Bhabha scattering at future colliders with BHLUMI/BHWIDE

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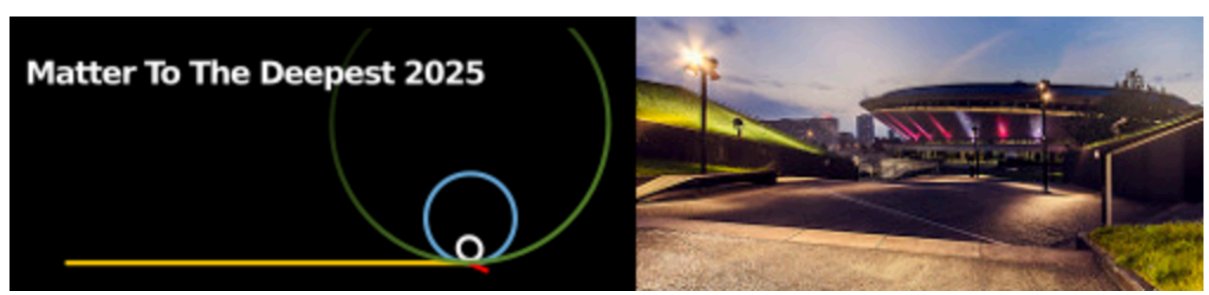


Conference ``Matter To The Deepest”, Katowice, 15-19 September 2025

Congratulations on 50th anniversary! Happy Golden Jubilee!

Outline

- Luminosity measurement
- Small-angle Bhabha (SABH) process with BHLUMI
- Large-angle Bhabha (LABH) process with BHWIDE
- Conclusions and outlook

Luminosity measurement

- **Luminosity** measurement at e^+e^- colliders:

cross section → σ, event count → N, luminosity → $\mathscr{L}$:

$$\sigma = \frac{N}{\mathscr{L}},$$

experiment → $N_{\rm ref}$, theory → $\sigma_{\rm ref}$:

$$\mathscr{L} = \frac{N_{\rm ref}}{\sigma_{\rm ref}},$$

luminosity error = experimental error ⊕ theory error:

$$\frac{\delta\mathscr{L}}{\mathscr{L}} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta\sigma_{\rm ref}}{\sigma_{\rm ref}}$$

- Criteria for **reference** process:

1. **Experiment**: high event statistics $N_{\rm ref}$, low backgrounds, good control of systematics.

2. **Theory**: possible high-precision calculations of $\sigma_{\rm ref}$, negligible ``new physics" contributions, precise and efficient Monte Carlo event generator.

$$\frac{\delta\sigma_{\rm ref}}{\sigma_{\rm ref}} \lesssim \frac{\delta N_{\rm ref}}{N_{\rm ref}}$$

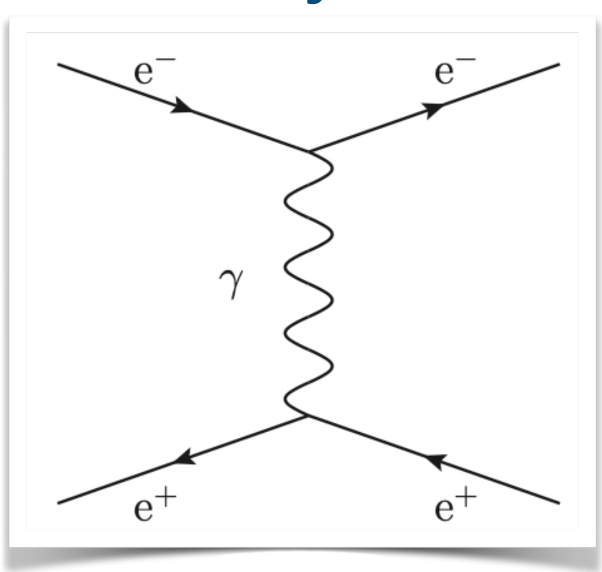
→ **luminosity precision should not be limited by theory!**

Luminosity measurement

- LEP: **small-angle Bhabha (SABH)** scattering ($\theta_e \lesssim 100$ mrad):

$$e^+ + e^- \longrightarrow e^+ + e^-$$

➡ dominated by t-channel γ exchange:



$$\rightarrow \quad \frac{d\sigma}{d\theta_e} \sim \frac{1}{\theta_e^3}$$

✓pure QED process (in principle) → high-precision theoretical calculation possible!

Luminosity measurement

- LEP $\to$ achieved precision:

$$\delta\mathscr{L}/\mathscr{L} \sim 5 \times 10^{-4}$$

➡All four LEP experiments used MC event generator **BHLUMI** for computing σ_{ref}

