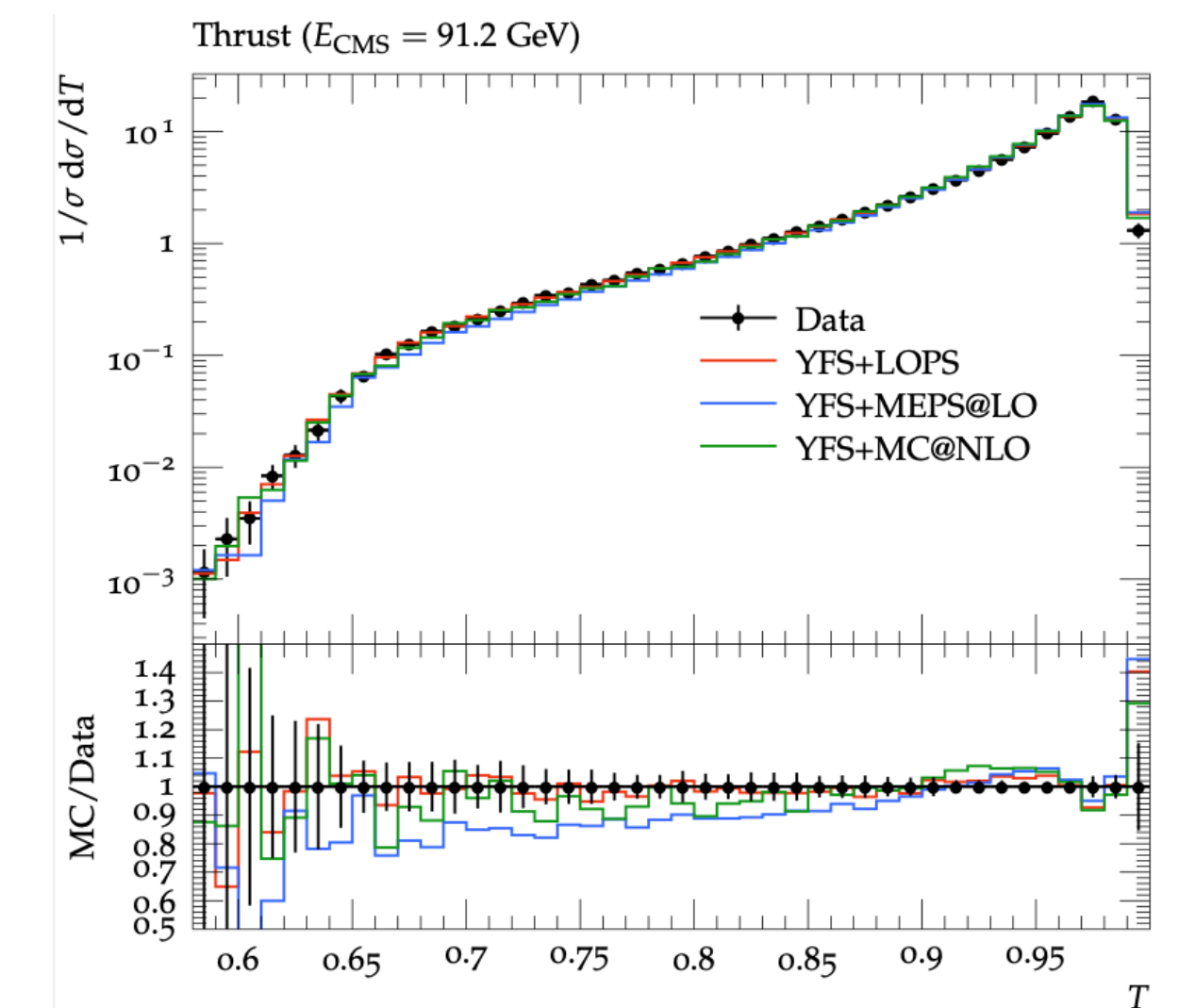
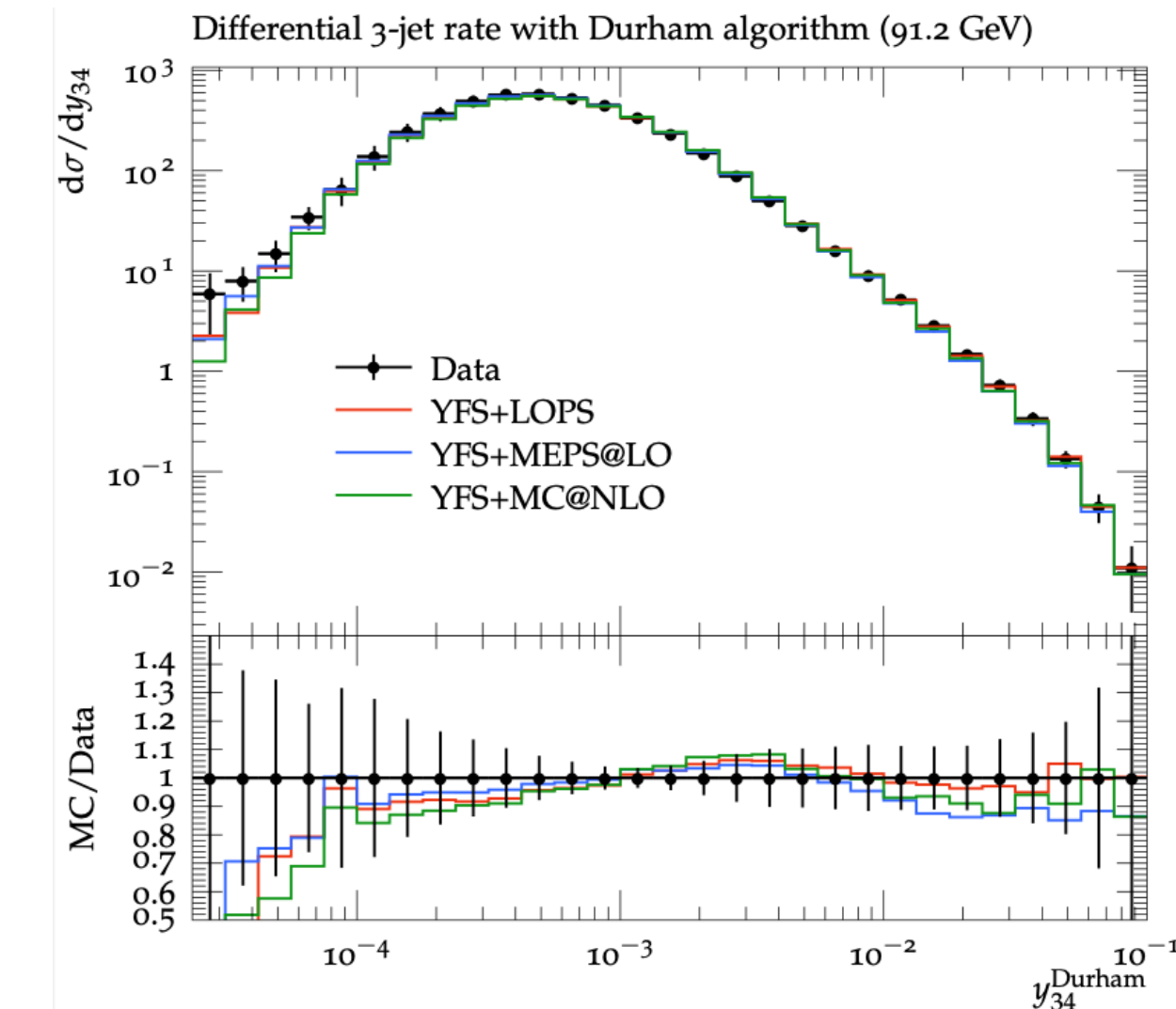
$$\tilde{\beta}_0^0(\Phi_n) = \mathcal{B}(\Phi_n)$$

- ❖ Automated within Sherpa for both initial and final dipoles [\[Krauss, Schönherr, Price 2022\]](#)
- ❖ Supplemented with collinear logs up to $\mathcal{O}(\alpha^3 L^3)$
- ❖ Complete treatment of multi-photon phasespace
:Explicit photons, no simplified electron PDF
- ❖ Amplitudes are provided by modern ME tools
COMIX/AMEGIC



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COMIX/AMEGIC
- ❖ We can combine Sherpa's state of the art QED
ISR and QCD FSR.



Virtual Emissions

Taking the soft limit allows us to factorise out amplitude

$$\tilde{\beta}_0^1(\Phi_n) = \mathcal{V}(\Phi_n) - \sum_{ij} \mathcal{D}_{ij}(\Phi_{ij})$$

IR Finite one-loop contribution

YFS Subtraction term

Full One-loop amplitude

Virtual Emissions

Unlike the CEEX approach we do our subtraction at the amplitude squared level

$$\tilde{\beta}_0^1(\Phi_n) = \mathcal{V}(\Phi_n) - \sum_{ij} \mathcal{D}_{ij}(\Phi_{ij})$$

❖ Calculation of one-loop EW(QCD) corrections is essentially automated

❖ OpenLoops [Eur.Phys.J.C 79 \(2019\) 10, 866](#)

❖ Recola [Comput.Phys.Comm. 214 \(2017\) 140-173](#)

❖ MadLoop [JHEP 05 \(2011\) 044](#)

❖ GoSam [Eur.Phys.J.C 74 \(2014\) 8, 3001](#)

❖ BlackHat [Phys.Rev.D 78 \(2008\) 036003](#)

I can't calculate one-loop amplitudes but I can interface them
- Anonymous MC Author

Full One-loop amplitude

Virtual Emissions

Automatically constructed in Sherpa

using YFS algorithm

Convolution of the YFS form-factor with the
born amplitude

$$\tilde{\beta}_0^1(\Phi_n) = \mathcal{V}(\Phi_n) - \sum_{ij} \mathcal{D}_{ij}(\Phi_{ij})$$

[SciPost Phys. 13 \(2022\) 026](#)

$$B_{ij} = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4 k}{k^2} \left(\frac{2p_i \theta_i - k}{k^2 - 2(k \cdot p_i) \theta_i} + \frac{2p_j \theta_j + k}{k^2 + 2(k \cdot p_j) \theta_j} \right)^2$$

