What to learn from $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu A \to eA$?

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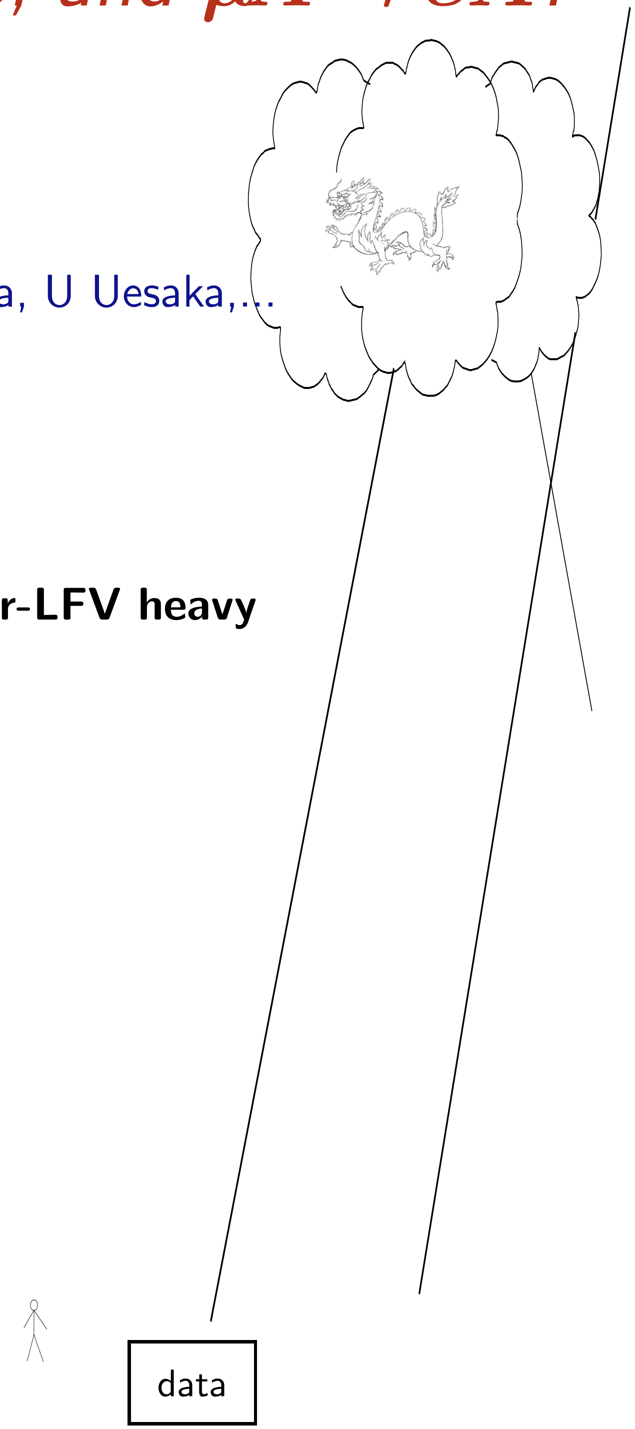
+**Marco Ardu**, B Echenard, M Gorbahn, Y Kuno, M Yamanaka, U Uesaka,...



1: leptons are nice... (no strong interactions)

- generate Baryon Asym without proton decay
- interact with New Physics; $[m_\nu]$ says so! **assume that NP-for-LFV heavy**

2: why focus on $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\mu A \to eA$?

3: data is 12 operator coefficients!

(**4**: from data $\to$ models: why and how?)

5: ask questions...*eg* distinguish models with data?

- considered 3 models —all could be ruled out
- ...