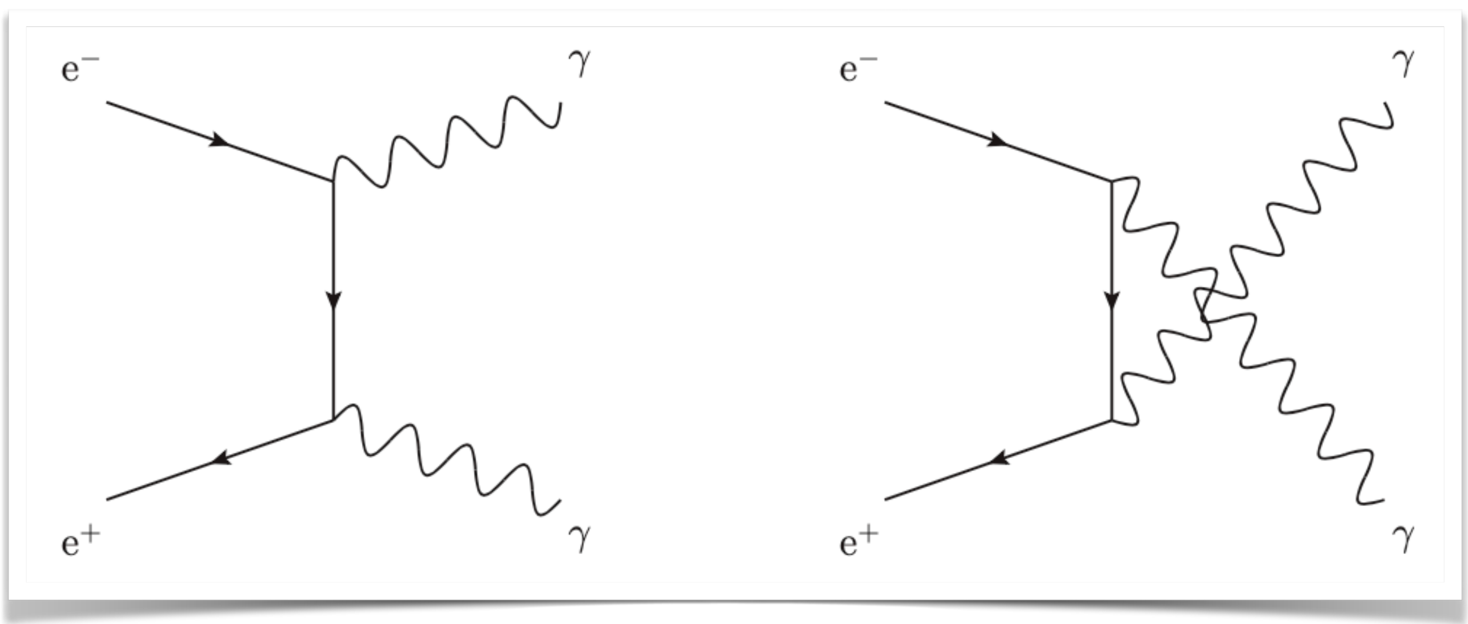
- Expected precision $\delta\mathscr{L}/\mathscr{L}$ at future e^+e^- colliders:
    - at Z pole: $\lesssim 10^{-4}$
    - for higher energies: $\mathcal{O}(10^{-3})$

Luminosity measurement

✳ Complementary process at future e^+e^- colliders

$$e^+ + e^- \to \gamma + \gamma$$

at wide angles:



- **Pros**: low-angle acceptance less critical, hadronic vacuum polarisation less important (NNLO)
- **Cons**: lower cross section, high background from large-angle Bhabha process, weak corrections needed at NLO, …

SABH with BHLUMI

- **BHLUMI 4.04** — MC event generator for **SABH** process including $\mathcal{O}(\alpha^2)^{\rm exp}_{\rm prag}$ QED corrections based on Yennie-Frautschi-Suura (YFS) exclusive exponentiation
  [S. Jadach, WP, E. Richter-Wąs, B.F.L. Ward, Z. Wąs, CPC 102 (1997) 229]
  →https://github.com/KrakowHEPSoft/BHLUMI

➡ Size of canonical coefficients in QED perturbative expansion:

Canonical coefficients					
		$\theta_{min} = 30$ mrad		$\theta_{min} = 60$ mrad	
		LEP1	LEP2	LEP1	LEP2
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi}4L$	137×10^{-3}	152×10^{-3}	150×10^{-3}	165×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2 L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	9.4×10^{-3}	11×10^{-3}	11×10^{-3}	14×10^{-3}
$\mathcal{O}(\alpha^2 L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.31×10^{-3}	0.35×10^{-3}	0.35×10^{-3}	0.38×10^{-3}
$\mathcal{O}(\alpha^3 L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.42×10^{-3}	0.58×10^{-3}	0.57×10^{-3}	0.74×10^{-3}

[S. Jadach *et al.*, "Event generators for Bhabha scattering", DOI: 10.5170/CERN-1996-001-V-2.229, hep-ph/9602393]

Table 2: The canonical coefficients indicating the generic magnitude of various leading and subleading contributions up to third-order. The big-log $L = \ln(|t|/m_e^2) - 1$ is calculated for $\theta_{min} = 30$ mrad and $\theta_{min} = 60$ mrad and for two values of the center of mass energy: at LEP1 ($\sqrt{s} = M_Z$), where the corresponding $|t| = (s/4)\theta_{min}^2$ are 1.86 and 7.53 GeV^2, and at LEP2 energy ($\sqrt{s} = 200$ GeV), where the corresponding $|t|$ are 9 and 36 GeV^2, respectively.

SABH with BHLUMI

✤ Pragmatic QED perturbative expansion:

$\mathcal{O}(\alpha^2)_{\text{prag}}$

1				
αL	α			
$\alpha^2 L^2$	$\alpha^2 L$	α^2		
$\alpha^3 L^3$	$\alpha^3 L^2$	$\alpha^3 L$	α^3	
$\alpha^4 L^4$	$\alpha^4 L^3$	$\alpha^4 L^2$	$\alpha^4 L^1$	α^4
$\vdots$	$\vdots$	$\vdots$		

BHLUMI 4.04

✓This was sufficient for LEP

$\mathcal{O}(\alpha^3)_{\text{prag}}$

1				
αL	α			
$\alpha^2 L^2$	$\alpha^2 L$	α^2		
$\alpha^3 L^3$	$\alpha^3 L^2$	$\alpha^3 L$	α^3	
$\alpha^4 L^4$	$\alpha^4 L^3$	$\alpha^4 L^2$	$\alpha^4 L^1$	α^4
$\vdots$	$\vdots$	$\vdots$		

To be done

(components available)