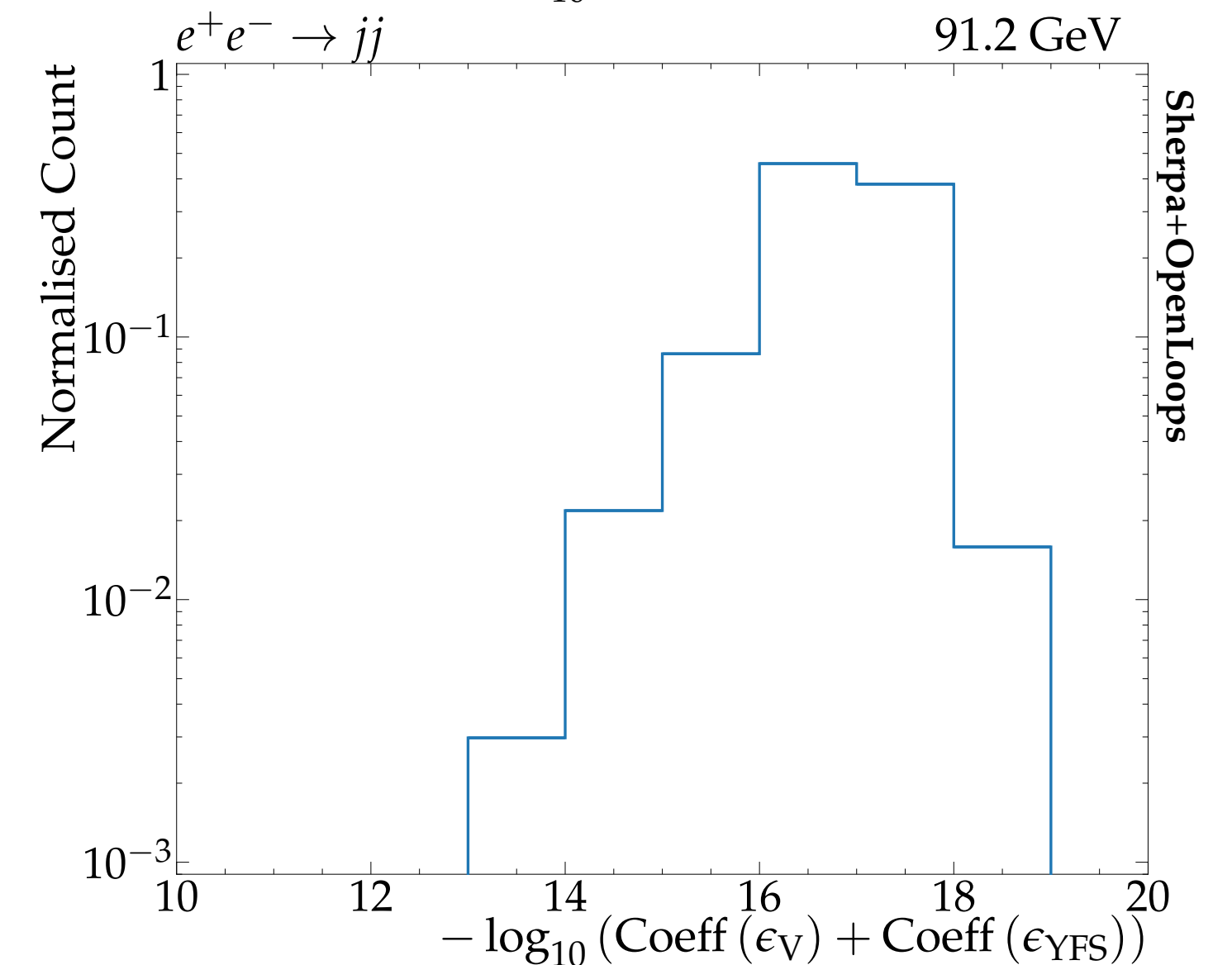
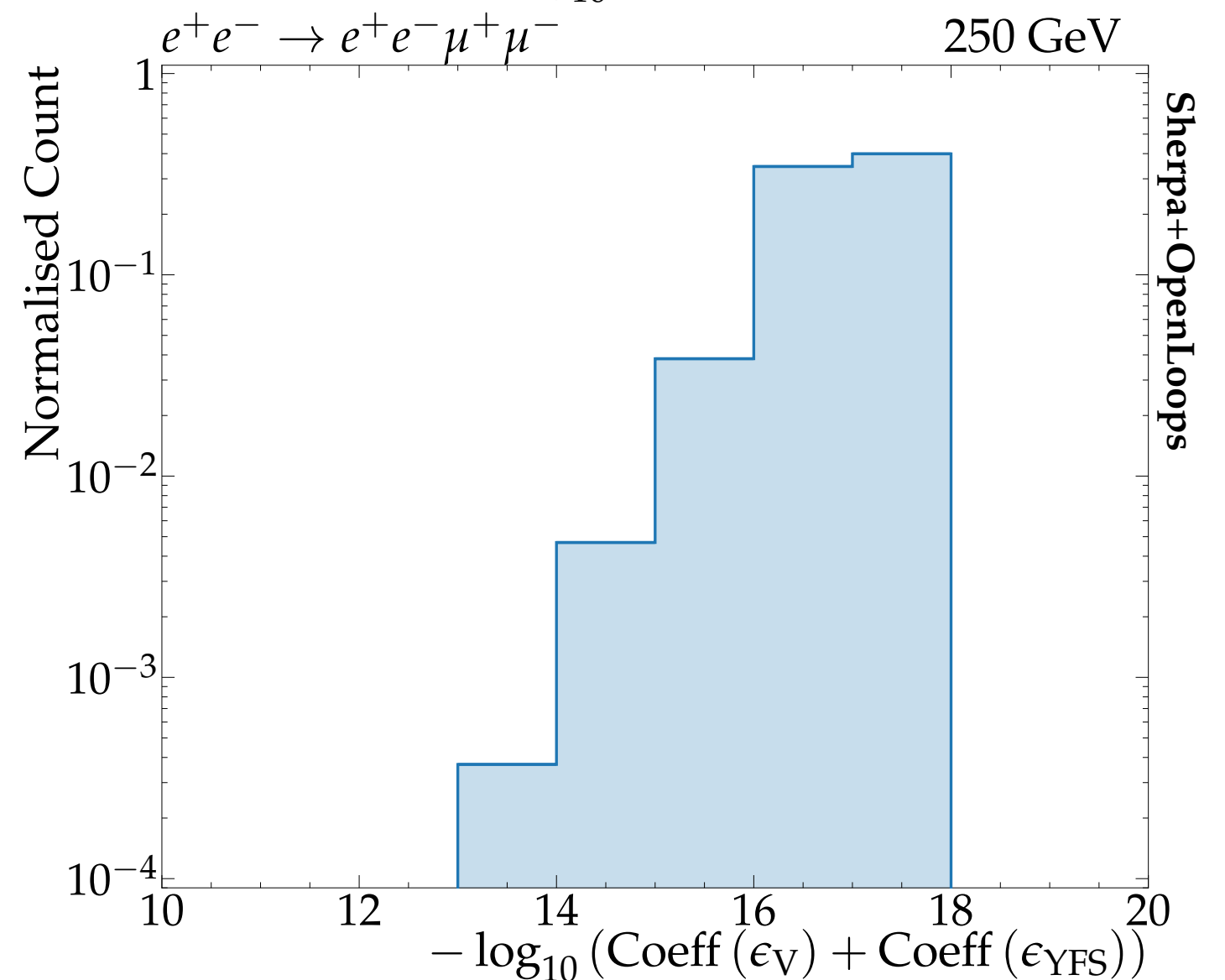
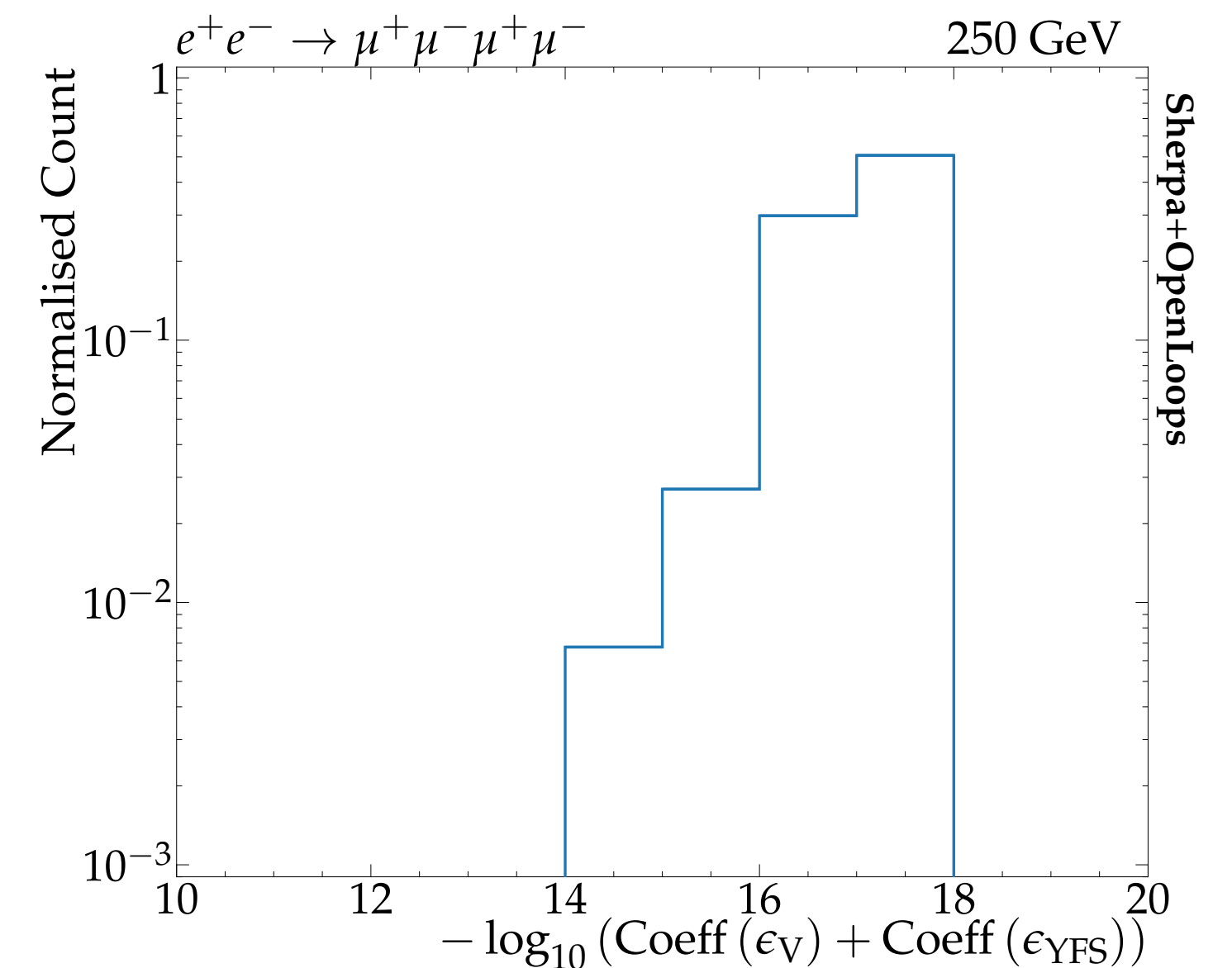
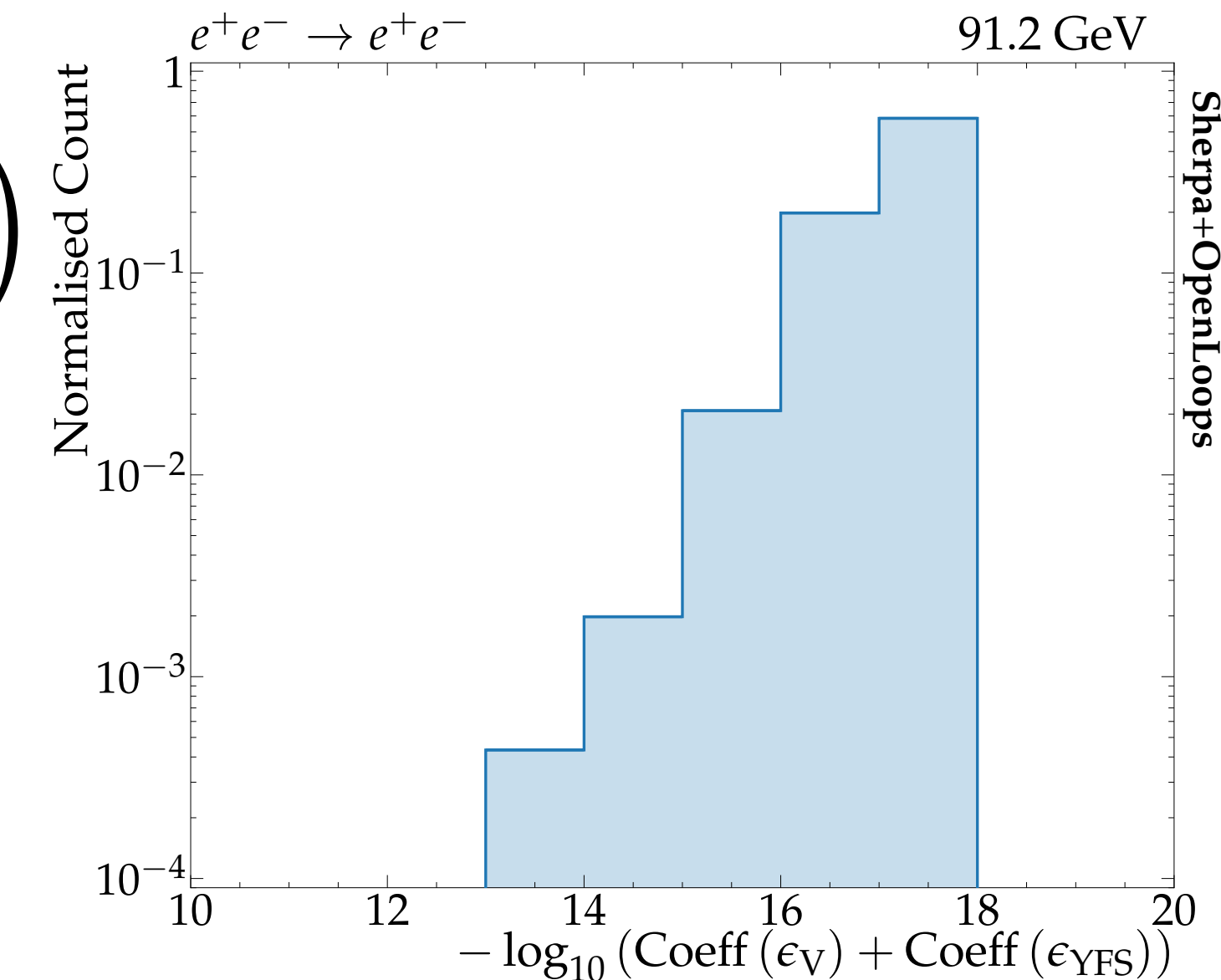
YFS Subtraction term

Virtual Corrections

$$\tilde{\beta}_0^1(\Phi_n) = \mathcal{V}(\Phi_n) - \sum_{ij} \tilde{\mathcal{D}}_{ij}(\Phi_{ij} \otimes \Phi_n)$$

Sherpa automatically constructs the subtraction terms while external tools provide the **IR divergent** one-loop amplitude

Pole cancellation for the virtual occur locally without input from the real emissions i.e Not via KLN theorem



Real Emissions

$$\tilde{\beta}_1^1(\Phi_{n+1}) = \frac{1}{2(2\pi)^3} \left| \mathcal{M}_0^{\frac{1}{2}}(\Phi_{n+1}) \right|^2 - \sum_{ij} \tilde{\mathcal{D}}_{ij}(\Phi_{ij+1} \otimes \Phi_n)$$

IR Finite contribution for real corrections

Real emission squared amplitude

YFS Subtraction term

Real Emissions

$$\tilde{\beta}_1^1(\Phi_{n+1}) = \frac{1}{2(2\pi)^3} \left| \mathcal{M}_0^{\frac{1}{2}}(\Phi_{n+1}) \right|^2 - \sum_{ij} \tilde{\mathcal{D}}_{ij}(\Phi_{ij+1} \otimes \Phi_n)$$