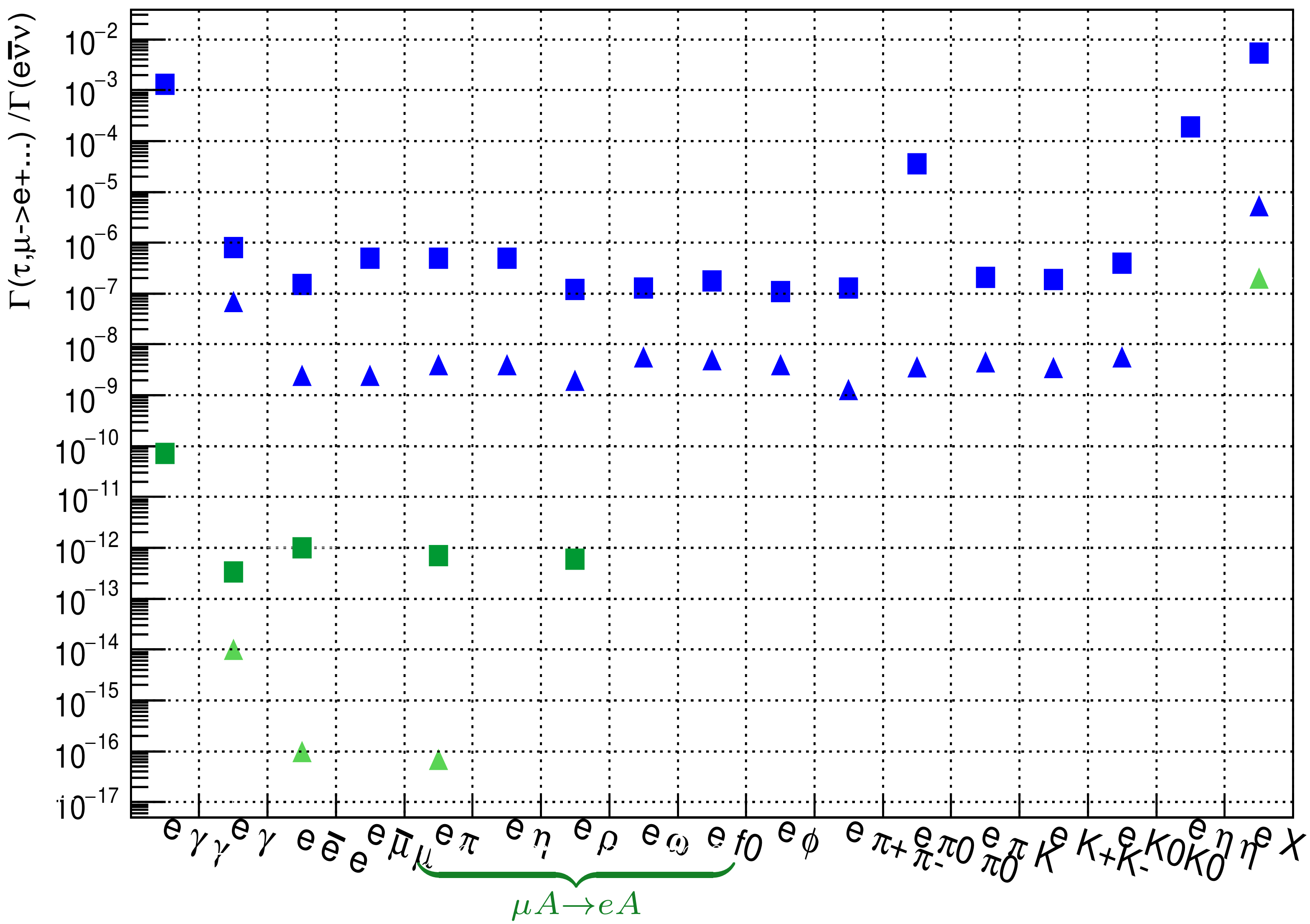
LFV $\equiv$ Lepton Flavour Violation

excellent LFV sensitivity of τ, μ decays

$\tau \to e + ..,\ \mu \to e + ..$ ■=bounds, ▲ = upcoming

plagiarised from BelleII

BanerjeeEtal Snowmass 2203.14919



⋆ $\tau \to l$ for distinguishing models ⋆

why $\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA$: summary

1. excellent sensitivity to NP-scale $\Lambda_{\rm LFV}$ compared to $\tau \leftrightarrow l$, $\Delta QF \neq 0$, collider...

   $\star$ μ decay via weak interactions: $BR(\mu \to e\bar{e}e) \leq 10^{-12} \Rightarrow \Lambda_{\rm LFV} > 10^3 m_W$.

   vs QED-induced $\pi_0, \Upsilon \to \mu\bar{e}$. Or $(g-2)_\mu$: 10 sig figs probe $\gtrsim m_W$.

   $\star$ maybe NP prefers 3rd gen? If NP couples $\propto (y_l)^n$, need n >1 to discover LFV in τs

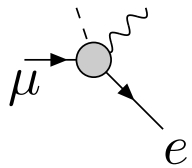
2. big leap in sensitivity in expts starting/under construction (with upgrades?)

   Mu3eI+II, Mu2e, COMETI+II, Mu2eII...?PRISM/PRIME?

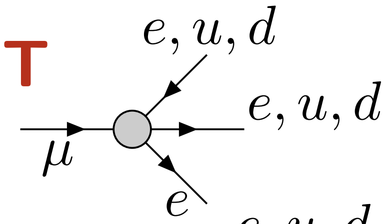
3. $\mu \leftrightarrow e$ is peculiar:few restrictive bds, vs $\left\{ \begin{array}{l} \sim 90 \Delta QF = 0 \text{ op.s in EFT} \\ \text{many param.s in models} \end{array} \right.$

$\Rightarrow$ *relating data to models is interesting*

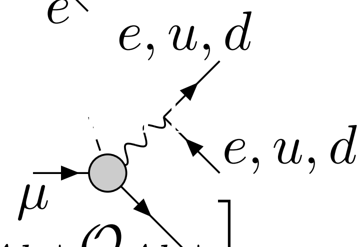
(I will use EFT)



Parametrising $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\mu A \to eA$ in EFT



$$\delta\mathcal{L}_{e_L} = \frac{1}{v^2}\Big[C_D(m_\mu \overline{e}\sigma^{\alpha\beta}P_R\mu)F_{\alpha\beta} + C_S(\overline{e}P_R\mu)(\overline{e}P_Re)$$



$$+C_{VR}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Re) + C_{VL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Le) + C_{Al}\mathcal{O}_{Al} + C_{Ah\perp}\mathcal{O}_{Ah\perp}\Big]$$

parametrise with 12 (also e_R) dimless $\{C\}$

O_{Al} =combo of $2l2q$ ops probed by light targets (Al,Ti)

$O_{Ah\perp}$= indep. combo of $2l2q$ ops probed by heavy targets (Au)

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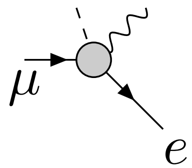

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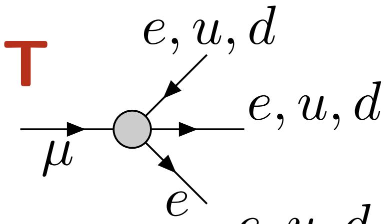