✓Necessary for future e^+e^- colliders: FCC-ee, …

SABH with BHLUMI

- SABH process in BHLUMI 4.04:

$$e^-(p_1) + e^+(q_1) \to e^-(p_2) + e^+(q_2) + n\gamma(k_j) + n'\gamma(k'_l)$$

➡Master formula $(r = 0,1,2)$:

$$\sigma^{(r)} = \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\frac{1}{n!}\frac{1}{n'!}\int\frac{d^3p_1}{p_1^0}\int\frac{d^3q_1}{q_1^0}$$
$$\prod_{j=1}^{n}\int_{k_j\notin\Omega_U}\frac{d^3k_j}{k_j^0}\tilde{S}_p(k_j)\prod_{l=1}^{n'}\int_{k'_l\notin\Omega_L}\frac{d^3k'_l}{k'^0_l}\tilde{S}_q(k'_l)$$
$$\delta^{(4)}\left(p_1 - p_2 + q_1 - q_2 - \sum_{j=1}^{n}k_j - \sum_{l=1}^{n'}k'_l\right)e^{Y_p(\Omega_U)+Y_q(\Omega_L)}$$
$$\left\{\bar{\beta}_0^{(r)} + \sum_{j=1}^{n}\frac{\bar{\beta}_{1U}^{(r)}(k_j)}{\tilde{S}_p(k_j)} + \sum_{l=1}^{n'}\frac{\bar{\beta}_{1L}^{(r)}(k'_l)}{\tilde{S}_q(k'_l)} + \sum_{n\geq j>k\geq 1}\frac{\bar{\beta}_{2UU}^{(r)}(k_j,k_k)}{\tilde{S}_p(k_j)\tilde{S}_p(k_k)}\right.$$
$$\left. + \sum_{n'\geq l>m\geq 1}\frac{\bar{\beta}_{2LL}^{(r)}(k_l,k_m)}{\tilde{S}_q(k'_l)\tilde{S}_q(k'_m)} + \sum_{j=1}^{n}\sum_{l=1}^{n'}\frac{\bar{\beta}_{2UL}^{(r)}(k_j,k'_l)}{\tilde{S}_p(k_j)\tilde{S}_q(k'_l)}\right\}.$$

SABH with BHLUMI

- ## YFS form factor:

$$Y(\Omega_U; p_1, p_2) = 2\alpha\tilde{B}(\Omega, p_1, p_2) + 2\alpha\Re B(p_1, p_2)$$

$$= -\frac{\alpha}{4\pi^2}\int_{k\in\Omega_U}\frac{d^3k}{k^0}\left(\frac{p_1}{kp_1}-\frac{p_2}{kp_2}\right)^2$$

$$+ 2\alpha\Re\int\frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left(\frac{2p_1-k}{2kp_1-k^2}-\frac{2p_2-k}{2kp_2-k^2}\right)^2$$

← real and virtual-photon IR contributions

- ## Non-IR beta-functions:

$$\bar{\beta}_0^{(r)} = \left\{D_{[0,0]}^{(r)}\exp(-Y_p(\Omega_U)-Y_q(\Omega_L)\right\}\Big|_{\mathcal{O}(\alpha^r)},$$

← virtual corrections up to 2 loops

$$\bar{\beta}_{1U}^{(r)}(k_i) = \left\{D_{[1,0]}^{(r)}(k_i)\exp(-Y_p(\Omega_U)-Y_q(\Omega_L)\right\}\Big|_{\mathcal{O}(\alpha^r)} - \tilde{S}_p(k_i)\,\bar{\beta}_0^{(r-1)},$$

← 1 real-photon emission up to $\mathcal{O}(\alpha^2)$

$$\bar{\beta}_{2UU}^{(2)}(k_i,k_j) = D_{[2,0]}^{(2)}(k_i,k_j) - \bar{\beta}_{1U}^{(1)}(k_i)\tilde{S}_p(k_j) - \bar{\beta}_{1U}^{(1)}(k_j)\tilde{S}_p(k_i) - \bar{\beta}_0^{(0)}\tilde{S}_p(k_i)\tilde{S}_p(k_j),$$

← 2 real-photon emissions from upper line (e^-)

$$\bar{\beta}_{2UL}^{(2)}(k_i,k'_j) = D_{[1,1]}^{(2)}(k_i,k'_j) - \bar{\beta}_{1U}^{(1)}(k_i)\tilde{S}_q(k'_j) - \bar{\beta}_{1L}^{(1)}(k'_j)\tilde{S}_p(k_i) - \bar{\beta}_0^{(0)}\tilde{S}_p(k_i)\tilde{S}_q(k'_j).$$

← 2 real-photon emissions: 1 from upper line (e^-) and 1 from lower line (e^+)

$$\tilde{S}_p(k_1) = \frac{\alpha}{4\pi^2}\left(\frac{2p_1p_2}{(kp_1)(kp_2)}-\frac{m^2}{(k_1p_1)^2}-\frac{m^2}{(k_1p_2)^2}\right)$$

➡More details in: S. Jadach, B.F.L. Ward, Acta Phys. Pol. B **28** (1997) 1907

SABH with BHLUMI

❖ $\mathcal{O}(\alpha^2)^{\text{exp}}_{\text{prag}}$ BHLUMI at LEP:

Table 2 [S. Jadach *et al.*, Phys. Lett. B **790** (2019) 314]

Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric LEP luminosity detector within the generic angular range of 18–52 mrad. Total error is summed in quadrature.

Type of correction/error	1999	Update 2018
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% [26]
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) Z and s-channel γ exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [28]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038%

main changes (rows (c) and (d))

➡ Based mainly on **internal** tests with MC generator **OLDBIS ⊕ LUMLOG** ($\mathcal{O}(\alpha) \oplus \mathcal{O}(L_e^3\alpha^3)$) and **semi-analytical** calculations as well as **external** cross-checks with independent MC generator **SABSPV** [M. Cacciari, G. Montagna, O. Nicrosini, F. Piccinini, CPC **90** (1995) 301].

SABH with BHLUMI

- $\mathcal{O}(\alpha^3)^{\text{exp}}_{\text{prag}}$ BHLUMI for future e^+e^- colliders:

$$\sigma_{\rm Bh} \simeq 4\pi\alpha^2 \left(\frac{1}{t_{\min}} - \frac{1}{t_{\max}}\right) = 4\pi\alpha^2 \left(\frac{t_{\max} - t_{\min}}{\bar{t}^2}\right), \quad \bar{t} = \sqrt{t_{\min} t_{\max}}$$

Machine	$\theta_{\min}-\theta_{\max}$ (mrad)	$\sqrt{s}$ (GeV)	$\bar{t}/s$	$\sqrt{\bar{t}}$ (GeV)
LEP	28–50	M_Z	3.5×10^{-4}	1.70
FCCee	64–86	M_Z	13.7×10^{-4}	3.37
FCCee	64–86	350	13.7×10^{-4}	13.0
ILC	31–77	500	6.0×10^{-4}	12.2
ILC	31–77	1000	6.0×10^{-4}	24.4
CLIC	39–134	3000	13.0×10^{-4}	108

[S. Jadach, WP, M. Skrzypek, B.F.L. Ward, Eur. Phys. J. C **81** (2021) 1047]

Forecast study for FCCee$_{M_Z}$		
Type of correction / Error	Published [2]	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_e^0)$		0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.70×10^{-4}

Forecast			
Type of correction / Error	ILC$_{500}$	ILC$_{1000}$	CLIC$_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

[B.F.L. Ward *et al.*, *PoS* ICHEP2024 (2025) 554, arXiv:2410.09115]

→ See also: M. Skrzypek *et al.*, Acta Phys. Pol. B Proc. Supp. **17** (2024) 2-A4

SABH with BHLUMI

How to reach the requisite precisions?