❖ Tree level amplitudes calculations fully automated

❖ Sherpa: Comix/Amegic

❖ Madgraph

❖ Whizard: O`Mega

❖

Real emission squared amplitude

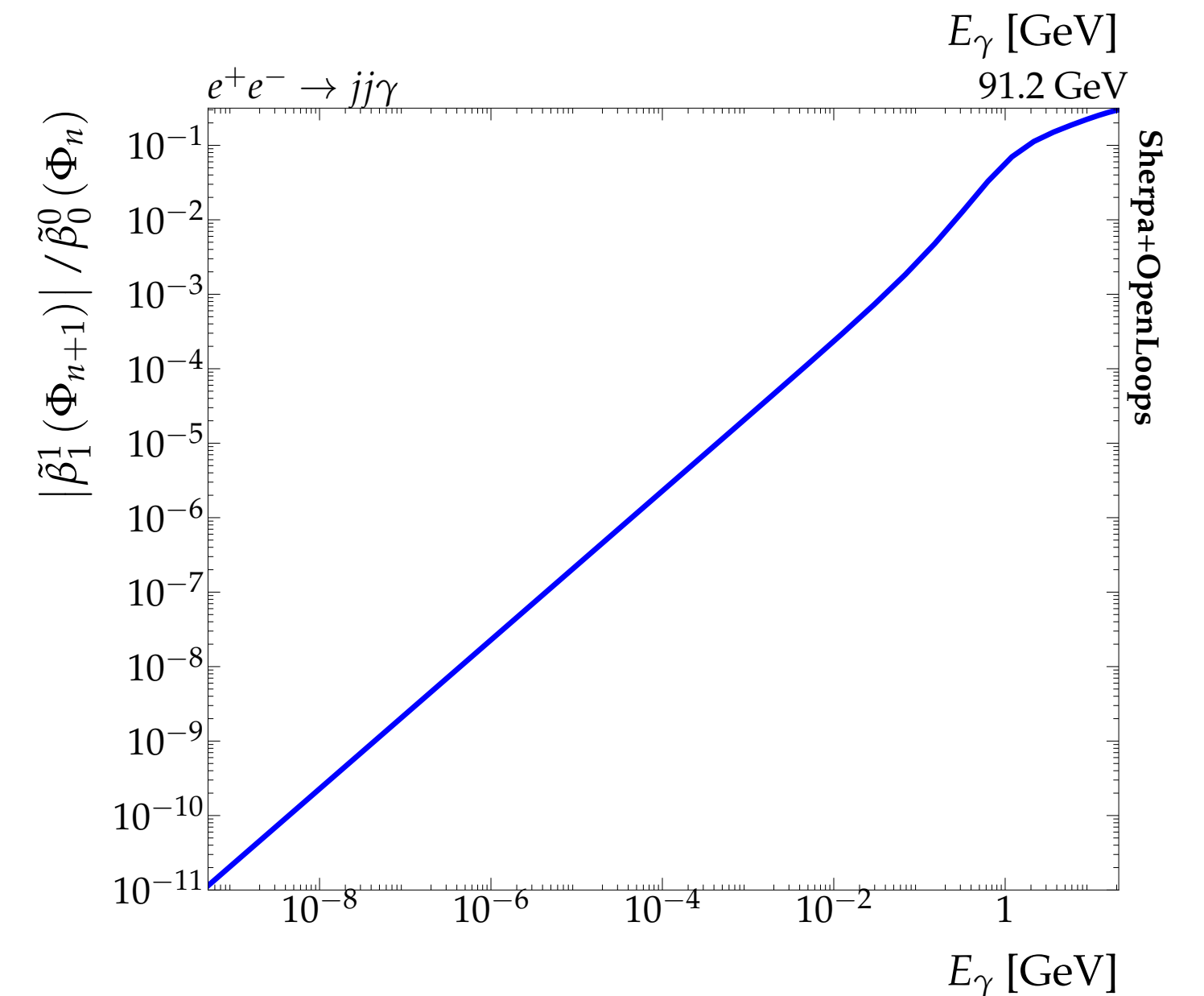
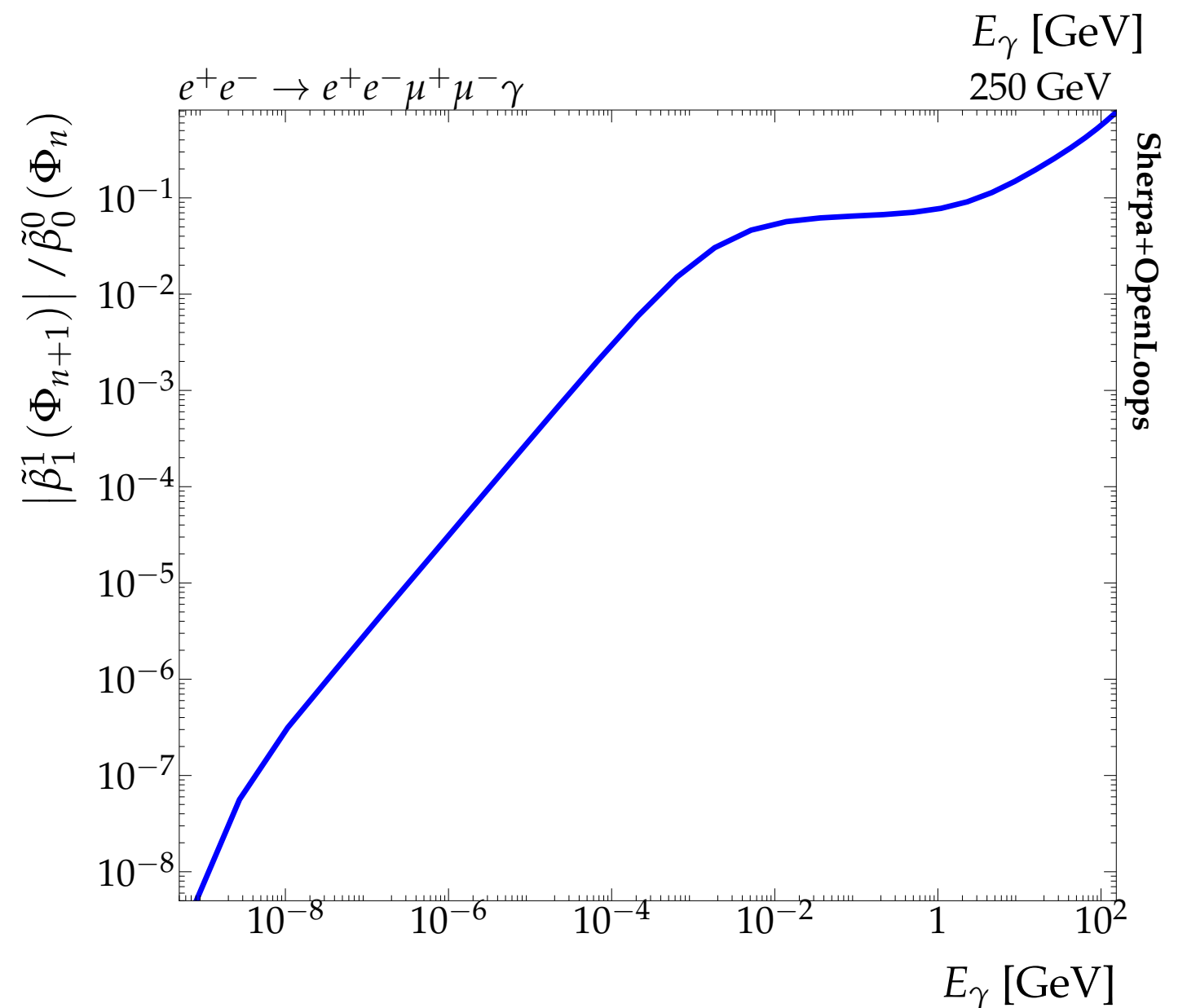
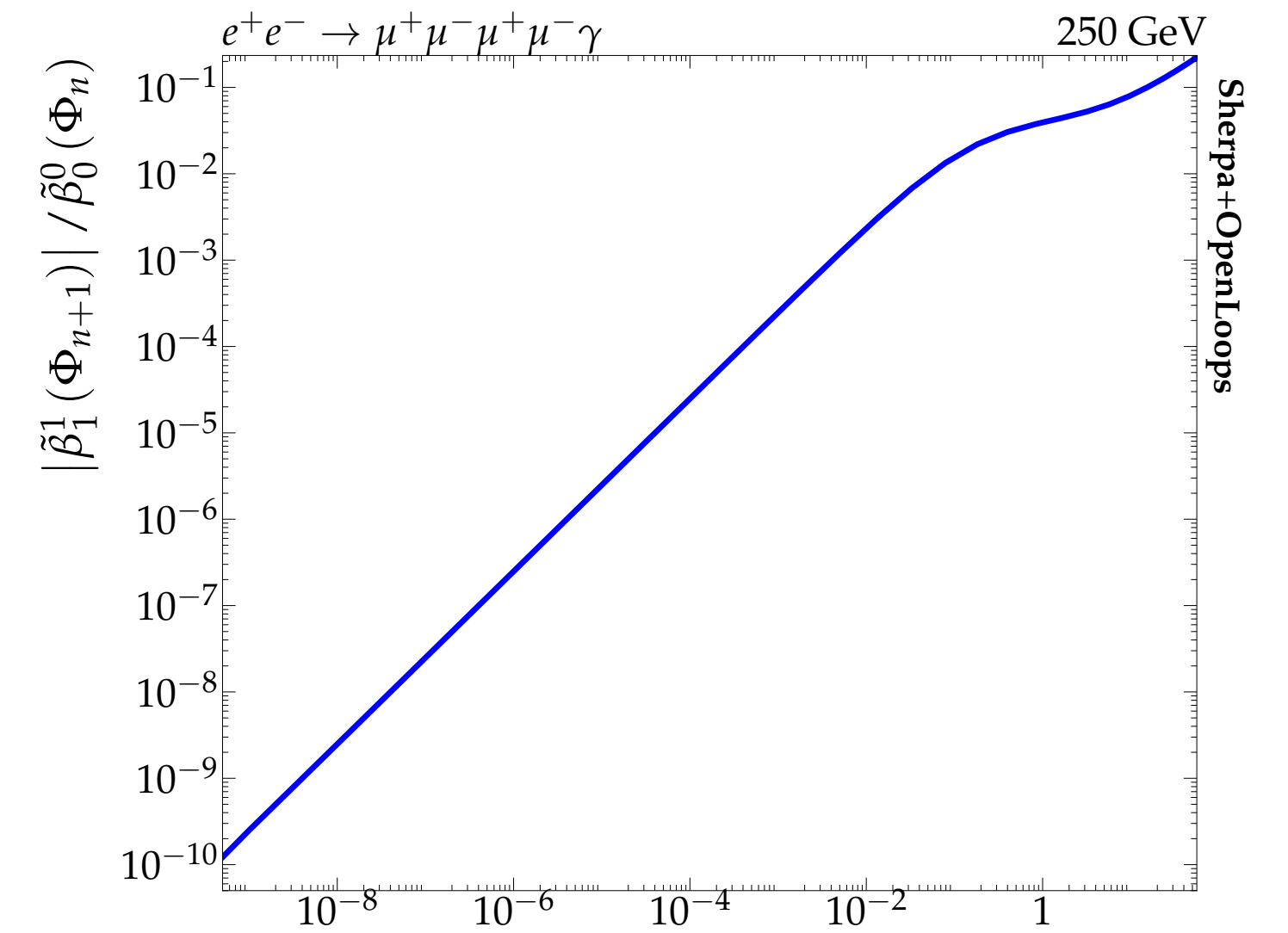
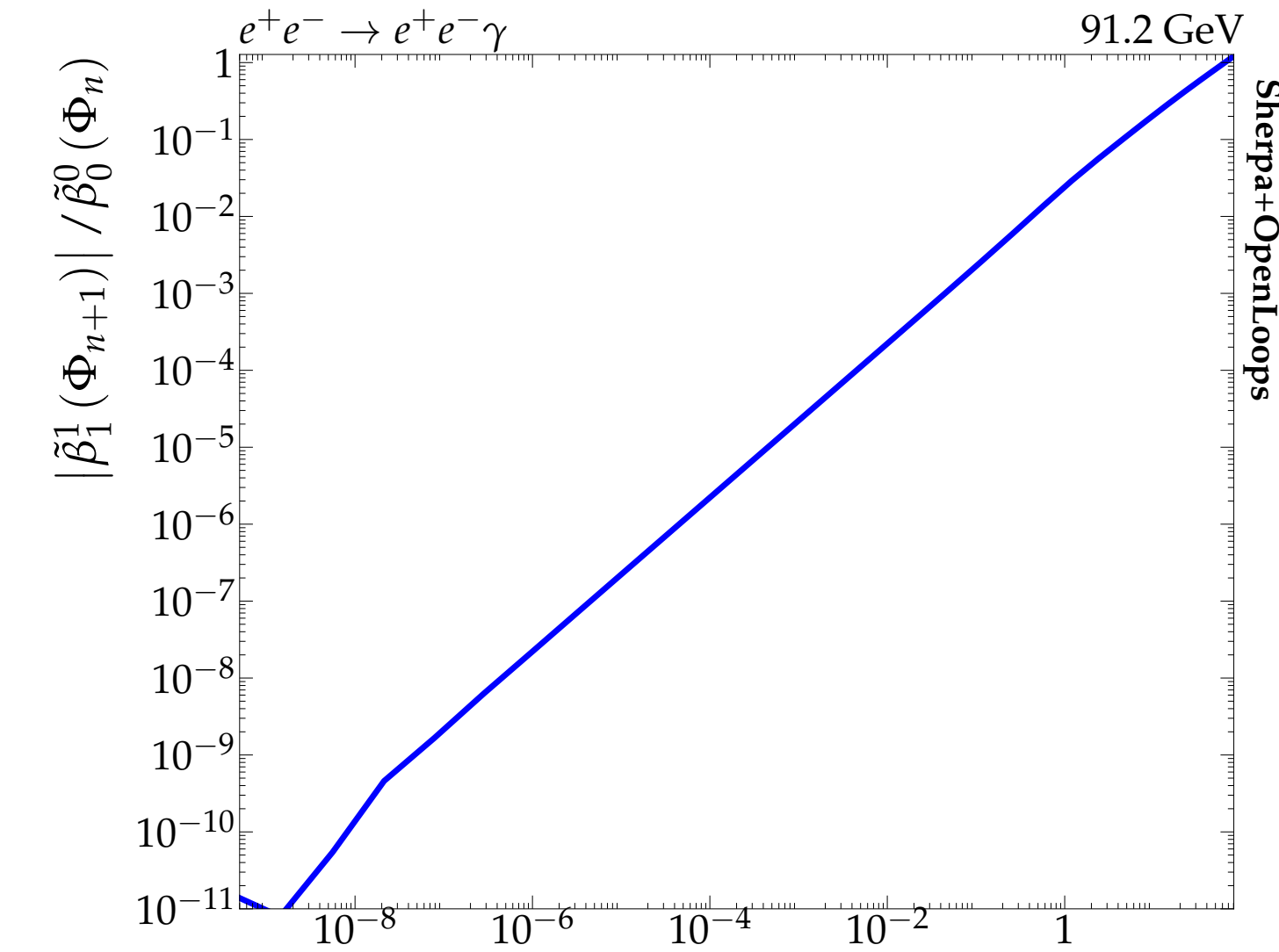
$$\tilde{\mathcal{D}}_{ij}(\Phi_{ij+1} \otimes \Phi_n) = \tilde{S}_{ij}(k) \tilde{\beta}_0^0(\Phi_n)$$