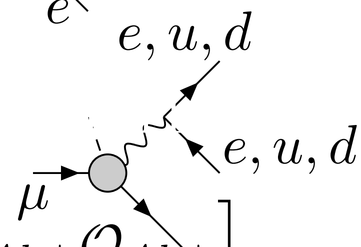

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$\mu \to e\gamma$: $\quad BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ $\quad$ **$\Rightarrow$ 2 constraints**

$\mu \to e\bar{e}e$: $\quad$ (e relativistic $\approx$ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2$$

$$+|C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

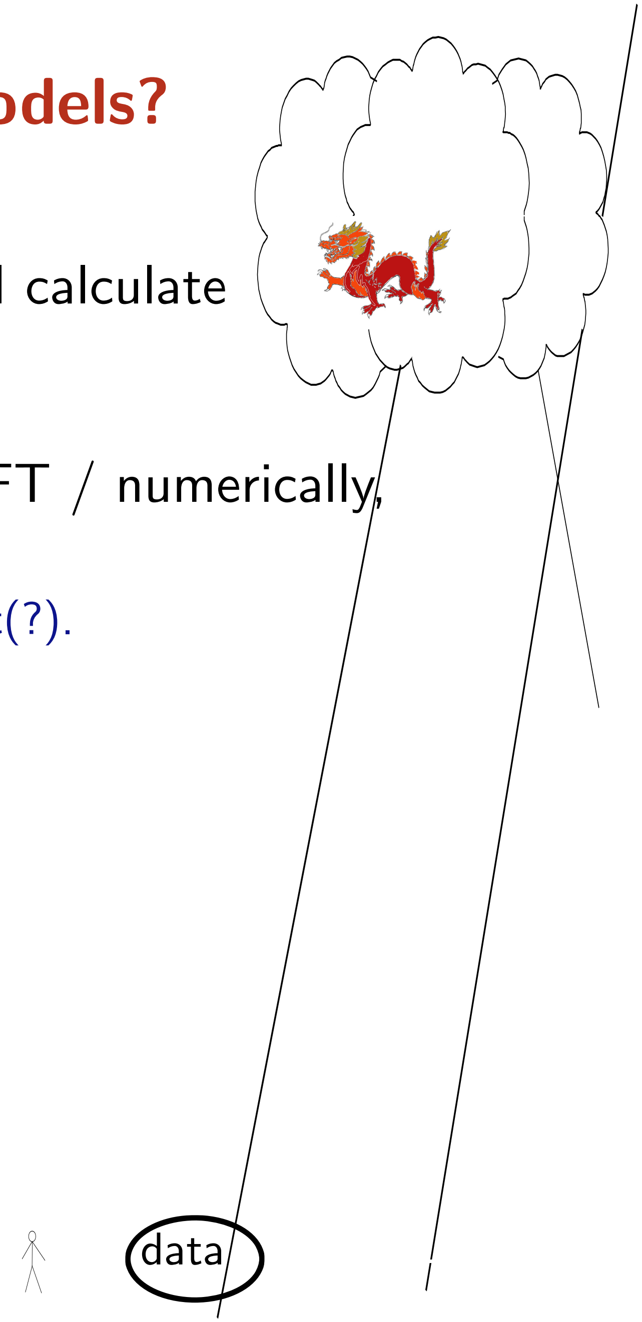
$\Rightarrow$ 6 more constraints

$\mu A \to eA$ ($= \mu^-$ in $1s$ of A, turns into e^-) nucl. phys. $\approx$ WIMP scattering,
SpinIndep coherent $\sim A^2$-enhanced, mediated by S,V,and D
all ops interfere$\Rightarrow$define $2l2q$ op for light (Al)/heavy (Ah) targets for $\{e_L, e_R\}$

$\Rightarrow$ 4 more constraints

$\Rightarrow \star\star$ indep constraints on the 12 coefficients $\star\star$

I want to relate (12) observables $\leftrightarrow$ models?



• "physical" question: $\approx$ unique answer, indep of how I calculate
so skip caln details...

but uncertainties may care how I calculate: model / EFT / numerically,
top$\rightarrow$down or bottom$\rightarrow$ up...

ex: EFT include "2-loop-EW" of $(g-2)$ for $\mu \rightarrow e\gamma$; models not(?).
breaks "lower bound" on $BR(\mu \rightarrow e\gamma)$ in TypeII seesaw.

• use EFT : can estimate SM and NP uncertainties
calculate once for many models

• data$\rightarrow$models: to be different...

...many top$\rightarrow$down model studies since decades

Sorry to colleagues I neglect to cite.

We arrived at $\Lambda_{\rm LFV}$! summary so far:

•expt gives constraints $|C_i| < \epsilon_i$ on 12(complex) coefficients (out of $\approx$ 90),

eg $$BR(\mu \to e\gamma) = 384\pi^2(|C_{DL}|^2 + |C_{DR}|^2)$$

•RGEs + matching "mix" operator coefficients:

$$C_D(m_\mu) = C_D(\Lambda_{\rm LFV}) + \sum_I C_I(\Lambda_{\rm LFV}) \frac{\gamma_{ID}}{16\pi^2} \log() + ...$$

$\Rightarrow$ 12-d ellipse's axes are dilated/rotated in 90-d space, by evolving $\to \Lambda_{\rm LFV}$

Could rotate basis rather than ellipse, to stay with 12 coefficients...

(eqn for ellipse $\approx 90 \times 90$ correlation matrix).

so now : questions

1. are the observables (the 12 coefficients) still orthogonal?