✴ Stage 1:

(a) Photonic $\mathcal{O}(L_e^3\alpha^3)$ - available in LUMLOG MC generator [see e.g. S. Jadach, B.F.L. Ward, PLB 389 (1996) 129 and APPB 28 (1997) 1907] → to be implemented in BHLUMI

(b) Photonic $\mathcal{O}(L_e\alpha^2)$ - available [S. Jadach, M. Melles, B.F.L. Ward, S.A. Yost: PLB 377 (1996) 168 and PLB 450 (1999) 262] and tested numerically → to be implemented in BHLUMI

(c) Vacuum polarisation (hadronic) - to be improved in near future: Lattice QCD computations, new low-energy e^+e^- experiments, MUonE (?)

(d) *Z* and *s*-channel γ exchange - available in BHWIDE MC generator, including YFS $\mathcal{O}(\alpha^1)_{\rm exp}$ EW corrections → to be computed and combined with BHLUMI results

(e) Light-fermion pairs - available in KoralW MC generator (subset of $e^+e^- \to 4f$) → to be combined with BHLUMI results, if needed

➡ **BHLUMI ⊕ BHWIDE ⊕ KoralW**

SABH with BHLUMI

✳ Stage 2:

- *Z* and *s*-channel γ exchange - implementing BHWIDE matrix element in BHLUMI, restoring up-down interferences in YFS exponentiation ($\mathcal{O}(\alpha)$ EW corrections can also be included, if necessary)
- Light-fermion pairs - implementing corresponding matrix elements in BHLUMI, if necessary

➡ **EEX BHLUMI**

✳ Stage 3 (ultimate solution):

- Applying **CEEX** (**C**oherent **E**xclusive **EX**ponentiation) formalism based on **spin amplitudes**, instead of EEX based matrix elements squared - like in KKMC MC generator for $e^+e^- \rightarrow 2f \; (f \neq e)$
- Using C++ (and Python?) for programming instead of Fortran

➡ **CEEX BHLUMI**

❖ **Tests** are important - both for technical and physical precision!

- **Internal** tests - upgrade of BHLUMI 4.04 testing software needed: both **semi-analytical** calculations and LUMLOG+OLDBIS-type **MC** program
- **External** - independent MC generator (like SABSPV) needed (BabaYaga?)

LABH with BHWIDE

❖ **Large-angle Bhabha (LABH)** scattering:

$$\theta_e \gtrsim 100\,\mathrm{mrad}$$

- at ***Z*-boson peak**:

$$\sigma_e^{\mathrm{peak}} = \frac{12\pi\Gamma_e^2}{M_Z^2\Gamma_Z^2}$$

➡ direct **measurement** of Γ_e - high sensitivity: $\propto \Gamma_e^2$

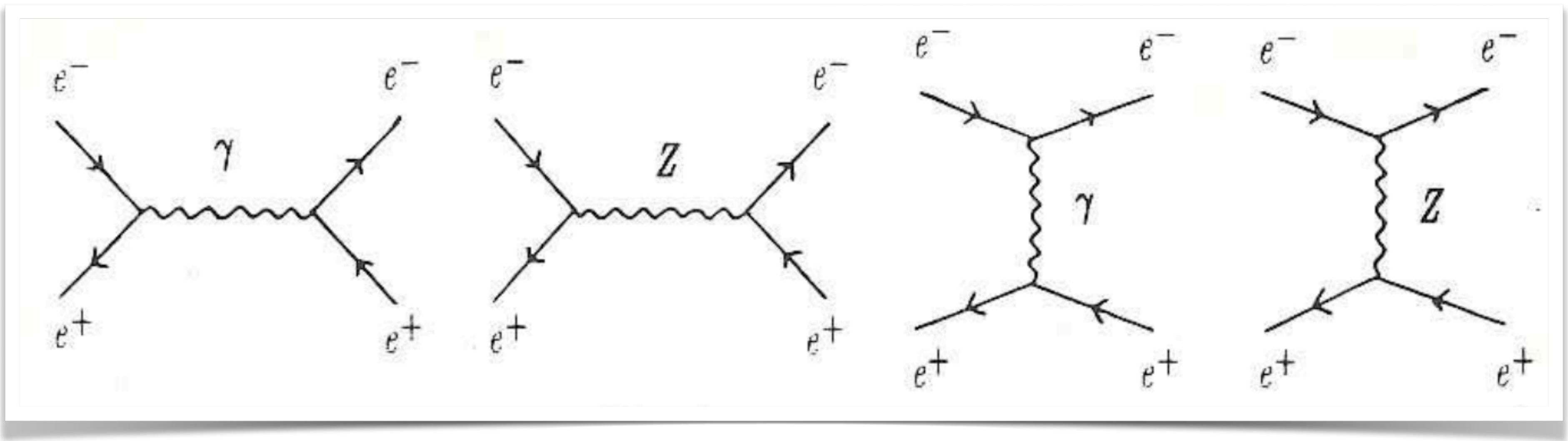
➡ needed for measurement of other Γ_f $\left[\sigma_f^{\mathrm{peak}} \propto \Gamma_e \Gamma_f\right]$

- at ***Z*-peak and above** - large **background** for other processes, in particular $e^+e^- \to \gamma\gamma$ (complementary for LUMI)