Real Corrections

$$\tilde{\beta}_1^1(\Phi_{n+1}) = \mathcal{R}(\Phi_{n+1}) - \sum_{ij} \mathcal{D}_{ij}(\Phi_{ij+1} \otimes \Phi_n)$$

The real emissions are simple tree level amplitudes which can be calculated using standard methods in Sherpa

In the soft limit we see this contribution vanishes



NNLO: Real-Virtual Emissions

YFS defined to all orders so we
can look at NNLO corrections

$$\tilde{\beta}_1^2(\Phi_{n+1}) = \mathcal{RV}(\Phi_{n+1}) - \sum_{ij} \mathcal{D}_{ij}^{(1)}(\Phi_{ij+1} \otimes \Phi_n)$$

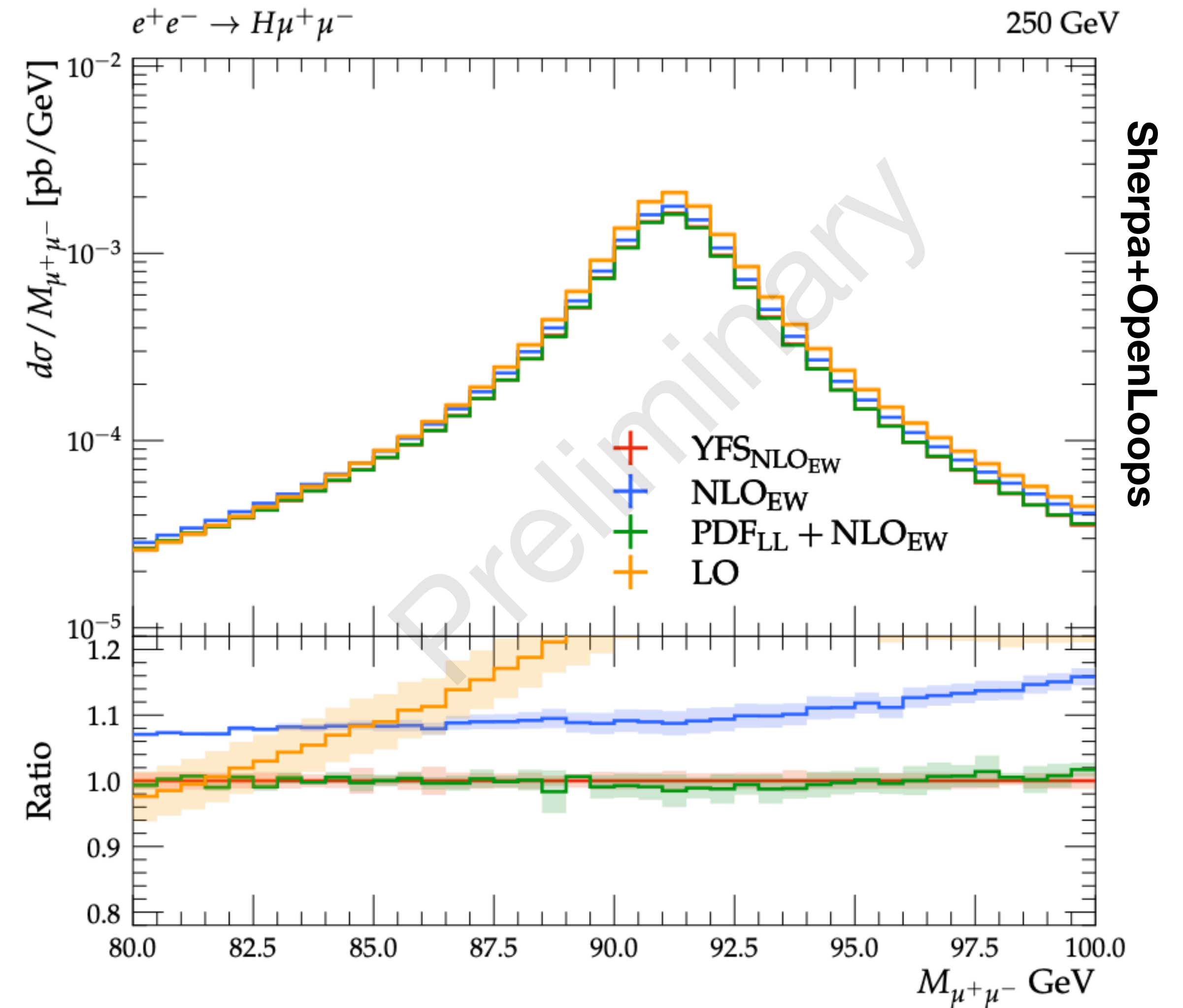
IR Finite one-loop contribution
with additional real emission

Full One-loop amplitude

YFS Subtraction term

Case Study: Higgsstrahlung

- ❖ $\text{YFS}_{\text{NLO}_{\text{EW}}}$ results are compatible with Sherpa's collinear (**CS**) based NLO calculations
- ❖ Great consistency check as both approaches are based on very different algorithms
- ❖ Error bands are from varying the input scheme