   Important exptal question: each $\mu \leftrightarrow e$ process has dedicated expt, do we need all? yes! ($\approx$)

   (there exist distinct observables measuring same $\sum C(\Lambda_{\rm LFV})$ within uncertainty...but not case here)

2. if $\mu \leftrightarrow e$ is there, will we see it?

   90-d vector space of coefficients, only constraints on 12 directions? probably?

3. if observe $\mu \leftrightarrow e$, can one distinguish models?

   $\Leftrightarrow$ are there portions of exptally allowed ellipse that models *cannot* fill? yes!

if observe $\mu \leftrightarrow e$ in 3 processes, can models be distinguished?

$\Leftrightarrow$ are there parts of the ellipse that models cannot fill?

ArduDLavignac

We considered three models:
1) type II seesaw (= add triplet Higgs)
m_ν model, LFV unsuppressed by m_ν, reputed "predictive" (new couplings $\propto [m_\nu]$)
2) "inverse" typeI seesaw
m_ν model, LFV unsuppressed by m_ν, unknown new Yukawa matrices
3) leptoquark to address R_{D,D^*} anomaly
generate $\mu - e$ conv. at tree level, NP interacts with singlets

All models at TeV-scale :... we though "models can do anything" $\Leftrightarrow$ anticipated to need collider constraints to prevent models from filling ellipse...

Inverse Seesaw

- $\approx$ add extra heavy singlet fermion S_a, to each gen. of type I seesaw:

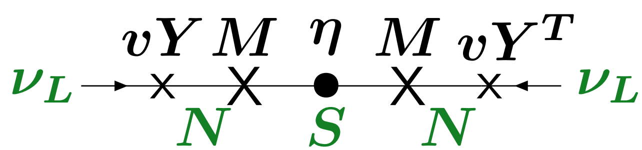
$$\delta\mathcal{L} \quad \supset \quad -\left(Y_\nu^{\alpha a}(\overline{\ell}_\alpha \tilde{H} N_a) + M_{ab}\overline{S}_a N_b + \frac{1}{2}\eta_{ab}\overline{S_a}S_b^c + \mathrm{h.c} \right)$$

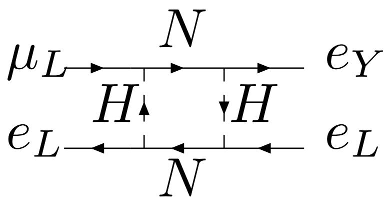
Majorana $\eta \ll M$
M is L-conserving Dirac
mass for singlets

gives neutral fermion mass matrix

$$\mathcal{M}_{\nu N} \approx \overline{(\nu_L \; N^c \; S)} \begin{bmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M^T \\ 0 & M & \eta \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \\ S^c \end{pmatrix}$$

gives $m_\nu \approx m_D(M^{-1})\eta(M^T)^{-1}m_D^T$, so flavour-changing $Y_\nu^{\alpha a}$ unrelated to m_ν.





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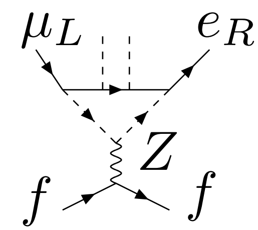

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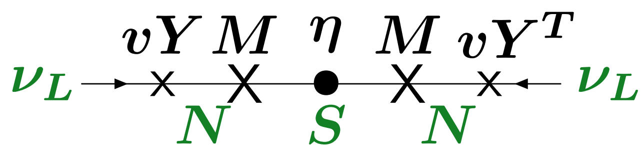

- LFV via loops with EW bosons; controlled by 2→4 "invariants" $(\Delta M : 0 \to> v)$

$$\left[Y_\nu[M^\dagger M]^{-1}Y_\nu^\dagger\right]_{e\mu} \quad , \quad \left[Y_\nu[M^\dagger M]^{-1}\log(M^\dagger M)Y_\nu^\dagger\right]_{e\mu}$$

$$\left[Y_\nu f_1(M^\dagger M)Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right]_{e\mu} \quad , \quad \left[Y_\nu f_2(M^\dagger M)Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right]_{e\mu}$$

despite many parameters in Y_ν! (?no need to scan).

- 7 operators with e_R, or scalar, have $C \propto y_e, y_\mu$ =unobservable. Remain 5Cs.
  $\Rightarrow$ model predicts 3→1 relations.