- at **lower-energy** e^+e^- colliders (flavour factories: $\sqrt{s} \lesssim 10\,\mathrm{GeV}$) - **luminosity** measurement with typical precision 1% ÷ 0.1%

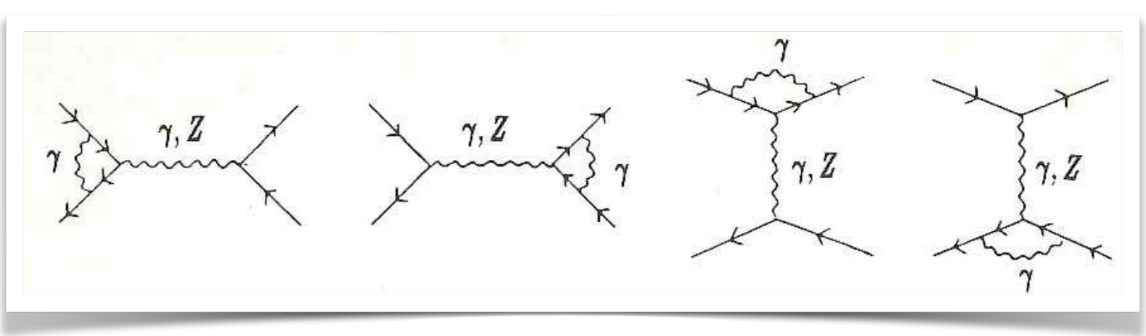
LABH with BHWIDE

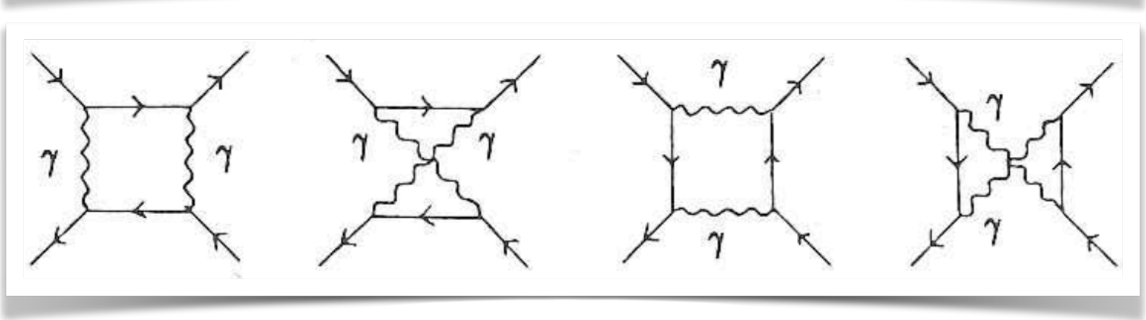
- Born-level diagrams [M. Boehm, A. Denner, W. Hollik, NPB 304 (1988) 687]

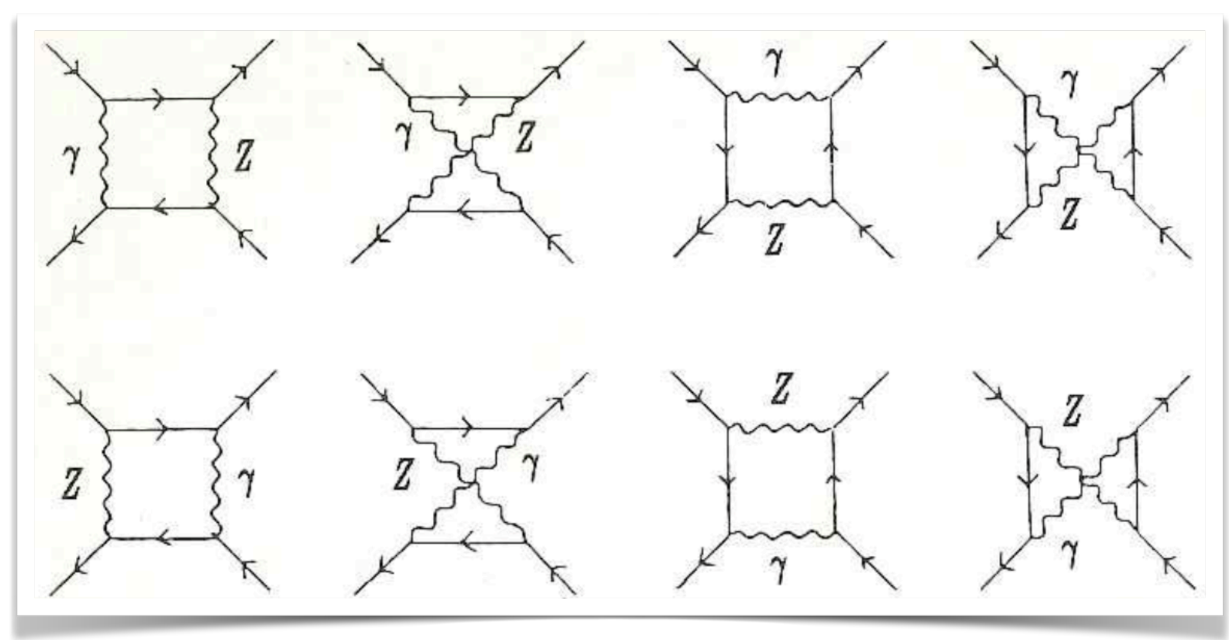


➡ s and t channel γ and Z contributions

- 1-loop QED corrections:

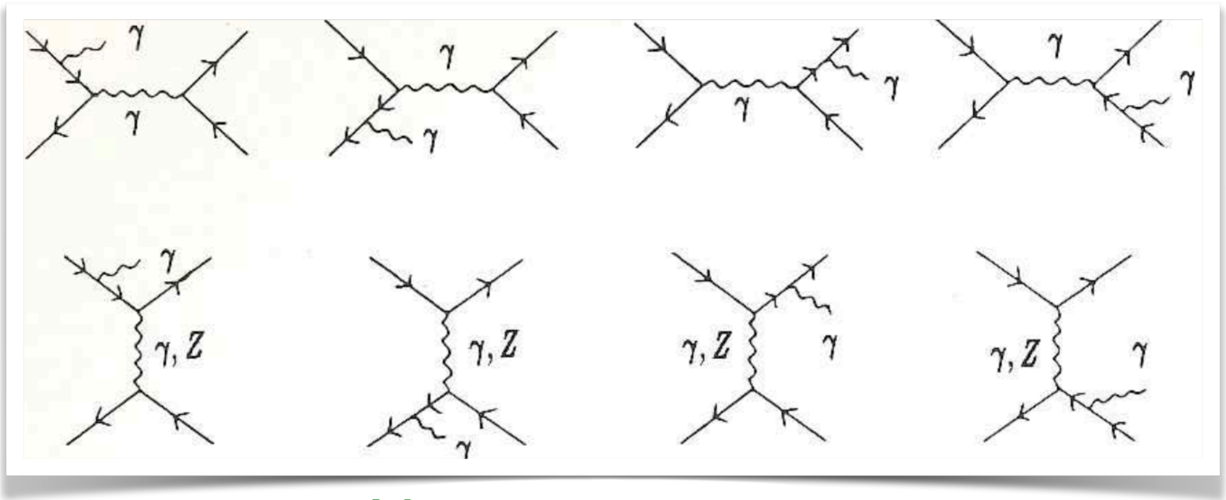




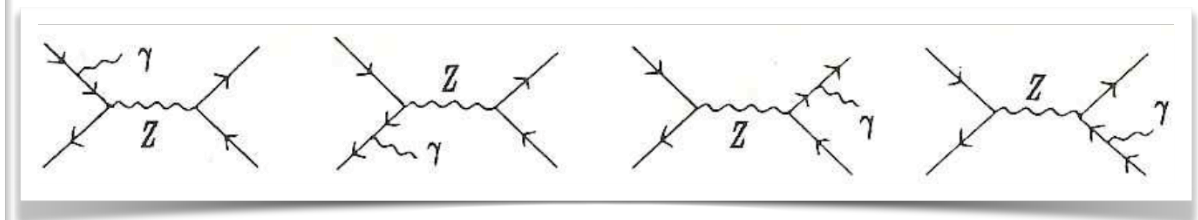


LABH with BHWIDE

- 1-real photon radiation:

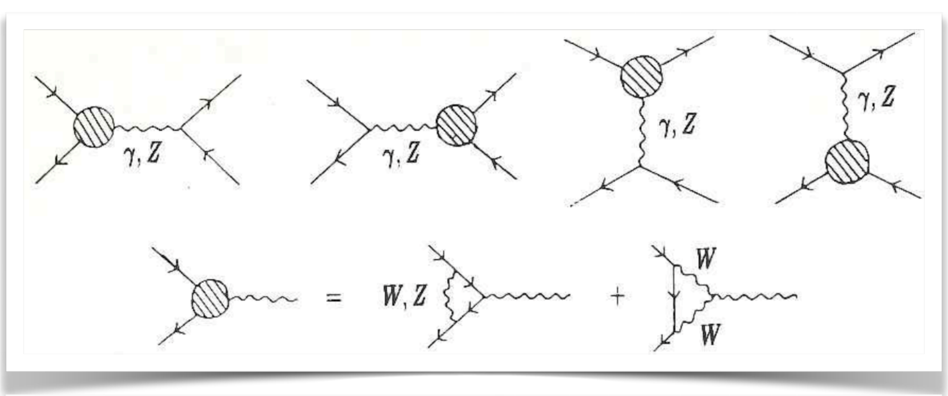


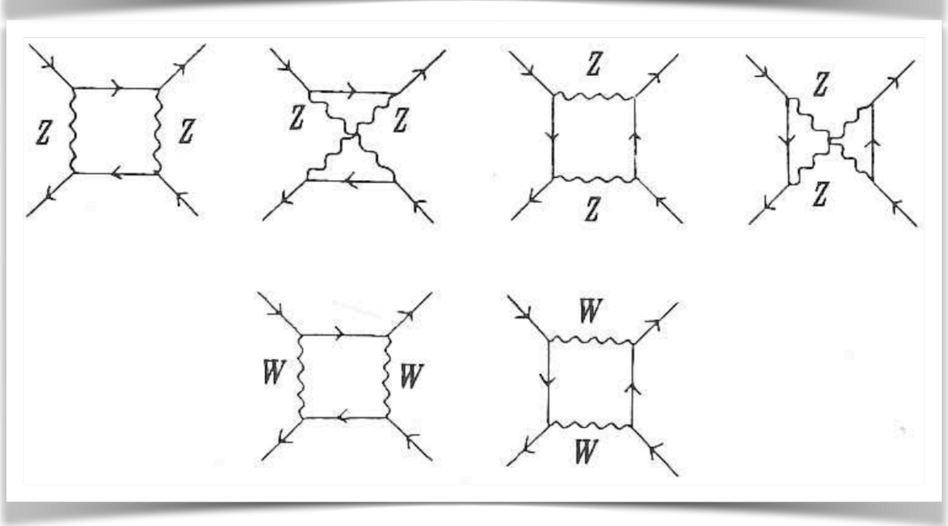
Non-resonant

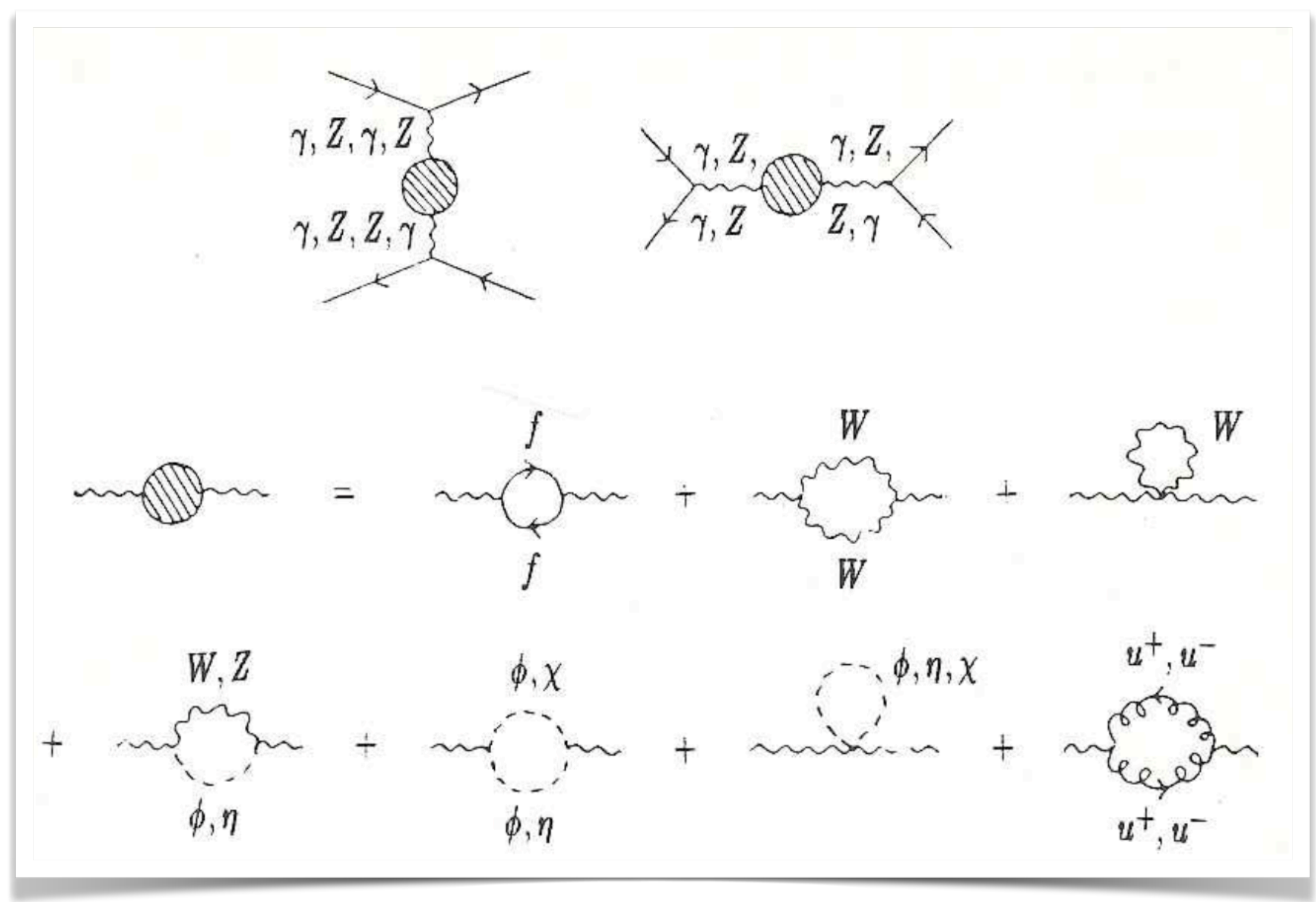


Resonant

- 1-loop weak corrections:







LABH with BHWIDE

- **BHWIDE** - Monte Carlo event generator for **wide-angle Bhabha** scattering with **YFS EEX** including $\mathcal{O}(\alpha)$ EW (electroweak) corrections →https://placzek.web.cern.ch/bhwide
[S. Jadach, WP, B.F.L. Ward, PLB 390 (1997) 298; WP *et al.*, hep-ph/9903381]

$$e^-(p_1) + e^+(q_1) \rightarrow e^-(p_2) + e^+(q_2) + \gamma(k_1) + \ldots + \gamma(k_n)$$

$$d\sigma = e^{2\alpha \operatorname{Re} B + 2\alpha \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D} \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

$$2\alpha\tilde{B} = \int^{k \leq K_{\max}} \frac{d^3 k}{k_0} \tilde{S}(k), \quad D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{\max} - k)\right)$$

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{f'} \left(\frac{p_1}{p_1 \cdot k} - \frac{q_1}{q_1 \cdot k}\right)^2 + \ldots\right]$$

- Non IR-divergent perturbative terms up to $\mathcal{O}(\alpha)$ EW:

$$\frac{1}{2}\bar{\beta}_0 = \frac{d\sigma^{1\text{-loop}}}{d\Omega} - 2\alpha \operatorname{Re} B \frac{d\sigma_{\mathrm{Born}}}{d\Omega}$$

← 0 real-photon emission: 1-loop EW corrections

$$\frac{1}{2}\bar{\beta}_1 = \frac{d\sigma^{B1}}{k\,dk\,d\Omega_\gamma d\Omega} - \tilde{S}(k) \frac{d\sigma_{\mathrm{Born}}}{d\Omega}$$

← 1 real-photon emission: single-bremsstrahlung matrix element

LABH with BHWIDE

- **Two one-loop EW libraries:**
  1. M. Boehm, A. Denner, W. Hollik, NPB 304 (1988) 687 (BABAMC)
  2. W. Beenakker, F.A. Berends, S.C. van der Marck, NPB 349 (1991) 323 (ALIBABA)

- **Two single-bremsstrahlung matrix elements:**
  1. Our calculation based on spin amplitudes - using methods of Z. Xu, D.-H, Zang, L. Cheng NPB 291 (1987) 392
  2. F.A. Berends *et al.* (CALKUL), NPB 206 (1982) 61 - for matrix element squared

- **Monte Carlo algorithm:**
  - similar to that of BHLUMI
  - all **up-down interferences restored** by MC weights in LAB frame

LABH with BHWIDE

❖ Vacuum polarisation

- **Self-energy** corrections to γ and Z propagators contain **light-fermion** loop contribution which is **sizeable** for LABH at high energies: $\sim 10\,\%$ at $\mathcal{O}(\alpha)$
  - ✓ **Dyson resummation** need $\rightarrow$ running couplings
  - ➡ Not easy for LABH because of mix of *s* and *t* channel γ and Z contributions - **running coupling** does **not** factorise!
- In EW library of Beenakker *et al*. - resumed propagators:

$$p_{s,\lambda} = -\left[\left(Q_e + F^V_{\gamma,e}(s)\right) - \lambda F^A_{\gamma,e}(s)\right]\left[\chi_\gamma(s)\right]^{1/2},$$