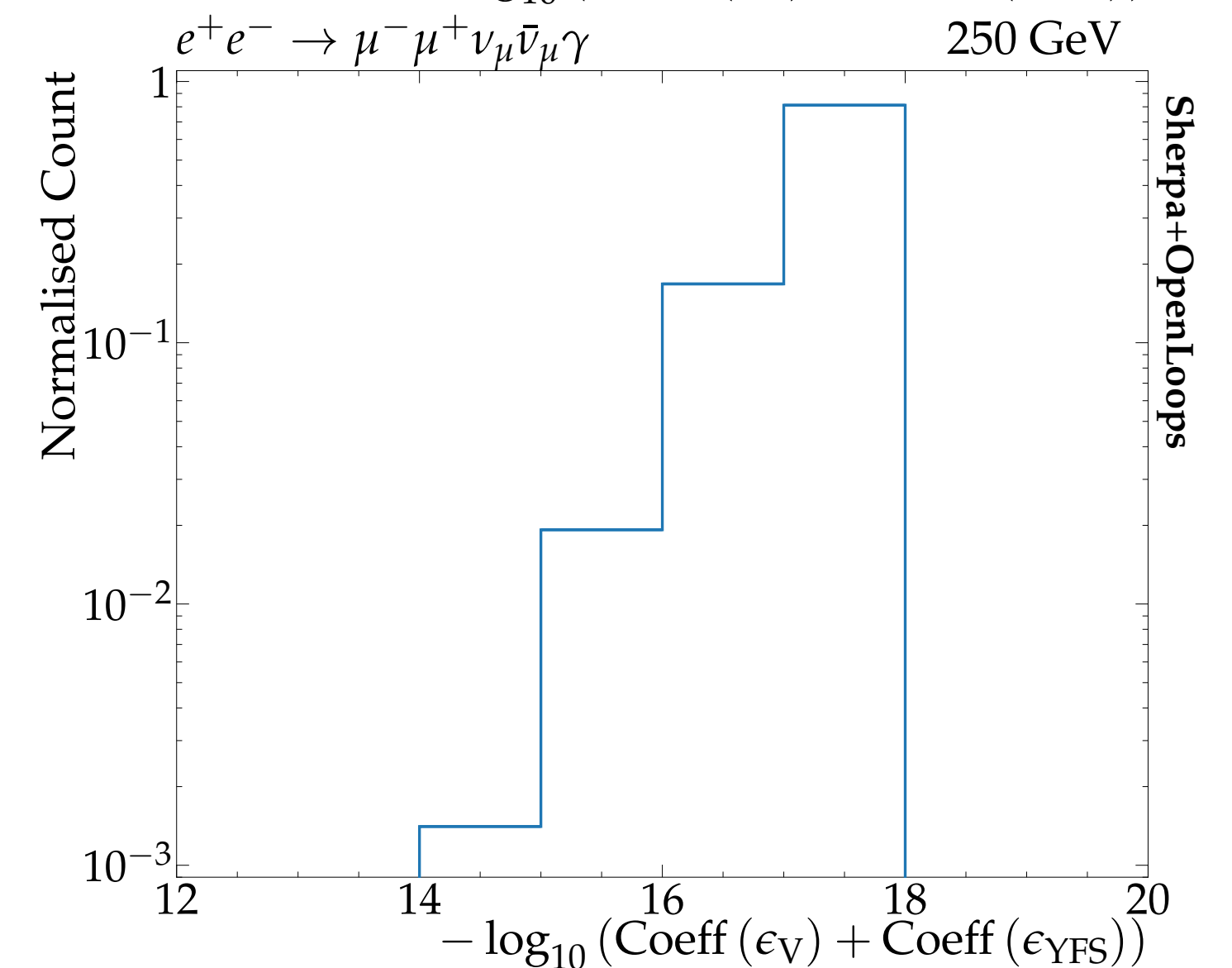
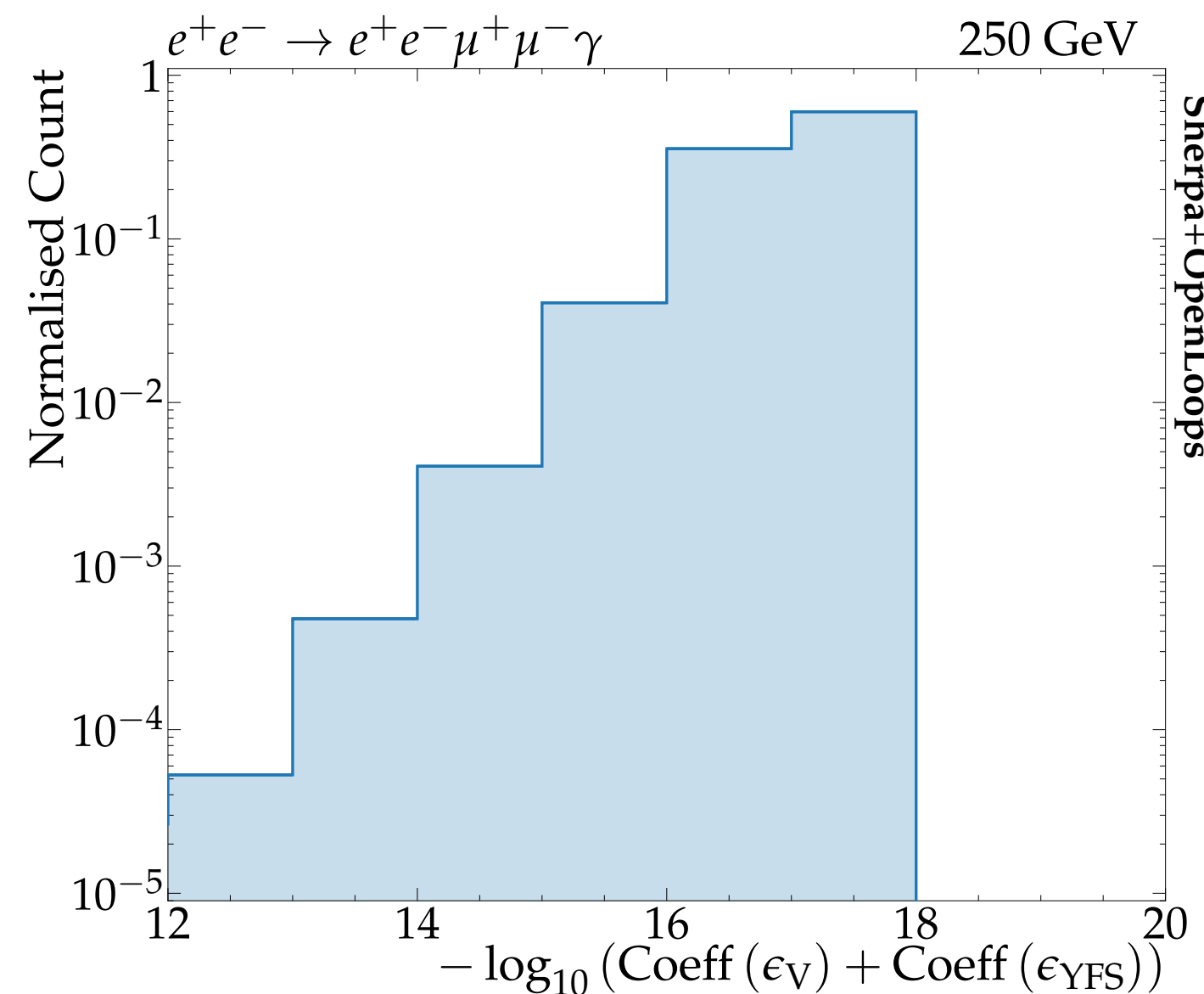
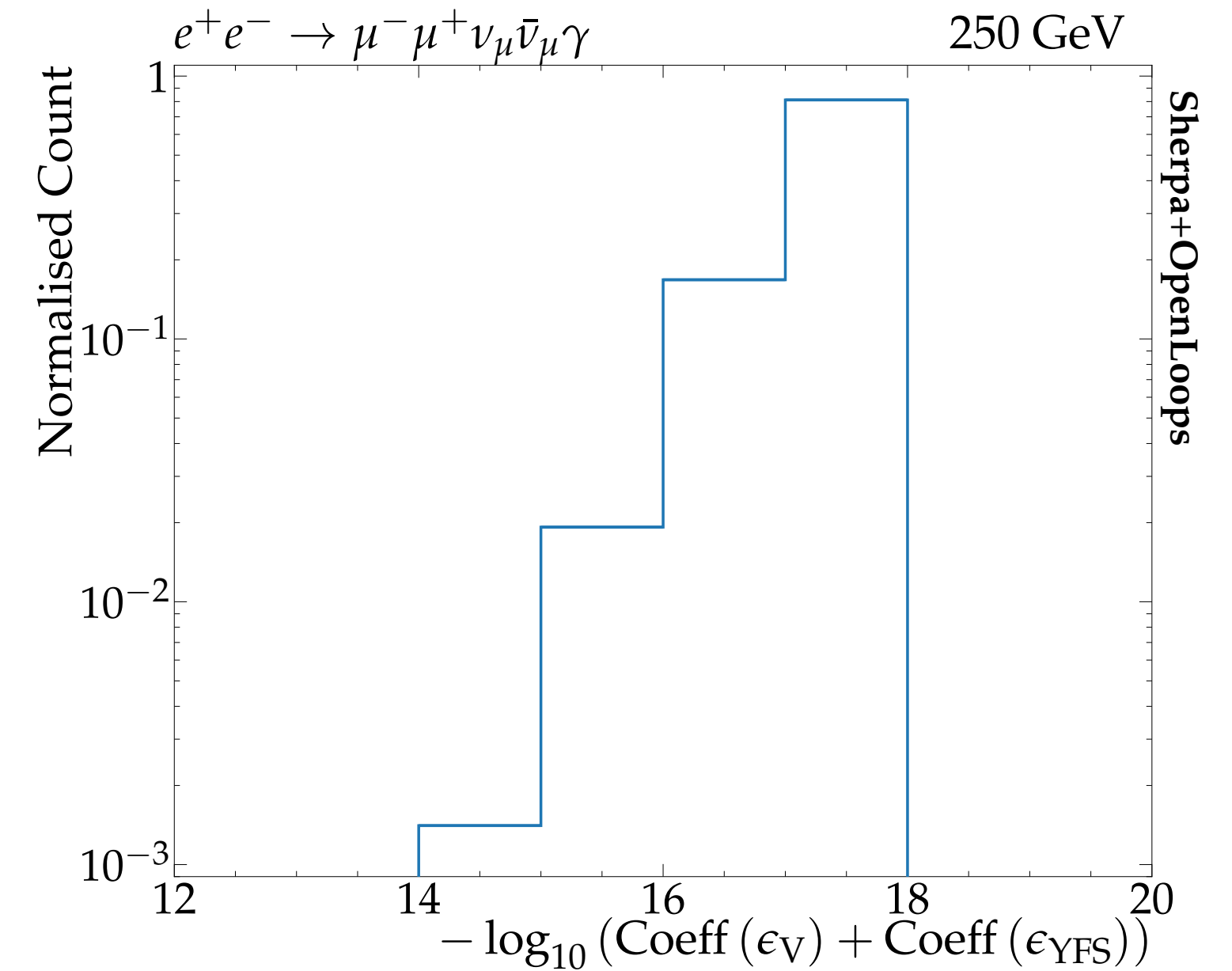
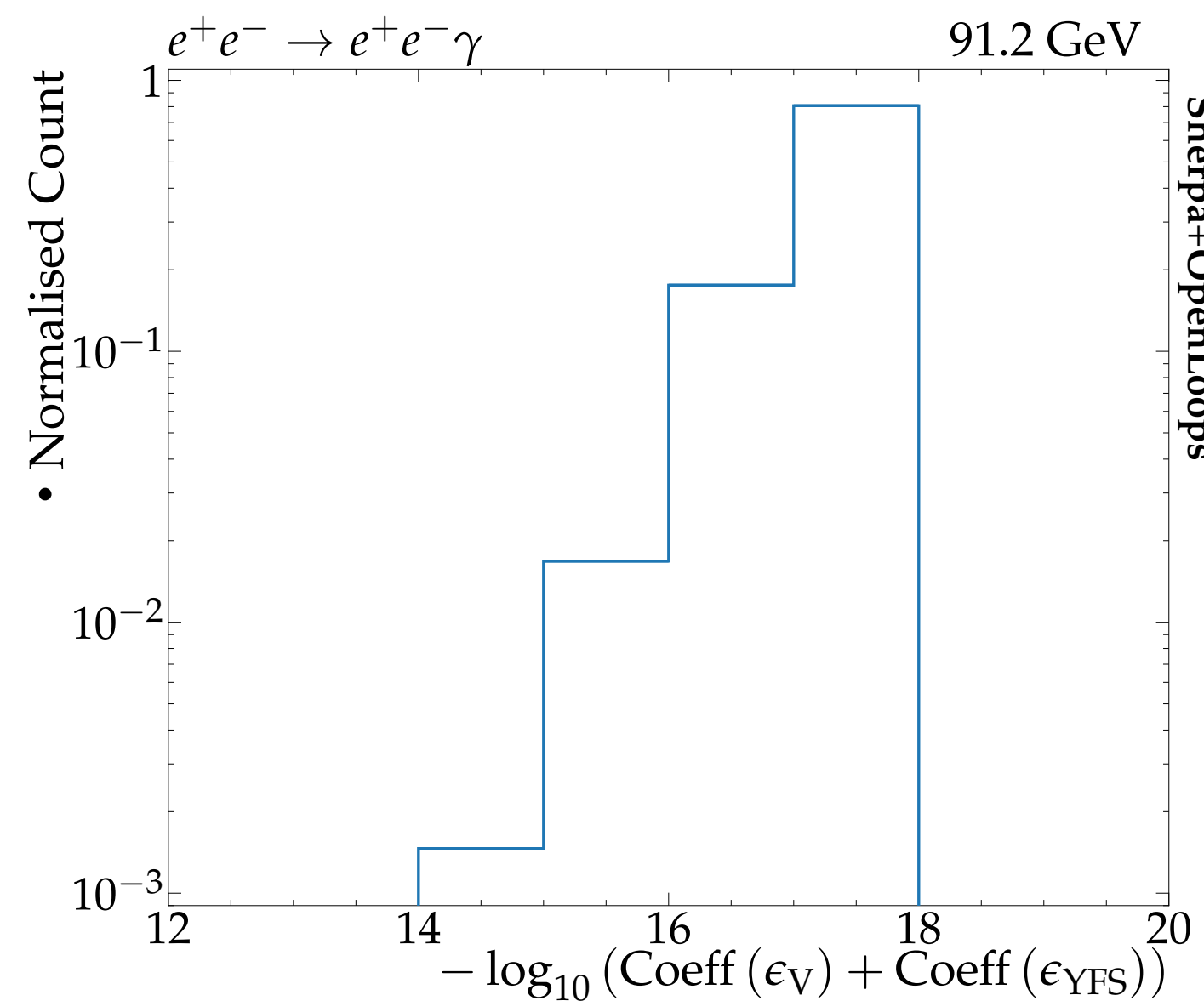
[A.Price, F.Krauss et al 25XY.abcd](#)

Real-Virtual Corrections

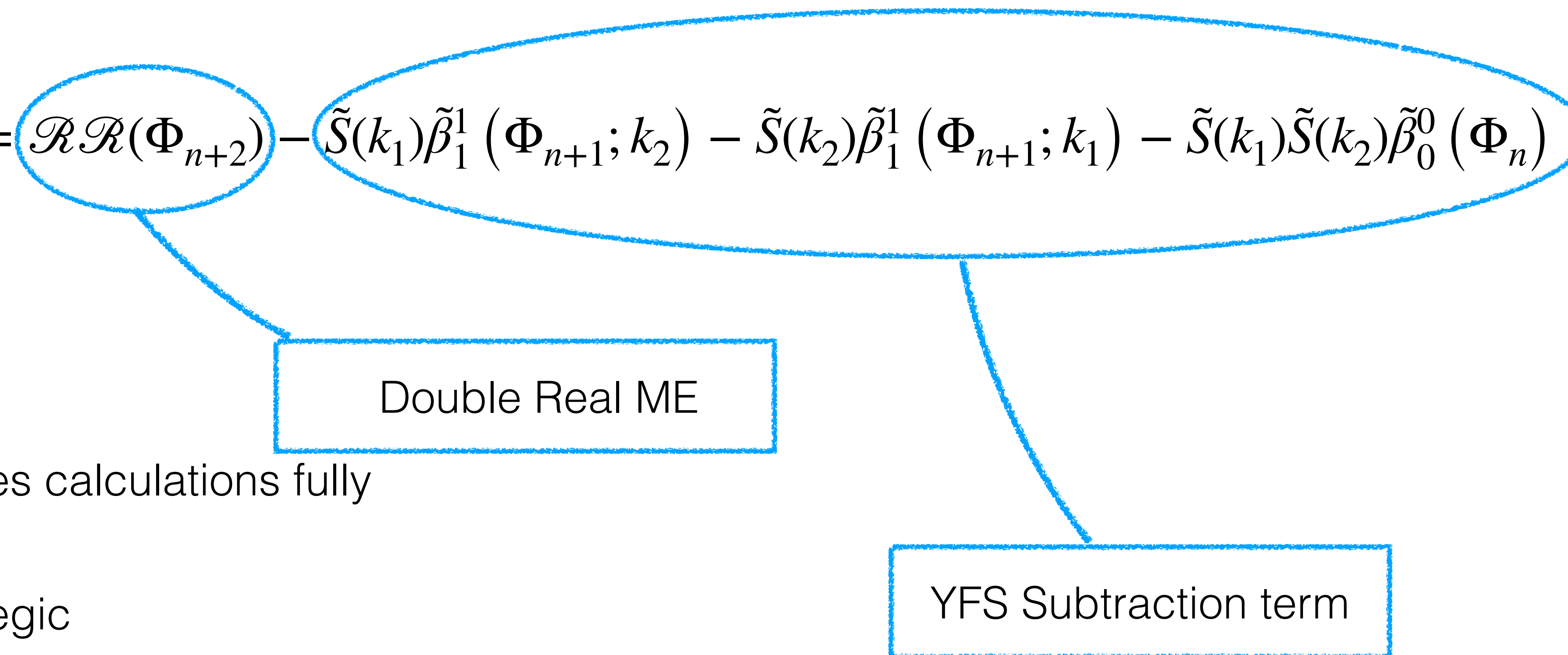
$$\tilde{\beta}_1^2(\Phi_{n+1}) = \mathcal{RV}(\Phi_{n+1}) - \sum_{ij} \mathcal{D}_{ij}^{(1)}(\Phi_{ij+1} \otimes \Phi_n)$$

One-loop amplitudes again provided by external tool.
Sherpa again automatically constructs the subtraction term

These corrections contain no approximations e.g
all masses are kept



Double Real Corrections

$$\tilde{\beta}_2^2(\Phi_{n+2}) = \mathcal{R}\mathcal{R}(\Phi_{n+2}) - \tilde{S}(k_1)\tilde{\beta}_1^1(\Phi_{n+1}; k_2) - \tilde{S}(k_2)\tilde{\beta}_1^1(\Phi_{n+1}; k_1) - \tilde{S}(k_1)\tilde{S}(k_2)\tilde{\beta}_0^0(\Phi_n)$$