plot dipole vs $C^{e\mu ee}_{VLR}$ vs 4f coeff for $\mu Al \to eAL$

for degen singlets $\Leftrightarrow$ 3 relations among 5 Cs

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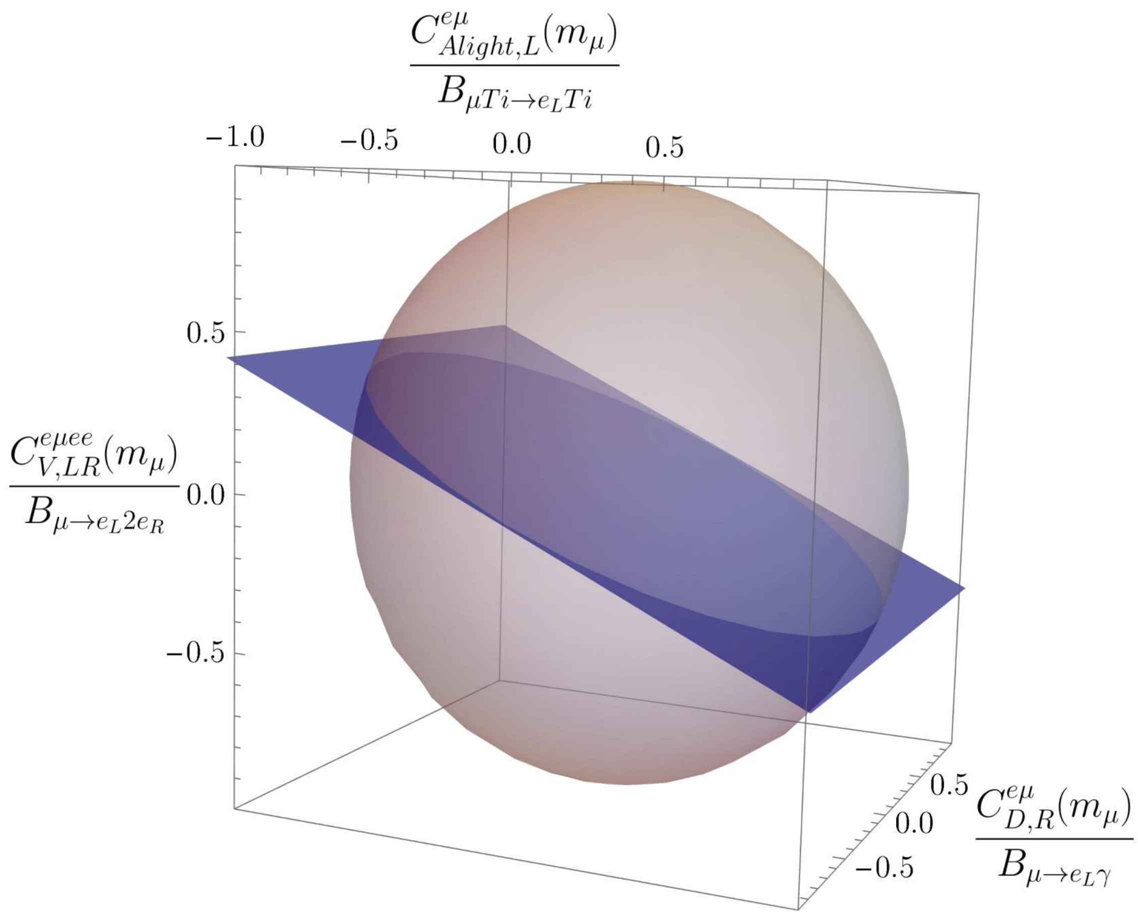


The light-gray sphere illustrate the experimentally allowed ellipse in the $C^{e\mu}_{D,R}(m_\mu)$, $C^{e\mu ee}_{Alight,L}(m_\mu)$, $C^{e\mu ee}_{V,LR}(m_\mu)$ space. We plotted the real parts of the coefficients, normalised to the current upper bound. If the sterile neutrinos are nearly degenerate, the model can cover the blue plane.

Summary

The neutrino mass matrix says there is NP among leptons inducing Lepton Flavour Violating processes. Some of the most sensitive LFV transitions are $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\mu A \to eA$ — and exptal reach should soon improve by $\sim 10^{-4}$.

These processes independently constrain (measure) 12 (10) Wilson coefficients $\{C\}$.

We studied two $[m_\nu]$ models and a leptoquark model; none can fill the exptally allowed, 12-d ellipse in C-space. So observations of $\mu \to e\gamma$,$\mu \to e\bar{e}e$ and $\mu A \to eA$ could out rule these models (!crucial to use 12 Cs, not BRs, for this result).

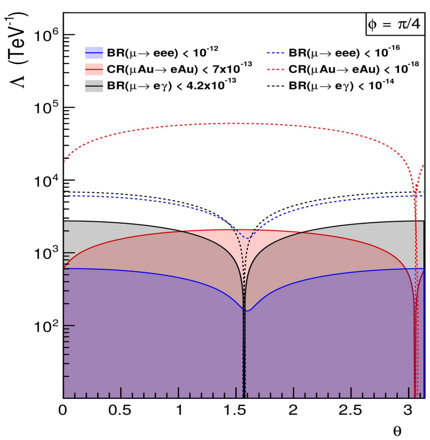
puzzle: model predictions for flavour-changing coefficients C involve only a few combinations of model parameters (hence the correlations): eg $[YY^\dagger]_{e\mu}$, $[YY^\dagger YY^\dagger]_{e\mu}$... ?Artifact of Leading Order? Or interesting stepping-stone from EFT to models?

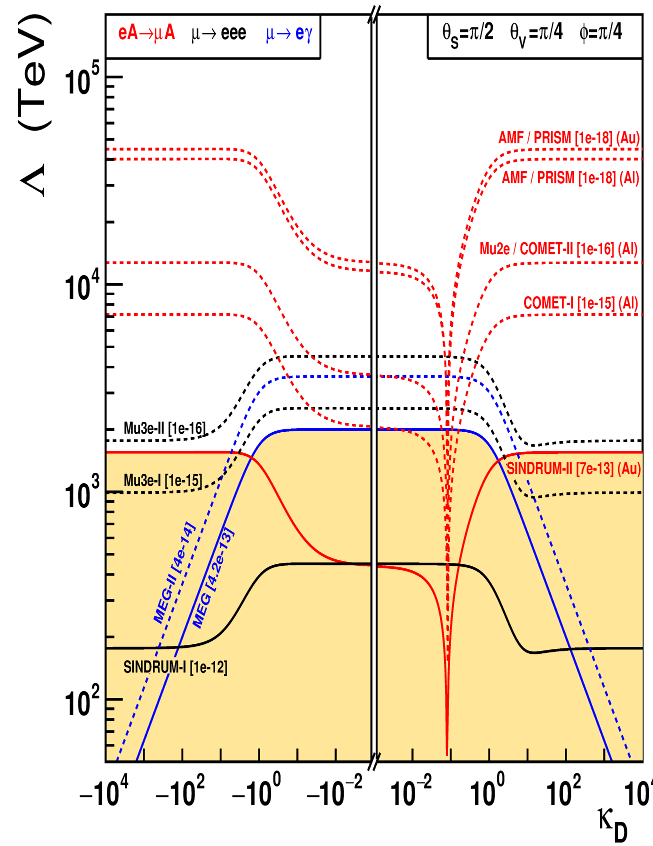
Backup

how independent are the "observables" at $\Lambda_{\rm LFV}$?