← γ propagator in *s*-channel

$$z_{s,\lambda} = \left[\left(v_e + F^V_{Z,e}(s) + Q_e \frac{\hat{\Sigma}_{\gamma Z}(s)}{s}\right) - \lambda\left(a_e + F^A_{Z,e}(s)\right)\right]\left[\chi_Z(s)\right]^{1/2}$$

← Z propagator in *s*-channel

$$p_{t,\lambda} = -\left[\left(Q_e + F^V_{\gamma,e}(t)\right) - \lambda F^A_{\gamma,e}(t)\right]\left[\frac{t}{t + \hat{\Sigma}_\gamma(t)}\right]^{1/2},$$

← γ propagator in *t*-channel

$$z_{t,\lambda} = \left[\left(v_e + F^V_{Z,e}(t) + Q_e \frac{\hat{\Sigma}_{\gamma Z}(t)}{t}\right) - \lambda\left(a_e + F^A_{Z,e}(t)\right)\right]\left[\frac{t - M_Z^2}{t - M_Z^2 + \hat{\Sigma}_Z(t)}\right]^{1/2}$$

← Z propagator in *t*-channel

LABH with BHWIDE

✤ Tests and cross-checks:

- **Small angles:** BHWIDE was cross-check with BHLUMI 4.04 for $1^{\mathrm{o}} < \theta_e < 10^{\mathrm{o}}$ - agreement within 0.1% was found
- **Large angles:** BHWIDE was compared other MC and semi-analytical programs for LABH, e.g. with ALIBABA, TOPAZ0 and SABSPV - based on this its precision was assessed:
  - for **LEP1**: 0.3%
  - for **LEP2**: 1.5%

  $\rightarrow$ sufficient for these experiments

- **Flavour factories:** BHWIDE was/is used in **low-energy** e^+e^- experiments, such as BaBar, Belle, VEPP, BES, KLOE, in particular for **luminosity** measurement → precision $\mathcal{O}(0.1\%)$
  - comparisons with other MC programs: BabaYaga, BHAGENF, BKQED, MCGPJ → see: S. Actis *et al.*, EPJC 66 (2010) 585.

LABH with BHWIDE

❖ BHWIDE for **future e^+e^- colliders**

- **FCC-ee/CEPC** will require precision of theory predictions for LABH at least a factor of **10** better than **LEP**

➡ $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections will be necessary (perhaps $\mathcal{O}(L^3\alpha^3)$?), and at *Z*-peak probably also NNLO weak corrections

- 2-loop QED corrections with massive leptons, e.g. M. Delto, C. Duhr, L. Tancredi, Y.J. Zhu, PRL 132 (2024) 231904
- 1-loop corrections to single bremsstrahlung: software packages for automated 1-loop calculations, e.g. OpenLoops, Recola
- tree-level double bremsstrahlung matrix element: calculation ``by hand" using spin amplitudes or using automated software packages
- NNLO weak corrections at *Z*-peak: e.g. GRIFFIN EW library - pole expansion about *Z*-resonance with NNLO corrections for leading-pole terms and NLO for subleasing ones

LABH with BHWIDE

✦ Modern 1-loop EW libraries:

- OpenLoops [F. Buccioni, J.M. Lindert, S. Pozzorini, H. Zhang, M. Zoller]
  - ✓ many processes, numerical stability, Fortran and C/C++ interfaces, massive fermions in recent versions - looks OK for BHWIDE (to be tested)
- Recola [A Denner, J.-N. Lang, S. Uccirati]
  - ✓ many processes, massive fermions, IR regularisation with “photon mass” - looks **best** for BHWIDE (to be tested)
- GRIFFIN [A. Freitas, L. Chen]
  - ✓ weak correction up to NNLO at Z-pole and NLO off-pole (QED subtracted)
  - ◉ not clear how to combine with QED corrections for Bhabha scattering (to be discussed with authors)

➡ **Dyson resummation** of **light-fermion** self-energy correction should be possible - with **hadronic vacuum-polarisation** contribution to be provided by **external** dedicated routine

Conclusions and outlook

❖ **BHLUMI 4.04** - MC event generator for SABH process including YFS $\mathcal{O}(\alpha^2)^{\rm exp}_{\rm prag}$ QED corrections

- **state-of-the art** at LEP for **LUMI** measurement
  - ✓ precision of **0.038%** at *Z*-peak
- **starting point** for **future** e^+e^-colliders luminometry: FCC-ee, CEPC, ILC/CLIC
  - ✓ at ***Z*-peak**, precision of **0.01%** or **better** can be achieved
  - ✓ at **higher** energies **sub-permille** precision possible
  - ➡ Needed:
    - upgrade to (CEEX) YFS $\mathcal{O}(\alpha^3)^{\rm exp}_{\rm prag}$ photonic corrections
    - inclusion of *Z* and *s*-channel γ contributions (possibly with NLO EW correction)
    - perhaps light-fermion pairs
  - ✓ Components available

Conclusions and outlook

- **BHWIDE** - MC event generator for wide-angle Bhabha process including YFS $\mathcal{O}(\alpha)_{\rm exp}$ EW corrections
  - used by LEP experiments for LABH
    - ✓ precision of **0.3%** at LEP1 and **1.5%** at LEP2 energies
  - used by flavour factories (BaBar, Belle, BEPC, VEPP, KLOE, …), e.g. for LUMI $\rightarrow \mathcal{O}(0.1\%)$ precision
  - **starting point** for LABH at **future** e^+e^-colliders: FCC-ee/CEPC, ILC/CLIC
    - ✓ precision requirements at least **10** times higher than at LEP, particularly at FCC-ee/CEPC
    - ➡ Needed:
      - upgrade to (CEEX) YFS $\mathcal{O}(\alpha^2)_{\rm exp}$ QED corrections (EW at *Z*-peak?)
      - new 1-loop EW library, e.g. Recola, OpenLoops
      - 2-loop QED corrections, e.g. M. Delto *et al.*, PRL 132 (2024) 231904, J. Gluza et al. (?); 2-loop EW at Z-pole, e.g. GRIFFIN