Double Real ME

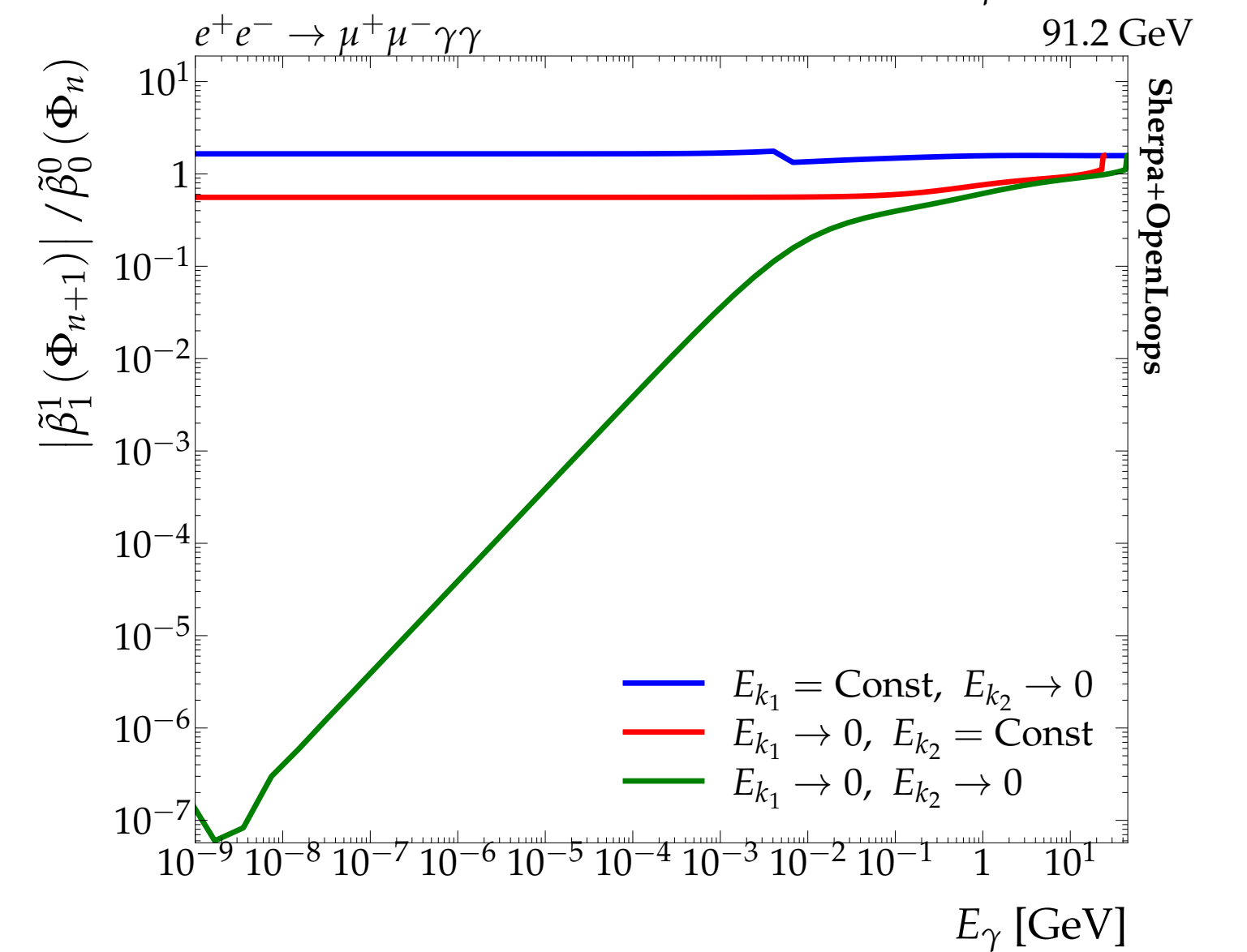
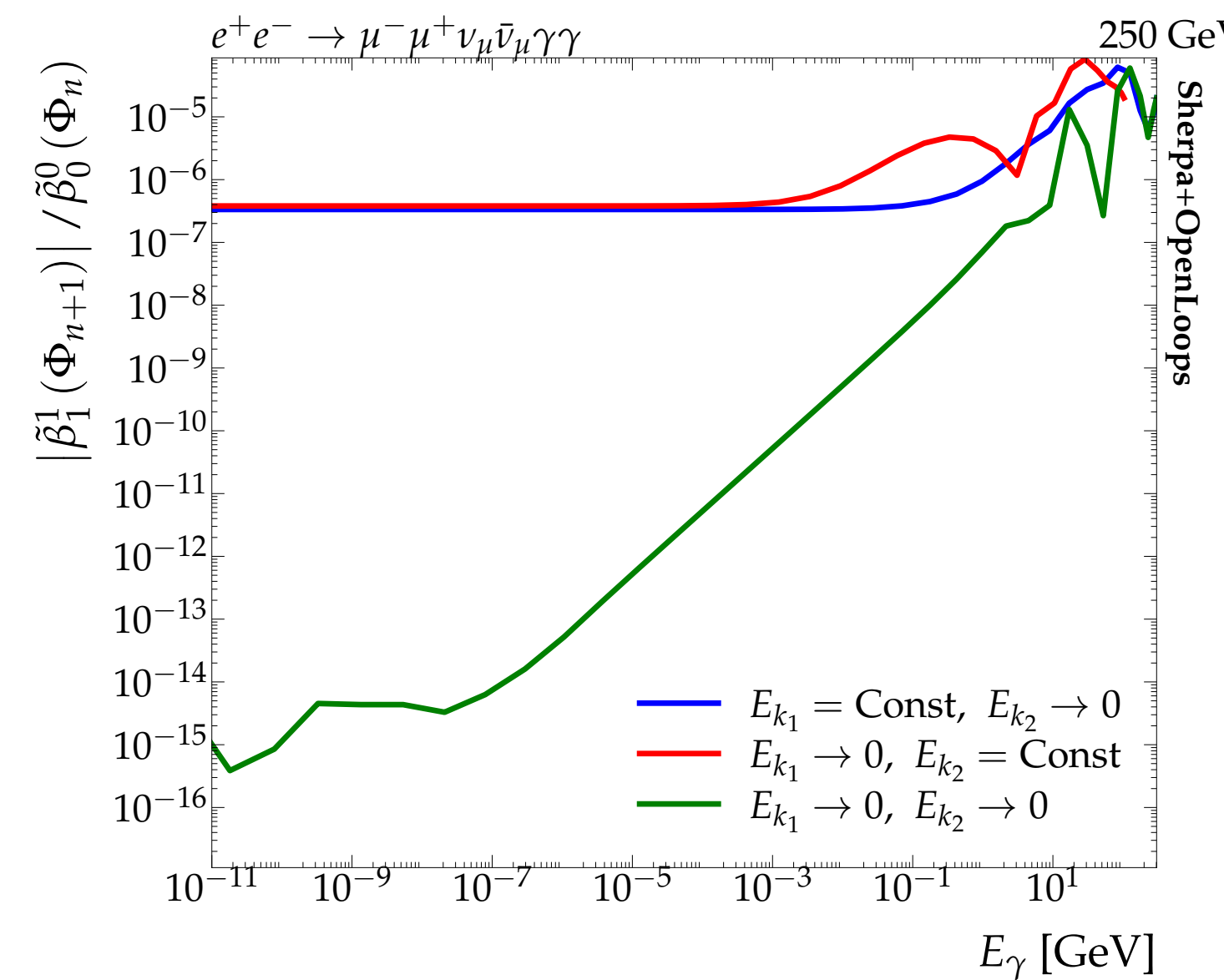
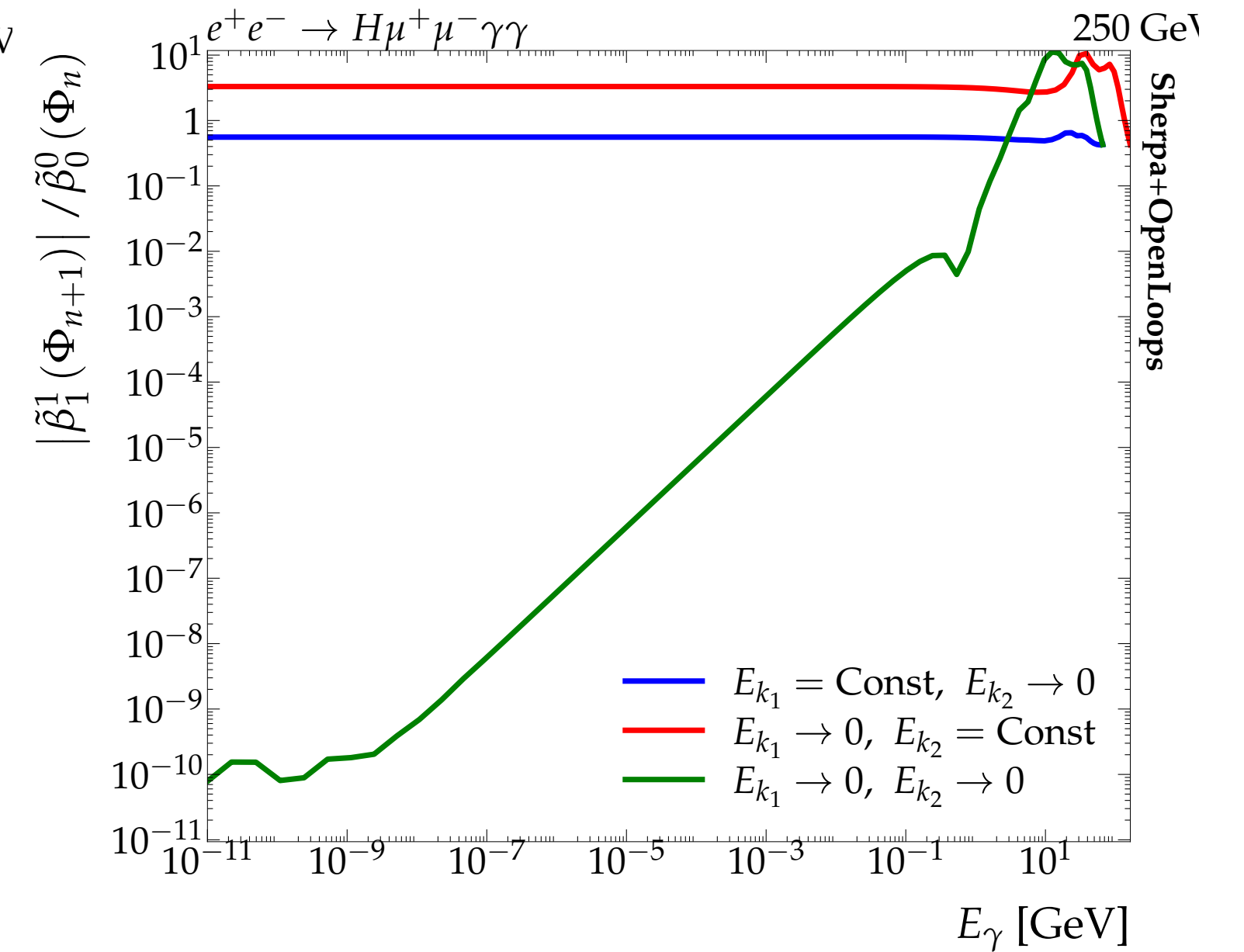
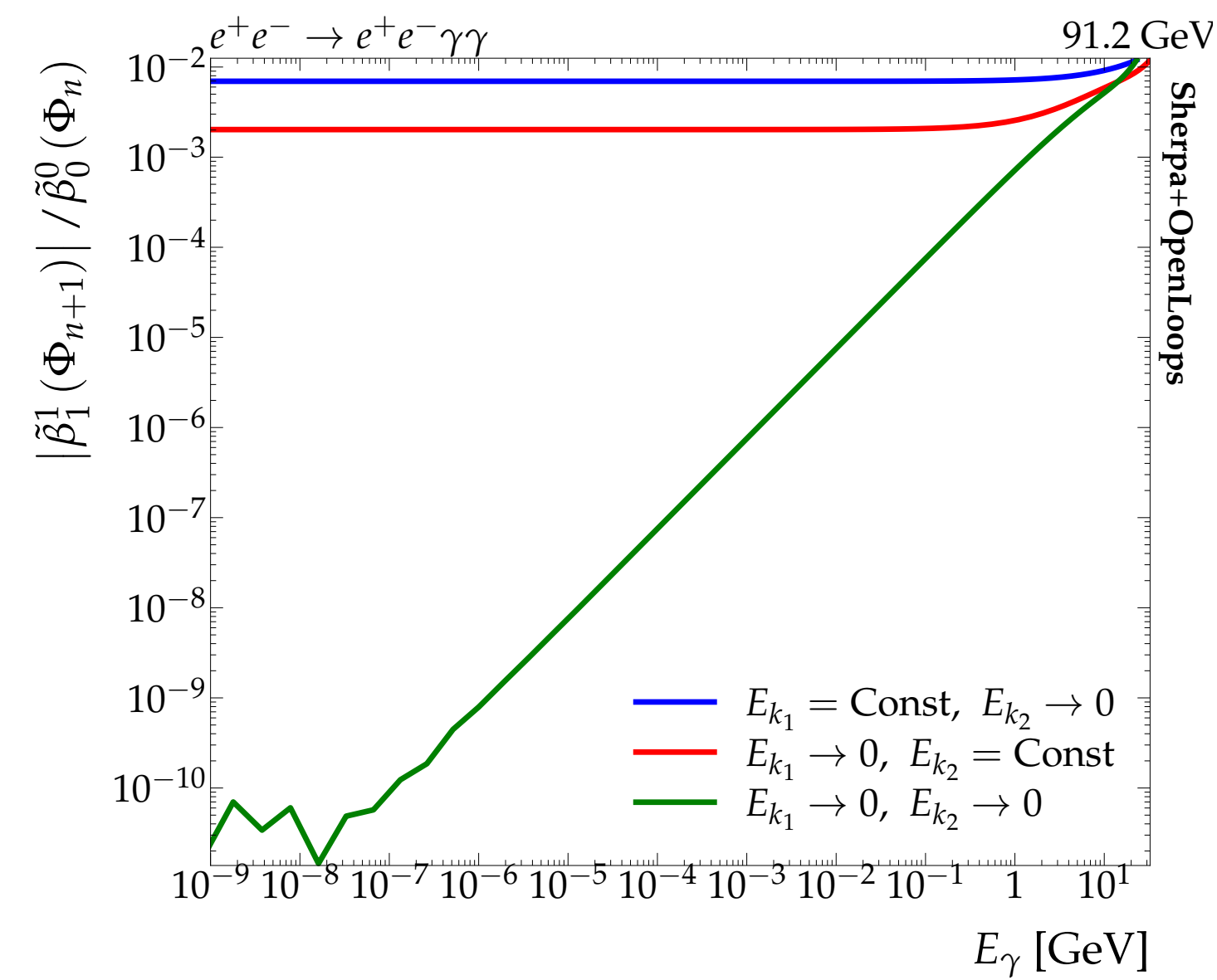
YFS Subtraction term

- ❖ Tree level amplitudes calculations fully automated
- ❖ Sherpa: Comix/Amegic
- ❖ Madgraph
- ❖ Whizard: O`Mega
- ❖

Double Real Corrections

Double real is by far the most complicated structure

We can examine its behaviour in various limits and find our approach is quite stable



Double Virtual Corrections

Unfortunately, there is not automated tool for the calculation of double virtual correction.