Easy to check:exptally probed directions at $\Lambda_{\rm LFV}$ are orthogonal up of $\mathcal{O}(10^{-3})$.
Yippee: $\mu \to e\gamma$,$\mu \to e\bar{e}e$ and $\mu A \to eA$ are complementary probes.

Experimentalists like plots to show complementary reach in $\Lambda_{\rm LFV}$:
take 2d slice through the box,
plot ≈ "surface vs angle" for each bd
except: vertical=$\Lambda_{\rm LFV}$ for $|C| = \sqrt{\frac{v^2}{\Lambda^2_{\rm LFV}}}$
$\kappa = \mathrm{cotg}(\theta - \pi/2)$





Are $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\mu A \to eA$ sufficient for discovery?

$\sim$100 ops: if $\Delta F_Q = 0$, $\mu \to e$ occurs, will it contribute to $\mu \to e\gamma, \mu \to e\bar{e}e$ or $\mu A \to eA$?

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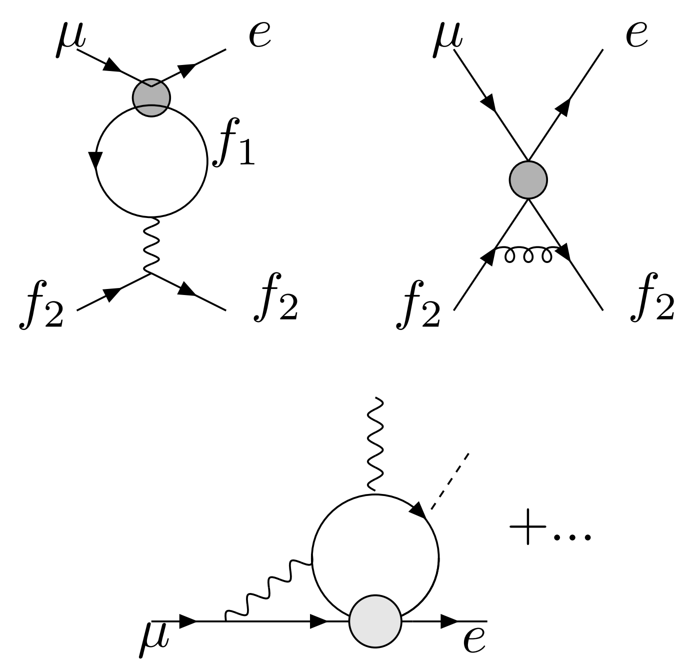


Probably yes: SM loops ensure almost every $\Delta QF = 0$, $\mu \to e$ interaction with ≤ 4 legs, contributes $\gtrsim \mathcal{O}(10^{-3})$ to amplitudes $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and/or $\mu A \to eA$

(not $\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)

What about cancellations?

Model could choose to sit along one of those $\sim$75 orthogonal directions?

...not occur in models I know...(not simple to arrange?)

“Invariants” we met in bottom-up-EFT studies of LFV

• find op. coefficients at the exptal scale proportional to a product of NP/SM matrices, *eg*:

$$C^{e\mu dd}_{V,LR} \quad \propto \quad \left[[m_\nu^*] \ln \frac{[m_e m_e^\dagger]}{M_\Delta^2} [m_\nu] \right]_{\mu e}$$

$+$ combine practical realism of S-matrix element($=$ function of parameters with scheme and scale), with functional elegance of Lagrangian invariants

- not Traces! Element of a matrix in flavour space; eg in mass eigenstate basis.

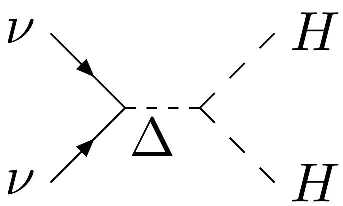
In addition:

1) no need for model parameter scans; "invariants" are complex $\#$s $\lesssim 1$. Simple to see which part of observable ellipse the model can, or not, fill.

2) simple to count $\#$ invariants that control $\mu \leftrightarrow e$. Find that none of our models can fill the whole ellipse!

Type II seesaw — add SU(2) triplet scalar $\vec{\Delta}$, with M_Δ

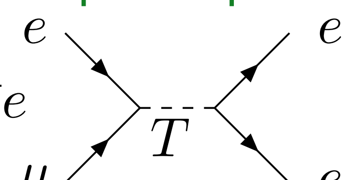
$$\mathcal{L} \supset f_{\alpha\beta}\,\ell_\alpha \vec{\tau} \ell_\beta \cdot \vec{\Delta} + M_\Delta \lambda_H\, H^T \vec{\tau} H \cdot \vec{\Delta}^* + \ldots$$



$$[m_\nu]_{\alpha\beta} \sim \frac{[f]^*_{\alpha\beta}\lambda_H M_\Delta v^2}{M_\Delta^2} \sim 0.03 \text{ eV} \times [f]_{\alpha\beta}\frac{\lambda_H}{10^{-12}}\frac{\text{TeV}}{M_\Delta}$$

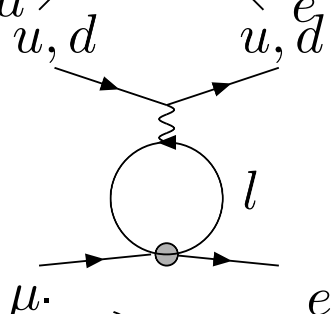
model reputed predictive, because $f_{\alpha\beta} \propto [m_\nu]_{\alpha\beta}$ (known up to $m_{min}, e^{i\phi_1}, e^{i\phi_2}$)

$\mu \to e\bar{e}e$



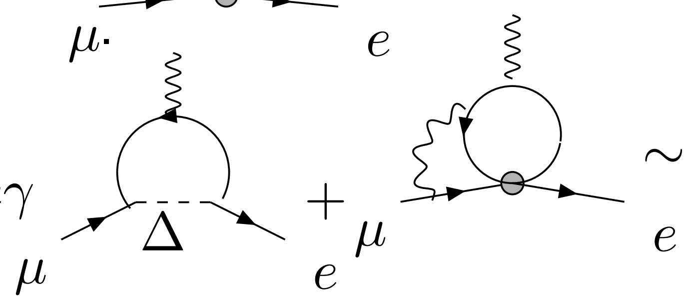
$$\sim \frac{[m_\nu^*]_{\mu e}[m_\nu]_{ee}}{2\lambda_H^2 v^2} \quad , \quad [m_\nu^*]_{\mu e}, [m_\nu]_{ee} \sim f(m_{min}, \phi_i)$$

$\mu A \to e A$



$$\sim \frac{\alpha_e}{6\pi\lambda^2 v^2}\left[m_\nu^* \log\frac{[m_e m_e^\dagger]}{M_\Delta^2} m_\nu\right]_{\mu e}, f(m_{min}, \phi_i,),$$

$\mu \to e\gamma$



$$\sim \frac{e}{128\pi^2}\left(\frac{[m_\nu^* m_\nu]_{\mu e}}{\lambda^2 v^2} + \frac{2\alpha}{\pi\lambda^2 v^2}\left[m_\nu \log\frac{[m_e m_e^\dagger]}{M_\Delta^2} m_\nu\right]_{\mu e}\right)$$

3 "invariants" determine the 5 non-negligeable Cs; 1 invar known from ν osc.

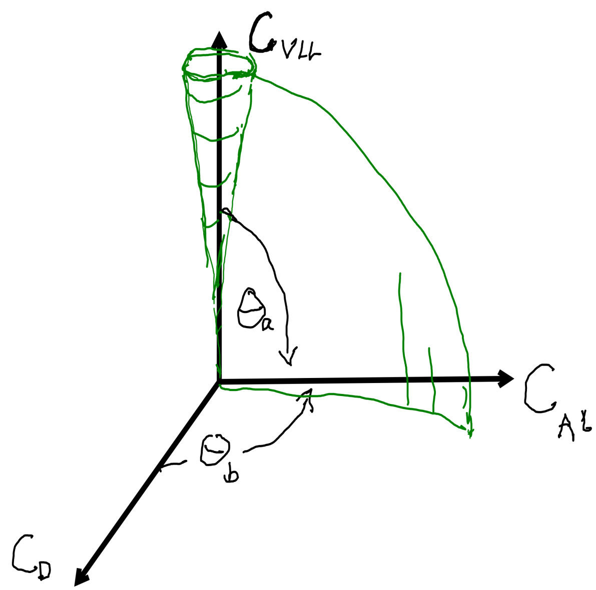
7 C with e_R, and scalars, are negligeable ($\propto y_{e,\mu}$) because LFV in doublets

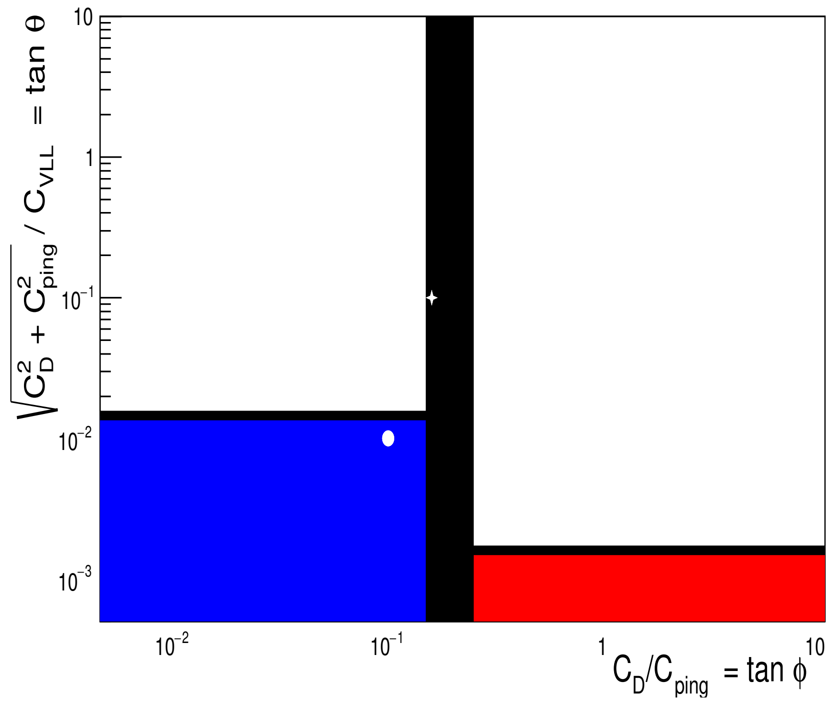
Type II seesaw: predictions

recall 12 (complex) operator coefficients $\left\{ \begin{array}{l} C_{DR},\ C^{e\mu ee}_{VLL},\ C^{e\mu ee}_{VLR},\ C^{e\mu ee}_{SRR},\ C_{AlightL},\ C_{AheavyR} \\ C_{DL},\ C^{e\mu ee}_{VRL},\ C^{e\mu ee}_{VRR},\ C^{e\mu ee}_{SLL},\ C_{AlightL},\ C_{AheavyR} \end{array} \right.$