But there is significant work underway

❖ Two-Loop QED Corrections to the Scattering of Four Massive Leptons

[Phys.Rev.Lett. 132 \(2024\) 23](#)

❖ Two-loop radiative corrections to $e^+e^- \rightarrow \gamma\gamma^*$ cross section

[JHEP 11 \(2023\) 148](#)

❖ Lepton-pair scattering with an off-shell and an on-shell photon at two loops
in massless QED

[JHEP 11 \(2023\) 041](#)

❖ Two-Loop Electroweak Corrections with Fermion Loops to $e^+e^- \rightarrow ZH$

[Phys.Rev.Lett. 130 \(2023\) 3](#)

Double Virtual Corrections

Unfortunately, there is not automated tool for the calculation of double virtual correction.

GRIFFIN v1.0: $f\bar{f} \rightarrow f'\bar{f}'$ with NLO EW corrections and h.o. @ Z-pole

Future upgrades:

- Bhabha scattering ($f = f'$)
- Higher-order off-resonance corrections, e.g.
 $\mathcal{O}(\alpha\alpha_s)$, Heller, v.Manteuffel, Schabinger, Spiesberger '20
 $\mathcal{O}(N_f\alpha^2)$ Bonciani et al. '21
- SMEFT $d=6$ operator effects
- W production and decay (a.k.a. charged-current DY)

Try out the code: github.com/lisongc/GRIFFIN/releases

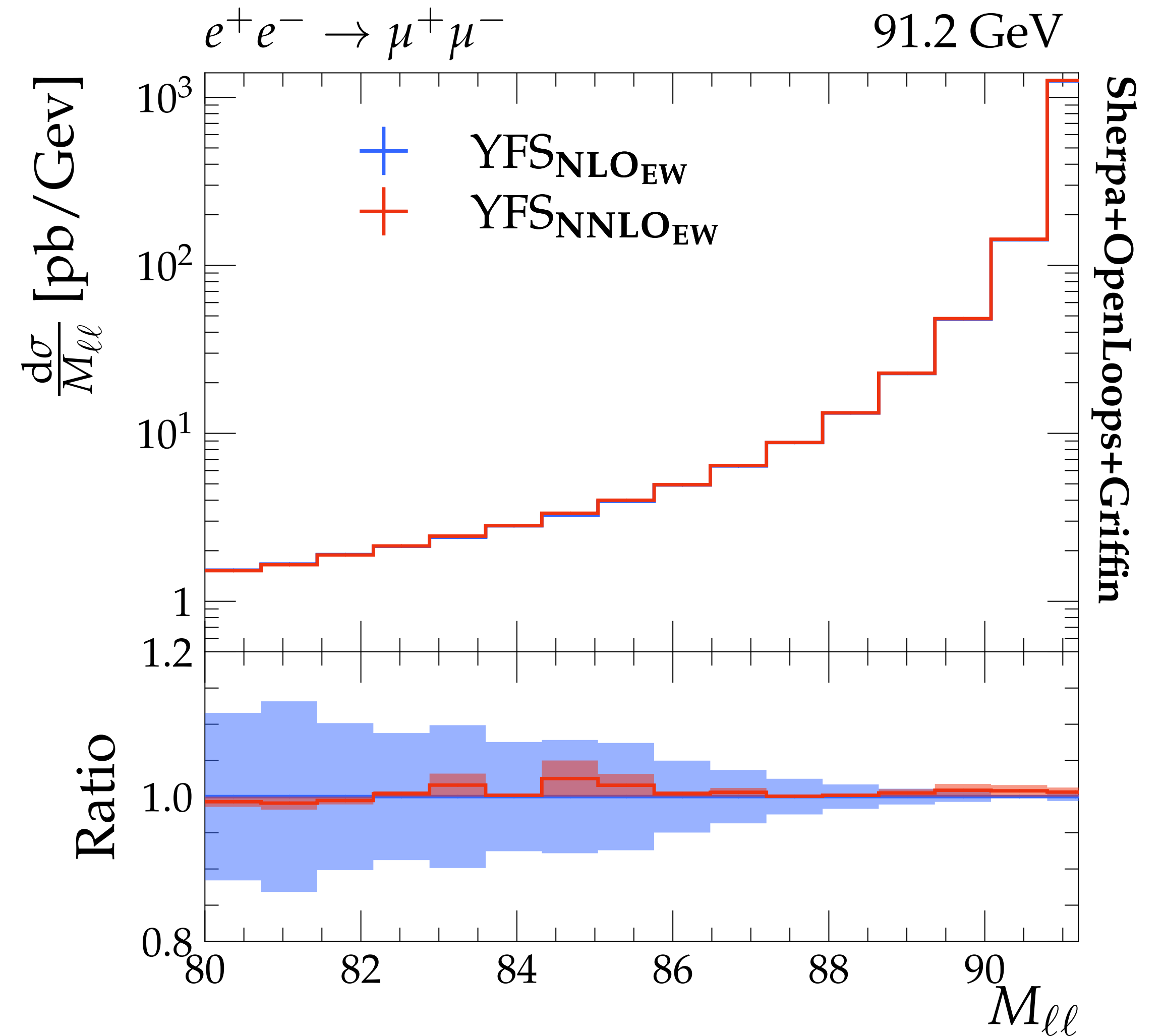
Feedback welcome!



Interfaced to the new Sherpa3 generator

[Ayres Freitas LoopFest 2024](#)

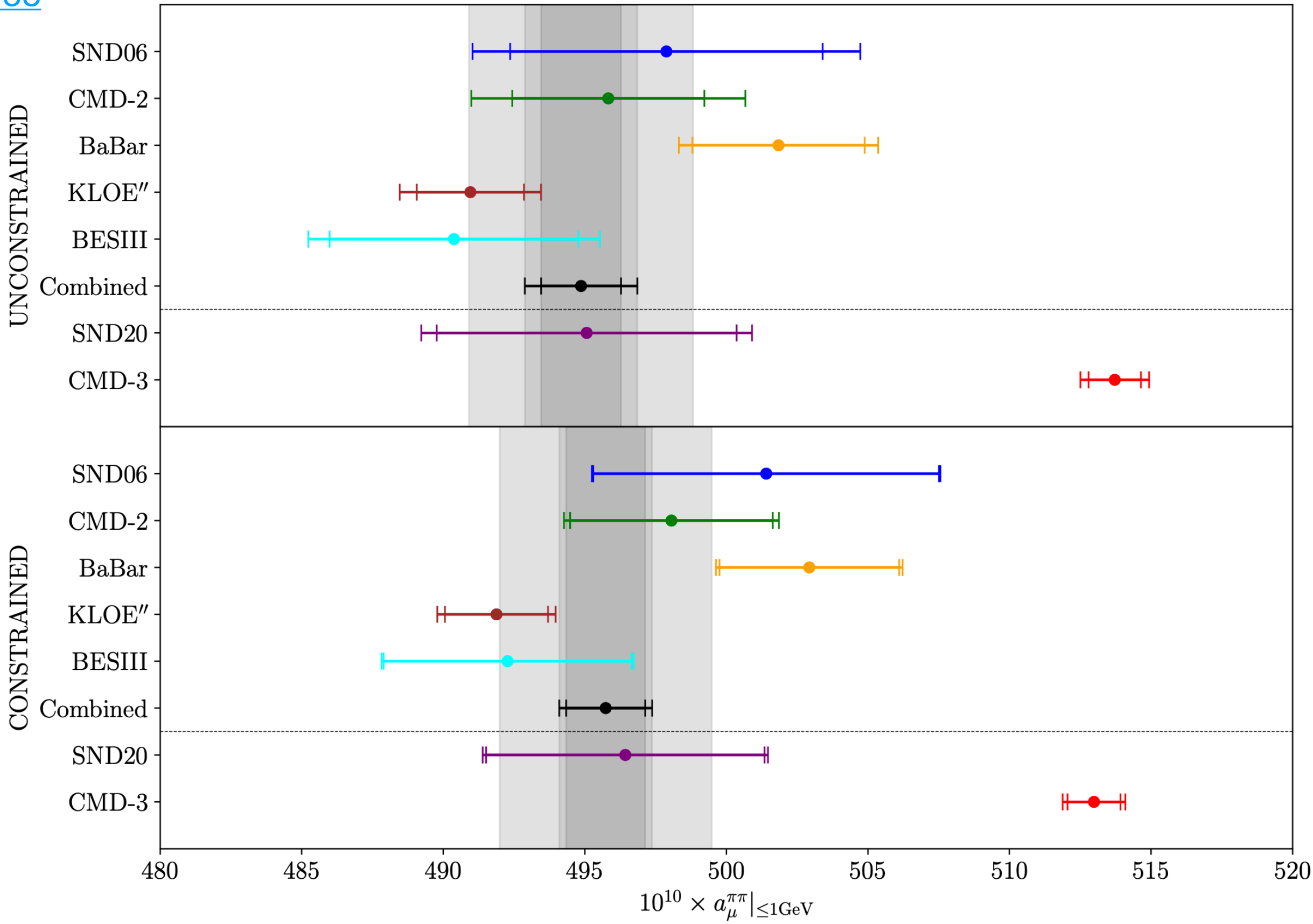
- ❖ Sherpa has a new fully automated NLO_{EW} matching scheme
- ❖ NNLO_{EW} matching is underway
 - ❖ All subtractions have been implemented
 - ❖ Limited by missing two-loop amplitudes
- ❖ YFS is defined to all orders
 - ❖ We know how to do the subtraction we are just missing the amplitudes



Motivation: $e^+e^- \rightarrow \pi^+\pi^-$

“high priority for all the developers of MC tools is the inclusion of structure-dependent corrections for pion production”

[Phys.Rept. 1143 \(2025\) 1-158](#)



YFS Calculation

A.Price, A.Siodmok: **UJ**

S.Delorme, J.Gluza, N. Salone

[Leplumey, Stoffer 2501.09643](#)

YFS for $e^+e^- \rightarrow \pi^+\pi^-$

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i) \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{s}(k_j)} + \sum_{\substack{j,k=1 \\ j < k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{s}(k_j)\tilde{s}(k_k)} + \dots \right)$$

❖ Soft Resummation is blind to the spin of particle: **Algorithm unchanged for scalars**

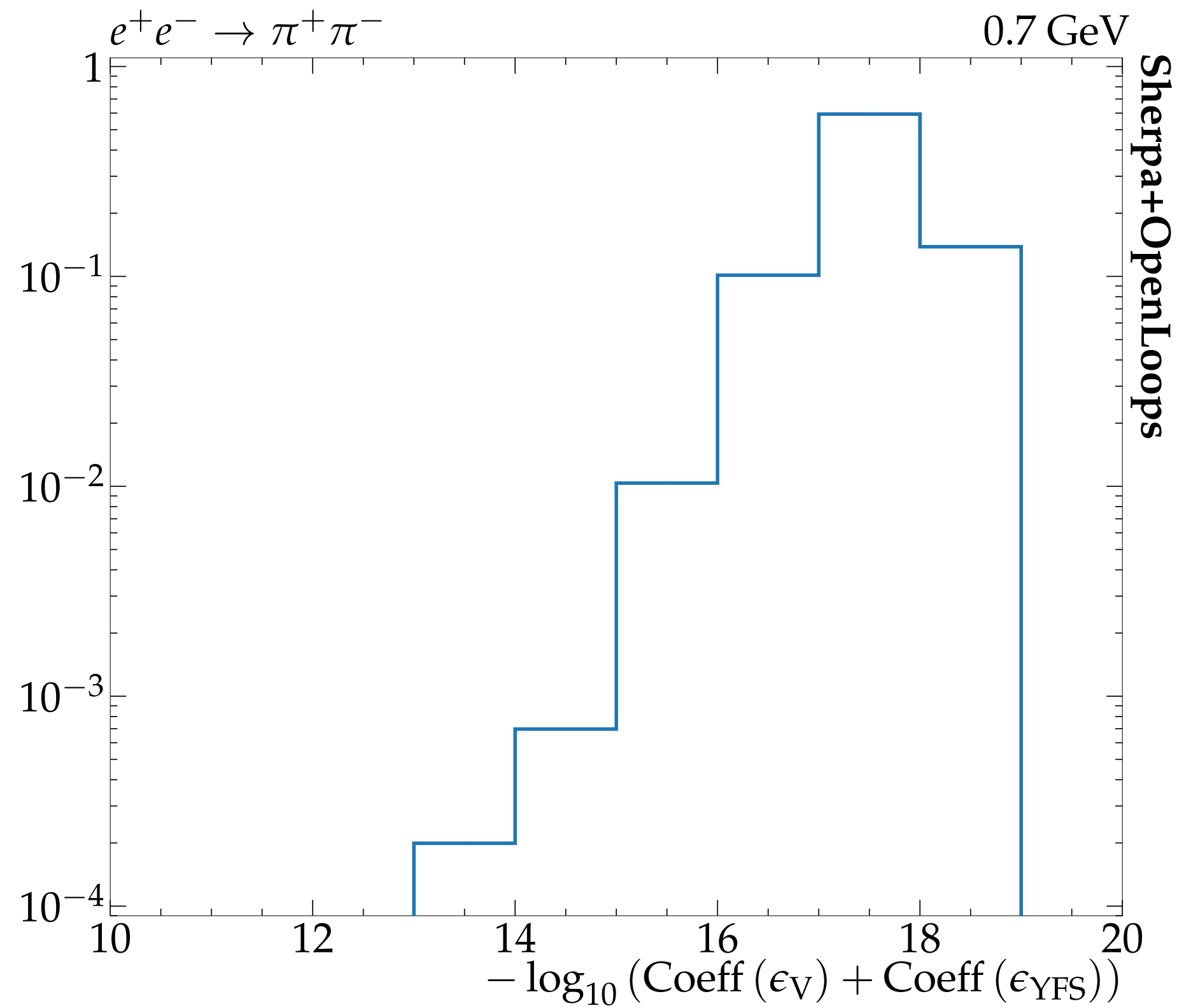
❖ Matching to real and virtual corrections have to be supplemented with SQED amplitudes