- seven coefficients for LFV-involving-singlet-leptons are negligeable
- $C^{e\mu ee}_{VLL}$ $(\mu \to e\bar{e}e)$ or $C_{Al,L}(\mu A \to eA)$ can vanish (also any of C_{DR} for $m_\nu \gg$)

- $C^{e\mu ee}_{VLL}$ $(\mu \to e\bar{e}e)$ "naturally" large: predict $C_{DR}/C_{Al,L}$ for small $C^{e\mu ee}_{VLL}$.





model lives in not-while areas expt can probe whole plot: $\tan\theta_{a,b} : 10^{-3} \to 10$

vert. axis $\sim$ loop/tree ; horiz. axis $\sim$ $|C_D|/|C_{Al}|$

What are $\mathcal{O}_{Alight}, \mathcal{O}_{Aheavy\perp}$?

$$\mathcal{O}_{Alight,X} \sim 0.7(\bar{e}P_X\mu)\Big[(\overline{u}u) + (\overline{d}d) + ...\Big] + 0.13(\bar{e}\gamma^\alpha P_X\mu)\Big[(\overline{u}\gamma_\alpha u) + (\overline{d}\gamma_\alpha d)\Big]$$

$$\mathcal{O}_{Aheavy\perp,X} \simeq (\bar{e}\gamma^\alpha P_X\mu)\Big[0.56(\overline{u}\gamma_\alpha u) + 0.8(\overline{d}\gamma_\alpha d)\Big] + ...$$

obtained by matching nucleons to quarks, then writing

$$\mathcal{O}_{Aheavy,X} = \mathcal{O}_{Alight,X} + \epsilon\mathcal{O}_{Aheavy\perp,X}$$

where ϵ calculable misalignement $\simeq 5\%$.

problem1: scalar density of u quarks in $N \in \{n,p\} \simeq$ scalar density of d quarks $\Rightarrow$ with current theory uncertainties in $\mu A \to eA$, measuring C_S^n and C_S^p only allows to determine $C_S^u + C_S^d$ (but not $C_S^u - C_S^d$).

problem2: caIn uses nuclear matrix elements of KitanoKoikeOkada from ~ 20yrs ago. There are recent shell-model (EtalHaxton) and ab-initio(EtalHeinz) results. ? ab-initio may agree with KKO for SpinIndep on light nuclei?

Counting constraints in space of ∼100 operators $_{\text{DKunoYamanaka}}$

Count constraints: (write $\delta\mathcal{L} = \frac{C^{flavour}_{Lorentz,XY}}{v^n}\,\mathcal{O}^{flav}_{Lorentz,XY}$ $,X,Y \in \{L,R\}$)

$\mu\to e\gamma$: $BR(\mu\to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ $\Rightarrow$ **2 constraints**

$\mu \to e\bar{e}e$: (e relativistic ≈ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

$\Rightarrow$ **6 more constraints**

$\mu A\to eA$:(S^N_A,V^N_A=integral over nucleus A of N distribution×lepton wavefns, **different** for diff. A)

$$BR_{SI} \sim Z^2|V^p_A\tilde{C}^p_{V,L} + S^p_A\tilde{C}^p_{S,R} + V^n_A\tilde{C}^n_{V,L} + S^b_A\tilde{C}^n_{S,R} + D_AC_{D,R}|^2 + |L\leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) $\Rightarrow$ **4 + 2 more constraints**

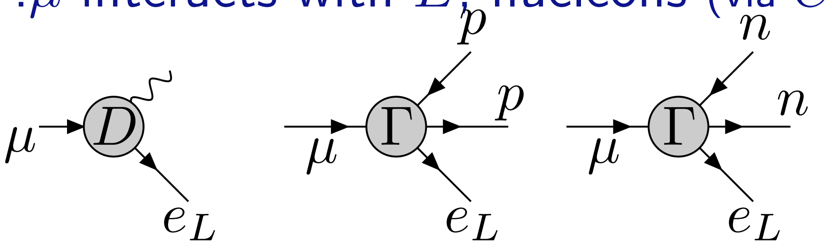
future: improved theory, 3SI+2SD targets $\Rightarrow$ **6+4** constraints

is 12-20 constraints on ~ 100 operators a problem?

$\mu A \to eA$: most sensitive process, expt + th

- μ^- captured by Al nucleus, tumbles down to $1s$. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV:μ interacts with $\vec{E}$, nucleons (via $\widetilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

($E_e \approx m_\mu$ so e_L/e_R)



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

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$$BR_{SI} \sim \quad Z^2 |\sum ...\tilde{C}_{SI}|^2 \quad , \quad \tilde{C}_{SI} \in \{\tilde{C}^p_V, \tilde{C}^p_S, \tilde{C}^n_V, \tilde{C}^n_S, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over N $\propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim ...|\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

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