Two YFS Calculations: KKMC(CEEX) & Sherpa

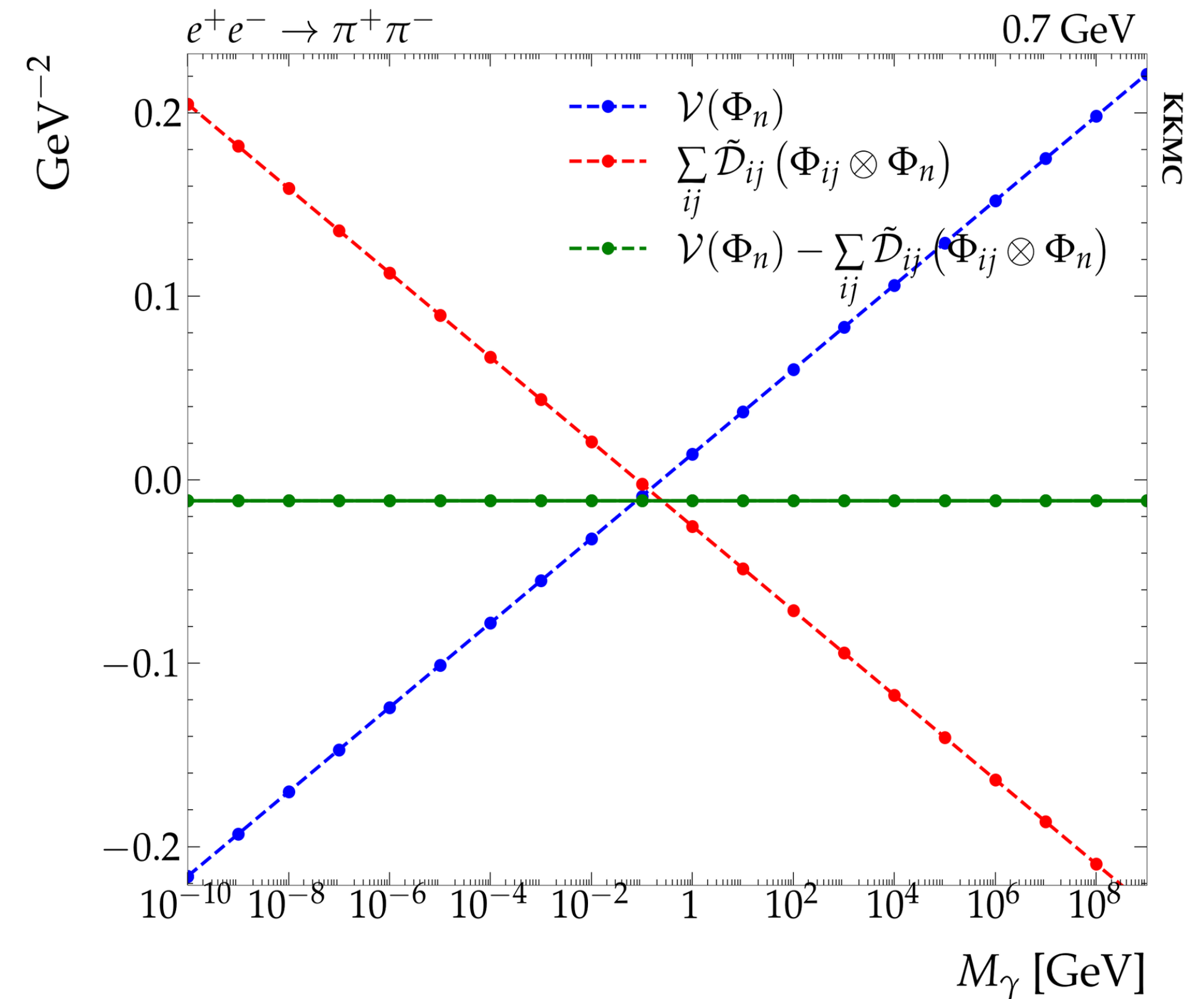
A.Price, A.Siodmok: Jagiellonian University

S.Delorme, J.Gluza, N. Salone: Silesian University

One-Loop $e^+e^- \rightarrow \pi^+\pi^-$

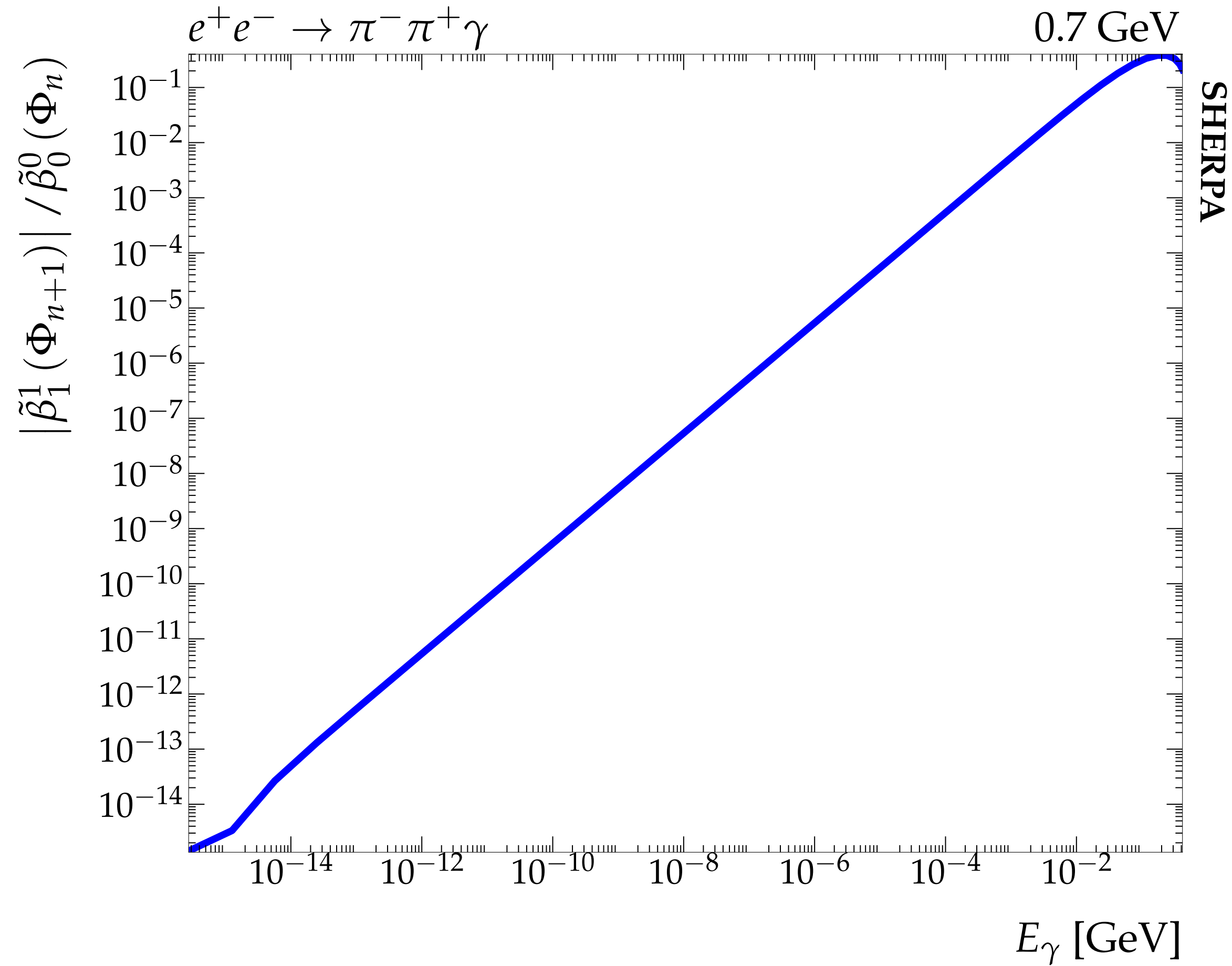


IR Subtraction: Dimensional Regularisation in Sherpa



IR Subtraction: Massive Regularisation in KKMC

$$e^+e^- \rightarrow \pi^+\pi^-\gamma$$

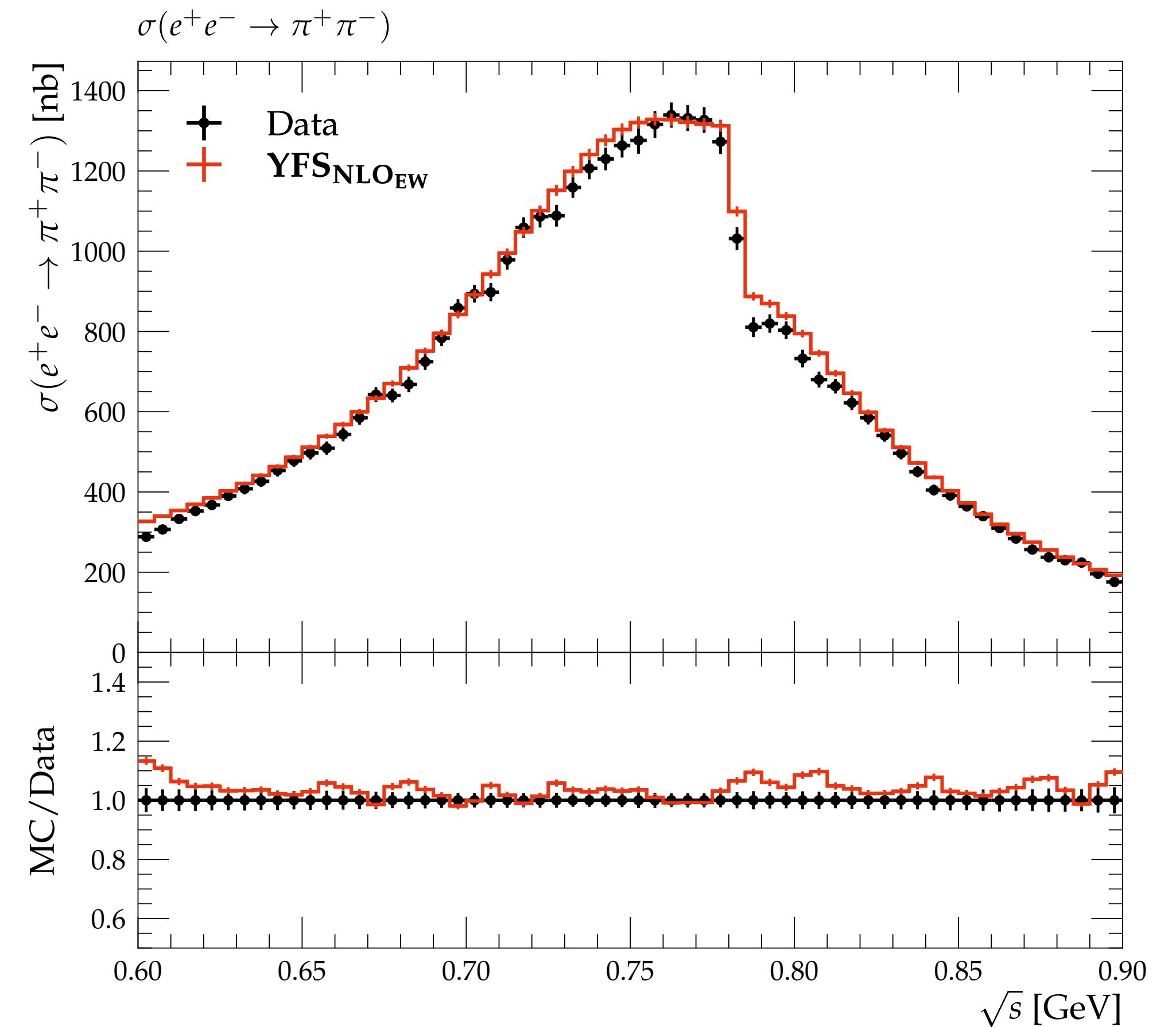


The real emissions are simple tree level amplitudes which can be calculated using standard methods in Sherpa

**We can probe very soft limit
at low energies**

YFS for $e^+e^- \rightarrow \pi^+\pi^-$

- ❖ In addition to the perturbative effects we also have
 - ❖ Pion Form Factor: Flexible parametrisation
 - ❖ HVP: Different options e.g Jegerlehner and Teubner
 - ❖ All of which should enter into the error calculation
- ❖ Missing ingredient for **NNLO_{QED}** are essentially the two loop amplitudes



**We have a lot of work to do to ensure
our theory calculations do not hold back**

$$e^+e^-$$

**I can see a path for the theory community to
achieve this but it will take huge effort by the
community**

But we have plenty of time !



